Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2025 (43)** : 1–9.
ISSN-0037-8712
doi:10.5269/bspm.76477

Optimizing Electricity Networks with Selje Topological Space, tda and R Programming

AHMAD AL-OMARI, JEYANTHI.V and SELVA NANDHINI.N*

ABSTRACT: Optimizing energy consumption in electricity networks requires identifying critical transformers that significantly impact operational efficiency. This optimization approach integrates Topological Data Analysis (TDA) to process field-collected data, providing a precise method for analyzing network structure. Selje Topological Space is utilized to detect key transformers and evaluate their influence on energy distribution and system resilience. R programming is employed to generate visualizations that clarify critical network components. This method enhances the management of energy distribution, particularly during peak demand or challenging conditions, offering a scalable solution adaptable to various network sizes.

Key Words: Energy optimization, Topological Data Analysis (TDA), Selje topological space, critical transformers, R programming.

Contents

1 Introduction **1**

2 Preliminaries **2**

3 Critical Transformer Identification using Selje Topological Space **3**

4 Methodology for Integrating SELJE Topology with Topological Data Analysis and R Programming **4**

5 On Applying SELJE Topological space for sorting Transformers **4**

6 Results and Discussion **7**

7 Conclusion **7**

1. Introduction

The evolution of topology has introduced concepts with far-reaching implications, particularly in fields such as energy optimization. Levine's work on generalized closed sets expanded the notion of closed sets in topology by comparing the closure of a subset with its open supersets (9). This foundational research laid the groundwork for further developments in topological spaces and their applications. In this context, Zadeh'sintroduction of fuzzy sets and Chang's exploration of fuzzy topological spaces have contributed to flexible and adaptive frameworks that support energy management(15)(2). These advances, along with contributions by Hamlett and Jankovic, Kelly and Mashhour in bitopological and supra-topological spaces, demonstrate the growing role of topology in optimizing energy systems(4)(8)(10). Recent developments in fine topological spaces by Powar and Rajak and M.L. Thivagar's work on nano topology have redefined spatial relationships in complex systems, with applications extending even to fields like medicine (12)(14)(13).

Optimizing electricity networks has become a critical focus in ensuring efficient energy distribution and minimizing power losses. In this context, Selje Topological Space, in combination with Topological Data Analysis (TDA) and R programming, offers a novel approach for addressing complex challenges in energy networks. This methodology enables more precise identification of key network components, particularly transformers, by leveraging both topological structures and advanced machine learning techniques. By

* Corresponding author.
2010 *Mathematics Subject Classification:* 54A05, 54B05.
Submitted April 21, 2025. Published September 23, 2025

Typeset by $\mathcal{B}^{S}\mathcal{P}_{\mathcal{M}}$ style.
© Soc. Paran. de Mat.

integrating these tools, Selje Topology enhances the analysis and management of energy distribution, providing solutions that scale with the demands of modern electricity networks.

Energy optimization has increasingly relied on advanced mathematical frameworks to enhance the efficiency of electricity networks. The application of Topological Data Analysis (TDA) and R programming has further transformed the approach to optimizing energy consumption. By leveraging machine learning and topological techniques, TDA allows for efficient data processing and visualization, as demonstrated by Perkins et al. and Ibáñez et al. (11)(5). These methods, supported by R programming, enable the effective identification of key network components through sophisticated data analysis techniques.

Comparatively, while traditional energy optimization methods such as graph-based analysis or big data approaches have contributed to understanding energy networks, they often lack the precision required for identifying transformers with the highest impact(16). Alipour et al. stress the importance of energy efficiency in smart grid environments, but conventional methods often struggle to pinpoint critical network components with the accuracy achieved through Selje topology(1).

Building on the progress of nano and micro topologies, Selje topological space, introduced by Jeyanthi.V and Selva Nandhini.N , offers a novel approach to identify critical transformers within electricity networks(6). In particular, transformers located in housing units with Housing Service Connection (HSC), where the voltage ranges between 16KVA to 500KVA, are a focal point of this analysis. The study takes into account transformers associated with low-voltage complaints, providing a more targeted approach to improving voltage stability in areas with frequent issues. By incorporating over seven nano open sets, Selje topology refines the analysis of energy systems by combining the influences of topology, TDA and R programming. This method provides a more precise means of identifying core transformers, particularly under high demand or adverse environmental conditions. This paper demonstrates how integrating Selje topological space with TDA and R programming outperforms traditional techniques, offering a scalable and precise solution for transformer identification and energy distribution management.

In Section 3, we focus on critical transformer identification using the Selje Topological Space, detailing the methodology and insights derived from this approach. Section 4 presents the methodology for integrating SELJE Topology with Topological Data Analysis (TDA) and R Programming, outlining the techniques and procedures used for this integration. Section 5 discusses the application of SELJE Topological Space for sorting transformers and presents the results obtained from this analysis, followed by a comprehensive discussion.

2. Preliminaries

Definition 2.1 *[13]*

Let U denote a non-empty finite set of objects referred to as the universe and let $\mathcal{R}$ represent an equivalence relation on U known as the indiscernibility relation. Elements within the same equivalence class are considered indiscernible from each other. This pair, denoted as $(U, \mathcal{R})$, constitutes the approximation space.
Let $\mathfrak{X}$ be a subset of U.

1. *The lower approximation of $\mathfrak{X}$ with respect to $\mathcal{R}$, denoted as $\mathcal{L}_{\mathcal{R}}(\mathfrak{E})$, consists of all objects that can definitively be classified as belonging to $\mathfrak{X}$ under the influence of $\mathcal{R}$. In mathematical terms, $\mathcal{L}_{\mathcal{R}}(\mathfrak{E}) =\cap\{\mathcal{R}(\mathcal{X}) : \mathcal{R}(\mathcal{X}) \subseteq \mathfrak{E}\}$ where $\mathcal{R}$ signifies the equivalence class determined by $\mathfrak{X}$.*

2. *The upper approximation of $\mathfrak{X}$ with respect to $\mathcal{R}$, denoted as $\mathcal{U}_{\mathcal{R}}(\mathfrak{E})$, comprises all objects that could potentially be classified as $\mathfrak{X}$ under the influence of $\mathcal{R}$. Mathematically, $\mathcal{U}_{\mathcal{R}}(\mathfrak{E}) =\cap\{\mathcal{R}(\mathcal{X}) : \mathcal{R}(\mathcal{X}) \cap \mathfrak{E} \neq \Phi\}$*

3. *The boundary region of $\mathfrak{X}$ with respect to $\mathcal{R}$, denoted as $\mathcal{B}_{\mathcal{R}}(\mathfrak{E})$, includes all objects that cannot be definitively classified as either belonging to $\mathfrak{X}$ or not belonging to $\mathfrak{X}$ under the influence of $\mathcal{R}$. In mathematical terms, $\mathcal{B}_{\mathcal{R}}(\mathfrak{E}) = \mathcal{U}_{\mathcal{R}}(\mathfrak{E}) - \mathcal{L}_{\mathcal{R}}(\mathfrak{E})$*

Definition 2.2 *[3] $(U, \mathfrak{T}_{\mathcal{R}}(\mathfrak{X}))$ creates a nanotopological space. In this case, the set $\mu_{\mathcal{R}}(\mathfrak{X})$ consists of two groups, namely $\{\mathfrak{N} \cup (\mathfrak{N}' \cap \mu) : \mathfrak{N}, \mathfrak{N}' \in \mathfrak{T}_{\mathcal{R}}(\mathfrak{E})\}$.*

Definition 2.3 *[6] Consider the microtopological space (U, $\mu_{\mathcal{R}}(\mathfrak{X})$) and Selje topology be defined as $SJ_{\mathcal{R}}(\mathfrak{X})$ = {(S-J)∪(S-$J^{'}$): S∈ $\mu_{\mathcal{R}}(\mathfrak{X})$ and for fixed J,$J^{'}$∉ $\mu_{\mathcal{R}}(\mathfrak{X})$),J∪$J^{'}$=U}*

Definition 2.4 *[6] The Selje topology $SJ_{\mathcal{R}}(\mathfrak{X})$ satisfies the following axioms*

1. *Both the universal set U and the empty set Φ are elements of $SJ_{\mathcal{R}}(\mathfrak{X})$.*
2. *Any subset of the union of elements from $SJ_{\mathcal{R}}(\mathfrak{X})$ remains within $SJ_{\mathcal{R}}(\mathfrak{X})$.*
3. *Any finite subset of the intersection of elements within $SJ_{\mathcal{R}}(\mathfrak{X})$ is contained within $SJ_{\mathcal{R}}(\mathfrak{X})$.*

The triplet ($\mathfrak{X}$, $\mu_{\mathcal{R}}(\mathfrak{X})$,$SJ_{\mathcal{R}}(\mathfrak{X})$) is labeled as Selje topological space. Then, the components of Selje topology are Selje-Open(SJ-Open) sets and their complements are Selje-closed(SJ-closed) sets. The collection of Selje closed sets of Selje topology is denoted as SJCL($\mathfrak{X}$).

3. Critical Transformer Identification using Selje Topological Space

In this section, two theorems are presented to demonstrate the application of Selje Topological Space in identifying critical transformers within an electricity network. The theorems explore how the removal of one or more critical transformers alters the structure of the topological space, providing insights into their significance for network stability and optimization.

Theorem 3.1 *Let $E = \{T_1, T_2, \ldots, T_n\}$ represent a set of transformers and $SJ(E)$ be the Selje topology defined on E. If $T_k \in E$ is a critical transformer, then the removal of T_k from E results in a topological space $SJ(E - \{T_k\})$ such that $SJ(E - \{T_k\}) \subset SJ(E)$ and the change in topology satisfies the condition $SJ(E) \neq SJ(E - \{T_k\})$.*

Proof: Let $E = \{T_1, T_2, \ldots, T_n\}$ be the set of transformers. Define the Selje topology on E as: $SJ(E) = \{\emptyset, E, U_1, U_2, \ldots, U_m\}$, where $U_1, U_2, \ldots, U_m$ are the Selje-open sets in $SJ(E)$.
Suppose $T_k \in E$ is a critical transformer. Remove T_k from E,
so: $E - \{T_k\} = \{T_1, \ldots, T_{k-1}, T_{k+1}, \ldots, T_n\}$.
The Selje topology on $E-\{T_k\}$ becomes: $SJ(E-\{T_k\}) = \{\emptyset, E-\{T_k\}, U'_1, U'_2, \ldots, U'_m\}$, where $U'_1, U'_2, \ldots, U'_m$ are the Selje-open sets formed by the removal of T_k.
By definition of criticality, removing T_k alters the structure of the Selje topology, meaning: $SJ(E - \{T_k\}) \subset SJ(E)$.
Additionally, since T_k is critical, $SJ(E) \neq SJ(E - \{T_k\})$.
Therefore, the removal of T_k reduces the number of Selje-open sets and the condition $SJ(E - \{T_k\}) \subset SJ(E)$ holds. Hence, the removal of a critical transformer T_k alters the Selje topology, proving the theorem. □

Theorem 3.2 *Let $E = \{T_1, T_2, \ldots, T_n\}$ be a set of transformers and $SJ(E)$ represent the Selje topology on E. If T_i and T_j are both critical transformers, then the simultaneous removal of T_i and T_j results in a topological space $SJ(E - \{T_i, T_j\})$ such that $SJ(E - \{T_i, T_j\}) \subset SJ(E)$, and $SJ(E - \{T_i, T_j\}) \neq SJ(E - \{T_i\}) \cap SJ(E - \{T_j\})$.*

Proof: Let $E = \{T_1, T_2, \ldots, T_n\}$ be the set of transformers and $SJ(E)$ denote the Selje topology on E defined as: $SJ(E) = \{\emptyset, E, U_1, U_2, \ldots, U_m\}$, where $U_1, U_2, \ldots, U_m$ are Selje-open sets in $SJ(E)$.
Suppose T_i and T_j are both critical transformers. Remove T_i and T_j from E, so: $E - \{T_i, T_j\} = \{T_1, \ldots, T_{i-1}, T_{i+1}, \ldots, T_{j-1}, T_{j+1}, \ldots, T_n\}$.
The Selje topology on $E - \{T_i, T_j\}$ is defined as:
$SJ(E - \{T_i, T_j\}) = \{\emptyset, E - \{T_i, T_j\}, U'_1, U'_2, \ldots, U'_m\}$.
Since both T_i and T_j are critical, the removal of each transformer individually alters the topology: $SJ(E - \{T_i\}) \subset SJ(E)$ and $SJ(E - \{T_j\}) \subset SJ(E)$.
However, the simultaneous removal of T_i and T_j results in a topological space that is not merely the intersection of the individual removals, i.e., $SJ(E - \{T_i, T_j\}) \neq SJ(E - \{T_i\}) \cap SJ(E - \{T_j\})$. Moreover,

since both transformers are critical, their removal causes a reduction in the number of open sets in the Selje topology, leading to: $SJ(E - \{T_i, T_j\}) \subset SJ(E)$.
Thus, the simultaneous removal of T_i and T_j alters the Selje topology in a manner distinct from removing each individually, proving the theorem. □

4. Methodology for Integrating SELJE Topology with Topological Data Analysis and R Programming

1. **Data Preprocessing**
   The transformer data is loaded, cleaned, and normalized to handle missing values and outliers (data points that deviate significantly from the normal range), and relevant features such as load, consumption, and environmental factors are extracted.
2. **Selje Topology Application**
   The Selje topological space $SJR(E)$ is defined and for each transformer, lower and upper approximations are computed to identify Selje-open and Selje-closed sets, which are used to evaluate transformer stability.
3. **Critical Transformer Identification**
   Transformers are ranked based on their stability impact and those with the highest influence are selected as critical transformers.
4. **Visualization and Analysis**
   Plots are generated to visualize the topological analysis and highlight the critical transformers on the network graph for easy identification.
5. **Output of Critical Transformers**
   The critical transformers identified are returned as the final output, representing the key transformers for maintaining network stability.

5. On Applying SELJE Topological space for sorting Transformers

The combined data table provides a comprehensive view of various metrics related to Transformer T001 to T005. It includes hourly recordings of electricity consumption, with values ranging from 80 kWh to 145 kWh throughout the day. The data also includes temperature measurements, varying from 22°C to 30° C and humidity levels, which fluctuate between 80% and 95%. Additionally, the table captures the load in megawatts (MW), which ranges from 5 MW to 9 MW and highlights a recorded fault of overload that occurred at 03:00, lasting for 2 hours and was subsequently repaired. The energy generated from solar and thermal sources is reported, with 50 MWh from solar and 150 MWh from thermal sources. The table omits columns related to longitude, latitude, rainfall, wind speed, energy source and actions taken, focusing on the available data for Transformer T001 to T005.

The total data collected encompassed various parameters related to transformer performance and environmental factors. This vast dataset was processed using Topological Data Analysis (TDA) and R programming to clean, normalize and transform the data, addressing missing values and inconsistencies. R programming is also used to explore and visualize the data, ensuring the dataset is rectified from the large volume of information collected. Selje Topology subsequently applied to identify and isolate the critical transformers, offering a precise analysis of the components that have the most significant impact on the electricity distribution network. Let $\mathfrak{E} = \{\mathrm{T001}, \mathrm{T002}, \mathrm{T003}, \mathrm{T004}, \mathrm{T005}\}$
$\mathfrak{G} = \{\mathrm{C}, \mathrm{L}, \mathrm{T}, \mathrm{H}, \mathrm{E}\}$, $\mathfrak{H} = \{\mathrm{C}, \mathrm{L}, \mathrm{T}, \mathrm{H}\}$ and $\mathfrak{I} = \{\mathrm{E}\}$
$\mathfrak{V}/\mathfrak{H} = \{\{\mathrm{T001}\}, \{\mathrm{T002}, \mathrm{T004}\}, \{\mathrm{T003}, \mathrm{T005}\}\}$

CASE 1: Electricity Distribution of Transformers with Natural Effect

Identify and analyze the electricity distribution patterns among transformers, considering the influence of natural factors such as weather and environmental conditions.
$\mathfrak{E} = \{\mathrm{T003}, \mathrm{T004}\}$
$\mathfrak{T}_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{T002, T003, T004, T005\}\}$
$\mu_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{T003\}, \{T002, T003, T004, T005\}\}$

Table 1: Data Summary for Transformers T001 to T005

Data Type	T001	T002	T003	T004	T005
Consumption (kWh)	146 (G)	147 (L)	148 (L)	149 (G)	150 (G)
Load (MW)	6 (G)	7 (L)	8 (L)	9 (L)	9.5 (L)
Temperature (°C)	26 (G)	27 (L)	28 (L)	29 (G)	30 (G)
Humidity (%)	85 (L)	86 (G)	87 (L)	88 (L)	89 (L)
Energy Generated (MWh)	50 (G)	52 (L)	53 (L)	54 (G)	55 (G)

If $J = \{\mathrm{T001}, T003, T004\}$ and $J' = \{T002, T005\}$
$SJ_{\mathcal{R}(\mathfrak{J})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{\mathrm{T001}, T003, T004\},$
$\{T003\}, \{T003, T004\}, \{T002, T005\},$
$\{T002, T003, T005\}, \{T002, T003, T004, T005\}\}$
PHASE I: T001 is removed
$\mathfrak{T}_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{001})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{T004\}, \{T003, T004, T005\}, \{T003, T005\}\}$
$\mu_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{001})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{T003\}, \{T004\}, \{T003, T004\},$
$\{T003, T004, T005\}, \{T003, T005\}\}$
If $J = \{\mathrm{T001}, T003, T004\}$ and $J' = \{T002, T005\}$
$SJ_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{001})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{\mathrm{T001}, T003, T004\}, \{T004\},$
$\{T003\}, \{T003, T004\}, \{T002, T005\}, \{T002, T003, T005\},$
$\{T002, T004, T005\}, \{T002, T003, T004, T005\},$
$\{T005\}, \{\mathrm{T001}, T003, T004, T005\},$
$\{T003, T005\}, \{T004, T005\}, \{T003, T004, T005\}\}$
$SJ_{\mathcal{R}(\mathfrak{J})}(\mathfrak{E}) \neq SJ_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{001})}(\mathfrak{E})$
PHASE II: T002 is removed
$\mathfrak{T}_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{002})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{T003, T004\}\}$
$\mu_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{002})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{T003\}, \{T003, T004\}\}$
If $J = \{\mathrm{T001}, T003, T004\}$ and $J' = \{T002, T005\}$
$SJ_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{002})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{\mathrm{T001}, T003, T004\},$
$\{T003, T004\}, \{T002, T005\}, \{T002, T003, T005\},$
$\{T003\}, \{T002, T003, T004, T005\}\}$
$SJ_{\mathcal{R}(\mathfrak{J})}(\mathfrak{E}) \neq SJ_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{001})}(\mathfrak{E})$
PHASE III: T003 is removed
$\mathfrak{T}_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{003})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}\}$
$\mu_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{003})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{T003\}\}$
If $J = \{\mathrm{T001}, T003, T004\}$ and $J' = \{T002, T005\}$
$SJ_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{003})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{\mathrm{T001}, T003, T004\},$
$\{T003\}, \{T002, T005\}, \{T002, T003, T005\}\}$
$SJ_{\mathcal{R}(\mathfrak{J})}(\mathfrak{E}) \neq SJ_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{001})}(\mathfrak{E})$
PHASE IV: T004 is removed
$\mathfrak{T}_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{004})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{T002, T003, T004, T005\}\}$
$\mu_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{004})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{T003\}, \{T002, T003, T004, T005\}\}$
If $J = \{\mathrm{T001}, T003, T004\}$ and $J' = \{T002, T005\}$
$SJ_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{004})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{\mathrm{T001}, T003, T004\},$
$\{\mathrm{T001}\}, \{T003\},$
$\{\mathrm{T001}, T003\}, \{T003, T004\},$
$\{T002, T005\}, \{\mathrm{T001}, T002, T005\},$
$\{T003, T002, T005\}, \{\mathrm{T001}, T002, T003, T005\},$
$\{T002, T003, T004, T005\}\}$
$SJ_{\mathcal{R}(\mathfrak{J})}(\mathfrak{E}) = SJ_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{001})}(\mathfrak{E})$
PHASE V: T005 is removed
$\mathfrak{T}_{\mathcal{R}(\mathfrak{J}-\mathfrak{T}_{005})}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{T002, T003, T004, T005\}\}$

$\mu_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}005)}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{T003\}, \{T002, T003, T004, T005\}\}$
If $J = \{\text{T001}, T003, T004\}$ and $J' = \{T002, T005\}$
$SJ_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}005)}(\mathfrak{E}) = \{\Phi, \mathfrak{V}, \{\text{T001}, T003, T004\}, \{T001\},$
$\{T003\}, \{1, T003\}, \{T003, T004\}, \{T002, T005\},$
$\{T001, T002, T005\}, \{T003, T002, T005\},$
$\{1, T002, T003, T005\}, \{T002, T003, T004, T005\}\}$
$SJ_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E}) = SJ_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}001)}(\mathfrak{E})$

CASE 2: Electricity Distribution And Consumption on Transformer

Examine the distribution of electricity across transformers and evaluate consumption patterns to optimize load management and efficiency.

$\mathfrak{X}$ ={T003,T004}
$\mathfrak{T}_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E})$ = {Φ,U,{T001},{T002,T003,T004,T005}}
$\mu_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E})$={Φ,U,{T001},{T003},{T001,T003},
{T002,T003,T004,T005}}
If J={T001,T003,T004} and $J^{'}$={T002,T005}
$SJ_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E})$={Φ,U,{T001,T003,T004},{T001},{T003},
{T001,T003},{T003,T004},{T002,T005},{T001,T002,T005},
{T003,T002,T005},{T001,T002,T003,T005},
{T002,T003,T004,T005}}

PHASE I: T001 is removed

$\mathfrak{T}_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}001)}(\mathfrak{E})$ = {Φ,U,{T001},{T002},{T004},{T003,T005}}
$\mu_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}001)}(\mathfrak{E})$={Φ,U,{T001,T002},{T003},
{T001,T002,T003},{T001,T002,T003,T005},{T003,T005}}
If J={T001,T003,T004} and $J^{'}$={T002,T005}
$SJ_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}001)}(\mathfrak{E})$={Φ,U,{T001,T003,T004},{T001},{T003},
{T001,T003},{T002,T005},{T001,T002,T005},{T002,T003,T005},
{T001,T002,T003,T005},{T002},{T001,T002,T003,T004},{T001,T002},{T002,T003},{T001,T002,T003},{T005},{T001,
{T001,T005},{T003,T005},{T001,T003,T005}}
$SJ_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E}) \neq SJ_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}001)}(\mathfrak{E})$

PHASE II: T002 is removed

$\mathfrak{T}_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}002)}(\mathfrak{E})$ = {Φ,U,{T001,T002,T005}}
$\mu_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}002)}(\mathfrak{E})$={Φ,U,{T003},{T001,T002,T005},
{T001,T002,T003,T005}}
If J={T001,T003,T004} and $J^{'}$={T002,T005}
$SJ_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}002)}(\mathfrak{E})$={Φ,U,{T001,T003,T004},{T003},{T001},
{T001,T003},{T002,T005},{T002,T003,T005},{T001,T002,T005},
{T001,T002,T003,T005}}
$SJ_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E}) \neq SJ_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}002)}(\mathfrak{E})$

PHASE III:T003 is removed

$\mathfrak{T}_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}003)}(\mathfrak{E})$ = {Φ,U}
$\mu_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}003)}(\mathfrak{E})$={Φ,U,{T003}}
If J={T001,T003,T004} and $J^{'}$={T002,T005}
$SJ_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}003)}(\mathfrak{E})$={Φ,U,{T001,T003,T004},{T003},{T002,T005},
{T002,T003,T005}
$SJ_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E}) \neq SJ_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}003)}(\mathfrak{E})$

PHASE IV:T004 is removed

$\mathfrak{T}_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}004)}(\mathfrak{E})$ = {Φ,U,{T002,T003,T004,T005}}
$\mu_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}004)}(\mathfrak{E})$={Φ,U,{T003},{T002,T003,T004,T005}}
If J={T001,T003,T004} and $J^{'}$={T002,T005}
$SJ_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}004)}(\mathfrak{E})$={Φ,U,{T001,T003,T004},{T001},{T003},
{T001,T003},{T003,T004},{T002,T005},{T001,T002,T005},
{T003,T002,T005},{T001,T002,T003,T005},{T002,T003,T004,T005}}
$SJ_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E}) = SJ_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}004)}(\mathfrak{E})$

PHASE V:T005 is removed
$\mathfrak{T}_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E}) = \{\Phi, U, \{T002, T003, T004, T005\}\}$
$\mu_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E}) = \{\Phi, U, \{T003\}, \{T002, T003, T004, T005\}\}$
If $J = \{T001, T003, T004\}$ and $J' = \{T002, T005\}$
$SJ_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E}) = \{\Phi, U, \{T001, T003, T004\}, \{T001\}, \{T003\},$
$\{T001, T003\}, \{T003, T004\}, \{T002, T005\},$
$\{T001, T002, T005\}, \{T003, T002, T005\},$
$\{T001, T002, T003, T005\}, \{T002, T003, T004, T005\}\}$
$SJ_{\mathcal{R}(\mathfrak{I})}(\mathfrak{E}) = SJ_{\mathcal{R}(\mathfrak{I}-\mathfrak{T}_{005})}(\mathfrak{E})$

6. Results and Discussion

This study extensively analyzed data collected from the Tamil Nadu Generation and Distribution Corporation (TANGEDCO), focusing on hourly electricity consumption, transformer locations, weather data and network analysis. The primary goal was to gain deeper insights into the energy distribution network, identify critical components and optimize overall efficiency. The data underwent rigorous preprocessing and analysis using R programming, which facilitated the visualization of significant patterns and trends across the network.

Initially, traditional graph-based methods were employed to evaluate the network and identify transformers that significantly impact the system's efficiency. However, these methods revealed limitations, particularly in distinguishing the transformers that play a crucial role in maintaining network stability. The inability of conventional methods to accurately identify these key transformers underscored the need for a more advanced analytical approach.

To overcome these challenges, Selje topological space was integrated with Topological Data Analysis (TDA), a combination that provided a more sophisticated means of network analysis. This approach was instrumental in pinpointing transformers T004 and T005 as critical components of the network. The network diagrams clearly depicted how T004 and T005 serve as central hubs in the electricity distribution system. These transformers were identified as crucial not only during routine operations but also in scenarios involving natural disruptions, load shifts, or other operational challenges.

The application of Selje topological space analysis revealed that transformers T004 and T005 are pivotal, particularly when influenced by natural events. These transformers play a significant role in electricity distribution and consumption and given their importance, they require focused attention and monitoring. By applying Selje Topological Analysis, the critical transformers were determined and visual representations were created to facilitate a thorough evaluation and interpretation of the results. T004 and T005 were identified and examined through data visualization and comparative analysis using R programming.

The graphs generated using R programming are presented below. Figure 1 illustrates that distinguishing between the crucial transformers is challenging due to overlapping lines, making differentiation difficult. However, based on the results obtained from the Selje topological space, points are plotted and Figure 2 clearly highlights the critical transformers, facilitating easier identification based on their distinct separation in the visualization.

The analysis revealed that any disturbance or failure in these transformers could significantly impact the network's overall stability and efficiency. Therefore, the study strongly recommends prioritizing these transformers for maintenance and load management, as their proper functioning is integral to the network's resilience.

7. Conclusion

The innovative application of Selje topology provided a clear advantage in pinpointing crucial network components, offering a scalable and adaptable approach to energy management. This study not only improves operational efficiency but also establishes a robust framework that can be applied to larger and more complex energy networks. Unlike standard methods, which failed to recognize the importance of these transformers, the application of Selje topology highlighted their pivotal role in the network. This discovery is particularly noteworthy as it demonstrates the limitations of conventional graph-based

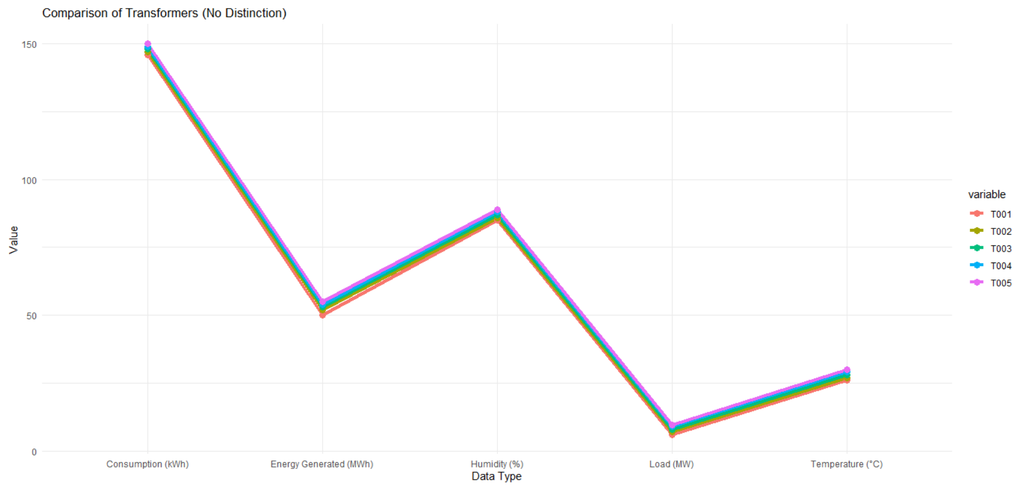

Figure 1: distinguishing between the crucial transformers is challenging due to overlapping lines

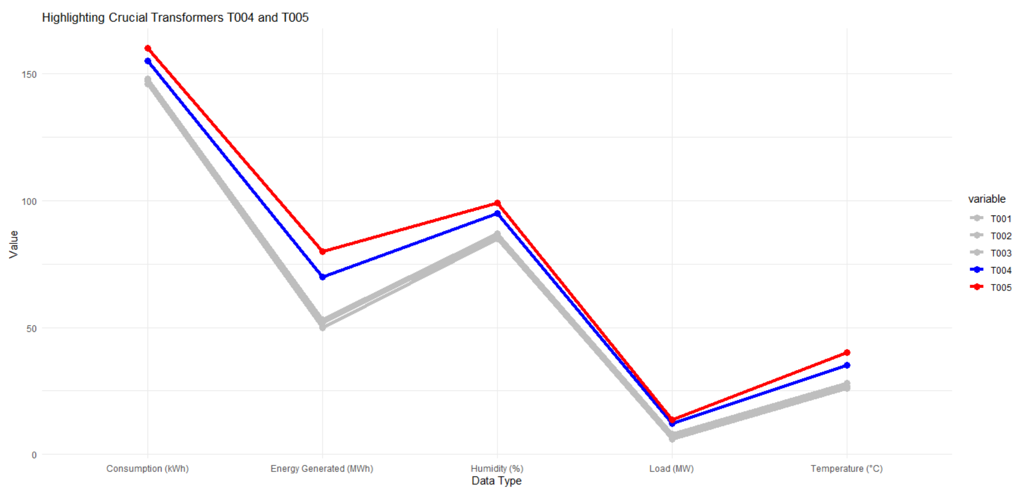

Figure 2: highlights the critical transformers, facilitating easier identification

methods and the advantages of incorporating topological analysis for a more detailed understanding of the network's structure.The insights gained from this research are invaluable for energy policymakers, utility companies and researchers, paving the way for more resilient and efficient electricity distribution systems. This methods versatility allows for broader application, potentially transforming how energy networks are optimized and maintained across different regions.

Acknowledgments

The authors express deep gratitude to the reviewers for their invaluable support, enhancing the research paper quality with insightful comments and valuable suggestions.

References

1. Alipour M, et al. Improving energy efficiency in smart grid environments. IEEE Trans Sustain Comput. (2018) ,4(3):246–57.

2. Chang CL. Fuzzy topological spaces. J Math Anal Appl. (1968) ,22:182–90.
3. Chandrasekar S. On micro topological spaces. J New Theory. (2019) ,26:23–31.
4. Hamlett TR, Rose D. Topological properties. Int J Math Math Sci. (1990) ,13(3):507–12.
5. Ibáñez JD, et al. Energy consumption optimization using topological data analysis and machine learning. IEEE Access. (2019) ,7:13030–40.
6. Jeyanthi V, Selva Nandhini N. On SELJE topological spaces. Indian J Nat Sci. (2023) ,14(80):64187–93.
7. Kashyap GR, Sajeed IG. On Kasaj topological spaces. Malaya J Math. (2020) ,8(4):1766–70.
8. Kelly JC. Bitopological spaces. Proc Lond Math Soc. (1963) ,13(3):71–89.
9. Levine N. Generalized closed sets in topology. Rend Circ Math Palermo. (1970) ,19(2):89–96.
10. Mashhour AS, Abd El-Monsef ME, El-Deeb SN. On pre-continuous mappings and weak pre-continuous mappings. Proc Math Phys Soc Egypt. (1982) ,53:47–53.
11. Perkins S, et al. An overview of the R package TDA for topological data analysis. arXiv preprint arXiv. (2014), 1411-1433
12. Powar PL, Rajak K. Fine irresolute mappings. J Adv Stud Topol. (2012) ,3(4):125–39.
13. Thivagar ML, Richard C, Paul NR. Mathematical innovations of a modern topology in medical events. Int J Inf Sci. (2012) ,2(4):33–6.
14. Thivagar ML, Richard C. Nano forms of weakly open sets. Int J Math Stat Invention. (2013) ,1(1):31–7.
15. Zadeh LA. Fuzzy sets. Inf Control. (1965) ,8:338–53.
16. Zou C, et al. Smart grid big data analytics: An overview. IEEE Trans Smart Grid. (2019) ,10(4):4567–83.

Ahmad Al-Omari,
Department of Mathematics, Faculty of Sciences,
Al al-Bayt University,
Jordan.
E-mail address: omarimutah1@yahoo.com

and

Jeyanthi V,
Department of Mathematics,
Sri Krishna Arts and Science College,
India.
E-mail address: jeyanthivenkatapathy@gmail.com

and

Selva Nandhini.N,
Department of Mathematics,
Sri Krishna Arts and Science College,
India.
E-mail address: nandhininagaraj230@gmail.com