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Neutrosophic Soft Metric Spaces and Fixed Point Theorems

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ABSTRACT: This paper introduces a new concept i.e. neutrosophic soft metric space. Within this framework, open balls, closed balls and several other fundamental notions are defined. Moreover, new fixed point theorems (FPTs) are established and novel applications are presented to demonstrate the effectiveness and applicability of NSMS in various mathematical contexts.

Key Words: Soft set, Neutrosophic Soft Metric Space (NSMS), fixed point.

Contents

1. Introduction

Fuzzy set (FS) was formulated by Zadeh [1] and then the theory of metric space (MS) was extended to FS and fuzzy metric space (FMS) came into existence in [2]. Further, George and Veeramani gave their own version of FMS in [3]. Later on, Park [4] developed intuitionistic fuzzy metric space (IFMS) with new FPT's. FS and intuitionistic fuzzy sets (IFS) do not give any knowledge of the indeterminacy degree (I). Hence, Smarandache [5] formulated neutrosophic set (NS) including "'I"'. Thereafter, the NS concept was applied to MS in [6] by Kirisci et al and hence developing neutrosophic metric space (NMS).
Molodtsov [7] came out with soft set for uncertainties having parameters. Later on, many soft versions of various MS were developed. Neutrosophic soft set (NSS) was developed by Karaaslan [10] by applying the concept of soft sets to neutrosophic sets. In this paper, we are extending FPT's given in [9] to NSMS.

2. Preliminaries

Now we are presenting basic notions and results, already given and proved, that have been utilized for developing our results.

Definition 2.1 *[5] NS is represented by $\Gamma = \{<\wp', \Xi_\Upsilon(\wp'), \Pi_\Upsilon(\wp'), \Sigma_\Upsilon(\wp')>: \wp \in \chi'\}$, with χ' being any non-empty set, $\Xi_\Upsilon(\wp'), \Pi_\Upsilon(\wp')$ and $\Sigma_\Upsilon(\wp')$ depicts the "T", "F" and "I" degree respectively of every $\wp' \in \chi'$.*

Definition 2.2 *[6] NMS is represented as $(\chi', \Gamma, *, \diamond)$ with $*$ and $\diamond$ being the continuous t-norms and t-conorms and the below assertions hold for all $\wp', \kappa', \gamma' \in \chi'$ and $\varrho', \mu' > 0$:*

NMS (a) $0 \leq \Xi(\wp', \kappa', \varrho') \leq 1$, $0 \leq \Pi(\wp', \kappa', \varrho') \leq 1$, $0 \leq \Sigma(\wp', \kappa', \varrho') \leq 1$;

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NMS (b) $\Xi(\wp',\kappa',\varrho')+\Pi(\wp',\kappa',\varrho')+\Sigma(\wp',\kappa',\varrho')\leq 3$;

NMS (c) $\Xi(\wp',\kappa',\varrho')=1$, $\Pi(\wp',\kappa',\varrho')=0$ *and* $\Sigma(\wp',\kappa',\varrho')=0$ *if and only if* $\wp'=\kappa'$;

NMS (d) $\Xi(\wp',\kappa',\varrho')=\Xi(\kappa',\wp',\varrho')$, $\Pi(\wp',\kappa',\varrho')=\Pi(\kappa',\wp',\varrho')$ *and* $\Sigma(\wp',\kappa',\varrho')=\Sigma(\kappa',\wp',\varrho')$;

NMS (e) $\Xi(\wp',\kappa',\varrho')*\Xi(\kappa',\gamma',\mu')\leq\Xi(\wp',\gamma',\varrho'+\mu')$, $\Pi(\wp',\kappa',\varrho')\diamond\Pi(\kappa',\gamma',\mu')\geq\Pi(\wp',\gamma',\varrho'+\mu')$ *and* $\Sigma(\wp',\kappa',\varrho')\diamond\Sigma(\kappa',\gamma',\mu')\geq\Sigma(\wp',\gamma',\varrho'+\mu')$;

NMS (f) $\Xi(\wp',\kappa',.),\Pi(\wp',\kappa',.)$ *and* $\Sigma(\wp',\kappa',.):[0,\infty)\to[0,1]$ *possesses continuity;*

NMS (g) $lim_{\varrho'\to\infty}\Xi(\wp',\kappa',\varrho')=1$, $lim_{\varrho'\to\infty}\Pi(\wp',\kappa',\varrho')=0$ *and* $lim_{\varrho'\to\infty}\Sigma(\wp',\kappa',\varrho')=0$;

NMS (h) $\Xi(\wp',\kappa',\varrho')=0$, $\Pi(\wp',\kappa',\varrho')=1$ *and* $\Sigma(\wp',\kappa',\varrho')=1$ *if* $\varrho'<0$.

Definition 2.3 *[10] Consider χ' a non-empty set, then a NSS over χ' is represented as $\varsigma:U'\to\Gamma(\chi')$, where set U' consists of parameters and $\Gamma(\chi')$ contains NS on χ', where ς is defined by*

$$\varsigma=\{(u,\{\wp',\Xi_{\varsigma(u)}(\wp'),\Pi_{\varsigma(u)}(\wp'),\Sigma_{\varsigma(u)}(\wp'):\wp'\in\chi'\}):u\in U'\}$$

3. Neutrosophic Soft Metric Space

Now, we are going to define and formulate new results in NSMS altogether with examples.

Definition 3.1 *Take $\bar{\chi'}$ be soft set which is absolute and $\varpi'=\{(e,T_{\varpi'(e)}(\bar{\wp'}),I_{\varpi'(e)}(\bar{\wp'}),F_{\varpi'(e)}(\bar{\wp'}):\bar{\wp'}\in\bar{\chi'}\}$ be NSS defined from $SP(\bar{\chi'})\times SP(\bar{\chi'})\times R(U')^*$ to $[0,1]$. Let $*$ and $\diamond$ representing the continuous t-norms and t-conorms respectively, then four tuple $(\bar{\chi'},\varpi',*,\diamond)$ is known as NSMS when the following assertions are satisfied for all $\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{r'}_{e_3}\in SP(\bar{\chi'})$; $\bar{\varrho'},\bar{\mu'}\in R(U')^*$:*

i. $0\leq T_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})\leq 1$, $0\leq I_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})\leq 1$, $0\leq F_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})\leq 1$;

ii. $T_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})+I_{\varpi(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})+F_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})\leq 3$;

iii. $T_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=1$, $I_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=0$ *and* $F_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=0$ *if and only if* $\bar{p'}_{e_1}=\bar{q'}_{e_2}$;

iv. $T_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=T_{\varpi'(e)}(\bar{q'}_{e_2},\bar{p'}_{e_1},\bar{\varrho'})$, $I_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=I_{\varpi'(e)}(\bar{q'}_{e_2},\bar{p'}_{e_1},\bar{\varrho'})$ *and* $F_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=F_{\varpi'(e)}(\bar{q'}_{e_2},\bar{p'}_{e_1},\bar{\varrho'})$;

v. $T_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})*T_{\varpi'(e)}(\bar{q'}_{e_2},\bar{r'}_{e_3},\bar{\mu'})\leq T_{\varpi'(e)}(\bar{p'}_{e_1},\bar{r'}_{e_3},\bar{\varrho'}+\bar{\mu'})$, $I_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})\diamond I_{\varpi'(e)}(\bar{q'}_{e_2},\bar{r'}_{e_3},\bar{\mu'})\geq I_{\varpi'(e)}(\bar{p'}_{e_1},\bar{r'}_{e_3},\bar{\varrho'}+\bar{\mu'})$ *and* $F_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})\diamond F_{\varpi'(e)}(\bar{q'}_{e_2},\bar{r'}_{e_3},\bar{\mu'})\geq F_{\varpi'(e)}(\bar{p'}_{e_1},\bar{r'}_{e_3},\bar{\varrho'}+\bar{\mu'})$;

vi. $T_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},.)$ $I_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},.)$ *and* $F_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},.):R(U')^*\to[0,1]$ *is continuous;*

vii. $lim_{\bar{\varrho'}\to\infty}T_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=1$, $lim_{\bar{\varrho'}\to\infty}I_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=0$ *and* $lim_{\bar{\varrho'}\to\infty}F_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=0$;

viii. $T_{\varpi'(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=0$, $I_{\varpi(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=1$, $F_{\varpi(e)}(\bar{p'}_{e_1},\bar{q'}_{e_2},\bar{\varrho'})=1$ *if* $\bar{\varrho'}\leq\bar{0}$.

Now, we will give an example of NSMS:

Example 3.1 *Consider $\bar{\chi}=N$, define $*$ and $\diamond$ as $\bar{\zeta'}*\bar{\eta'}=max\{0,\bar{\zeta'}+\bar{\eta'}-1\}$; $\bar{\zeta'}\diamond\bar{\eta'}=\bar{\zeta'}+\bar{\eta'}-\bar{\zeta'}\bar{\eta'}$ where $\bar{\zeta'},\bar{\eta'}\in\bar{\chi}$, $\bar{\varrho}>\bar{0}$. Let*

$$T_{\varpi(e)}(\bar{\zeta'},\bar{\eta'},\bar{\varrho})=\begin{cases}\frac{\bar{\zeta'}}{\bar{\eta'}}, & if\bar{\zeta'}\leq\bar{\eta'},\\ \frac{\bar{\eta'}}{\bar{\zeta'}}, & if\bar{\eta'}\leq\bar{\zeta'},\end{cases}$$

$$I_{\varpi(e)}(\bar{\zeta'},\bar{\eta'},\bar{\varrho})=\begin{cases}\frac{\bar{\eta'}-\bar{\zeta'}}{\bar{x}}, & if\bar{\zeta'}\bar{x}\leq\bar{\eta'},\\ \frac{\bar{\zeta'}-\bar{\eta'}}{\bar{x}}, & if\bar{\eta'}\bar{x}\leq\bar{\zeta'},\end{cases}$$

$$F_{\varpi(e)}(\bar{\zeta'},\bar{\eta'},\bar{\varrho})=\begin{cases}\bar{\eta'}-\bar{\zeta'}, & if\bar{\zeta'}\leq\bar{\eta'},\\ \bar{\zeta'}-\bar{\eta'}, & if\bar{\eta'}\leq\bar{\zeta'}.\end{cases}$$

Then, $(\bar{\chi},\varpi,*,\diamond)$ is NSMS.

Definition 3.2 *Let* $(\bar{\chi}', \varpi', *, \diamond)$ *be NSMS,* $\bar{0} < \bar{\hbar} < \bar{1}$, $\bar{\xi} > \bar{0}$ *and* $\bar{\wp} \in \bar{\chi}'$. *Then,* $O(\bar{\wp}, \bar{\hbar}, \bar{\xi}) = \{\bar{\iota} \in \bar{\chi} : T_{\varpi'(e)}(\bar{\wp}, \bar{\iota}, \bar{\xi}) > \bar{1} - \bar{\hbar}, I_{\varpi'(e)}(\bar{\wp}, \bar{\iota}, \bar{\xi}) < \bar{\hbar}, F_{\varpi'(e)}(\bar{\wp}, \bar{\iota}, \bar{\xi}) < \bar{\hbar}\}$ *is an open ball with* $\bar{\wp}$ *and* $\bar{\hbar}$ *being the center and radius respectively in relation to* $\bar{\xi}$.

Theorem 3.2 *In a NSMS every open ball* $O(\bar{\zeta}, \bar{\hbar}, \bar{\xi})$ *is an open set.*

Proof: Consider, $O(\bar{\zeta}, \bar{\hbar}, \bar{\xi})$ be an open ball. Let $\bar{\eta} \in O(\bar{\zeta}, \bar{\hbar}, \bar{\xi})$, thus $T_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\xi}) > \bar{1} - \bar{\hbar}'$, $I_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\xi}) < \bar{\hbar}'$, $F_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\xi}) < \bar{\hbar}'$, then there exists $\bar{\xi}_o \in (\bar{0}, \bar{\xi})$ so that $T_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\xi}_o) > \bar{1} - \bar{\hbar}'$, $I_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\xi}_o) < \bar{\hbar}'$, $F_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\xi}_o) < \bar{\hbar}'$. Considering $\bar{\hbar}'_o = T_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\xi}_o)$, then $\bar{\hbar}'_o > \bar{1} - \bar{\hbar}'$ there exists $\bar{\varsigma} \in (\bar{0}, \bar{1})$, so that $\bar{\hbar}'_o > \bar{1} - \bar{\varsigma} > \bar{1} - \bar{\hbar}'$.
For given $\bar{\hbar}'_o$ and $\bar{\varsigma}$ satisfying $\bar{\hbar}'_o > \bar{1} - \bar{\varsigma}$, there exists $\bar{\hbar}'_1, \bar{\hbar}'_2, \bar{\hbar}'_3 \in (\bar{0}, \bar{1})$, so that $\bar{\hbar}'_o * \bar{\hbar}'_1 > \bar{1} - \bar{\varsigma}$, $(\bar{1} - \bar{\hbar}'_o) \diamond (\bar{1} - \bar{\hbar}'_2) \leq \bar{\varsigma}$ and $(\bar{1} - \bar{\hbar}'_o) \diamond (\bar{1} - \bar{\hbar}'_3) \leq \bar{\varsigma}$. Taking $\bar{\hbar}'_4 = max\{\bar{\hbar}'_1, \bar{\hbar}'_2, \bar{\hbar}'_3\}$. Now, consider open ball $O(\bar{\eta}, \bar{1} - \bar{\hbar}'_4, \bar{\xi} - \bar{\xi}_o)$.
Claim that, $O(\bar{\eta}, \bar{1} - \bar{\hbar}'_4, \bar{\xi} - \bar{\xi}_o) \subset O(\bar{\zeta}, \bar{\hbar}', \bar{\varsigma})$. Let $\bar{\theta} \in O(\bar{\theta}, \bar{1} - \bar{\hbar}'_4, \bar{\xi} - \bar{\xi}_o)$, then $T_{\varpi(e)}(\bar{\eta}, \bar{\theta}, \bar{\xi} - \bar{\xi}_o) > \bar{\hbar}'_4$, $I_{\varpi(e)}(\bar{\eta}, \bar{\theta}, \bar{\xi} - \bar{\xi}_o) < \bar{1} - \bar{\hbar}'_4$ and $F_{\varpi(e)}(\bar{\eta}, \bar{\theta}, \bar{\xi} - \bar{\xi}_o) < \bar{1} - \bar{\hbar}'_4$. Thus

$$\begin{aligned} T_{\varpi(e)}(\bar{\zeta}, \bar{\theta}, \bar{\xi}) &\geq T_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\xi}_o) * T_{\varpi(e)}(\bar{\eta}, \bar{\theta}, \bar{\xi} - \bar{\xi}_o) \\ &\geq \bar{\hbar}'_o * \bar{\hbar}'_4 \geq \bar{\hbar}'_o * \bar{\hbar}'_1 \geq \bar{1} - \bar{\varsigma} > \bar{1} - \bar{\hbar}', \end{aligned}$$

$$\begin{aligned} I_{\varpi(e)}(\bar{\zeta}, \bar{\theta}, \bar{\xi}) &\leq I_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\xi}_o) \diamond I_{\varpi(e)}(\bar{\eta}, \bar{\theta}, \bar{\xi} - \bar{\xi}_o) \\ &\leq (\bar{1} - \bar{\hbar}'_o) \diamond (\bar{1} - \bar{\hbar}'_4) \leq (\bar{1} - \bar{\hbar}'_o) \diamond (\bar{1} - \bar{\hbar}'_2) \leq \bar{\varsigma} < \bar{\hbar}', \end{aligned}$$

$$\begin{aligned} F_{\varpi(e)}(\bar{\zeta}, \bar{\theta}, \bar{\xi}) &\leq F_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\xi}_o) \diamond F_{\varpi(e)}(\bar{\eta}, \bar{\theta}, \bar{\xi} - \bar{\xi}_o) \\ &\leq (\bar{1} - \bar{\hbar}'_o) \diamond (\bar{1} - \bar{\hbar}'_4) \leq (\bar{1} - \bar{\hbar}'_o) \diamond (\bar{1} - \bar{\hbar}'_2) \leq \bar{\varsigma} < \bar{\hbar}'. \end{aligned}$$

Thus, $\bar{\theta} \in B(\bar{\zeta}, \bar{\hbar}', \bar{\xi})$ and hence $O(\bar{\eta}, \bar{1} - \bar{\hbar}'_4, \bar{\xi} - \bar{\xi}_o) \subset O(\bar{\zeta}, \bar{\hbar}', \bar{\xi})$. □

Theorem 3.3 *Every NSMS is a Hausdorff space.*

Proof: Take a NSMS $(\bar{\chi}', \varpi, *, \diamond)$. Let $\bar{\iota}, \bar{\kappa} \in \bar{\chi}'$ so that $\bar{\iota} \neq \bar{\kappa}$. Thus, $\bar{0} < T_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) < \bar{1}$, $\bar{0} < I_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) < \bar{1}$, $\bar{0} < F_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) < \bar{1}$. Let $\bar{\psi}_1 = T_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\lambda})$, $\bar{\psi}_2 = I_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\lambda})$, $\bar{\psi}_3 = F_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\lambda})$ and $\bar{\psi} = max\{\bar{\psi}_1, \bar{1} - \bar{\psi}_2, \bar{1} - \bar{\psi}_3\}$. If we consider $\bar{\psi}_o \in (\bar{\hbar}, \bar{1})$, then there exists $\bar{\psi}_4, \bar{\psi}_5, \bar{\psi}_6$ so that $\bar{\psi}_4 * \bar{\psi}_4 \geq \bar{\psi}_o$, $(\bar{1} - \bar{\psi}_5) \diamond (\bar{1} - \bar{\psi}_5) \leq \bar{1} - \bar{\psi}_o$ and $(\bar{1} - \bar{\psi}_6) \diamond (\bar{1} - \bar{\psi}_6) \leq \bar{1} - \bar{\psi}_o$. Take $\bar{\psi}_7 = max\{\bar{\psi}_4, \bar{\psi}_5, \bar{\psi}_6\}$.
Consider, open balls $O(\bar{\iota}, \bar{1} - \bar{\psi}_7, \frac{\bar{\lambda}}{2})$ and $O(\bar{\kappa}, \bar{1} - \bar{\psi}_7, \frac{\bar{\lambda}}{2})$. Claim that, $O(\bar{\iota}, \bar{1} - \bar{\psi}_7, \frac{\bar{\lambda}}{2}) \cap O(\bar{\kappa}, \bar{1} - \bar{\psi}_7, \frac{\bar{\lambda}}{2}) = \phi$.
Let $O(\bar{\iota}, \bar{1} - \bar{\psi}_7, \frac{\bar{\lambda}}{2}) \cap O(\bar{\kappa}, \bar{1} - \bar{\psi}_7, \frac{\bar{\lambda}}{2}) \neq \phi$, then there exists $\bar{r} \in O(\bar{\iota}, \bar{1} - \bar{\psi}_7, \frac{\bar{\lambda}}{2}) \cap O(\bar{\kappa}, \bar{1} - \bar{\psi}_7, \frac{\bar{\lambda}}{2})$, thus

$$\begin{aligned} \bar{\psi}_1 = T_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) &\geq T_{\varpi(e)}(\bar{\iota}, \bar{r}, \frac{\bar{\lambda}}{2}) * T_{\varpi(e)}(\bar{r}, \bar{\kappa}, \frac{\bar{\lambda}}{2}) \\ &\geq \bar{\psi}_7 * \bar{\psi}_7 \geq \bar{\psi}_4 * \bar{\psi}_4 \geq \bar{\psi}_o > \bar{\psi}_1, \end{aligned}$$

$$\begin{aligned} \bar{\psi}_2 = I_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) &\leq I_{\varpi(e)}(\bar{\iota}, \bar{r}, \frac{\bar{\lambda}}{2}) \diamond I_{\varpi(e)}(\bar{r}, \bar{\kappa}, \frac{\bar{\lambda}}{2}) \\ &\leq (\bar{1} - \bar{\psi}_7) \diamond (\bar{1} - \bar{\psi}_7) \leq (\bar{1} - \bar{\psi}_5) \diamond (\bar{1} - \bar{\psi}_5) \leq \bar{1} - \bar{\psi}_o < \bar{\psi}_2, \end{aligned}$$

$$\begin{aligned} \bar{\psi}_3 = F_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\lambda}) &\leq F_{\varpi(e)}(\bar{\iota}, \bar{r}, \frac{\bar{\lambda}}{2}) \diamond F_{\varpi(e)}(\bar{r}, \bar{\kappa}, \frac{\bar{\lambda}}{2}) \\ &\leq (\bar{1} - \bar{\psi}_7) \diamond (\bar{1} - \bar{\psi}_7) \leq (\bar{1} - \bar{\psi}_6) \diamond (\bar{1} - \bar{\psi}_6) \leq \bar{1} - \bar{\psi}_o < \bar{\psi}_3, \end{aligned}$$

that is contradictory to our assumption. Hence, $(\bar{\chi}', \varpi, *, \diamond)$ is Hausdorff. □

Definition 3.3 *Take, $\Theta \subseteq \bar{\chi'}$ in a NSMS $(\bar{\chi'}, \varpi, *, \diamond)$. Then, Θ is Neutrosophic bounded, if for all $\bar{\zeta}, \bar{\eta} \in \bar{\chi'}$ implies the existence of $\bar{\varrho'} > \bar{0}$ and $\bar{\hbar} \in (\bar{0}, \bar{1})$, so that $T_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho'}) > \bar{1} - \bar{\hbar}, I_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho'}) < \bar{\hbar}$ and $F_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho'}) < \bar{\hbar}$.*

Definition 3.4 *Take, $\Theta \subseteq \bar{\chi'}$ in NSMS $(\bar{\chi'}, \varpi, *, \diamond)$ and $\Xi_\varpi = \{\Delta : \Theta \subseteq \bigcup_{\Delta \in \Xi_\varpi} \Delta\}$. The collection Ξ_ϖ is an open cover for Θ.*

Definition 3.5 *Any subspace Δ in NSMS $(\bar{\chi'}, \varpi, *, \diamond)$ is compact, if every open cover of Δ possesses a subcover which is finite.*

Definition 3.6 *Any subspace Δ of NSMS $(\bar{\chi'}, \varpi, *, \diamond)$ is compact sequentially if every sequence in Δ possesses a subsequence that is convergent as well.*

Theorem 3.4 *Any subset $\mho$ which is compact in a NSMS $(\bar{\chi'}, \varpi, *, \diamond)$ is Neutrosophic bounded.*

Proof: Consider, $\mho \subseteq \bar{\chi}$ be compact in NSMS $(\bar{\chi}, \varpi, *, \diamond)$. Let $K = \{B(\bar{p}_e, \bar{\hbar}, \bar{\varrho}) : \bar{p}_e \in \mho$ for $\bar{\varrho} > \bar{0}, \bar{\hbar} \in (\bar{0}, \bar{1})\}$ be the open cover of $\mho$. As $\mho$ is compact, there exists $\bar{p}_{e_1}, \bar{p}_{e_2}, \bar{p}_{e_3}, ..., \bar{p}_{e_n} \in \mho$, so that $\mho \subseteq \bigcup_{k=1}^{n} B(\bar{p}_{e_k}, \bar{\hbar}, \bar{\varrho})$. Consider $\bar{p}, \bar{q} \in \mho$, thus there exists $i, j \in N$ so that $\bar{p} \in B(\bar{p}_{e_i}, \bar{\hbar}, \bar{\varrho})$ and $\bar{q} \in B(\bar{p}_{e_j}, \bar{\hbar}, \bar{\varrho})$. Thus, we have

$$T_{\varpi(e)}(\bar{p}, \bar{p}_{e_i}, \bar{\varrho}) > \bar{1} - \bar{\hbar}, I_{\varpi(e)}(\bar{p}, \bar{p}_{e_i}, \bar{\varrho}) < \bar{\hbar}, F_{\varpi(e)}(\bar{p}, \bar{p}_{e_i}, \bar{\varrho}) < \bar{\hbar}$$

and

$$T_{\varpi(e)}(\bar{q}, \bar{p}_{e_j}, \bar{\varrho}) > \bar{1} - \bar{\hbar}, I_{\varpi(e)}(\bar{q}, \bar{p}_{e_j}, \bar{\varrho}) < \bar{\hbar}, F_{\varpi(e)}(\bar{q}, \bar{p}_{e_j}, \bar{\varrho}) < \bar{\hbar}.$$

Consider $\lambda = min\{T_{\varpi(e)}(\bar{p}_{e_i}, \bar{p}_{e_j}, \bar{\varrho}) : 1 \leq i, j \leq n\}$, $\mu = max\{I_{\varpi(e)}(\bar{p}_{e_i}, \bar{p}_{e_j}, \bar{\varrho}) : 1 \leq i, j \leq n\}$ and $\sigma = max\{F_{\varpi(e)}(\bar{p}_{e_i}, \bar{p}_{e_j}, \bar{\varrho}) : 1 \leq i, j \leq n\}$, then $\lambda, \mu, \sigma > 0$. Thus, for some $\bar{0} < \bar{\zeta}_1, \bar{\zeta}_2, \bar{\zeta}_3 < \bar{1}$,

$$\begin{aligned} T_{\varpi(e)}(\bar{p}, \bar{q}, 3\bar{\varrho}) &\geq T_{\varpi(e)}(\bar{p}, \bar{p}_{e_i}, \bar{\varrho}) * T_{\varpi(e)}(\bar{p}_{e_i}, \bar{p}_{e_j}, \bar{\varrho}) * T_{\varpi(e)}(\bar{p}_{e_j}, \bar{q}, \bar{\varrho}) \\ &\geq (\bar{1} - \bar{\hbar}) * (\bar{1} - \bar{\hbar}) * \lambda > \bar{1} - \bar{\zeta}_1, \end{aligned}$$

$$\begin{aligned} I_{\varpi(e)}(\bar{p}, \bar{q}, 3\bar{\varrho}) &\leq I_{\varpi(e)}(\bar{p}, \bar{p}_{e_i}, \bar{\varrho}) \diamond I_{\varpi(e)}(\bar{p}_{e_i}, \bar{p}_{e_j}, \bar{\varrho}) \diamond I_{\varpi(e)}(\bar{p}_{e_j}, \bar{q}, \bar{\varrho}) \\ &\leq \bar{\hbar} \diamond \bar{\hbar} \diamond \mu < \bar{\zeta}_2, \end{aligned}$$

$$\begin{aligned} F_{\varpi(e)}(\bar{p}, \bar{q}, 3\bar{\varrho}) &\leq F_{\varpi(e)}(\bar{p}, \bar{p}_{e_i}, \bar{\varrho}) \diamond F_{\varpi(e)}(\bar{p}_{e_i}, \bar{p}_{e_j}, \bar{\varrho}) \diamond F_{\varpi(e)}(\bar{p}_{e_j}, \bar{q}, \bar{\varrho}) \\ &\leq \bar{\hbar} \diamond \bar{\hbar} \diamond \sigma < \bar{\zeta}_3. \end{aligned}$$

Let $\bar{\zeta} = max\{\bar{\zeta}_1, \bar{\zeta}_2, \bar{\zeta}_3\}$ and $\bar{\varrho}_o = 3\bar{\varrho}$, then we have $T_{\varpi(e)}(\bar{p}, \bar{q}, \bar{\varrho}_o) > \bar{1} - \bar{\zeta}$, $I_{\varpi(e)}(\bar{p}, \bar{q}, \bar{\varrho}_o) < \bar{\zeta}$ and $F_{\varpi(e)}(\bar{p}, \bar{q}, \bar{\varrho}_o) < \bar{\zeta}$, for all $\bar{p}, \bar{q} \in \mho$. Thus, $\mho$ is Neutrosophic bounded. □

Theorem 3.5 *Take, τ_ϖ be topology in a NSMS $(\bar{\chi}, \varpi, *, \diamond)$, then a sequence $\{\bar{p}_{e_n}\}$ in $\bar{\chi}$ converges to $\bar{p}$ if and only if $T_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) \to 1$, $I_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) \to 0$ and $F_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) \to 0$, when $n \to \infty$.*

Proof: Consider $\bar{\varrho} > \bar{0}$. Let $\bar{p}_{e_n} \to \bar{p}$. If $\bar{0} < \bar{\epsilon} < \bar{1}$, then there exist $m \in N$ so that $\bar{p}_{e_n} \in B(\bar{p}, \bar{\epsilon}, \bar{\varrho})$ for all $n \geq m$. Thus, $\bar{1} - T_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) < \bar{\epsilon}$, $I_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) < \bar{\epsilon}$ and $F_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) < \bar{\epsilon}$. Thus, we have $T_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) \to 1$, $I_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) \to 0$ and $F_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) \to 0$ as $n \to \infty$. Hence, proved.
Conversely: Let $T_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) \to 1$, $I_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) \to 0$ and $F_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) \to 0$ as $n \to \infty$ for every $\bar{\varrho} > \bar{0}$. Thus, for $\bar{0} < \bar{\epsilon} < \bar{1}$, there exists $m \in N$ so that $\bar{1} - T_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) < \bar{\epsilon}$, $I_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) < \bar{\epsilon}$ and $F_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) < \bar{\epsilon}$ for all $n \geq m$. Therefore, $T_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) > \bar{1} - \bar{\epsilon}$, $I_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) < \bar{\epsilon}$ and $F_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) < \bar{\epsilon}$, for each $n \geq m$. Thus $\bar{p}_{e_n} \in B(\bar{p}, \bar{\epsilon}, \bar{\varrho})$, for each $n \geq m$. Hence, $\bar{p}_{e_n} \to \bar{p}$. □

Definition 3.7 *Consider a NSMS $(\bar{\chi}', \varpi', *, \diamond)$, then any sequence $\{\bar{\iota}_{e_n}\}$ in $\bar{\chi}'$ is Cauchy whenever for all $\bar{\hbar} > \bar{0}$ and $\jmath_o \in N$; $T_{\varpi'(e)}(\bar{p}_{e_m}, \bar{p}_{e_n}, \frac{\bar{\varrho}}{2}) > \bar{1} - \bar{\hbar}$, $I_{\varpi'(e)}(\bar{p}_{e_m}, \bar{p}_{e_n}, \frac{\bar{\varrho}}{2}) < \bar{\hbar}$ and $F_{\varpi'(e)}(\bar{p}_{e_m}, \bar{p}_{e_n}, \frac{\bar{\varrho}}{2}) < \bar{\hbar}$, for every $n, m \geq \jmath_o$.*

Theorem 3.6 *Take a NSMS $(\bar{\chi}, \varpi, *, \diamond)$. Then, it is complete if every Cauchy sequence in $\bar{\chi}$ has a convergent subsequence.*

Proof: Consider, a Cauchy sequence $\{\bar{p}_{e_n}\}$ in $\bar{\chi}$ and let $\{\bar{p}_{e_{n_i}}\}$ be any subsequence of $\{\bar{p}_{e_n}\}$ converging to $\bar{p}$. Consider $\bar{\varrho} > \bar{0}$ and $\bar{\sigma} \in (\bar{0}, \bar{1})$. Let $\bar{0} < \bar{\hbar} < \bar{1}$, satisfying $(\bar{1} - \bar{\hbar}) * (\bar{1} - \bar{\hbar}) \geq \bar{1} - \bar{\sigma}$, $\bar{\hbar} \diamond \bar{\hbar} \leq \bar{\sigma}$. Since $\{\bar{p}_{e_n}\}$ is Cauchy, thus there exists $n_o \in N$ so that $T_{\varpi(e)}(\bar{p}_{e_m}, \bar{p}_{e_n}, \frac{\bar{\varrho}}{2}) > \bar{1} - \bar{\hbar}$, $I_{\varpi(e)}(\bar{p}_{e_m}, \bar{p}_{e_n}, \frac{\bar{\varrho}}{2}) < \bar{\hbar}$ and $F_{\varpi(e)}(\bar{p}_{e_m}, \bar{p}_{e_n}, \frac{\bar{\varrho}}{2}) < \bar{\hbar}$, for each $n, m \geq n_o$.
Since $\bar{p}_{e_{n_i}} \to \bar{p}$, there exists positive integer i_k so that $i_k > n_o$. For $n \geq n_o$, we have

$$\begin{aligned} T_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) &\geq T_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}_{e_{i_k}}, \frac{\bar{\varrho}}{2}) * T_{\varpi(e)}(\bar{p}_{e_{i_k}}, \bar{p}, \frac{\bar{\varrho}}{2}) > (\bar{1} - \bar{\hbar}) * (\bar{1} - \bar{\hbar}) \geq \bar{1} - \bar{\sigma}, \\ I_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) &\leq I_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}_{e_{i_k}}, \frac{\bar{\varrho}}{2}) \diamond I_{\varpi(e)}(\bar{p}_{e_{i_k}}, \bar{p}, \frac{\bar{\varrho}}{2}) < \bar{\hbar} \diamond \bar{\hbar} \leq \bar{\sigma}, \\ F_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}, \bar{\varrho}) &\leq F_{\varpi(e)}(\bar{p}_{e_n}, \bar{p}_{e_{i_k}}, \frac{\bar{\varrho}}{2}) \diamond F_{\varpi(e)}(\bar{p}_{e_{i_k}}, \bar{p}, \frac{\bar{\varrho}}{2}) < \bar{\hbar} \diamond \bar{\hbar} \leq \bar{\sigma} \end{aligned} \tag{3.1}$$

Hence, $\bar{p}_{e_n} \to \bar{p}$ and $(\bar{\chi}, \varpi, *, \diamond)$ is a complete space. □

Definition 3.8 *Take $(\bar{\chi}, \varpi, *, \diamond)$ be NSMS, then it possesses property ⋇ if for every $\bar{\iota}, \bar{\kappa} \in \bar{\chi}$ (3.2) holds*

$$lim \bar{\varrho} \to \infty T_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\varrho}) = 1, lim \bar{\varrho} \to \infty I_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\varrho}) = 0, lim \bar{\varrho} \to \infty F_{\varpi(e)}(\bar{\iota}, \bar{\kappa}, \bar{\varrho}) = 0. \tag{3.2}$$

Definition 3.9 *Take a NSMS $(\bar{\chi}', \varpi', *, \diamond)$ and $\Pi : \bar{\chi}' \to \bar{\chi}'$ be a map, then Π possesses **t-uniform continuity** if for every $\bar{\hbar}' \in (\bar{0}, \bar{1})$ implicates the existence of $\bar{0} < \bar{\xi} < \bar{1}$, such that for every $\bar{\zeta}, \bar{\eta} \in \bar{\chi}'$ and $\bar{\varrho} > 0$ the following inequalities hold*

$$T_{\varpi'(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho}) \geq 1 - \bar{\xi}, I_{\varpi'(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho}) \leq \bar{\xi}, F_{\varpi'(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho}) \leq \bar{\xi}$$

implies,

$$T_{\varpi'(e)}(\Pi(\bar{\zeta}), \Pi(\bar{\eta}), \bar{\varrho}) \geq 1 - \bar{\hbar}', I_{\varpi'(e)}(\Pi(\bar{\zeta}), \Pi(\bar{\eta}), \bar{\varrho}) \leq \bar{\hbar}', F_{\varpi'(e)}(\Pi(\bar{\zeta}), \Pi(\bar{\eta}), \bar{\varrho}) \leq \bar{\hbar}'.$$

Definition 3.10 *Take a NSMS $(\bar{\chi}', \varpi', *, \diamond)$ and $\Pi : \bar{\chi}' \to \bar{\chi}'$ be a map, then Π is **neutrosophic soft fuzzy contractive** if it implicates the existence of $\sigma \in (0, 1)$, so that (3.3) is asserted for every $\bar{\iota}, \bar{\kappa} \in \bar{\chi}'$ and $\bar{\varrho} > 0$*

$$\begin{aligned} \frac{1}{T_{\varpi'(e)}(\Pi(\bar{\iota}), \Pi(\bar{\kappa}), \bar{\varrho})} - 1 &\leq \sigma(\frac{1}{T_{\varpi'(e)}(\bar{\iota}, \bar{\kappa}, \bar{\varrho})} - 1) \\ \frac{1}{I_{\varpi'(e)}(\Pi(\bar{\iota}), \Pi(\bar{\kappa}), \bar{\varrho})} - 1 &\geq \frac{1}{\sigma}(\frac{1}{I_{\varpi'(e)}(\bar{\iota}, \bar{\kappa}, \bar{\varrho})} - 1) \\ \frac{1}{F_{\varpi'(e)}(\Pi(\bar{\iota}), \Pi(\bar{\kappa}), \bar{\varrho})} - 1 &\leq \frac{1}{\sigma}(\frac{1}{F_{\varpi'(e)}(\bar{\iota}, \bar{\kappa}, \bar{\varrho})} - 1). \end{aligned} \tag{3.3}$$

Definition 3.11 *Take a NSMS $(\bar{\chi}', \varpi', *, \diamond)$, then a sequence $\{\bar{p}_{e_n}\}$ in $\bar{\chi}'$ is neutrosophic soft fuzzy contractive if it implicates the existence of $\hbar \in (0, 1)$, so that (3.4) holds for every $\bar{\varrho} > 0$ and $n \in N$*

$$\begin{aligned} \frac{1}{T_{\varpi'(e)}(\bar{p}_{e_{n+1}}, \bar{p}_{e_{n+2}}, \bar{\varrho})} - 1 &\leq \hbar(\frac{1}{T_{\varpi'(e)}(\bar{p}_{e_n}, \bar{p}_{e_{n+1}}, \bar{\varrho})} - 1) \\ \frac{1}{I_{\varpi'(e)}(\bar{p}_{e_{n+1}}, \bar{p}_{e_{n+2}}, \bar{\varrho})} - 1 &\geq \frac{1}{\hbar}(\frac{1}{I_{\varpi'(e)}(\bar{p}_{e_n}, \bar{p}_{e_{n+1}}, \bar{\varrho})} - 1) \\ \frac{1}{F_{\varpi'(e)}(\bar{p}_{e_{n+1}}, \bar{p}_{e_{n+2}}, \bar{\varrho})} - 1 &\leq \frac{1}{\hbar}(\frac{1}{F_{\varpi'(e)}(\bar{p}_{e_n}, \bar{p}_{e_{n+1}}, \bar{\varrho})} - 1). \end{aligned} \tag{3.4}$$

Theorem 3.7 *Take a NSMS $(\bar{\chi}', \varpi, *, \diamond)$ which is complete possessing ⋇ property and having every neutrosophic soft fuzzy contractive sequence Cauchy. Take $\Pi : \bar{\chi}' \to \bar{\chi}'$ a neutrosophic soft fuzzy contractive map and $0 < \nu < 1$ be contractive constant, then Π possesses a fixed point that is unique also.*

4. Fixed point theorems

Theorem 4.1 *Take a complete NSMS* $(\bar{\chi}, \varpi, *, \diamond)$ *possessing* $\circledast$ *property and having every neutrosophic soft fuzzy contractive sequence Cauchy. Take* $\Pi : \bar{\chi} \to \bar{\chi}$ *a neutrosophic soft fuzzy contractive map on closed ball* $S(\bar{p'}_o, \bar{\vartheta}, \bar{\varrho})$ *with contractive constant* ν*. Furthermore, consider*

$$\frac{1}{T_{\varpi(e)}(\bar{p'}_o, \Pi(\bar{p'}_o), \bar{\varrho})} - 1 < (1-\nu)(\frac{1}{1-\bar{\vartheta}} - 1)$$
$$\frac{1}{I_{\varpi(e)}(\bar{p'}_o, \Pi(\bar{p'}_o), \bar{\varrho})} - 1 > \frac{1}{1-\nu}(\frac{1}{\bar{\vartheta}} - 1)$$
$$\frac{1}{F_{\varpi(e)}(\bar{p'}_o, \Pi(\bar{p'}_o), \bar{\varrho})} - 1 > \frac{1}{1-\nu}(\frac{1}{\bar{\vartheta}} - 1).$$

Then, Π *possesses a fixed point that is unique also in* $S(\bar{p'}_o, \bar{\vartheta}, \bar{\varrho})$*.*

Proof: Take $\bar{\zeta}_1 = \Pi(\bar{\zeta}_o)$, $\bar{\zeta}_2 = \Pi(\bar{\zeta}_1) = \Pi^2(\bar{\zeta}_o)$,..., $\bar{\zeta}_n = \Pi(\bar{\zeta}_{n-1})$. Then (4.1), (4.2) and (4.3) holds

$$\begin{aligned} &\frac{1}{T_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_1, \bar{\varrho})} - 1 < (1-\nu)(\frac{1}{1-\bar{\vartheta}} - 1) = (1-\nu)(\frac{\bar{\vartheta}}{1-\bar{\vartheta}}) \\ &T_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_1, \bar{\varrho}) > 1-\bar{\vartheta}, \end{aligned} \tag{4.1}$$

$$\begin{aligned} &\frac{1}{I_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_1, \bar{\varrho})} - 1 > \frac{1}{1-\nu}(\frac{1-\bar{\vartheta}}{\bar{\vartheta}}) > \frac{1-\bar{\vartheta}\nu}{\bar{\vartheta}(1-\nu)} \\ &I_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_1, \bar{\varrho}) < \bar{\vartheta}, \end{aligned} \tag{4.2}$$

$$\begin{aligned} &\frac{1}{F_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_1, \bar{\varrho})} - 1 > \frac{1}{1-\nu}(\frac{1-\bar{\vartheta}}{\bar{\vartheta}}) > \frac{1-\bar{\vartheta}\nu}{\bar{\vartheta}(1-\nu)} \\ &F_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_1, \bar{\varrho}) < \bar{\vartheta} \end{aligned} \tag{4.3}$$

Then, $\bar{\zeta}_1 \in S(\bar{\zeta}_o, \bar{\vartheta}, \bar{\varrho})$.
Let $\bar{\zeta}_1, \bar{\zeta}_2, ..., \bar{\zeta}_n \in S(\bar{\zeta}_o, \bar{\vartheta}, \bar{\varrho})$. Then

$$\begin{aligned} &\frac{1}{T_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}_2, \bar{\varrho})} - 1 = \frac{1}{T_{\varpi(e)}(\Pi(\bar{\zeta}_o), \Pi(\bar{\zeta}_1), \bar{\varrho})} - 1 < \nu(\frac{1}{1-\bar{\vartheta}} - 1) \\ &T_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}_2, \bar{\varrho}) > \frac{1-\bar{\vartheta}}{1-\bar{\vartheta}(1-\nu)} > 1-\bar{\vartheta}, \end{aligned}$$

$$\begin{aligned} &\frac{1}{I_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}_2, \bar{\varrho})} - 1 = \frac{1}{I_{\varpi(e)}(\Pi(\bar{\zeta}_o), \Pi(\bar{\zeta}_1), \bar{\varrho})} - 1 > \frac{1}{\nu}(\frac{1-\bar{\vartheta}}{\bar{\vartheta}}) \\ &I_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}_2, \bar{\varrho}) < \frac{\nu\bar{\vartheta}}{1-\bar{\vartheta}(1-\nu)} < \bar{\vartheta}, \end{aligned}$$

$$\begin{aligned} &\frac{1}{F_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}_2, \bar{\varrho})} - 1 = \frac{1}{F_{\varpi(e)}(\Pi(\bar{\zeta}_o), \Pi(\bar{\zeta}_1), \bar{\varrho})} - 1 > \frac{1}{\nu}(\frac{1-\bar{\vartheta}}{\bar{\vartheta}}) \\ &F_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}_2, \bar{\varrho}) < \frac{\nu\bar{\vartheta}}{1-\bar{\vartheta}(1-\nu)} < \bar{\vartheta}. \end{aligned}$$

Similarly, we can claim that $T_{\varpi(e)}(\bar{\zeta}_{n-1}, \bar{\zeta}_n, \bar{\varrho}) > 1 - \bar{\vartheta}$, $I_{\varpi(e)}(\bar{\zeta}_{n-1}, \bar{\zeta}_n, \bar{\varrho}) < \bar{\vartheta}$ and $F_{\varpi(e)}(\bar{\zeta}_{n-1}, \bar{\zeta}_n, \bar{\varrho}) < \bar{\vartheta}$. Therefore,

$$\begin{aligned}
&T_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_n, \bar{\varrho}) \geq T_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_1, \frac{\bar{\varrho}}{n}) * T_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}_2, \frac{\bar{\varrho}}{n}) * \dots * T_{\varpi(e)}(\bar{\zeta}_{n-1}, \bar{\zeta}_n, \frac{\bar{\varrho}}{n})\\
&T_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_n, \bar{\varrho}) > (1 - \bar{\vartheta}) * (1 - \bar{\vartheta}) * \dots * (1 - \bar{\vartheta}) = 1 - \bar{\vartheta}\\
&T_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_n, \bar{\varrho}) > (1 - \bar{\vartheta}),
\end{aligned}$$

$$\begin{aligned}
&I_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_n, \bar{\varrho}) \leq I_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_1, \frac{\bar{\varrho}}{n}) \diamond I_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}_2, \frac{\bar{\varrho}}{n}) \diamond \dots \diamond I_{\varpi(e)}(\bar{\zeta}_{n-1}, \bar{\zeta}_n, \frac{\bar{\varrho}}{n})\\
&I_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_n, \bar{\varrho}) < \bar{\vartheta} \diamond \bar{\vartheta} \diamond \dots \diamond \bar{\vartheta} = \bar{\vartheta}\\
&I_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_n, \bar{\varrho}) < \bar{\vartheta},
\end{aligned}$$

$$\begin{aligned}
&F_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_n, \bar{\varrho}) \leq F_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_1, \frac{\bar{\varrho}}{n}) \diamond F_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}_2, \frac{\bar{\varrho}}{n}) \diamond \dots \diamond F_{\varpi(e)}(\bar{\zeta}_{n-1}, \bar{\zeta}_n, \frac{\bar{\varrho}}{n})\\
&F_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_n, \bar{\varrho}) < \bar{\vartheta} \diamond \bar{\vartheta} \diamond \dots \diamond \bar{\vartheta} = \bar{\vartheta}\\
&F_{\varpi(e)}(\bar{\zeta}_o, \bar{\zeta}_n, \bar{\varrho}) < \bar{\vartheta}.
\end{aligned}$$

Thus, $\bar{\zeta}_n \in S(\bar{\zeta}_o, \bar{\vartheta}, \bar{\varrho})$ and Π possesses a unique fixed point. □

Definition 4.1 *Take a NSMS $(\bar{\chi}', \varpi, *, \diamond)$ and $\Pi : \bar{\chi}' \to \bar{\chi}'$ a map, then Π is $\Pi S-$**neutrosophic soft fuzzy contractive** if it implicates the existence of $\hbar \in (0,1)$ so that (4.4) is asserted for every $\bar{\zeta}, \bar{\eta} \in \bar{\chi}'$ and $\bar{\varrho} > 0$*

$$\begin{aligned}
&\hbar T_{\varpi(e)}(\Pi(\bar{\zeta}), \Pi(\bar{\eta}), \bar{\varrho}) \geq T_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho})\\
&\frac{1}{\hbar} I_{\varpi(e)}(\Pi(\bar{\zeta}), \Pi(\bar{\eta}), \bar{\varrho}) \leq I_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho})\\
&\frac{1}{\hbar} F_{\varpi(e)}(\Pi(\bar{\zeta}), \Pi(\bar{\eta}), \bar{\varrho}) \leq F_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho}).
\end{aligned} \tag{4.4}$$

Theorem 4.2 *Take a complete NSMS $(\bar{\chi}', \varpi, *, \diamond)$ possessing $\circledast$ property. Take $\Pi : \bar{\chi}' \to \bar{\chi}'$ a $\Pi S-$neutrosophic soft fuzzy contractive map and $0 < \nu < 1$ be contractive constant, then Π possesses a fixed point in $\bar{\chi}'$ that is unique also.*

Proof: Take $\bar{\zeta} \in \bar{\chi}'$ and $\bar{\zeta}_n \in \Pi^n(\bar{\zeta})$ for every $n \in \mathbb{N}$. Then for $\bar{\varrho} > 0$, we have

$$\nu T_{\varpi(e)}(\bar{\zeta}_2, \bar{\zeta}_1, \bar{\varrho}) = \nu T_{\varpi(e)}(\Pi(\bar{\zeta}_1), \Pi(\bar{\zeta}), \bar{\varrho}) \geq T_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}, \bar{\varrho}),$$

$$\frac{1}{\nu} I_{\varpi(e)}(\bar{\zeta}_2, \bar{\zeta}_1, \bar{\varrho}) = \frac{1}{\nu} I_{\varpi(e)}(\Pi(\bar{\zeta}_1), \Pi(\bar{\zeta}), \bar{\varrho}) \leq I_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}, \bar{\varrho})$$

and

$$\frac{1}{\nu} F_{\varpi(e)}(\bar{\zeta}_2, \bar{\zeta}_1, \bar{\varrho}) = \frac{1}{\nu} F_{\varpi(e)}(\Pi(\bar{\zeta}_1), \Pi(\bar{\zeta}), \bar{\varrho}) \leq F_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}, \bar{\varrho}).$$

Now,

$$\begin{aligned}
&\nu T_{\varpi(e)}(\bar{\zeta}_3, \bar{\zeta}_2, \bar{\varrho}) = \nu T_{\varpi(e)}(\Pi(\bar{\zeta}_2), \Pi(\bar{\zeta}_1), \bar{\varrho}) \geq T_{\varpi(e)}(\bar{\zeta}_2, \bar{\zeta}_1, \bar{\varrho})\\
&\nu^2 T_{\varpi(e)}(\bar{\zeta}_3, \bar{\zeta}_2, \bar{\varrho}) \geq \nu T_{\varpi(e)}(\bar{\zeta}_2, \bar{\zeta}_1, \bar{\varrho}) \geq T_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}, \bar{\varrho}),
\end{aligned}$$

$$\begin{aligned}
&\frac{1}{\nu} I_{\varpi(e)}(\bar{\zeta}_3, \bar{\zeta}_2, \bar{\varrho}) = \frac{1}{\nu} I_{\varpi(e)}(\Pi(\bar{\zeta}_2), \Pi(\bar{\zeta}1), \bar{\varrho}) \leq I_{\varpi(e)}(\bar{\zeta}_2, \bar{\zeta}_1, \bar{\varrho})\\
&\frac{1}{\nu^2} I_{\varpi(e)}(\bar{\zeta}_3, \bar{\zeta}_2, \bar{\varrho}) \geq \frac{1}{\nu} I_{\varpi(e)}(\bar{\zeta}_2, \bar{\zeta}_1, \bar{\varrho}) \leq I_{\varpi(e)}(\bar{\zeta}_1, \bar{\zeta}, \bar{\varrho})
\end{aligned}$$

and

$$\frac{1}{\nu}F_{\varpi(e)}(\bar{\zeta_3},\bar{\zeta_2},\bar{\varrho}) = \frac{1}{\nu}F_{\varpi(e)}(\Pi(\bar{\zeta_2}),\Pi(\bar{\zeta_1}),\bar{\varrho}) \leq F_{\varpi(e)}(\bar{\zeta_2},\bar{\zeta_1},\bar{\varrho})$$
$$\frac{1}{\nu^2}F_{\varpi(e)}(\bar{\zeta_3},\bar{\zeta_2},\bar{\varrho}) \leq \frac{1}{\nu}F_{\varpi(e)}(\bar{\zeta_2},\bar{\zeta_1},\bar{\varrho}) \leq F_{\varpi(e)}(\bar{\zeta_1},\bar{\zeta},\bar{\varrho}).$$

Thus, for $n \in \mathbb{N}$

$$\nu^2 T_{\varpi(e)}(\bar{\zeta}_{n+1},\bar{\zeta_n},\bar{\varrho}) \geq T_{\varpi(e)}(\bar{\zeta_1},\bar{\zeta},\bar{\varrho}),$$
$$\frac{1}{\nu^2}I_{\varpi(e)}(\bar{\zeta}_{n+1},\bar{\zeta_n},\bar{\varrho}) \leq I_{\varpi(e)}(\bar{\zeta_1},\bar{\zeta},\bar{\varrho}),$$
$$\frac{1}{\nu^2}F_{\varpi(e)}(\bar{\zeta}_{n+1},\bar{\zeta_n},\bar{\varrho}) \geq F_{\varpi(e)}(\bar{\zeta_1},\bar{\zeta},\bar{\varrho}).$$

Now, claim that $\bar{\zeta_n}$ is Cauchy in $(\bar{\chi'},\varpi,*,\diamond)$. Consider, $\bar{\varrho'} = \frac{\varrho}{\lambda}$, then

$$\begin{aligned}
T_{\varpi(e)}(\bar{\zeta_n},\bar{\zeta}_{n+\lambda},\bar{\varrho'}) &\geq T_{\varpi(e)}(\bar{\zeta_n},\bar{\zeta}_{n+1},\bar{\varrho'}) * ... * T_{\varpi(e)}(\bar{\zeta}_{n+\lambda-1},\bar{\zeta}_{n+\lambda},\bar{\varrho'})\\
&\geq (\frac{1}{\nu^n}\nu^n T_{\varpi(e)}(\bar{\zeta_n},\bar{\zeta}_{n+1},\bar{\varrho'})) * ... * (\frac{1}{\nu^{n+\lambda-1}}\nu^{n+\lambda-1}T_{\varpi(e)}(\bar{\zeta}_{n+\lambda-1},\bar{\zeta}_{n+\lambda},\bar{\varrho'}))\\
&\geq (\frac{1}{\nu^n}T_{\varpi(e)}(\bar{\zeta_1},\bar{\zeta},\bar{\varrho'})) * ... * (\frac{1}{\nu^{n+\lambda-1}}T_{\varpi(e)}(\bar{\zeta_1},\bar{\zeta},\bar{\varrho'}))\\
&\geq (\frac{1}{\nu^n}T_{\varpi(e)}(\bar{\zeta_1},\bar{\zeta},\bar{\varrho'})) * (\frac{1}{\nu^n}T_{\varpi(e)}(\bar{\zeta_1},\bar{\zeta},\bar{\varrho'})) = (\frac{1}{\nu^n}T_{\varpi(e)}(\bar{\zeta_1},\bar{\zeta},\bar{\varrho'})).
\end{aligned}$$

Therefore, $lim_{n\to\infty}T_{\varpi(e)}(\bar{\zeta_n},\bar{\zeta}_{n+\lambda},\bar{\varrho'}) = 1$.
On similar lines, we can claim that $lim_{n\to\infty}I_{\varpi(e)}(\bar{\zeta_n},\bar{\zeta}_{n+\lambda},\bar{\varrho'}) = 0$ and $lim_{n\to\infty}F_{\varpi(e)}(\bar{\zeta_n},\bar{\zeta}_{n+\lambda},\bar{\varrho'}) = 0$ implying that $\{\bar{\zeta_n}\}$ is Cauchy in NSMS.
The completeness of $(\bar{\chi'},\varpi,*,\diamond)$ implicates the existence of $\bar{\gamma} \in \bar{\chi'}$, so that $\bar{\zeta_n} \to \bar{\gamma}$ when $n \to \infty$.
Thus,

$$\nu T_{\varpi(e)}(\Pi(\bar{\zeta_n}),\Pi(\bar{\zeta}),\bar{\varrho}) \geq T_{\varpi(e)}(\bar{\zeta_n},\bar{\gamma},\bar{\varrho})$$
$$lim_{n\to\infty}T_{\varpi(e)}(\Pi(\bar{\zeta_n}),\Pi(\bar{\gamma}),\bar{\varrho}) \geq \frac{1}{\nu} > 1$$
$$1 < lim_{n\to\infty}T_{\varpi(e)}(\Pi(\bar{\zeta_n}),\Pi(\bar{\gamma}),\bar{\varrho}) \leq 1$$
$$lim_{n\to\infty}T_{\varpi(e)}(\Pi(\bar{\zeta_n}),\Pi(\bar{\gamma}),\bar{\varrho}) = 1,$$

$$I_{\varpi(e)}(\Pi(\bar{\zeta_n}),\Pi(\bar{\gamma}),\bar{\varrho}) \leq \nu I_{\varpi(e)}(\bar{\zeta_n},\bar{\gamma},\bar{\varrho})$$
$$lim_{n\to\infty}I_{\varpi(e)}(\Pi(\bar{\zeta_n}),\Pi(\bar{\gamma}),\bar{\varrho}) \leq lim_{n\to\infty}\nu I_{\varpi(e)}(\bar{\zeta_n},\bar{\gamma},\bar{\varrho}) = 0$$
$$lim_{n\to\infty}I_{\varpi(e)}(\Pi(\bar{\zeta_n}),\Pi(\bar{\gamma}),\bar{\varrho}) = 0,$$

and

$$F_{\varpi(e)}(\Pi(\bar{\zeta_n}),\Pi(\bar{\gamma}),\bar{\varrho}) \leq \nu F_{\varpi(e)}(\bar{\zeta_n},\bar{\gamma},\bar{\varrho})$$
$$lim_{n\to\infty}F_{\varpi(e)}(\Pi(\bar{\zeta_n}),\Pi(\bar{\gamma}),\bar{\varrho}) \leq lim_{n\to\infty}\nu F_{\varpi(e)}(\bar{\zeta_n},\bar{\gamma},\bar{\varrho}) = 0$$
$$lim_{n\to\infty}F_{\varpi(e)}(\Pi(\bar{\zeta_n}),\Pi(\bar{\gamma}),\bar{\varrho}) = 0.$$

Therefore,

$$lim_{n\to\infty}\Pi(\bar{\zeta_n}) = \Pi(\bar{\gamma}) \Rightarrow lim_{n\to\infty}\bar{\zeta_n} = \Pi(\bar{\gamma}).$$

Therefore, Π possesses $\bar{\gamma}$ as its fixed point.
Uniqueness: Take $\bar{\delta} \in \bar{\chi'}$ such that $\Pi(\bar{\delta}) = \bar{\delta}$. Now, we have

$$\begin{aligned}
T_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) &= T_{\varpi(e)}(\Pi(\bar{\gamma}), \Pi(\bar{\delta}), \bar{\varrho}) \\
&\geq \frac{1}{\nu} T_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) \\
&\geq \frac{1}{\nu} T_{\varpi(e)}(\Pi(\bar{\gamma}), \Pi(\bar{\delta}), \bar{\varrho}) \\
&\geq \frac{1}{\nu^2} T_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) \geq ... \\
&\geq \frac{1}{\nu^n} T_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) \to 1.
\end{aligned}$$

Thus, $T_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) = 1$. Similarly, we have (4.5) and (4.6)

$$\begin{aligned}
I_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) &= I_{\varpi(e)}(\Pi(\bar{\gamma}), \Pi(\bar{\delta}), \bar{\varrho}) \\
&\leq \nu I_{\varpi(e)}(\Pi(\bar{\gamma}), \Pi(\bar{\delta}), \bar{\varrho}) \\
&\leq \nu^2 I_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) \leq ... \\
&\leq \nu^n I_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) \to 0,
\end{aligned} \tag{4.5}$$

$$\begin{aligned}
F_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) &= F_{\varpi(e)}(\Pi(\bar{\gamma}), \Pi(\bar{\delta}), \bar{\varrho}) \\
&\leq \nu F_{\varpi(e)}(\Pi(\bar{\gamma}), \Pi(\bar{\delta}), \bar{\varrho}) \\
&\leq \nu^2 F_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) \leq ... \\
&\leq \nu^n F_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) \to 0.
\end{aligned} \tag{4.6}$$

Hence, $I_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) = 0$ and $F_{\varpi(e)}(\bar{\gamma}, \bar{\delta}, \bar{\varrho}) = 0$. Therefore, $\bar{\gamma} = \bar{\delta}$ □

Example 4.3 *Take,* $\bar{\chi'} = \{\frac{1}{m'} : m' \in \mathbb{N}\} \cup \{0\}$. *Take*

$$T_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho}) = \frac{\bar{\varrho}}{\bar{\varrho}+ \mid \bar{\zeta} - \bar{\eta} \mid}, I_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho}) = \frac{\mid \bar{\zeta} - \bar{\eta} \mid}{\bar{\varrho}+ \mid \bar{\zeta} - \bar{\eta} \mid}, F_{\varpi(e)}(\bar{\zeta}, \bar{\eta}, \bar{\varrho}) = \frac{\mid \bar{\zeta} - \bar{\eta} \mid}{\bar{\varrho}}.$$

Then, $(\bar{\chi'}, \varpi, *, \diamond)$ *is a complete NSMS with* $*$ *and* $\diamond$ *defined as* $\phi * \upsilon = \phi\upsilon$, $\phi \diamond \upsilon = min\{1, \phi + \upsilon\}$. *Let*

$$\Pi(\iota) = \begin{cases} 1, & \text{if } \iota \text{ is rational} \\ 0, & \text{if } \iota \text{ is irrational} \end{cases}$$

Then, Π *satisfies theorem (4.2) and possesses a fixed point* 1 *that is unique also.*

5. Application

Now, here we are going to utilize our results to solve a differential equation.
Take the fractional differential equation

$$\Psi^{\upsilon}_{0+}\varphi(\iota) + \ell(\iota, \varphi(\iota)) = 0, \iota \in (0, 1) \tag{5.1}$$

with the following boundary conditions:

$$\varphi(0) + \varphi'(0) = 0, \varphi(1) + \varphi'(1) = 0.$$

Define, $T_{\varpi(e)}$, $I_{\varpi(e)}$ and $F_{\varpi(e)}$ for $\varrho > 0$as

$$\begin{aligned}
T_{\varpi(e)}(\varphi(\iota'), \wp(\iota'), \varrho) &= sup_{\iota' \in [a,b]} \frac{\varrho}{\varrho + |\varphi(\iota') - \wp(\iota')|^2} \\
I_{\varpi(e)}(\varphi(\iota'), \wp(\iota'), \varrho) &= 1 - sup_{\iota' \in [a,b]} \frac{\varrho}{\varrho + |\varphi(\iota') - \wp(\iota')|^2} \\
F_{\varpi(e)}(\varphi(\iota'), \wp(\iota'), \varrho) &= sup_{\iota' \in [a,b]} \frac{|\varphi(\iota') - \wp(\iota')|^2}{\varrho}
\end{aligned}$$

with $*$ and $\diamond$ given as $\phi * \upsilon = \phi\upsilon$ and $\phi \diamond \upsilon = max\{\phi, \upsilon\}$.
Let $\Im : \chi' \times \chi' \to [1, \infty]$ defined as $\Im(\varphi, \wp) = \varphi + \wp + 1$.
Then, $(\bar{\chi'}, \varpi, *, \diamond)$ is complete. It is trivial that $\varphi \in \chi'$ solves (5.1) by Theorem (4.2).

6. Conclusion

In this paper, we have introduced new fixed point theorems (FPTs) in neutrosophic soft metric spaces (NSMS). The results obtained not only extend existing work but also provide a foundation for further investigations in this field. To substantiate the validity of our findings, illustrative examples have been presented together with an application that highlights their applicability. We believe that the new results established in this work will contribute significantly to the advancement of fixed point theory in NSMS and will encourage the development of further important results in the future.

References

1. L.A. Zadeh, *Fuzzy Sets*. Fuzzy Sets Information and Control 8, 338–353, (1965).
2. O. Kaleva, S. Seikkala, *On fuzzy metric spaces*. Fuzzy Sets Syst 12, 225–229, (1984).
3. A. George, P. Veeramani, *On some results in fuzzy metric spaces*. Fuzzy Sets Syst 64, 395–399, (1994).
4. JH. Park, *Intuitionistic fuzzy metric spaces*. Chaos, Solitons and Fractals 22, 1039–1046, (2004).
5. F. Samarandache, *Neutrosophy: Neutrosophic Probability, Set and Logic*. ProQuest Information and Learning: Ann Arbor, MI, USA, 1–105, (1998).
6. M. Kirisci, N. Simsek, *Neutrosophic metric spaces*. Mathematical Sciences 14(3), 1–11, (2020).
7. D. Molodtsov, *Soft set theory-First results*. Computers and Mathematics with Applications 37(4-5), 19–31, (1991).
8. S. Sowndrarajan, M. Jeyaraman, F. Smarandache, *Fixed Point Results for Contraction Theorems in Neutrosophic Metric Spaces*. Neutrosophic Sets and Systems 36(1), 1–12, (2020).
9. T.K. Samanta, Sumit Mohanta, Iqbal H. Jebril, *On fixed-point theorems in intuitionistic fuzzy metric space*. International Journal of Open Problems in Computer Science and Mathematics 5, DOI 10.12816/0006102.
10. F. Karaaslan, *Neutrosophic Soft Sets with Applications in Decision Making*. International Journal of Information Science and Intelligent System 4(2), 1–20, (2015).
11. Vishal Gupta, Aanchal Gondhi *Fixed Point Theorems in Neutrosophic Soft Metric Space*. Advances in Intelligent Systems and Computing, 25–37, (2023).
12. Vishal Gupta, Aanchal Gondhi, Naveen Mani *Contraction theorems in neutrosophic soft metric space*. AIP Conference Proceedings, DOI 10.1063/5.0140216, (2023).
13. Vishal Gupta, Aanchal Gondhi *Fixed points of weakly compatible maps on modified intuitionistic fuzzy soft metric spaces*. International Journal of Systems Assurance Engineering and Management 13(3), 1232–1238 (2022).
14. Vishal Gupta, Aanchal Gondhi *Soft Tripled Coincidence fixed point theorems in Soft Fuzzy metric space*. Journal of Siberian Federal University. Mathematics and Physics 16(3), 398–408 (2023).
15. Vishal Gupta, Aanchal Gondhi, R. K. Saini *Rational-type soft fuzzy contraction in soft fuzzy metric space and solution of nonlinear integral equation*. International Journal of Modern Physics B 38(2), 01–13 (2024).
16. Vishal Gupta, Aanchal Gondhi, Rahul Shukla *Fixed Point Results in Modified Intuitionistic Fuzzy Soft Metric Spaces with Application*. Mathematics 12(8), 01–11 (2024).
17. Vishal Gupta, Aanchal Gondhi *On Modified Intuitionistic Fuzzy Soft Metric Spaces and Application*. Miskolc Mathematical Notes 25(1), 255–269 (2024).
18. Vishal Gupta, Aanchal Gondhi *Existence of fixed points in soft metric spaces with application to boundary value problem*. Journal of Ambient Intelligence and Humanized Computing 15(4), 2557-2563 (2024).
19. Vishal Gupta, Manish Jain, Aanchal, Sunakshi Kapoor *New Fixed Point Results in Soft Fuzzy Metric Spaces Employing Soft Fuzzy Contractions. In: Anita Tomar, Manish Jain (eds)*. Banach Contraction Principle: A Centurial Journey, part of the book series: Industrial and Applied Mathematics (INAMA), Springer, Singapore Chapter 17, 291–303 (2025).
20. Vishal Gupta, Nitika Garg, Rahul Shukla *Subsequential Continuity in Neutrosophic Metric Space with Applications*. Computation 13(4), 01-21 (2025).

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