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Rational Type Intuitionistic Fuzzy Soft Contraction results and application to Integral problem

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ABSTRACT: This work aims to present the concept of intuitionistic fuzzy soft contraction maps of rational type in intuitionistic fuzzy soft metric spaces ($\mathbb{IFSMS}$). With thorough examples to support our outcomes, we provide some fixed point findings in $\mathbb{IFSMS}$ under rational type intuitionistic fuzzy soft contraction conditions. Additionally, we incorporate an application to solve a nonlinear integral problem for a unique solution in order to assist our efforts.

Key Words: Intuitionistic Fuzzy soft set; fixed points; rational type contraction.

Contents

1. Introduction

The idea of fuzzy sets was first presented by Zadeh [24] in 1965. These sets allow for a gradual evaluation of an element's membership in a set. Some fixed point theorems in fuzzy metric ($\mathbb{FM}$) spaces of contractive type were demonstrated in 2002 by Gregory and Sapena [14]. Several kinds of fixed point findings in intuitionistic fuzzy metric space were demonstrated by Park [18]. The theory of fixed point also contains some findings by using integral and triangular properties ([1], [4], [16], [19]).
Molodtsov [17] established the idea of soft set theory. Based on the soft sets, Das and Samanta [9] proposed the idea of soft metric space which was further extended by Beaula and Gunaseli to define fuzzy soft metric space [5]. The intuitionistic fuzzy set notion was expanded upon by Attansov [2]. The $\mathbb{IFSMS}$ was defined by Malleswari and Amarendra Babu [22]. (For additional details, see [15], [10], [11], [12]).

Using the idea of the triangle property, this study attempts to propose the new notion of rational type Intuitionistic fuzzy soft contraction mappings in $\mathbb{G}$-complete $\mathbb{IFSMS}$.

2. Preliminaries

The notations V, $\mathbb{P}$ and $\mathbb{P}(V)$ stand for the universal set, the collection of parameters, as well as the assembly of all V-subsets respectively.

Definition 2.1 [17] For $B \subset V$, a soft set over V is a pair $(\mathbb{S}, B)$ given by $\mathbb{S} : B \to \mathbb{P}(V)$. A class of subsets of V thus characterizes a soft set.

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Definition 2.2 [17] For a fuzzy soft set $\mathbb{S} = \{(b_\wr, \alpha(b_\wr)) : b_\wr \in B, \wr \in \mathbb{P}\}$ on B, the map $\alpha : B \to [0,1]$ represents the soft membership, and $\alpha(b_\wr)$ indicates the degree of soft membership of $b_\wr$.

Definition 2.3 [17] $F = \{b, \alpha_F(b), \beta_F(b) : b \in B\}$ is an intuitionistic fuzzy (IF) set that meets the criterion $0 \leq \alpha_F(b) + \beta_F(b) \leq 1$ where for every $b \in B$; $\alpha_F : B \to [0,1]$ represents the degree of association, expressed as $\alpha_F(b)$ and $\beta_F : B \to [0,1]$ expresses the degree of non association expressed as $\beta_F(b)$.

Definition 2.4 [9] A soft fuzzy metric (SFM) on B is determined by $\mathbb{S} : \mathbb{S}L(B) \times \mathbb{S}L(B) \times (0,\infty) \to [0,1]$, if the following conditions hold true:

$$
\begin{aligned}
&(i)\ \mathbb{S}(b_{\wr_1}, h_{\wr_2}, a) > 0, \forall\ b_{\wr_1}, h_{\wr_2} \in B \textit{ and } a > 0,\\
&(ii)\ \mathbb{S}(b_{\wr_1}, h_{\wr_2}, a) = 1 \iff b_{\wr_1} = h_{\wr_2} \forall\ b_{\wr_1}, h_{\wr_2} \in B \textit{ and } a > 0,\\
&(iii)\ \mathbb{S}(b_{\wr_1}, h_{\wr_2}, a) = \mathbb{S}(h_{\wr_2}, b_{\wr_1}, a), \forall\ b_{\wr_1}, h_{\wr_2} \in X \textit{ and } a > 0,\\
&(iv)\ \mathbb{S}(b_{\wr_1}, i_{\wr_3}, a \oplus s) = \mathbb{S}(b_{\wr_1}, h_{\wr_2}, a)\ \star\ \mathbb{S}(h_{\wr_2}, i_{\wr_3}, s),\ \forall\ b_{\wr_1}, h_{\wr_2}, i_{\wr_3} \in B \textit{ and } a > 0,\\
&(v)\ \mathbb{S}(b_{\wr_1}, h_{\wr_2}, .) : (0,\infty) \to [0,1] \textit{ is continuous.}
\end{aligned}
$$

$(B, \mathbb{S})$ is called a SFM space.

Definition 2.5 [21] If $\star : [0,1] \times [0,1] \to [0,1]$ fulfills the following axioms:

$$
\begin{aligned}
&(i)\ \theta_1 \star \theta_2 = \theta_2 \star \theta_1,\\
&(ii)\ (\theta_1 \star \theta_2) \star \theta_3 = \theta_1 \star (\theta_2 \star \theta_3),\\
&(iii)\ \star : [0,1] \times [0,1] \to [0,1] \textit{ is continuous},\\
&(iv)\ \theta_1 \star 1 = 1 \star \theta_1,\\
&(v)\ \theta_1 \star \theta_2 \leq \theta_3 \star \theta_4 \textit{ if } \theta_1 \leq \theta_3,\ \theta_2 \leq \theta_4\ \forall\ \theta_1, \theta_2, \theta_3, \theta_4 \in [0,1].
\end{aligned}
$$

Then $\star$ is considered as a continuous t-norm.

Definition 2.6 [21] If $\diamondsuit : [0,1] \times [0,1] \to [0,1]$ fulfills:

$$
\begin{aligned}
&(i)\ \theta_1 \diamondsuit \theta_2 = \theta_2 \diamondsuit \theta_1,\\
&(ii)\ (\theta_1 \diamondsuit \theta_2) \diamondsuit \theta_3 = \theta_1 \diamondsuit (\theta_2 \diamondsuit \theta_3),\\
&(iii)\ \diamondsuit :\ [0,1] \times [0,1] \to [0,1] \textit{ is continuous},\\
&(iv)\ \theta_1 \diamondsuit 0 = 0 \diamondsuit \theta_1,\\
&(v)\ \theta_1 \diamondsuit \theta_2 \leq \theta_3 \diamondsuit \theta_4 \textit{ if } \theta_1 \leq \theta_3,\ \theta_2 \leq \theta_4\ \forall\ \theta_1, \theta_2, \theta_3, \theta_4 \in [0,1].
\end{aligned}
$$

Then $\diamondsuit$ is considered as a continuous t-conorm.

Definition 2.7 [22] A 5-tuple $(\tilde{B}, \mathbb{S}, T, \tilde{\star}, \tilde{\diamondsuit})$, where $\tilde{B}$ is an arbitrary set, defines an IF soft metric space.
The t-norm and t-conorm are denoted by $\star$ and $\tilde{\diamondsuit}$ respectively. $\mathbb{S} : \mathbb{S}H(\tilde{B}) \times \mathbb{S}H(\tilde{B}) \times (0,\infty) \to [0,1]$, $T : \mathbb{S}H(\tilde{B}) \times \mathbb{S}H(\tilde{B}) \times (0,\infty) \to [0,1]$ are soft fuzzy sets satisfying the following axioms for all $\tilde{b_{\wr_1}},\ \tilde{h_{\wr_2}},\ \tilde{z_{\wr_3}} \in \mathbb{S}H(\tilde{B}),\ s,\ a > 0$:

(a) $\mathbb{S}(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a) + T(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a) \leq 1$;
(b) $\mathbb{S}(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a) > 0$;
(c) $\mathbb{S}(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a) = 1 \iff \tilde{b_{l_1}} = \tilde{h_{l_2}}$;
(d) $\mathbb{S}(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a) = \mathbb{S}(\tilde{h_{l_2}}, \tilde{b_{l_1}}, a)$;
(e) $\mathbb{S}(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a) \star (\mathbb{S}\tilde{h_{l_2}}, \tilde{i_{l_3}}, t) \leq \mathbb{S}(\tilde{b_{l_1}}, \tilde{i_{l_3}}, a+s)$;
(f) $\mathbb{S}(\tilde{b_{l_1}}, \tilde{h_{l_2}}, .) : (0, \infty) \to (0, 1]H$ *is continuous*;
(g) $T(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a) > 0$;
(h) $T(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a) = 0 \iff \tilde{b_{l_1}} = \tilde{h_{l_2}}$;
(i) $T(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a) = T(\tilde{h_{l_2}}, \tilde{b_{l_1}}, t)$;
(j) $T(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a) \mathbin{\tilde{\Diamond}} T(\tilde{h_{l_2}}, \tilde{i_{l_3}}, s) \geq T(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a+s)$;
(k) $T(\tilde{b_{l_1}}, \tilde{h_{l_2}}, .) : (0, \infty) \to (0, 1]H$ *is continuous*.

The 2-tuple $(\mathbb{S}, T)$ is referred to as a $\mathbb{IFSM}$ on $\tilde{B}$. The function $\mathbb{S}(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a)$ indicates the degree of closeness between $\tilde{b_{l_1}}, \tilde{h_{l_2}}$ w.r.t. a, while the function $T(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a)$ indicates the degree of non-closeness between $\tilde{b_{l_1}}, \tilde{h_{l_2}}$ w.r.t a.

3. Main Findings

Definition 3.1 Suppose $(\tilde{B}, \mathbb{S}, T, \tilde{\star}, \tilde{\Diamond})$ as a $\mathbb{IFSM}$ and $Y : \tilde{B} \to \tilde{B}$. Then, Y is recognized as an IF soft contraction, if there is $0 < N < 1$ in order that

$$\left(\frac{1}{\mathbb{S}(\tilde{b}_{l_{1_j}}, \tilde{b}_{l_{1_{j+1}}}, a)} - 1\right) \geq N\left(\frac{1}{\mathbb{S}(\tilde{b}_{l_{1_{j-1}}}, \tilde{b}_{l_{1_j}}, a)} - 1\right), \ \textit{and}$$

$$T(\tilde{b}_{l_{1_j}}, \tilde{b}_{l_{1_{j+1}}}, a) \leq N(T(\tilde{b}_{l_{1_{j-1}}}, \tilde{b}_{l_{1_j}}, a)), \forall\ a > 0,\ j \geq 1.$$

Definition 3.2 Consider $(\tilde{B}, \mathbb{S}, T, \tilde{\star}, \tilde{\Diamond})$ as an $\mathbb{IFSM}$. The maps $\mathbb{S}, T$ are called triangular if

$$\left(\frac{1}{\mathbb{S}(\tilde{b}_{l_1}, \tilde{h}_{f_2}, a)} - 1\right) \leq \left(\frac{1}{\mathbb{S}(\tilde{b}_{l_1}, \tilde{i}_{f_3}, a)} - 1\right) + \left(\frac{1}{\mathbb{S}(\tilde{i}_{f_3}, \tilde{h}_{f_2}, a)} - 1\right),$$

$$T(\tilde{b}_{l_1}, \tilde{h}_{f_2}, a) \ \leq\ T(\tilde{b}_{l_1}, \tilde{i}_{f_3}, a) + T(\tilde{i}_{f_3}, \tilde{h}_{f_2}, a))\ \forall\ \tilde{b}_{l_1}, \tilde{h}_{f_2}, \tilde{i}_{f_3} \in \mathbb{S}H(\tilde{B}),\ a > 0,\ j \geq 1.$$

Lemma 1 *Assume that* $(\tilde{B}, \mathbb{S}, T, \tilde{\star}, \tilde{\Diamond})$ *is an* $\mathbb{IFSM}$. *Let* $\{\tilde{b}_{l_{1_j}}\} \in \mathbb{S}H(\tilde{B}) \to w_{l_4} \in \mathbb{S}H(\tilde{B}) \iff \mathbb{S}(\tilde{b}_{l_{1_j}}, w_{l_4}, a) \to 1$ *and* $T(\tilde{b}_{l_{1_j}}, w_{l_4}, a) \to 0$ *as* $j \to \infty$, *for* $a > 0$.

Definition 3.3 Let $(\tilde{B}, \mathbb{S}, T, \tilde{\star}, \tilde{\Diamond})$ be a $\mathbb{IFSM}$. The map $Y : \mathbb{S}P(\tilde{B}) \to \mathbb{S}P(\tilde{B})$ is called a rational type IF soft-contraction map if there exists $n, t \in [0, 1)$ such that:

$$\left(\frac{1}{\mathbb{S}(Y\tilde{b_{l_1}}, Y\tilde{h_{l_2}}, a)} - 1\right) \geq n\left(\frac{1}{\mathbb{S}(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a)} - 1\right) + t\left(\frac{\mathbb{S}(\tilde{b_{l_1}}, \tilde{h_{l_2}}, a)}{\mathbb{S}(\tilde{b_{l_1}}, Y\tilde{h_{l_2}}, a) \star \mathbb{S}(\tilde{h_{l_2}}, Y\tilde{b_{l_1}}, 2a)} - 1\right), \tag{3.1}$$

$$T(Y\tilde{b_{l_1}}, Y\tilde{h_{l_2}}, a) \leq n(T(\tilde{b_{l_1}}, \tilde{h_{f_2}}, a)) + t\left(\frac{T(\tilde{b_{l_1}}, Y\tilde{b_{l_1}}, a) \Diamond T(\tilde{h_{l_2}}, Y\tilde{b_{l_1}}, 2a)}{T(\tilde{b_{l_1}}, \tilde{h_{f_2}}, a)}\right), \tag{3.2}$$

$\forall\ a > 0,\ \tilde{b_{l_1}},\ \tilde{h_{l_2}} \in \mathbb{S}H(\tilde{B})$.

Theorem 3.1 *Consider $(\tilde{B}, \mathbb{S}, T, \tilde{\star}, \tilde{\diamondsuit})$ as a $\mathbb{G}$-complete $\mathbb{IFSM}$ with $\mathbb{S}, T$ being triangular maps. We define a rational type IF-soft contraction as a mapping $Y : \tilde{B} \to \tilde{B}$ that satisfies (3.1) and (3.2) with $n+t<1$. Then, Y has a fixed point in $\mathbb{S}H(\tilde{B})$.*

Proof: Suppose $b_{\tilde{\iota_{1_j}}} \in \tilde{B}$ and $b_{\tilde{\iota_{1_{j+1}}}} = Yb_{\tilde{\iota_{1_j}}}$, $j \geq 0$. Then, by (3.1), for $a > 0$, $j \geq 0$

$$\begin{aligned}
\left(\frac{1}{\mathbb{S}(\tilde{b}_{\iota_{1_j}}, \tilde{b}_{\iota_{1_{j+1}}}, a)} - 1\right) &= \left(\frac{1}{\mathbb{S}(Y\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)} - 1\right) \\
&\geq n\left(\frac{1}{\mathbb{S}(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)} - 1\right) + t\left(\frac{\mathbb{S}(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)}{\mathbb{S}(\tilde{b}_{\iota_{1_{j-1}}}, Y\tilde{b}_{\iota_{1_{j-1}}}, a) \star \mathbb{S}(\tilde{b}_{\iota_{1_j}}, \tilde{b}_{\iota_{1_{j-1}}}, 2a)} - 1\right) \\
&= n\left(\frac{1}{\mathbb{S}(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)} - 1\right) + t\left(\frac{T(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)}{T(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a) \star T(\tilde{b}_{\iota_{1_j}}, \tilde{b}_{\iota_{1_j}}, 2a)} - 1\right)
\end{aligned}$$

and by (3.2),

$$\begin{aligned}
T(\tilde{b}_{\iota_{1_j}}, \tilde{b}_{\iota_{1_{j+1}}}, a) &= (T(Y\tilde{b}_{\iota_{1_{j-1}}}, Y\tilde{b}_{\iota_{1_j}}, a)) \\
&\leq n(T(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)) + t(\frac{T(\tilde{b}_{\iota_{1_{j-1}}}, Y\tilde{b}_{\iota_{1_{j-1}}}, a) \diamondsuit T(\tilde{b}_{\iota_{1_j}}, Y\tilde{b}_{\iota_{1_{j-1}}}, 2a)}{T(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)}) \\
&= n(T(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)) + t\left(\frac{T(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a) \diamondsuit T(\tilde{b}_{\iota_{1_j}}, Y\tilde{b}_{\iota_{1_j}}, 2a)}{T(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)}\right)
\end{aligned}$$

After generality,

$$\left(\frac{1}{\mathbb{S}(\tilde{b}_{\iota_{1_j}}, n\tilde{b}_{\iota_{1_{j+1}}}, a)} - 1\right) \geq n\left(\frac{1}{\mathbb{S}(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)} - 1\right), for (a > 0). \tag{3.3}$$

$$T(\tilde{b}_{\iota_{1_j}}, \tilde{b}_{\iota_{1_{j+1}}}, a) \leq n(T(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)), for (a > 0). \tag{3.4}$$

Correspondingly,

$$\left(\frac{1}{\mathbb{S}(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)} - 1\right) \geq n\left(\frac{1}{\mathbb{S}(\tilde{b}_{\iota_{1_{j-2}}}, \tilde{b}_{\iota_{1_{j-1}}}, a)} - 1\right), for (a > 0). \tag{3.5}$$

$$T(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a) \leq n(T(\tilde{b}_{\iota_{1_{j-2}}}, \tilde{b}_{\iota_{1_{j-1}}}, a)), for (a > 0). \tag{3.6}$$

Now, from (3.3), (3.4), (3.5) and (3.6) by induction, for $a > 0$, we have that

$$\begin{aligned}
\left(\frac{1}{\mathbb{S}(\tilde{b}_{\iota_{1_j}}, \tilde{b}_{\iota_{1_{j+1}}}, a)} - 1\right) \leq \Bigg\{ n\left(\frac{1}{\mathbb{S}(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)} - 1\right) &\leq n^2\left(\frac{1}{\mathbb{S}(\tilde{b}_{\iota_{1_{j-2}}}, \tilde{b}_{\iota_{1_{j-1}}}, a)} - 1\right) \\
\leq \ldots \leq n^j\left(\frac{1}{\mathbb{S}(\tilde{b}_{\iota_{1_0}}, \tilde{b}_{\iota_{1_1}}, a)} - 1\right) \Bigg\} &\to 0, \ as \ j \to \infty.
\end{aligned}$$

$$\begin{aligned}
T(\tilde{b}_{\iota_{1_j}}, \tilde{b}_{\iota_{1_{j+1}}}, a) \leq n(T(\tilde{b}_{\iota_{1_{j-1}}}, \tilde{b}_{\iota_{1_j}}, a)) &\leq n^2(T(\tilde{b}_{\iota_{1_{j-2}}}, \tilde{b}_{\iota_{1_{j-1}}}, a)). \\
&\leq \ldots \leq n^j(T(\tilde{b}_{\iota_{1_0}}, \tilde{b}_{\iota_{1_1}}, a))
\end{aligned}$$

Hence, $\{\tilde{b}_{\iota_{1_j}}\}$ is IF soft contractive sequence in $(\tilde{b}, \mathbb{S}, T, \tilde{\star}, \tilde{\diamondsuit})$, therefore

$$\lim_{j \to \infty} \mathbb{S}(\tilde{b}_{\iota_{1_j}}, \tilde{b}_{\iota_{1_{j-1}}}, a) = 1, for \ a > 0, \tag{3.7}$$

$$\lim_{j\to\infty} T(\tilde{b}_{\wr 1_j}, \tilde{b}_{\wr 1_{j-1}}, a) = 0, for\ a > 0, \tag{3.8}$$

Now, we show that $\{\tilde{b}_{\wr 1_j}\}$ is a $\mathbb{G}$-Cauchy sequence. Let $j \in N$, and there is a fixed $q \in \mathbb{N}$ such that

$$\begin{aligned}\left(\frac{1}{\mathbb{S}(\tilde{b}_{\wr 1_j}, \tilde{b}_{\wr 1_{j+1}}, a)} - 1\right) \geq \Bigg\{ &\left(\frac{1}{\mathbb{S}(\tilde{b}_{\wr 1_j}, \tilde{b}_{\wr 1_{j+1}}, \frac{a}{q})} - 1\right) \diamondsuit \left(\frac{1}{\mathbb{S}(\tilde{b}_{\wr 1_{j+1}}, \tilde{b}_{\wr 1_{j+2}}, \frac{a}{q})} - 1\right) \\ &\diamondsuit \ldots \diamondsuit \left(\frac{1}{\mathbb{S}(\tilde{b}_{\wr 1_{j+q-1}}, \tilde{b}_{\wr 1_{j+q}}, \frac{a}{q})} - 1\right)\Bigg\} \\ &\to 1 \diamondsuit 1 \diamondsuit \ldots \diamondsuit 1 = 1,\ as\ j \to \infty..\end{aligned}$$

and

$$\begin{aligned}T(\tilde{b}_{\wr 1_j}, \tilde{b}_{\wr 1_{j+q}}, a) \leq T &\left(\tilde{b}_{\wr 1_j}, \tilde{b}_{\wr 1_{j+1}}, \frac{a}{q}\right) \diamondsuit T \left(\tilde{b}_{\wr 1_{j+1}}, \tilde{b}_{\wr 1_{j+2}}, \frac{a}{q}\right) \\ &\diamondsuit \ldots \diamondsuit T \left(\tilde{b}_{\wr 1_{j+q-1}}, \tilde{b}_{\wr 1_{j+q}}, \frac{t}{q}\right) \\ &\to 0 \diamondsuit 0 \ \diamondsuit \ldots \diamondsuit 0 = 0,\ as\ j \to \infty.\end{aligned}$$

Hence, it is proved that $\{\tilde{b}_{\wr 1_j}\}$ is a $\mathbb{G}$-Cauchy sequence. Since $(\tilde{b}, \mathbb{S}, T, \tilde{\star}, \tilde{\diamondsuit})$ is $\mathbb{G}$-complete, $\exists\ \tilde{w_{\wr 4_1}} \in \tilde{b}$ such that $\tilde{b}_{\wr 1_j} \to 1$ as $j \to \infty$, *i.e.*

$$\lim_{j\to\infty} \mathbb{S}(\tilde{b}_{\wr 1_j}, \tilde{w_{\wr 4_1}}, a) = 0,\ and \tag{3.9}$$

$$\lim_{j\to\infty} T(\tilde{b}_{\wr 1_j}, \tilde{w_{\wr 4_1}}, a) = 0,\ for\ t > 0. \tag{3.10}$$

Since $\mathbb{S},\ T$ are triangular, from (3.1), (3.2), (3.7), (3.8), (3.9) and (3.10), for $a > 0$, we have

$$\begin{aligned}\left(\frac{1}{\mathbb{S}(\tilde{w}_{\wr 4_1}, Y\tilde{w}_{\wr 4_1}, a)} - 1\right) &\leq \left(\frac{1}{\mathbb{S}(\tilde{w}_{\wr 4_1}, \tilde{b}_{\wr 1_{j+1}}, a)} - 1\right) + \left(\frac{1}{\mathbb{S}(Y\tilde{b}_{\wr 1_j}, Y\tilde{w}_{\wr 4_1}, a)} - 1\right) \\ &\leq \left(\frac{1}{\mathbb{S}(\tilde{w}_{\wr 4_1}, \tilde{b}_{\wr 1_{j+1}}, a)} - 1\right) + n\left(\frac{1}{\mathbb{S}(\tilde{x_{\wr 1_j}}, \tilde{w}_{\wr 4_1}, a)} - 1\right) \\ &\quad + t\left(\frac{\mathbb{S}(\tilde{x_{\wr 1_j}}, \tilde{w}_{\wr 4_1}, a)}{\mathbb{S}(\tilde{b}_{\wr 1_j}, Y\tilde{b}_{\wr 1_j}, a) \star \mathbb{S}(\tilde{w}_{\wr 4_1}, Y\tilde{b}_{\wr 1_j}, 2a)}\right),\ as\ j \to \infty \\ &= \Bigg\{\left(\frac{1}{\mathbb{S}(\tilde{w}_{\wr 4_1}, \tilde{b}_{\wr 1_{j+1}}, a)} - 1\right) + n\left(\frac{1}{\mathbb{S}(\tilde{b}_{\wr 1_j}, \tilde{w}_{\wr 4_1}, a)} - 1\right) \\ &\quad + n\left(\frac{\mathbb{S}(\tilde{b}_{\wr 1_j}, \tilde{w}_{\wr 4_1}, a)}{\mathbb{S}(\tilde{b}_{\wr 1_j}, Y\tilde{b}_{\wr 1_{j+1}}, a) \diamondsuit \mathbb{S}(\tilde{w}_{\wr 4_1}, Y\tilde{b}_{\wr 1_{j+1}}, 2a)}\right)\Bigg\} \\ &\to 0,\ as\ j \to \infty.\end{aligned}$$

Also,

$$\begin{aligned}
T(\tilde{w}_{\mathfrak{l}_{4_1}}, Y\tilde{b}_{\mathfrak{l}_{4_1}}, a) &\leq T(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{b}_{\mathfrak{l}_{1_{j+1}}}, a) + T(Y\tilde{b}_{\mathfrak{l}_{1_j}}, Y\tilde{w}_{\mathfrak{l}_{4_1}}, a)\\
&\leq T(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{b}_{\mathfrak{l}_{1_{j+1}}}, a) + n(T(\tilde{b}_{\mathfrak{l}_{1_j}}, \tilde{w}_{\mathfrak{l}_{4_1}}, a)\\
&\quad + t\left(\frac{T(\tilde{b}_{\mathfrak{l}_{1_j}}, Y\tilde{b}_{\mathfrak{l}_{1_j}}, a) \diamondsuit T(\tilde{w}_{\mathfrak{l}_{4_1}}, Y\tilde{b}_{\mathfrak{l}_{1_j}}, 2a)}{T(\tilde{b}_{\mathfrak{l}_{1_j}}, \tilde{w}_{\mathfrak{l}_{4_1}}, a)}\right)\\
&= \left\{T\left(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{b}_{\mathfrak{l}_{1_{j+1}}}, a\right) + n\left(T(\tilde{b}_{\mathfrak{l}_{1_j}}, \tilde{w}_{\mathfrak{l}_{4_1}}, a)\right)\right.\\
&\quad \left. + t\left(\frac{T(\tilde{b}_{\mathfrak{l}_{1_j}}, Y\tilde{b}_{\mathfrak{l}_{1_{j+1}}}, a) \diamondsuit T\left(\tilde{w}_{\mathfrak{l}_{4_1}}, Y\tilde{b}_{\mathfrak{l}_{1_{j+1}}}, 2a\right)}{T(\tilde{b}_{\mathfrak{l}_{1_j}}, \tilde{w}_{\mathfrak{l}_{4_1}}, a)}\right)\right\}\\
&\to 0, \ as\ j \to \infty.
\end{aligned}$$

Now,

$$\begin{aligned}
\left(\frac{1}{\mathbb{S}(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}, a)} - 1\right) &= \left(\frac{1}{\mathbb{S}(Y\tilde{w}_{\mathfrak{l}_{4_1}}, Y\tilde{s_{\mathfrak{l}_{4_1}}}, a)} - 1\right)\\
&\leq n\left(\frac{1}{\mathbb{S}(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}, a)} - 1\right) + t\left(\frac{\mathbb{S}(\tilde{w}_{\mathfrak{l}_{1_j}}, \tilde{s_{\mathfrak{l}_{4_1}}}, a)}{\mathbb{S}(\tilde{w}_{\mathfrak{l}_{4_1}}, Y\tilde{w}_{\mathfrak{l}_{4_1}}, a)) \star \mathbb{S}(\tilde{s_{\mathfrak{l}_{4_1}}}, Y\tilde{w}_{\mathfrak{l}_{4_1}}, 2a)}\right)\\
&\leq n\left(\frac{1}{\mathbb{S}(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}, t)} - 1\right) + t\left(\frac{\mathbb{S}(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}, a)}{\mathbb{S}(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{w}_{\mathfrak{l}_{4_1}}, a)) \star \mathbb{S}\left(\tilde{s_{\mathfrak{l}_{4_1}}}, \tilde{w}_{\mathfrak{l}_{4_1}}, a\right)}\right)\\
&= n\left(\frac{1}{\mathbb{S}(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}, a)} - 1\right)\\
&= n\left(\frac{1}{\mathbb{S}(Y\tilde{w}_{\mathfrak{l}_{4_1}}, Y\tilde{s_{\mathfrak{l}_{4_1}}}, a)} - 1\right)\\
&\leq n^2\left(\frac{1}{\mathbb{S}(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}, a)} - 1\right)\\
&\quad .\\
&\quad .\\
&\quad .\\
&\leq n^j\left(\frac{1}{\mathbb{S}(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}, a)} - 1\right)\\
&\to 0, \ as\ j \to \infty.
\end{aligned}$$

and

$$\begin{aligned}
T(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}, a) &= T(Y\tilde{w}_{\mathfrak{l}_{4_1}}, Y\tilde{s_{\mathfrak{l}_{4_1}}}, a)\\
&\leq n(T(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}, a)) + n\left(\frac{(T\tilde{w}_{\mathfrak{l}_{1_j}}, Y\tilde{w}_{\mathfrak{l}_{4_1}}, a) \diamondsuit T(\tilde{s_{\mathfrak{l}_{4_1}}}, Y\tilde{w}_{\mathfrak{l}_{4_1}}, 2a)}{T(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}}, a\right)\\
&\leq n(T(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}, a) + n\left(\frac{T(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{w}_{\mathfrak{l}_{4_1}}, a) \diamondsuit T(\tilde{\mathbb{S}_{\mathfrak{l}_{4_1}}}, \tilde{w}_{\mathfrak{l}_{4_1}}, a)}{T(\tilde{w}_{\mathfrak{l}_{4_1}}, \tilde{s_{\mathfrak{l}_{4_1}}}, a)}\right),
\end{aligned}$$

$$
\begin{aligned}
&= n(T(\tilde{w}_{\wr 4_1}, \tilde{s_{\wr 4_1}}, a)) \\
&= n(T(Y\tilde{w}_{\wr 4_1}, Y\tilde{s_{\wr 4_1}}, a)) \\
&\leq n^2(T(\tilde{w}_{\wr 4_1}, \tilde{s_{\wr 4_1}}, a)) \\
&\ \ . \\
&\ \ . \\
&\ \ . \\
&\leq n^j(T(\tilde{w}_{\wr 4_1}, \tilde{s_{\wr 4_1}}, a)) \\
&\to 0 \ as \ j \to \infty.
\end{aligned}
$$

Hence, it is proved that $\mathbb{S}(\tilde{w}_{\wr 4_1}, \tilde{s_{\wr 4_1}}, a) = 1$ and $T(\tilde{w}_{\wr 4_1}, \tilde{s_{\wr 4_1}}, a) = 0$, and this implies that $\tilde{w}_{\wr 4_1} = \tilde{s_{\wr 4_1}}$. □

4. Illustration

Let $\tilde{b} = [0, \infty)$ and $\star$, $\diamondsuit$ be a continuous t-norm and t-conorm respectively. Let $\mathbb{S}, T : (\tilde{b})^2 \times (0, \infty) \to [0, 1]$ be defined as

$$
\begin{aligned}
\mathbb{S}(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}, a) &= \frac{a}{a + \left|\frac{(4\tilde{p_{\wr_1}} - 4\tilde{q_{\wr_2}})}{5}\right|} \ and \\
T(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}, a) &= \frac{\left|\frac{(4\tilde{p_{\wr_1}} - 4\tilde{q_{\wr_2}})}{5}\right|}{a + \left|\frac{(4\tilde{p_{\wr_1}} - 4\tilde{q_{\wr_2}})}{5}\right|}, \ \forall\ \tilde{p_{\wr_1}}, \tilde{q_{\wr_2}} \in \tilde{b},\ a > 0.
\end{aligned}
$$

The one can easily verify that $(\mathbb{S}, T)$ is triangular and $(\tilde{b}, \mathbb{S}, T, \star, \diamondsuit)$ is a $\mathbb{G}$-complete IFSM space. Now we define a mapping $Y : \tilde{b} \to \tilde{b}$ as

$$
Y(\tilde{p_{\wr_1}}) = \begin{cases} \frac{3\tilde{p_{\wr_1}}}{4} & if \quad \tilde{p_{\wr_1}} \in [0, 1], \\ \frac{2\tilde{p_{\wr_1}}}{4} + 8 & if \quad \tilde{p_{\wr_1}} \in (0, \infty). \end{cases}
$$

Then,

$$
\left(\frac{1}{\mathbb{S}(Y\tilde{p_{\wr_1}}, Y\tilde{q_{\wr_2}}, a)} - 1 \right) = \frac{3}{4} \left(\frac{1}{\mathbb{S}(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}, a)} - 1 \right)
$$

$$
\Rightarrow T(Y\tilde{p_{\wr_1}}, Y\tilde{q_{\wr_2}}, a) = \frac{3}{4} (T(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}, a)\,.
$$

Hence, Y is a IF soft contraction. Now, from Definition 2.7 (part i and j), for $t > 0$,

$$
\begin{aligned}
\left(\frac{\mathbb{S}(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}, a)}{\mathbb{S}(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, a) \star \mathbb{S}(\tilde{q_{\wr_2}}, G\tilde{x_{\wr_1}}, 2a)} - 1 \right) &\leq \left(\frac{\mathbb{S}(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}, a)}{\mathbb{S}(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, a) \star \mathbb{S}(\tilde{q_{\wr_2}}, Y\tilde{x_{\wr_1}}, 2a) \star \mathbb{S}(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, a)} \right) \\
&= \left(\frac{1}{\mathbb{S}(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, a) \star \mathbb{S}(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, a)} - 1 \right) \\
&= \left(\frac{1}{\mathbb{S}(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, a)^2} - 1 \right) \\
&= \frac{2\tilde{p_{\wr_1}}}{5t^2} (\frac{\tilde{p_{\wr_1}}}{5} + a).
\end{aligned}
$$

Also,

$$\left(\frac{T(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, a) \diamondsuit T(\tilde{q_{\wr_2}}, Y\tilde{p_{\wr_1}}, 2a)}{T(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}, a)\wr_2}\right) \leq \left(\frac{T(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, a) \diamondsuit T(\tilde{q_{\wr_2}}, Y\tilde{p_{\wr_1}}, 2\wr) \diamondsuit T(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, a)}{T\,(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}, a)}\right)$$
$$= T(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, a) \diamondsuit T(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, 2a)$$
$$= T(\tilde{p_{\wr_1}}, Y\tilde{p_{\wr_1}}, a)^2$$
$$= \frac{2\tilde{p_{\wr_1}}}{5t^2}\left(\frac{\tilde{p_{\wr_1}}}{5} + a\right),$$

Thus, all the conditions of Theorem 3.1 are satisfied with $n = \frac{3}{4},\ t = \frac{2}{9}$. Thus, Y admits a fixed point *i.e.*, $Y(24) = 24[0, \infty)$.

5. Application

To complement our work, we give an integral type application in this section.

Assume that the space containing all $\tilde{R}$-valued continuous functions on the interval $[0, \hbar]$, where $0 < \hbar \in \mathbb{R}$, is $\tilde{b} = \mathbb{C}([0, \hbar], \mathbb{R})$. Consider the nonlinear integral equation

$$\tilde{p_{\wr_1}}(\rho) = \int_0^\rho U(\rho, \nu, \tilde{p_{\wr_1}}(\nu))d\nu,\ \forall\ \tilde{p_{\wr_1}} \in \tilde{b},$$

where $\rho, \nu \in [0, \hbar]$ *and* $T : [0, \hbar] \times [0, \hbar] \times \mathbb{R} \to \mathbb{R}$. The induced metric $b : B^2 \to \mathbb{R}$ can be defined as

$$b(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}) = \sup_{\rho\in[0,\ \hbar]} |\tilde{p_{\wr_1}}(\rho) - \tilde{q_{\wr_2}}(\rho)| = ||\tilde{p_{\wr_1}} - \tilde{q_{\wr_2}}||,$$

where $\forall\ \tilde{p_{\wr_1}}, \tilde{q_{\wr_2}} \in \mathbb{C}[0, \hbar] = B$,

$$b(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}) = \inf_{\rho\in[0,\ \hbar]} |\tilde{p_{\wr_1}}(\rho) - \tilde{q_{\wr_2}}(\rho)| = ||\tilde{p_{\wr_1}} - \tilde{q_{\wr_2}}||,$$

where $\forall\ \tilde{p_{\wr_1}}, \tilde{q_{\wr_2}} \in \mathbb{C}[0, \hbar] = B$.

The binary operations $\star,\ \diamondsuit$ are defined by

$$\tilde{p_{\wr_1}} \star \tilde{q_{\wr_2}} = \tilde{p_{\wr_1}}\tilde{q_{\wr_2}}\ \textit{and}$$

$$\tilde{p_{\wr_1}} \diamondsuit \tilde{q_{\wr_2}} = \tilde{p_{\wr_1}} + \tilde{q_{\wr_2}} - \tilde{p_{\wr_1}}\tilde{q_{\wr_2}},\quad \tilde{p_{\wr_1}}, \tilde{q_{\wr_2}} \in [0, \hbar].$$

And $\mathbb{S}, T : B^2 \times (0, \infty) \to [0, 1]$ are defined as

$$\mathbb{S}(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}, a) = \frac{a}{a + b(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}})}\ \textit{and}$$

$$T(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}}, a) = \frac{b(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}})}{a + b(\tilde{p_{\wr_1}}, \tilde{q_{\wr_2}})},\ \forall\ a > 0,\ \forall\ \tilde{p_{\wr_1}}, \tilde{q_{\wr_2}} \in \mathbb{S}H(\tilde{b})$$

Subsequently, it becomes effortless to confirm that $(\mathbb{S}, T)$ is triangular and $(\tilde{b}, \mathbb{S}, T, \star, \diamondsuit)$ is a $\mathbb{G}$ - complete $\mathbb{IFSM}$.

6. Conclusion

We proposed the notion of rational type Intuitionistic fuzzy soft contraction maps in $\mathbb{IFSMS}$ and established rational type fixed point theorems in $\mathbb{G}$-complete $\mathbb{IFSMS}$ using the triangular property of $\mathbb{IFSMS}$. In the last section, we presented an integral type application for this class of maps.

7. Future scope

The presented results allow one for the demonstration of more logical Intuitionistic fuzzy soft contraction results in $\mathbb{G}$-complete $\mathbb{IFSMS}$ with a variety of applications.

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