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Metaheuristic Approach for Green Vehicle Routing Problem with Time Windows and Fuzzy Customer Demand

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ABSTRACT: Green Vehicle Routing Problem (GVRP) is one of the extensions of Vehicle Routing Problem. GVRP means that it is concerned with the environment. To fulfil the sustainable development goals, GVRP came into existence as the main motive is to minimize the CO_2 emissions in the environment along with optimizing the total travel cost and distance covered by the vehicles. Sustainable development goals (SDG) are also considered in this paper as 4 out of 17 SDG goals are being satisfied by our research. In this research article, the focus is to minimize the total operational costs and total number of vehicles to cover the routes and minimizing CO_2 emissions combined with satisfying the fuzzy customer demand. The mathematical model is developed that integrates fuzzy set theory with energy- efficient routing constraints and time windows. Solomon R101 dataset is considered to check the efficiency of the proposed model. For this task, Ant Colony Optimization algorithm along with Mamdani fuzzy inference system is applied to calculate the optimal values. It is found that the applied approach gives better results in case of minimizing the number of vehicles used and reducing CO_2 emissions along with satisfying the uncertain customer demand. Computational experiments are done to check the efficiency of the proposed model and it has given better results for the chosen Solomon dataset.

Key Words: Green vehicle routing, environment sustainability, nature inspired, logistics.

Contents

1. Introduction

Vehicle Routing is the basic need of industries these days as transporting goods and products from one place to another is important for people around the world. Vehicle routing problem first came into existence in 1959 by [1]. The main aim was to minimize the total travel cost. Therefore, VRP was considered as a main research area at that time and researchers are still working on it. Vehicle Routing problem has many extensions involving Capacitated Vehicle Routing problem (CVRP), Vehicle Routing problem with time windows (VRPTW), Vehicle routing problem with pickup and delivery (VRPPD) and many extensions are there based on the constraints and conditions considered in the problem. GVRP and Electric vehicle routing problem (EVRP) are two important extensions of VRP and all the different constraints can be used in these extensions as well. VRPs are extended to GVRP and EVRP to maintain the sustainability of the environment and minimize the fuel consumption along with minimizing the operational costs, distance and total travel time. GVRPs are different from EVRPs in a manner that in GVRP, the focus is to minimize the fuel consumption , vehicles can be either traditional or hybrid but in EVRPs the vehicles are electric and the concept of charging stations and battery swapping is included in there.

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GVRP came into existence by concerning the climate change, air pollution and sustainable development grow and logistics optimization linked to the environmental goals. The vehicles can be alternate fuel vehicles involving the vehicles running on compressed natural gas (CNG) or the hybrid vehicles that have limited driving ranges and need access to specific type of refueling. These constraints add complexity to the problem, because the planner does not need to optimize the path only but also to minimize the fuel consumption to maintain the environmental sustainability. Sustainable development goals [32] generated by United Nations includes 17 goals and 169 targets. These sustainable goals also include goals related to the environment and green vehicle routing problem fulfil those goals. Green vehicle routing problem fulfils 4 sustainable development goals including Goal numbers 7,9,11 and 13. Goal 7 is related to the affordable and clean energy which means that affordable, reliable and sustainable energy for all the people. Goal 9 is concerned with the Industry, Innovation and Infrastructure which means the development in industries and logistics including sustainable industrialization. Goal 11 is about Sustainable cities and Communities as it is clear from the heading that it means make the cities more sustainable for living , one factor that can make cities reliable is minimum pollution. Goal 13 is related to the Climate Action which means there is need to take prompt actions against the factors affecting the climate. GVRP fulfils all these goals and thus it is a very complex problem as it involves real- world actions. These complex problems are known to be NP-Hard [31] problems which means the problems that cannot be solved in the polynomial time. NP- Hard problems depicts a class of problems that are in a sense, at least as hard as the hardest problems in NP (Non-deterministic Polynomial time). NP- Hard problems often happen considering the real-world conditions including routing, optimization, scheduling and source allocation. Some examples involve the Traveling Salesman problem, Knapsack problem, and Graph Coloring. Such complex problems are hard to solve in the polynomial time as there are many constraints and conditions during their modeling. To solve such complex problems, having number of defined constraints based on the given conditions or data provided, many exact, heuristics and metaheuristics approaches are applied. Now, hybrid metaheuristics approaches are also applied on VRPs to obtain better optimal results as compared to the results obtained from exact or metaheuristics algorithms. In real- world transportation problems, especially the last- mile delivery, customers have uncertain demand due to many factors such as customer factors, fluctuation in orders or any other external conditions. This uncertainity can be removed by using fuzzy logic where the demand can be represented as fuzzy numbers and can be converted into a crisp value to obtain a proper value of units for fuzzy demand. This kind of real- life problems lead to green vehicle routing problem with fuzzy demand (GVRPFD). An extra constraint of time windows is also added in the proposed model, which means there is limited time for a delivery person to visit a particular customer, which converts this problem into green vehicle routing problem having time windows with fuzzy demand (GVRPTWFD). In GVRPTWFD, demand of each customer is formulated using a fuzzy set, that represents the uncertain demand in a more realistic way. The main motive is to minimize the total distance covered, total transportation costs, total carbon emissions and not violating the variations in customers' demands. This can be done only by adding the fuzzy constraints to the existing model of green vehicle routing problem and for this some advanced fuzzy optimization techniques are used. By combining environmental sustainability with demand uncertainty, GVRPFD gives a more realistic mathematical model for modern green logistics. In this research article, the research is done by considering the factors of green vehicle along with uncertain customer demand. **Figure 1** resembles VRP in a very refine way. Now, in this paper, a detailed information is provided in different sections about the research done in this research article. **Section 2** depicts the literature related to the green vehicle routing problems. **Section 3** is the mathematical model used having the objective function, the constraints and conditions and the decision variables. **Section 4** depicts the solution approaches applied to calculate the optimal solutions. **Section 5** is related to the experimental results and **Section 6** gives the conclusion of the research article.

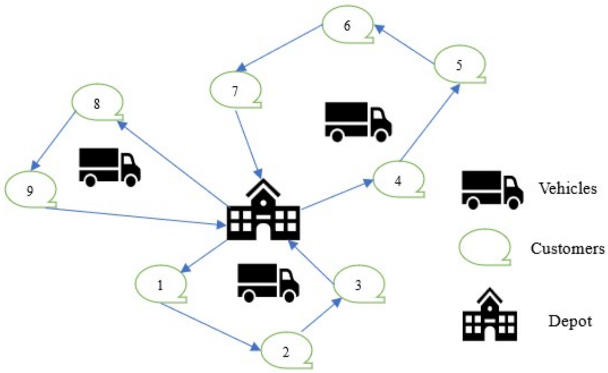

Figure 1: Vehicle Routing Problem

2. Literature Review

[1] were the first to focus on vehicle routing challenge as transporation was the base of importing and exporting things from one place to another place. The problem was truck dispatching problem where the gasoline was transported. The focus was to minimize the total transportation cost. Researchers considered it a topic of major research and started working on it. From 1959 to till now, many algorithms and methods were developed by the scholars and still it is under consideration and researchers are still working on it to get better optimal values for the problems. Many research papers were written on vehicle routing problem in the by the researchers after 1959. In this paper, the literature is reviewed from the past 25 years which is considered to be the recent research work done in this area, however , in the past 25 years, thousands of papers are there to explore , we have reviewed many of them. In 2002, [2] wrote a book on VRP where the authors described the VRP in detail and the algorithms and methods that can be applied to solve VRPs and obtain optimal results. Different variants of VRP were also discussed by the authors in that book. In the next year (2003), [3] used a genetic algorithm combined with neighbourhood search methods for VRP and found that the applied approach is comparable to tabu search and simulated annealing in aspect of solution time and quality of the results. In 2008, a book is written by [4] where they discussed about the latest challenges and advances of VRP. Moreover, algorithms to overcome the challenges , many solution methods are also discussed related to the respective vehicle routing problem. In the same year, [5] introduced multi objective vehicle routing problem where the routing problems are examined based on their objectives, definitions and multi objective algorithms preferred to solve them. [6] worked on reviewing dynamic vehicle routing problem where they provided a brief description of dynamic routing and information about the solution approches used to solve dynamic vehicle routing problems. After vehicle routing problem, green vehicle routing problem came into existence in 2012 introduced by [7]. The authors focused on minimizing the total operational cost during the completion of routes along with minimizing the carbon emissions, which was prime motive of the researchers as pollution was increasing day by day. They have applied Modified Clarke and Wright Savings heuristic and the Density-Based Clustering Algorithm with an improved customised technique to get the optimal results. Green VRP became the most focused topic of research at that time and till now, researchers are working on it. [8] did a survey on past and future trends of green vehicle routing problem and identified the classification of it and suggested future research directions related to this research area. An exact algorithm was applied by [9] to solve green VRP where the researchers have found that the algorithm works better for upto 110 customers. An evolutionary algorithm was also applied on green vehicle routing problem by [10] where the genetic algorithm was examined on benchmark instances including road speed and gradient data. The scholars found that the approach works better in minimizing the emissions without increasing the total operational costs. [11] worked on satisfying the customers . In this paper, a mixed integer linear programing model was developed and also provided numerical studies to show the efficiency of the proposed model. As, green vehicle routing problem was in progress and traditional vehicle routing problem

was also under consideration for research, fuzzy logic was also introduced in vehicle routing problems where there is uncertainty whether in delivery time or in customer demand. Vehicle routing problem with fuzzy customer demand was introduced by [12] where a bee system algorithm was applied to VRP with fuzzy demands. After that, many researchers worked on this topic as well. [13] applied a differential evolutionary algorithm to VRP with fuzzy demands. [14] used genetic ant colony optimization algorithm for fuzzy logic in vehicle routing problems.They did research on a case study considering garbage collection system. Ant colony optimization was combined with genetic algorithm and two local search methods: 2-opt and prim's algorithm and found that this algorithm gives better results as compared to the other existing algorithms. Moving futher, now the focus was on fuzzy customer responses regarding the delivery of the products. [23] included variable neighbourhood search algorithm for green vehicle routing problem and the constraints involved the two dimensional loading constraints and split delivery. The authors found 21 best solutions out of 60 instances in short computational times. Deap reinforcement learning method based on adaptive large search was used by [24] to solve time dependent green VRP. This approach worked in two different stages where in the first level hybrid initialization strategy was used to obtain high quality results and in the second stage adaptive search was designed and used for learning and searching. The experimental results showed better solution quantity obtained by proposed approach. Vehicle routing is one of the most noticable topics for research in the recent years. [25] worked on half-open multi- depot vehicle routing problem. half- open means the vehicles start their routes from multi depots and can return to any depot, it is not necessay that the vehicle has to return to the same depot. The authors used simulated annealing and tempering algorithm to calculate the results. The authors selected for different approaches for comparison and 24 groups of problem instances were analysed. The applied approach gave satisfactory results. [26] worked on small and medium cities for minimizing fuel consumption. The researchers proposed a heuristic algorithm based on variable neighbourhood search algorithm. The results concluded that proposed approach can minimize 25% fuel consumption as compared to manual routing and scheduling. Sensitivity analyses was also done on different vehicle capacities. Moreover, a Liquid Petroleum Gas (LPG) case study was done in Yogyakarta, Indonesia by [27]. Genetic Algorithm was applied to minimize the vehicle routing and reducing the costs. By including carbon taxes and detailed emission calculations into the objective function, the green vehicle routing problem model offered provided better results for real world logistics. Adaptive Large neighbourhood search method was used by [28] for GVRP where the depots were in sharing. Numerical experiments were computed on different instances to test the algorithm and it was concluded that depot sharing decreases the carbon emissions and achieved an average optimization rate of 10.1 % around all instances as compared to returning to the original depot. [29] used Q- learning butterfly optimization algorithm for green VRP focusing on customer preference where Q-learning is a reinforcement learning technique. 18 benchmark functions were used to compare the applied method with 5 classical metaheuristics algorithms and 3 butterfly optimization algorithm variable optimization methods. Ant Colony Optimization algorithm along with Student Psychology Based Optimization technique was implemented by [30] for electric vehicle routing problem. In the starting phase, Ant colony optimization technique was used as it is good at local search strategy and in the later stage, Student Psychology Based Optimization was more prominent to solve path planning issue. The proposed algorithm was applied in industrial setting, that reduces the electric loader's driving distance during raw material transporting and filling, which showed that it has prominent usage in real- world as well. [15] focused on it and worked on the fuzzy customer responses. The customer responses had a great impact on future demand of the products. This new approach helped in reducing the customer demands and avoid losing regular customers. VRP with Simultaneous pickup and delivery was considered by [16] where fuzzy logic was also applied on payloads. Prime objective was to minimize the fuel consumption as it was concerned with the environment sustainability. Ant colony optimization was prefereed by many researchers to find optimal results. Similarly, mamdani fuzzy inference system was used in many other fields like medical sciences. In 2015, [19] used Mamdani fuzzy inference system, for breast cancer risk detection. the authors calculated the results and mamdani with one another algorithm gave the results to 93%. Moreover, [20] Mamdani method was applied in calculating the green supply chain management performance. This paper targeted to minimize the uncertainty developed by human judgements in the process of supply chain management performance. Mamdani fuzzy method was also used by [21] to estimate the poultry weight keeping indoor weight, humidity and feed consumption as input variables. This fuzzy logic method has provided numerous benefits in the field of poultry. In short, it can be said that wherever is uncertainty, Mamdani fuzzy inference method can be applied to obtain better results considering different input variables depending on the conditions. This paper is concerned with using ant colony optimization algorithm and mamdani fuzzy inference system. As there is less research done on fuzzy logic in field of vehicle routing problems in the recent years. Therefore, in this paper, the concentration is on fuzzy logic along with minimizing carbon emissions and minimizing total operational costs.

3. Mathematical Model

A green vehicle routing problem with time windows and fuzzy demand focuses to minimize the fuel consumption and overall operational cost. The mathematical model of GVRPTWFD is summarized as follows:

- Loading capacity of vehicle is restricted.
- One customer is visited by exactly one vehicle.
- Fleet should be homogeneous.
- Vehicle must depart and arrive back to the depot after completing the route.

Notations, Parameters, objective function, constraints, sets, and decision variables are presented below:
W= Set of nodes where w_0= depot
E= N $\backslash w_0$ set of customers
B=Set of edges
T= Set of Vehicles
$m, n \in W$ = nodes
$q \in T$ = Vehicle Index
Q_{mn} = Cost from node m to n
j_{mn} = Tour Time from node m to node n
f_m, k_m =Time Window for node m
i_m = Servicing period at node m
d_m = Triangular fuzzy demand $\{d_1, d_2, d_3\}$ for customer m
S_q = Distance from node x to node y
L_k = Capacity of vehicle q
λ_{mn} = Emission cost per unit in an arc m, n
P = Constant applied to deactivate time window constraint when arcs are not in use $\{l_{mnq}$=0$\}$
γ = Weighing parameter between cost and emission
Decision Variables:

$$l_{mnq} = \begin{cases} 1, & \text{if vehicle } q \text{ visits node } n \text{ after node } m \\ 0, & \text{otherwise} \end{cases}$$

$b_{mq} \geq 0$, Arriving time of vehicle q at node m
$c_{mq} \geq 0$, Cummulative load of vehicle q after visiting node m
The methodology used to calculate results for GVRPTWFD is as follows: The objective function is to reduce the total operational cost and carbon emissions:

Objective Function:

$$\sum_{q \in T} \sum_{(m,n) \in B} l_{mnq}(\gamma \cdot Q_{mn} + (1-\gamma) \cdot \lambda_{mn} \tag{3.1}$$

$$\sum_{q \in T} \sum_{n \in W} l_{mnq} = 1, \forall m \in E \tag{3.2}$$

$$\sum_{q \in T \backslash \{0\}} l_{0nq} - \sum_{q \in T \backslash \{0\}} l_{m0q} = 0, \forall q \in T \tag{3.3}$$

$$\sum_{q\in T, n\neq m} l_{mnq} = \sum_{n\in W, n\neq m} l_{nmq}, \quad \forall m \in W, q \in D \tag{3.4}$$

$$b_{nq} \geq b_{mq} + i_m + j_{mn} - P(1 - l_{mnq}), \quad \forall m \neq n, \forall q \in T \tag{3.5}$$

$$f_m \leq b_{mq} \leq k_m, \quad \forall m \in W, q \in T \tag{3.6}$$

$$c_{nq} \geq c_{mq} + d_n - S_q(1 - l_m nq) \quad \forall m \neq n, \forall q \in T \tag{3.7}$$

$$c_{mq} \leq S_q, \forall n \in W, q \in T \tag{3.8}$$

$$c_{0q} = 0, \quad \forall q \in T \tag{3.9}$$

$$l_{mnq} \in [0,1], b_{mq} \geq 0, c_{mq} \geq 0 \tag{3.10}$$

The main objective of this mathematical model is to reduce the transportational cost and the carbon emissions which is properly described in equation (3.1). Equation (3.2) ensures that every vehicle must visit exactly one customer. Constraint (3.3) guarantees that vehicle starts and ends its tour at the depot. Condition (3.4) ensures the continuity of the route of vehicle. Constraint (3.5) is related to the time windows, which means that vehicle starts its tour from node m to node n. P is the constant that deactivates the time windows constraint when arcs are not in use. Equation (3.6) restricts that vehicle must must the customer in given time. Equation (3.7) is the load propagation after dealing with customer m. Constraint (3.8) limits the load of the vehicle. equation (3.9) ensures that at initial point i.e. the load of the vehicle is zero. Constraint (3.10) reflects the binary decision variables.

4. Solution Approach

In this research article, solution method applied to solve the problem includes the ant colony optimization algorithm and Mamdani fuzzy inference system. Ant colony optimization technique is used to find the optimal solution by minimizing the carbon emissions and total traveling costs and mamdani fuzzy inference system involves fuzzy logic that defuzzifies the fuzzy demand of customers which is taken to be as triangular fuzzy number. Ant colony optimization algorithm and mamdani fuzzy inference system is explained one by one below.

4.1. Ant Colony Optimization (ACO) Algorithm :

Ant colony optimization algorithm first came into existence in 1992 by [17] which was his thesis on optimization and natural algorithms where he discussed the behaivour of ants in search of their foods from their nests covering the shortest route by following a chemical compound released by them known as pheromone. ACO involves some steps that are followed by ants to reach to their foods. The steps are discussed below:

Step 1: Initializing Phase: Initial phase involves the positioning of ants at random positions. Some important key factors involves the pheromone evaporation rate, number of ants, and importance of pheromone versus distance are set.

Step 2: Foraging path: Ants choose their path depending on the level of pheromone on the routes and distance between the steps. A formula is used to choose the route based on pheromone level and distance.

$$P_{mn} = \frac{\tau_{mn}^{\alpha} d_{mn}^{\beta}}{\sum_{z\in allowed} \tau_{mz}^{\alpha} d_{mz}^{\beta}} \tag{4.1}$$

where,
P_{mn}= possibility of choosing the route from node m to node n
τ^{α}_{mn}= level of pheromone on edge *(m,n)*
d_{mn}= distance between nodes m and node n
α and β are parameters that control the importance of pheromone and distance between the nodes.
$\sum_{z \in allowed} \tau^{\alpha}_{mz} d^{\beta}_{mz}$ is the normalization factor to ensure the possibilities for all allowed moves from node m always sum to 1.
The set "allowed" contains all the nodes that ant has not visited yet.
Step 3: Updating the Pheromone: After completing the route, ants update the pheromone. As they got the better solution they go on updating the pheromone. On the routes having long distance, the pheromone gets evaporated and that route is then ignored by the ants. Formula to update pheromone is as follows:

$$\tau_{mn}(1-\rho) \cdot \tau_{mn} + \Delta\tau_{mn} \tag{4.2}$$

where
τ_{mn}= level of pheromone on edge *(m,n)*
ρ = pheromone evaporation rate
$\Delta\tau_{mn}$= pheromone deposited on the edges by ants
Step 4: Repetition of the process: Repeat the whole process until an optimal path is found by the ants. Multiple colonies and random path selection can be used by the ants to avoid blocking in local optima.

4.2. Mamdani Fuzzy Inference System (MFIS):

Mamdani fuzzy inference system was first came into existence in 1975 by [18] where the authors focused on a linguistic synthesis of a controller for a steam engine. Fuzzy logic was preferred to convert human operated heuristic control rues to a automatic control strategy. Moreover, Mamdani fuzzy inference system was used by many researchers in many fileds. There are some steps involved in Mamdani fuzzy inference system which are as follows:
Step 1: Fuzzification: Some input variables are considered and are converted to input fuzzy values. In our research article, we are considering two different input variables such as customer demand and second is time of day. These input fuzzy values are shown using membership functions as : "Low","Mid", "High". For first input variable which is customer demand, membership functions will be: "Small", "Medium", "Large". Similarly, for second input variable which is time of day, the membership functions will be: "Morning","Afternoon","Evening". From these input values, the output variable will be the whole fulfilled demand of the customer.
Step 2: Evaluation of Rules: Fuzzy logic rules are then applied to get output fuzzy sets. 9 rules will be generated after combining all the given conditions based on both the input variables. After evaluating the rules, the rule strength is calculated.
Step 3: Combination of Outputs: All the fuzzy outputs are than combined in a fuzzy set for whole fulfilled demand. This wil give one final result for fuzzy demand.
Step 4: Defuzzification: Defuzzification is applied to convert the fuzzy output value into a crisp value. All the values of output membership function are considered and mean of maxima is applied to calculate the crisp value.
To elaborate the whole concept of Mamdani Fuzzy Inference system , Let us consider an exampe having 2 input variables and 1 output variable. Similar to the steps written above, let us take demand and time of day as 2 input variables and whole fulfilled demand as the output variable. The membership function for Input 1 i.e. the demand of the cutomers is μ_x and membership function for Input 2 i.e. the time of day is μ_y and the membership function for output whole demand is μ_z.
A triangular fuzzy number for input 1(demand) ranges from 0 to 100. Three categories are there for demand, "Small", "Medium" and "Large". Minimum value for demand is 0, highest value is 100 and the medium value lies at 50. As triangular fuzzy number is implemented , therefore 3 different conditions

arise as :

$$\mu_{SD} = \frac{50 - x}{50}, \quad 0 \le x \le 50 \tag{4.3}$$

$$\mu_{MD} = \begin{cases} \frac{x}{50}, & 0 \le x \le 50 \\ \frac{100-x}{50}, & 50 \le x \le 100 \end{cases} \tag{4.4}$$

$$\mu_{LD} = \frac{x - 50}{50}, \quad 50 \le x \le 100 \tag{4.5}$$

Similar to demand, a triangular fuzzy number for input 2(Time of Day) ranges from 0 to 24. Three categories are there, “Morning”, “Afternoon” and “Evening”. The starting value for time of day is 0, middle value is 12 and the highest value is 24. Three different conditions arise here as:

$$\mu_{MT} = \frac{12 - y}{12}, \quad 0 \le y \le 12 \tag{4.6}$$

$$\mu_{AT} = \begin{cases} \frac{y}{12}, & 0 \le y \le 12 \\ \frac{24-y}{12}, & 12 \le y \le 24 \end{cases} \tag{4.7}$$

$$\mu_{ET} = \frac{y - 12}{12}, \quad 12 \le y \le 24 \tag{4.8}$$

Now, the next is the membership function μ_z for whole demand which is generated by generating rules by combining all the conditions. The equations for output membership function are as follows:

$$\mu_{VS} = \frac{10 - z}{10}, \quad 0 \le z \le 10 \tag{4.9}$$

$$\mu_{S} = \begin{cases} \frac{z}{10}, & 0 \le z \le 10 \\ \frac{25-z}{15}, & 10 \le z \le 25 \end{cases} \tag{4.10}$$

$$\mu_{M} = \begin{cases} \frac{z-10}{15}, & 10 \le z \le 25 \\ \frac{40-z}{15}, & 25 \le z \le 40 \end{cases} \tag{4.11}$$

$$\mu_{L} = \begin{cases} \frac{z-25}{15}, & 25 \le z \le 40 \\ \frac{60-z}{20}, & 40 \le z \le 60 \end{cases} \tag{4.12}$$

$$\mu_{VL} = \frac{z - 40}{20}, 40 \le z \le 60 \tag{4.13}$$

Now, Let us assume demand is 60 and Time is 17 (5pm),
then $\mu_{SD} = 0$, $\mu_{MD} = \frac{4}{5}$ and $\mu_{LD} = \frac{1}{5}$ and $\mu_{MT} = 0$, $\mu_{AT} = \frac{7}{12}$ and $\mu_{ET} = \frac{5}{12}$
Here, 2 values are zero and only 4 values are there and these 4 values generate 4 rules combining demand and time of day together. As for these 4 rules the strengths are calculated based on the given conditions. Rule 1 is when the demand is medium and the time of day is afternoon. Demand Medium and Afternoon Time (DMAT) ,
the strength for rule 1 is
$S_1 = min[\mu_{MD}, \mu_{AT}] = min[\frac{4}{5}, \frac{7}{12}] = \frac{7}{12}$
For rule 2, the condition is demand is medium and time of day is evening. Demand Medium and Evening Time (DMET),
the strength for rule 2 is
$S_2 = min[\mu_{MD}, \mu_{ET}] = min[\frac{4}{5}, \frac{5}{12}] = \frac{5}{12}$
For rule 3, the condition is that demand is large and time is afternoon. Demand Large and Afternoon Time (DLAT) ,
the strength is
$S_3 = min[\mu_{LD}, \mu_{AT}] = min[\frac{1}{5}, \frac{7}{12}] = \frac{1}{5}$

Lastly, for rule 4, the demand is large and time of day is evening. Demand Large and Evening Time (DLET),
the strength is
$S_4 = min[\mu_{LD}, \mu_{ET}] = min[\frac{1}{5}, \frac{5}{12}] = \frac{1}{5}$

Next is to calculate the maximum value of the membership function from all the values of the 4 strengths.
Maximum Strength $= max\{S_1, S_2, S_3, S_4\} = [\frac{7}{12}, \frac{5}{12}, \frac{1}{5}, \frac{1}{5}] = \frac{7}{12} \in$ DMAT Rule

Now, The value of membership function is calculated to be $\frac{7}{12}$.

Next and last step is the defuzzification of the calculated strengths to find a crisp value from all the data gathered. For this, it can be clearly seen that, according to the value of the membership function , the rule 1 is followed which means the condition when demand is medium and time is afternoon i.e. DMAT, which means to calculate the output value for whole demand the value to be used is $\frac{7}{12}$. Therefore, put the value of membership function in equation (4.14) given below which is equation for the output to calculate the crisp value.

$$\mu_M = \begin{cases} \frac{z-10}{15}, & 10 \leq z \leq 25 \\ \frac{40-z}{15}, & 25 \leq z \leq 40 \end{cases} \tag{4.14}$$

By putting the value of μ_z here, from the first part, the value of z is to be calculated as $z = 18.75$ and from the second part, the value $z = 31.25$. From these two values, the crisp value is computed by using the mean of maxima method, where the mean of the values is taken which means, $\frac{18.75+31.25}{2} = \frac{50}{2} = 25$. Hence , the crisp value or the output or the defuzzified demand for the given problem is 25 units.
This is a very small example used to show the working of Mamdani Fuzzy Inference System. In this paper, this same method is applied to compute the solutions for large dataset, where the constraints are related to CO_2 emissions, time windows, limited capacity and closed Vehicle routing that make it a complex problem. Therefore, metaheuristic approach along with MFIS is applied to compute the results shown in next section.

5. Experimental Results

As the above model is concerned with green vehicle routing problems and there are some constraints as well. The results are based on the constraints collectively used in the mathematical model which involves the time windows constraint, capacity constraint, visiting of customers by the vehicles and many more are there discussed in **Section 3**. Solomon R101 dataset [22] is considered to calculate the efficiency of the proposed approach. The route length is computed using the Euclidean distance, which is measured in unit of distance. Time taken to cover one unit distance is considered to be one unit time. The proposed approach is tested in the Python 3.6 programming language. To test its efficiency, a machine having 3 GHz i5 processor and 28 GB RAM was used. The final solution is that when 100 iterations are there, 24 vehicles are used to complete the routes and the mimimum CO_2 emissions are 481.81 kg. Similarly, the results are also calculated on 75 iterations where the number of vehicles used was 23 but the CO_2 emissions are more as compared to 100 iterations. The CO_2 emissions for 75 iterations are 488.51 kg. Therefore, it is observed that 100 iterations gives better optimized results as compared to the 75 iterations. Moreover, defuzzified demand is also calculated in both the cases. Defuzzified demand is 1040.85 units when iterations are 100 and 1037.85 units when the iterations are 75. **Table 1** provides the results for randomly picked 10 vehicles from 24 vehicles for 75 iterations and **Table 2** reveals the detailed results for randomly picked 10 vehicles out of 24 vehicles for 100 iterations. Both the tables include the vehicle number, their routes, total customers served and total CO_2 emissions during the delivery. **Table 3** and **Table 4** gives the value of defuzzified demand for randomly picked 10 customers out of 100 for 75 iterations and 100 iterations respectively. In **Table 5** it can be clearly seen that during 75 iterations total number of vehicles used is 23 which is least but CO_2 emissions are less during the 100 iterations which is 481.81 kg. The plots shown in **Figure 2** and **Figure 3** represents the routes of 24 vehicles and the Ant colony optimization algorithm convergence plot for 100 iterations respectively. **Figure 3** representing the convergence plot means that the best result came after 50 iterations. To be more precise, after 57 iterations, there were no changes in the values or it can be said that the result becomes constant and no chnages seen in the values of output for cost and CO_2 emissions. The tables below show all the results in a clear way.

Table 1: Output for 75 iterations

Vehicle Number (Randomly Picked)	Path Followed	Customers Attended	CO_2 Emissions
2	[0, 52, 18, 6, 96, 60, 89, 58, 0]	7	20.90 kg
6	[0, 28, 12, 76, 79, 3, 68, 80, 0]	7	17.83 kg
7	[0, 63, 64, 50, 24, 77, 0]	5	37.94 kg
11	[0, 14, 44, 38, 43, 0]	4	24.19 kg
15	[0, 31, 88, 7, 54, 0]	4	22.59 kg
17	[0, 72, 75, 41, 55, 0]	4	22.27 kg
19	[0, 33, 81, 34, 35, 0]	4	21.37 kg
20	[0, 5, 83, 61, 85, 0]	4	16.40 kg
21	[0, 29, 78, 0]	2	15.69 kg
23	[0, 59, 99, 0]	2	8.26 kg

Table 2: Output for 100 iterations

Vehicle Number (Randomly Picked)	Path Followed	Customers Attended	CO_2 Emissions
1	[0, 33, 81, 3, 68, 80, 0]	5	16.61 kg
6	[0, 27, 69, 30, 51, 20, 48, 0]	6	27.48 kg
9	[0, 28, 12, 76, 78, 34, 35, 77, 0]	7	26.56 kg
11	[0, 95, 94, 74, 0]	3	15.42 kg
13	[0, 42, 15, 41, 56, 0]	4	21.16 kg
16	[0, 36, 47, 19, 8, 0]	4	24.16 kg
17	[0, 92, 98, 61, 86, 0]	4	17.89 kg
20	[0, 29, 79, 50, 0]	3	15.20 kg
22	[0, 21, 73, 0]	2	9.09 kg
24	[0, 5, 16, 0]	2	13.41 kg

Table 3: Defuzzified Demand for 75 iterations

Customer Number (Randomly Picked)	Fuzzified Demand (Triangular Fuzzy Number)	Defuzzified Whole Demand
8	[6,12,18]	10.00
19	[8,11,16]	10.00
25	[9,15,21]	13.34
35	[8,12,15]	10.46
49	[7,12,18]	10.00
56	[6,11,16]	10.00
68	[7,10,15]	10.39
77	[10,15,20]	13.54
89	[9,15,20]	13.83
94	[8,11,13]	10.00

Table 4: Defuzzified Demand for 100 iterations

Customer Number (Randomly Picked)	Fuzzified Demand (Triangular Fuzzy Number)	Defuzzified Whole Demand
5	[6,12,18]	10.00
13	[10,13,16]	11.56
24	[9,12,15]	11.78
35	[8,12,15]	10.71
48	[7,13,19]	11.07
59	[5,9,16]	10.00
68	[7,11,15]	10.39
75	[6,12,17]	10.00
89	[9,15,20]	13.83
95	[8,11,13]	10.00

Table 5: Overall Results

Iterations	Total Vehicles Used	Total CO_2 Emissions	Total Defuzzified Demand (units)
75	23	$488.51kg$	1037.85
100	24	$481.81kg$	1040.85

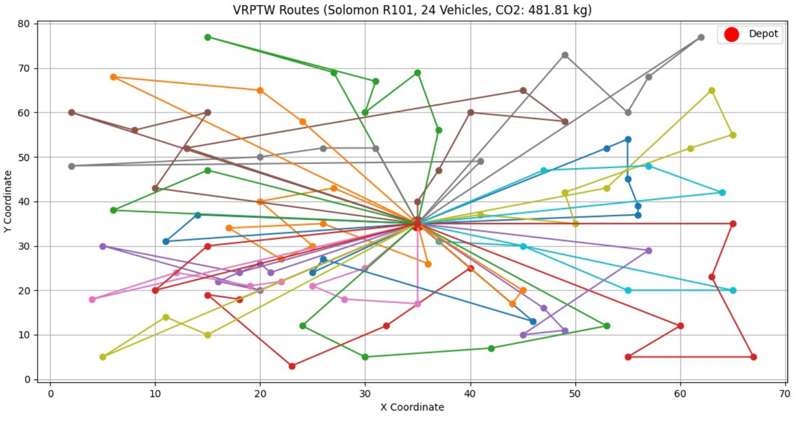

Figure 2: Routes of 24 Vehicles for 100 iterations

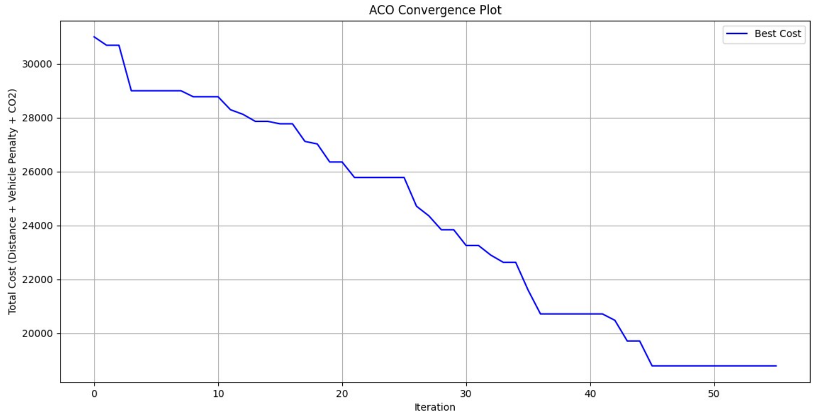

Figure 3: Convergence Graph for 100 iterations

6. Conclusion

From all the results and plots generated using the ant colony optimization algorithm for the proposed mathematical model, it is observed that ant colony optimization works better for vehicle routing problem as it has given minimum number of vehicles to serve all the customers along with minimizing the distance and overall operational costs and all the fuzzy demands of the customers are also satisfied. The example shown regarding mamdani fuzzy inference system also explains mamdani method in a prominent way. During 75 iterations, the number of vehicles was least but when the iterations is 100, the CO_2 emissions are least and the defuzzified demand in both the cases is also satisfying. To minimize the number of vehicles, penalty method is used. As this paper includes many constraints and conditions related to the real world problems in case of logistics, still the results are satisfactory and ant colony optimization algorithm along with mamdani fuzzy inference system works better and provides good results. From this paper, it can be concluded that green vehicle routing is one of the best ways to fulfill the sustainable development goals along with fulfilling the uncertain customer demands and it can also be helpful for the future innovations in logistics related to the real- world.

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