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Rida-Jassim integral transform: a tool for solving linear and non-linear differential equations

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ABSTRACT: In this paper, we introduce a new integral transform belonging to the class of Laplace transforms, called the Rida-Jassim Transform (RJ Transform). We explore its properties and compare it to the classical Laplace transform. Furthermore, we provide proofs for the key properties associated with this transform and demonstrate its application in solving differential equations. By employing this new transform, we can reduce the original problem to an algebraic equation that can be solved directly, followed by applying the inverse transform to obtain the solution to the original problem.

Key Words: Integral transform, differential equations, Laplace transform, Rida-Jassim transform.

Contents

1. Introduction

Integral transforms have served as fundamental mathematical instruments for streamlining the resolution of differential, integral, and fractional equations. Classical transforms such as the Laplace transform, for instance, facilitate the conversion of differential equations into algebraic forms, thereby simplifying their analytical and numerical treatment. This methodology typically involves three stages: applying the transform to the target equation, solving the resultant algebraic system, and inverting the transform to recover the solution in the original domain [1]. Recent advancements in this field have introduced a diverse array of specialized integral transforms, broadening the scope of problems amenable to such techniques. Notable contributions include the Sumudu, Elzaki, Natural, Aboodh, Pourreza, Mohand, Yang, Yasser-Jassim, and Kamal transforms, each tailored to address distinct classes of equations and boundary value problems [3,22,13,44,9,10,12,1]. These transforms have demonstrated significant utility across interdisciplinary domains, including cryptography [15], digital image processing [17], engineering mathematics [27,40], optical physics [2], and theoretical physics [33,38], as well as other scientific and engineering applications [22,28]. Their efficacy is particularly evident in solving ordinary differential equations (ODEs), partial differential equations (PDEs), and fractional-order differential equations (FDEs) [20,36,35,34,11,30,31,32,4,6,7,8,29,43,19,42,5,9]. In this work, we introduce a novel integral transform designed to overcome limitations in existing methodologies for analyzing differential and integral equations. Subsequent sections will rigorously establish its mathematical formulation, operational properties, and practical applications, with a focus on its capacity to address complex and previously intractable problems.

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2. Main Results

Definition 2.1 *Suppose f(t) is an integrable function defined for values $t > 0$, We define a new transformation for the function $f(t)$ denoted by the symbol $NTf(t$ or $RJ(\sqrt{a}, v)$ in the following form:*

$$NT\{f(t)\} = RJ(\sqrt{a}, v) = \int_0^\infty e^{-\sqrt{a}t} f(vt)\, dt, \quad a \geq 0. \tag{2.1}$$

Theorem 2.1 (Relationship: Laplace – NT Transform) *If the new transform of the function $f(t)$ is*

$$RJ(\sqrt{a}, v) = NT\{f(t)\},$$

then

$$NT\{f(t)\} = \int_0^\infty e^{-\sqrt{a}t} f(vt)\, dt.$$

Proof:

Let $vt = s$, which implies $t = s/v$. Substituting this into the integral, we obtain

$$NT\{f(t)\} = \int_0^\infty e^{-(\sqrt{a}s)/v} f(s) \frac{1}{v}\, ds.$$

We can express this as

$$NT\{f(t)\} = \frac{1}{v} F\left(\frac{\sqrt{a}}{v}\right).$$

□

Theorem 2.2 (Relationship: Sumudu – NT Transform) *If*

$$NT\{f(t)\} = \int_0^\infty e^{-\sqrt{a}t} f(vt)\, dt,$$

let $\sqrt{a}\,t = u$, which implies $t = u/\sqrt{a}$. Then

$$NT\{f(t)\} = \int_0^\infty e^{-u} f\left(\frac{vu}{\sqrt{a}}\right) \frac{1}{\sqrt{a}}\, du = \frac{1}{\sqrt{a}} G\left(\frac{v}{\sqrt{a}}\right).$$

Theorem 2.3 (Relationship: Elzaki – NT Transform) *Again,*

$$NT\{f(t)\} = \int_0^\infty e^{-\sqrt{a}t} f(vt)\, dt,$$

let $vt = u$, which implies $t = u/v$. Then

$$NT\{f(t)\} = \int_0^\infty e^{-(\sqrt{a}u)/v} f(u) \frac{1}{v}\, du.$$

We can express this as

$$NT\{f(t)\} = F\left(\frac{\sqrt{a}}{v}\right).$$

Some Useful Properties

1. $NT(k) = \frac{k}{\sqrt{a}}$, where k is constant.

$$NT(k) = \int_0^\infty e^{-\sqrt{a}t} k\, dt = k\left[-\frac{1}{\sqrt{a}}e^{-\sqrt{a}t}\right]_0^\infty = \frac{k}{\sqrt{a}}.$$

2. $NT\{t\} = \frac{v}{(\sqrt{a})^2}$

$$NT\{t\} = \int_0^\infty e^{-\sqrt{a}t} vt\, dt.$$

By parts: let $u = t$, $dv = e^{-\sqrt{a}t}dt$.

$$du = dt, \quad v = -\frac{1}{\sqrt{a}}e^{-\sqrt{a}t}.$$

Then

$$= v\left[-\frac{1}{\sqrt{a}}te^{-\sqrt{a}t}\Big|_0^\infty + \frac{1}{\sqrt{a}}\int_0^\infty e^{-\sqrt{a}t}dt\right]$$

$$= v \cdot \frac{1}{\sqrt{a}} \cdot \frac{1}{\sqrt{a}} = \frac{v}{(\sqrt{a})^2}.$$

3. $NT\{e^{bt}\} = \frac{1}{\sqrt{a} - bv}$

$$NT\{e^{bt}\} = \int_0^\infty e^{-\sqrt{a}t} e^{bvt}\, dt = \int_0^\infty e^{-t(\sqrt{a}-bv)}\, dt$$

$$= \frac{1}{\sqrt{a} - bv}.$$

4. $NT\{t^n\} = \frac{n!\, v^n}{(\sqrt{a})^{n+1}}$

5. $NT\{\sin(bt)\} = \frac{bv}{a + b^2v^2}$

6. $NT\{\sinh(bt)\} = \frac{bv}{a - b^2v^2}$

7. $NT\{\cos(bt)\} = \frac{\sqrt{a}}{a + b^2v^2}$

8. $NT\{\cosh(bt)\} = \frac{\sqrt{a}}{a - b^2v^2}$

Basic Properties of the New Integral Transform

In this section of the article, we present some key proofs that are directly used in the transform and can also be beneficial for its applications.

Theorem 2.4 (Linearity Property) *Suppose that*

$$NT\{f(t)\} = RJ_1(\sqrt{a}, v), \quad NT\{g(t)\} = RJ_2(\sqrt{a}, v),$$

then for constants ν *and* κ, *the linearity property holds:*

$$NT\{\nu f(t) + \kappa g(t)\} = \nu RJ_1(\sqrt{a}, v) + \kappa RJ_2(\sqrt{a}, v). \tag{1}$$

Proof: The proof is clear and easy to verify by the linearity of the integral. □

Theorem 2.5 (New Transform of Derivatives) *Let* $RJ(\sqrt{a}, v)$ *be the new integral transform of* $u(t)$. *Then:*

$$\begin{aligned}
&(1) \quad NT\{u'(t)\} = \frac{\sqrt{a}}{v} NT\{u(t)\} - \frac{u(0)}{v},\\
&(2) \quad NT\{u''(t)\} = \frac{a}{v^2} NT\{u(t)\} - \frac{\sqrt{a}\, u(0)}{v^2} - \frac{u'(0)}{v}.
\end{aligned}$$

Proof:

$$NT\{u'(t)\} = \int_0^\infty e^{-\sqrt{a}t}\, u'(vt)\, dt$$

Using integration by parts:

$$\begin{cases} u = e^{-\sqrt{a}t} & \Rightarrow \quad du = -\sqrt{a} e^{-\sqrt{a}t}\, dt, \\ dv = u'(vt)\, dt & \Rightarrow \quad v = \frac{1}{v} u(vt). \end{cases}$$

Then:

$$\int_0^\infty e^{-\sqrt{a}t} u'(vt)\, dt = \left[\frac{1}{v} e^{-\sqrt{a}t} u(vt)\right]_0^\infty + \frac{\sqrt{a}}{v} \int_0^\infty e^{-\sqrt{a}t} u(vt)\, dt.$$

By the limit behavior of $e^{-\sqrt{a}t}$:

$$= -\frac{u(0)}{v} + \frac{\sqrt{a}}{v} NT\{u(t)\}.$$

Thus:

$$NT\{u'(t)\} = \frac{\sqrt{a}}{v} NT\{u(t)\} - \frac{u(0)}{v}.$$

For the second derivative:

$$\text{Let } g(t) = u'(t),$$

then by applying (1):

$$NT\{u''(t)\} = NT\{g'(t)\} = \frac{\sqrt{a}}{v} NT\{g(t)\} - \frac{g(0)}{v}.$$

But from the first result:

$$NT\{g(t)\} = NT\{u'(t)\} = \frac{\sqrt{a}}{v} NT\{u(t)\} - \frac{u(0)}{v}.$$

Therefore:

$$NT\{u''(t)\} = \frac{\sqrt{a}}{v}\left(\frac{\sqrt{a}}{v}NT\{u(t)\} - \frac{u(0)}{v}\right) - \frac{u'(0)}{v}$$
$$= \frac{a}{v^2}NT\{u(t)\} - \frac{\sqrt{a}u(0)}{v^2} - \frac{u'(0)}{v}.$$

□

Theorem 2.6 (New Transform of Integral) *If*

$$NT\{f(t)\} = RJ(v),$$

then

$$NT\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{v}{\sqrt{a}}RJ(v).$$

Proof: Let

$$h(t) = \int_0^t f(\tau)\,d\tau.$$

Then by differentiation:

$$h'(t) = f(t).$$

Taking the new integral transform on both sides:

$$\frac{\sqrt{a}}{v}NT\{h(t)\} = RJ(v) \quad \Rightarrow \quad NT\{h(t)\} = \frac{v}{\sqrt{a}}RJ(v).$$

□

Theorem 2.7 (Convolution Theorem) *Let $F(s)$ and $G(s)$ be the Laplace transforms of $f(t)$ and $g(t)$, respectively. Similarly, let $RJ_1(\sqrt{a}, v)$ and $RJ_2(\sqrt{a}, v)$ be the new integral transforms of $f(t)$ and $g(t)$, respectively. Then the new integral transform of the convolution of f and g, given by:*

$$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)\,d\tau,$$

is expressed as:

$$NT\{(f * g)(t)\} = v\,RJ_1(\sqrt{a}, v)\,RJ_2(\sqrt{a}, v).$$

Proof: From the classical Laplace transformation property:

$$\mathcal{L}\{(f * g)(t)\} = F(s)G(s) = H(s).$$

Assume

$$NT\{(f * g)\} = RJ(\sqrt{a}, v).$$

By the duality relation with the Laplace transformation:

$$RJ(\sqrt{a}, v) = \frac{1}{v}H\left(\frac{\sqrt{a}}{v}\right) = \frac{1}{v}F\left(\frac{\sqrt{a}}{v}\right)G\left(\frac{\sqrt{a}}{v}\right).$$

But:

$$F\left(\frac{\sqrt{a}}{v}\right) = v\,RJ_1(\sqrt{a}, v), \quad G\left(\frac{\sqrt{a}}{v}\right) = v\,RJ_2(\sqrt{a}, v).$$

Thus:

$$RJ(\sqrt{a}, v) = v\, RJ_1(\sqrt{a}, v)\, RJ_2(\sqrt{a}, v).$$

□

3. Application

Example 1

Consider the first-order ODE:

$$y' + y = 1, \quad y(0) = 0.$$

Taking the transform on both sides:

$$NT\{y'\} + NT\{y\} = NT\{1\}.$$

Using the derivative property:

$$\frac{\sqrt{a}}{v} NT\{y\} - \frac{y(0)}{v} + NT\{y\} = \frac{1}{\sqrt{a}}.$$

Since $y(0) = 0$, we have:

$$\left(\frac{\sqrt{a}}{v} + 1\right) NT\{y\} = \frac{1}{\sqrt{a}}.$$

Thus:

$$NT\{y\} = \frac{v}{\sqrt{a}(\sqrt{a} + v)}.$$

Partial fraction decomposition:

$$\frac{v}{\sqrt{a}(\sqrt{a} + v)} = \frac{A}{\sqrt{a}} + \frac{B}{\sqrt{a} + v}.$$

Solving:

$$A + B = 0, \quad A = 1 \implies B = -1.$$

Hence:

$$NT\{y\} = \frac{1}{\sqrt{a}} - \frac{1}{\sqrt{a} + v}.$$

Taking the inverse transform:

$$y(t) = 1 - e^{-t}.$$

Example 2

$$y'' + y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

Transform:

$$NT\{y''\} + NT\{y\} = NT\{0\}.$$

$$\frac{a}{v^2} NT\{y\} - \frac{\sqrt{a}y(0)}{v^2} - \frac{y'(0)}{v} + NT\{y\} = 0.$$

$$\frac{a}{v^2} NT\{y\} - \frac{1}{v} + NT\{y\} = 0.$$

$$NT\{y\}\left(\frac{a}{v^2} + 1\right) = \frac{1}{v}.$$

$$NT\{y\} = \frac{v}{a+v^2}.$$

$$= \frac{\sqrt{a}}{a+v^2} + \frac{v}{a+v^2}.$$

Inverse:

$$y(t) = \cos(t) + \sin(t).$$

Example 3

$$y'' - 2y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 1.$$

Transform:

$$NT\{y''\} - 2NT\{y'\} + 2NT\{y\} = 0.$$

$$\frac{a}{v^2}NT\{y\} - \frac{\sqrt{a}y(0)}{v^2} - \frac{y'(0)}{v} - \frac{2\sqrt{a}}{v}NT\{y\} + \frac{2y(0)}{v} + 2NT\{y\} = 0.$$

Substitute $y(0) = 1, y'(0) = 1$:

$$\frac{a}{v^2}NT\{y\} - \frac{\sqrt{a}}{v^2} - \frac{1}{v} - \frac{2\sqrt{a}}{v}NT\{y\} + \frac{2}{v} + 2NT\{y\} = 0.$$

Collecting terms:

$$NT\{y\}\left(\frac{a}{v^2} - \frac{2\sqrt{a}}{v} + 2\right) = \frac{\sqrt{a}+v}{v^2}.$$

$$NT\{y\} = \frac{\sqrt{a}-v}{a - 2v\sqrt{a} + 2v^2}.$$

Inverse transform:

$$y(t) = e^t\cos(t).$$

Example 4

Consider the third-order ODE:

$$y''' + 2y'' + 2y' + 3y - \cos t = \sin t, \quad y(0) = y''(0) = 0, \quad y'(0) = 1.$$

Taking the transform:

$$NT\{y'''\} + 2NT\{y''\} + 2NT\{y'\} + 3NT\{y\} - NT\{\cos t\} = NT\{\sin t\}.$$

$$\frac{a\sqrt{a}}{v^3}NT\{y\} - \frac{ay(0)}{v^3} - \frac{\sqrt{a}y'(0)}{v^2} - \frac{y''(0)}{v}$$
$$+2\left(\frac{a}{v^2}NT\{y\} - \frac{\sqrt{a}y(0)}{v^2} - \frac{y'(0)}{v}\right)$$
$$+2\left(\frac{\sqrt{a}}{v}NT\{y\} - \frac{y(0)}{v}\right) + 3NT\{y\} - \frac{\sqrt{a}}{a+v^2} = \frac{v}{a+v^2}.$$

Substituting initial conditions:

$$NT\{y\} = \frac{V(3v^3 + 2v^2\sqrt{a} + a\sqrt{a} + 2av)}{(3v^3 + 2v^2\sqrt{a} + a\sqrt{a} + 2av)a + v^2}.$$

Taking the inverse transformation:

$$y(t) = \sin t.$$

4. Conclusion

This study introduced an innovative integral transform and demonstrated its applicability in solving ordinary differential equations (ODEs). The proposed methodology was successfully applied to derive novel solutions for this class of equations. The results underscore the transform's efficacy and computational efficiency, enabling the simplification of complex mathematical problems while reducing procedural complexities. Furthermore, this approach opens avenues for future development, particularly when integrated with numerical and iterative techniques to address broader classes of equations, such as partial differential equations (PDEs). Such advancements could enrich computational frameworks in applied mathematics and engineering, supporting the design of effective methodologies for nonlinear systems. The transform's flexibility highlights its potential as a promising research tool in theoretical and applied studies, with possibilities for extension to multidisciplinary contexts requiring robust analytical or computational solutions.

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