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Binary Soft Fuzzy Points Extraction

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ABSTRACT: The main objective of this work is to present a new type of soft fuzzy point called Binary Soft Fuzzy-point via the concept of a binary soft fuzzy set (that contains the concept of a binary sets of parameters that can be applied in one of the essential scientific disciplines in the future). In this work, we study binary soft fuzzy point, we, define new binary soft point and binary soft fuzzy point from combination fuzzy point and binary soft point give example of each case and apply the binary soft fuzzy point in MATLAB and represented figure different of each point.

Key Words: binary soft point, extraction of binary soft fuzzy point, binary soft fuzzy point.

Contents

1. Introduction:

Fuzzy set developed by Zadeh and R. Al. Mohammad in [10,15,16]. Following the semblance of the concept of fuzzy set, the researcher has been giving much attention to the development of fuzzy set theory. Molodtsov [14,3] proposed a new theory called the soft set theory for the representation of fuzziness and non-certainty, which is problem-free of the mentioned methods. The soft set is considered the parameterized family of subsets of the universal set. It is very convenient to apply in practice, and this theory has excellent application to such areas as the smoothness of functions, game theory, Riemann integration, Perron integration, probability theory, and measurement theory in [7]. Soft set theory is a comparatively recent concept. For the past few years, much work has been done, and marked progress has been made in [9,10,11,12]. However, all of the above works are developed from classical soft-set theory.

In this study, a soft fuzzy was first developed by A. Sabri [1], but after that, many researchers expanded this study and produced new results [5]. Tanay and Kandemir stated and introduced the soft fuzzy topology on a soft fuzzy set and presented a preliminary theoretical background for further evolution of this area of study. S. Roy and T. K. Samanta also investigated soft fuzzy topology in [17,2]. LA AL-Swidi, MH, Hadi, RD, Ali about the dual soft set theory in [11]

In a recent work [19], the author tried for the first time to propose a connection between Binary, Ternary, and N-ary relation Soft-sets and Fuzzy-set respect to the power set for arising the new sorts of fuzzy soft, which he named Binary relation-fuzzy -soft-set, Ternary Relation-Fuzzy-Soft-set, and N-ary relation-Fuzzy-Soft-set. The reader can find the application of the above novel concepts in digital image similarity and related ideas in [8,18].

The same authors of this work a new type of soft fuzzy set called Binary Soft Fuzzysets [4] and we looked at the primary and essential operations on this set with examples.

The main objective of this work is to present a new type of soft fuzzy point called Binary Soft Fuzzy-point via the concept of a binary soft fuzzy set (that contains the concept of a binary sets of parameters that can be applied in one of the essential scientific disciplines in the future).

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This article is grouped into three sections, as outlined below. Section 1 introduces our new idea, while Section 2 mentions some central concepts in binary soft fuzzy set that we needed in this work. In section $3, 4, 5$ we present a new idea (definition of the binary soft point and extraction binary soft fuzzy point via binary soft fuzzy set). In this same section, we also illustrate the essential and central examples.

2. Preliminaries

In this part, we view the important notions that are important and necessary for subsequent visions and which are the central in this our paper.

Definition 2.1 *[4] Let μ be an initial universe, $\pounds_1, \pounds_2$ are two sets of parameters. And $\mathcal{P}(\mu)$ represents μ 's power set. A binary soft fuzzy set (BSF-set) ψ on μ with member-ship mapping f_ψ representing by a combination between binary- soft set and a member-ship mapping of fuzzy set with respect to power set $f : \mu \to \nVdash = [0, 1]$; together with binary-soft set $S : \pounds_1 \times \pounds_2 \to \mathcal{P}(\mu)$, Then $f_\psi : \pounds_1 \times \pounds_2 \to l_7^m$ (the set of all fuzzy sets in seventh family), is a member-ship mapping of BSF-set ψ where;*

$$\psi = \{(\mathrm{e_i}, \mathrm{e_j}), \mathrm{f}_\psi(\mathrm{e_i}, \mathrm{e_j})), \forall (\mathrm{e_i}, \mathrm{e_j}) \in \pounds_1 \times \pounds_2, \text{ where } \mathrm{e_i} \in \pounds_1 \text{ and } \mathrm{e_j} \in \pounds_2, \mathrm{f} : \pounds_1 \times \pounds_2 \to l_m^7, \mathrm{i} = l, \ldots, \mathrm{v}; \mathrm{j} = l, \ldots, \mathrm{u}\}.$$

Definition 2.2 *[11] 1-Type I: for some $\mathrm{x} \in \mu, 0 < \alpha \le 1, p_\alpha^\times$ is classical fuzzy point on the fuzzy point of type I*

$$p_\alpha^\times(y) = \begin{cases} \alpha & \text{if } y = \times \\ 0, & y \ne \times \end{cases}$$

2-Type II: Let A μ and $0 < \alpha \le 1, p_\alpha^A$ is the fuzzy point of type II

$$p_\alpha^\times(y) = \begin{cases} p_y^\alpha & \text{if } y \in A \\ 0 & y \notin A \end{cases}$$

3-Type III: For any $0 < \alpha < 1, p_\alpha$ is fuzzy point of type III
Such that $p_\alpha(y) = \alpha, \forall y \in \mu$.

3. Binary soft point.

In [11] define soft point, in this sec. define new kind of binary soft point.
1-the first point of binary soft point,

$$sF(S) = \left\{ \begin{array}{cc} \{S\}, & \text{if } S = (\mathrm{e}_i, \mathrm{e}_j) \\ \emptyset, & S \ne (\mathrm{e}_i, \mathrm{e}_j) \end{array} \right.$$

Example 3.1

$$BS = \{((\mathrm{e}_1, \mathrm{e}_3), \{\mathrm{x}_3, \emptyset\}), ((\mathrm{e}_1, \mathrm{e}_4), \{\mathrm{x}_1, \mathrm{x}_4\}), ((\mathrm{e}_2, \mathrm{e}_3), \{\mathrm{x}_3, \mathrm{x}_5\}), ((\mathrm{e}_2, \mathrm{e}_4), \{\mathrm{x}_4, \mathrm{x}_5\})\}$$
$$sF_{(\mathrm{e}_1, \mathrm{e}_3)}^{\times_1} = \{(\mathrm{e}_1, \mathrm{e}_3), \{\mathrm{x}_1\}), ((\mathrm{e}_1, \mathrm{e}_4), \{\varnothing\}), ((\mathrm{e}_2, \mathrm{e}_3), \{\varnothing\}), ((\mathrm{e}_2, \mathrm{e}_4), \{\varnothing\})$$

2-the second binary soft point

$$sF_\times = \{(\mathrm{e}_i, \mathrm{e}_j), \{\times\}, (\mathrm{e}_i, \mathrm{e}_j) \in \pounds_i \times \pounds_j\}$$

Example 3.2

$$sF_i(S) = \{(\mathrm{e}_i, \mathrm{e}_j), \{\mathrm{x}_1\}, (\mathrm{e}_i, \mathrm{e}_j) \in \pounds_i \times \pounds_j\}$$
$$sF_{\times_1} = \{(\mathrm{e}_1, \mathrm{e}_3), \{\times_1\}), ((\mathrm{e}_1, \mathrm{e}_4), \{\times_1\}), ((\mathrm{e}_2, \mathrm{e}_3), \{\times_1\}), ((\mathrm{e}_2, \mathrm{e}_4), \{\times_1\})$$

3-the third binary soft point.

$$sF_{(\mathrm{e}_i,\mathrm{e}_j)}(S) = \begin{cases} F(\mathrm{e}_i,\mathrm{e}_j) \neq \emptyset & \text{if } S = (\mathrm{e}_i,\mathrm{e}_j) \\ \emptyset, & S \neq (\mathrm{e}_i,\mathrm{e}_j) \end{cases}$$
$$, \forall (\mathrm{e}_i,\mathrm{e}_j) \in \pounds_i \times \pounds_j$$

Example 3.3

$$A = \{\mathrm{e}_1,\mathrm{e}_3,\mathrm{e}_4\}, \pounds_1 = \{\mathrm{e}_1,\mathrm{e}_2\}, \pounds_2 = \{\mathrm{e}_3,\mathrm{e}_4\}$$
$$sF(\mathrm{e}_1,\mathrm{e}_3) = \{\{(\mathrm{e}_1,\mathrm{e}_3), \{\times_1\}), ((\mathrm{e}_1,\mathrm{e}_4), \{\varnothing\})$$

Note: we belief from definition binary soft fuzzy set that binary soft fuzzy set is the final form and not binary fuzzy soft set since in the final form the set appears binary soft set and not fuzzy set so our belief is binary soft fuzzy hence the points are said binary soft fuzzy points and then via the combination of three different types of fuzzy point and three different types of binary soft points we get seven different binary soft fuzzy points as in section following:

4. Binary Soft Fuzzy Point.

The combination of binary soft point and fuzzy point get new binary soft fuzzy point.

1-The first binary soft fuzzy point

The combination of binary soft point of type $sF_{\times_i}(s)$ with fuzzy point $p_\alpha^{\times i}$ for some $\times_i \in \mu$, for $\bar{\bar{0}} < \alpha < \bar{\bar{1}}$, we get the binary soft fuzzy point of type $sfFp_\alpha^{\times i}$

$$sfFp_\alpha^{\times i} = \left\{\left((\mathrm{e}_i,\mathrm{e}_j), p_\alpha^{\times i}\right), (\mathrm{e}_i,\mathrm{e}_j) \in \pounds_i \times \pounds_j\right\}$$

Example 4.1 *let* $\mu = \{\times_1, \times_2, \times_3, \times_4, \times_5\}$,

$$p_{0.2}^{\times i}, i = 1,3$$
$$p_{0.2}^{\times_{1,3}} = \{(\times_1, 0.2), (\times_2, 0), (\times_3, 0.2), (\times_4, 0), (\times_5, 0)\}$$
$$p_{0.2}^{\times i}, i = 2,3$$
$$p_{0.2}^{\times_{2,3}} = \{(\times_1, 0), (\times_2, 0.2), (\times_3, 0.2), (\times_4, 0), (\times_5, 0)\}$$
$$p_{0.2}^{\times i}, i = 1,4$$
$$p_{0.2}^{\times_{1,4}} = \{(\times_1, 0.2), (\times_2, 0), (\times_3, 0), (\times_4, 0.4), (\times_5, 0)\}$$
$$\mathrm{sfF}(\mathrm{e}_1,\mathrm{e}_3) = \Big\{(\mathrm{e}_1,\mathrm{e}_3), \{(\times_1, 0.2), (\times_2, 0), (\times_3, 0.2), (\times_4, 0), (\times_5, 0)\} = p_{0.2}^{\times_{1,3}},$$
$$\mathrm{sfF}\Big((\mathrm{e}_1,\mathrm{e}_4) = \Big\{(\mathrm{e}_1,\mathrm{e}_4), \{(\times_1, 0.2), (\times_2, 0), (\times_3, 0), (\times_4, 0.2), (\times_5, 0)\} = p_{0.2}^{\times_{1,4}};$$
$$\mathrm{sfF}(\mathrm{e}_2,\mathrm{e}_3) = \Big\{(\mathrm{e}_2,\mathrm{e}_3), \{(\times_1, 0), (\times_2, 0.2), (\times_3, 0.2), (\times_4, 0), (\times_5, 0)\} = p_{0.2}^{\times_{2,3}},$$
$$\mathrm{sfF}\Big((\mathrm{e}_2,\mathrm{e}_4) = \Big\{(\mathrm{e}_2,\mathrm{e}_4), \{(\times_1, 0), (\times_2, 0.2), (\times_3, 0), (\times_4, 0.2), (\times_5, 0)\} = p_{0.2}^{\times_{2,4}}$$

In this figure 1 represented in MATLAB by depend on $sfFp_\alpha^{\times i}$ *of example above*

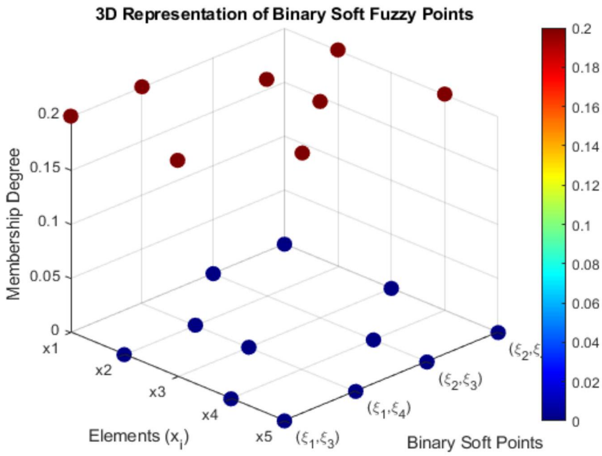

Figure 1: (3D representation of the first binary soft fuzzy point)

- *each color represented the degree of belonging, the closer it is to red, the higher the degree for every color.*
- *axis X element from* $\times_1$ *to* $\times_5$*.*
- *axis Y binary point* $(\mathrm{e}_i, \mathrm{e}_j)$*.*
- *axis Z belong degree.*

2- the second Binary soft fuzzy point The combination of binary soft point of type $sF_{(\mathrm{e}_i,\mathrm{e}_j)}(s)$ with fuzzy point $p_\alpha^{\times i}$ for some $\times_i \in \mu$, $(\mathrm{e}_i, \mathrm{e}_j) \in \pounds_i \times \pounds_j$ for $\overline{\overline{0}} < \alpha < \overline{\overline{1}}$, where,

$$sF_{(\mathrm{e}_i,\mathrm{e}_j)}(S) = \begin{cases} F(\mathrm{e}_i, \mathrm{e}_j) \neq \emptyset & \text{if } S = (\mathrm{e}_i, \mathrm{e}_j) \\ \emptyset, & S \neq (\mathrm{e}_i, \mathrm{e}_j) \end{cases}$$
$$, \forall (\mathrm{e}_i, \mathrm{e}_j) \in \pounds_i \times \pounds_j$$

Then the following:
1- if $x \in sF_{(\mathrm{e}_i,\mathrm{e}_j)}$ and through it binary soft fuzzy point if of type $sfF^{(\mathrm{e}_i,\mathrm{e}_j)}_{p^{\times i}_{\alpha i}}$.
2-if $\times \notin sF_{(\mathrm{e}_i,\mathrm{e}_j)}$ and through it binary soft fuzzy point if of type $sfF^{(\mathrm{e}_i,\mathrm{e}_j)}_{p^{\times i}_{\alpha i}} = \overline{\overline{0}}$.

$$sfF^{(\mathrm{e}_i,\mathrm{e}_j)}_{p^{\times i}_{\alpha i}} = \begin{cases} F^{(\mathrm{e}_i,\mathrm{e}_j)}_{p^{\times i}_{\alpha i}} & \text{if } \times \in sF_{(\mathrm{e}_i,\mathrm{e}_j)} \\ \overline{\overline{0}} & \text{if } \times \notin sF_{(\mathrm{e}_i,\mathrm{e}_j)} \end{cases}$$

Example 4.2 $sF(\mathrm{e}_1, \mathrm{e}_3) = \{((\mathrm{e}_1, \mathrm{e}_3), \{\times_3, \times_1\}), ((\mathrm{e}_1, \mathrm{e}_4), \{\varnothing\}), ((\mathrm{e}_2, \mathrm{e}_3), \{\varnothing\}), ((\mathrm{e}_2, \mathrm{e}_4), \{\varnothing\})\}$
$p^{\times_1}_{0.2} = \{(\times_1, 0.2), (\times_2, 0), (\times_3, 0), (\times_4, 0), (\times_5, 0)\}$
If $A = \{\times_1, \times_3\}$
$\mathrm{sfF}(\mathrm{e}_1, \mathrm{e}_3) = \{(\mathrm{e}_1, \mathrm{e}_3), \{(\times_1, 0.2), (\times_2, 0), (\times_3, 0), (\times_4, 0)\} = p^{\times_1}_{0.2}$,
$sfF\left((\mathrm{e}_1, \mathrm{e}_4) = fF(\mathrm{e}_2, \mathrm{e}_3) = \mathrm{sfF}\left((\mathrm{e}_2, \mathrm{e}_4) = \overline{\overline{0}}\right.\right.$.
$sfF^{(\mathrm{e}_1,\mathrm{e}_3)}_{p^{\times 1}_{0.2}} = \left\{(\mathrm{e}_1, \mathrm{e}_3), p^{\times_1}_{0.2}\right), \left\{(\mathrm{e}_1, \mathrm{e}_4), \overline{\overline{0}}\right), \left((\mathrm{e}_2, \mathrm{e}_3), \overline{\overline{0}}\right), \left(\left((\mathrm{e}_2, \mathrm{e}_4), \overline{\overline{0}}\right)\right\}$

Let $B = \{\times_2, \times_3, \times_4\}$ *then* $sfF\left((\mathrm{e}_1, \mathrm{e}_3) = sfF\left((\mathrm{e}_1, \mathrm{e}_4) = fF(\mathrm{e}_2, \mathrm{e}_3) = sfF\left((\mathrm{e}_2, \mathrm{e}_4) = \overline{\overline{0}}\right.\right.\right.$. *In this figure 2 represented in MATLAB by depend on* $sfF^{(\mathrm{e}_1,\mathrm{e}_3)}_{p^{\times\times 1}_{0.2}}$ *of example above*

0

- represented every point in 3D.

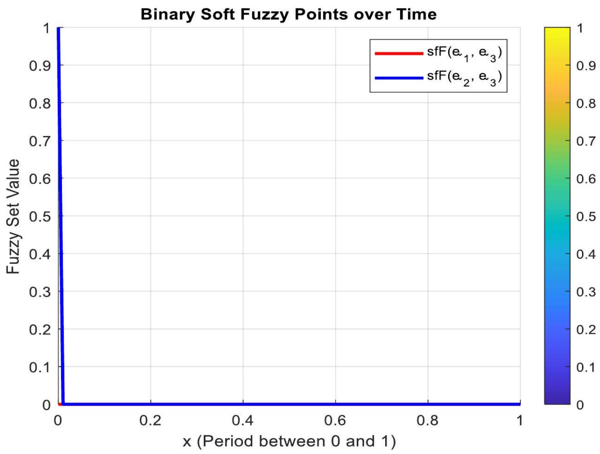

Figure 2: (the second of binary soft fuzzy point)

3-The third binary soft fuzzy point

Let $sF^{\times}_{(\mathrm{e}_i,\mathrm{e}_j}$ be binary soft point and $p^{\times}_{\alpha}$be fuzzy point, then binary soft fuzzy point which is combination of binary soft point of type $sF^{\times}_{(\mathrm{e}_i,\mathrm{e}_j}$ and fuzzy point type $p^{\times i}_{\alpha}$ we get binary soft fuzzy point of types $fF^{(\mathrm{e}_i,\mathrm{e}_j)}_{p^{\alpha}_{\alpha i}}$

$$sfF^{(\mathrm{e}_i,\mathrm{e}_j)}_{p^{\times i}_{\alpha i}}\left(\mathrm{C}_i\times\mathrm{C}_j\right)=\begin{cases}p^{\times i}_{\alpha i} & \text{if } \mathrm{C}_i\times\mathrm{C}_j=(\mathrm{e}_i,\mathrm{e}_j)\\ \overline{\overline{0}} & \text{if } \mathrm{C}_i\times\mathrm{C}_j\neq(\mathrm{e}_i,\mathrm{e}_j)\end{cases}$$

Example 4.3

$$\mu=\{\times_1,\times_2,\times_3,\times_4,\times_5\},\pounds=\{\mathrm{e}_1,\mathrm{e}_2,\mathrm{e}_3,\mathrm{e}_4\}\ \textit{such that}\ \pounds_1=\{\mathrm{e}_1,\mathrm{e}_2\},\pounds_2=\{\mathrm{e}_3,\mathrm{e}_4\}$$

$$sF^{\times_1,\times_3}_{(\mathrm{e}_1,\mathrm{e}_3)}=\{(\mathrm{e}_1,\mathrm{e}_3),\{\times_1,\times_3\}),((\mathrm{e}_1,\mathrm{e}_4),\{\varnothing\}),((\mathrm{e}_2,\mathrm{e}_3),\{\varnothing\}),((\mathrm{e}_2,\mathrm{e}_4),\{\varnothing\})$$

$p^{\times_{1,3}}_{0.2}=\{(\times_1,0.2),(\times_2,0),(\times_3,0.2),(\times_4,0),(\times_5,0)\}$

$$sfF^{(\mathrm{e}_i,\mathrm{e}_j)}_{p^{\times i}_{\alpha i}}\left(\pounds_i\times\pounds_j\right)$$

$\mathrm{sfF}\left(\mathrm{e}_1,\mathrm{e}_3\right)=\{(\times_1,0.2),(\times_2,0),(\times_3,0.2),(\times_4,0),(\times_5,0)\}=p^{\times_{1,3}}_{0.2}$
$\mathrm{sfF}\Big((\mathrm{e}_1,\mathrm{e}_4)=\{(\times_1,0),(\times_2,0),(\times_3,0),(\times_4,0),(\times_5,0)\}=\overline{\overline{0}};$
$\mathrm{sfF}\left(\mathrm{e}_2,\mathrm{e}_3\right)=\{(\times_1,0),(\times_2,0),(\times_3,0),(\times_4,0),(\times_5,0)\}=\overline{\overline{0}},$
$\mathrm{sfF}\Big((\mathrm{e}_2,\mathrm{e}_4)=\{(\times_1,0),(\times_2,0),(\times_3,0),(\times_4,0),(\times_5,0)\}=\overline{\overline{0}}$
$sfF^{(\mathrm{e}_1,\mathrm{e}_3)}_{p^{1,3}_{0.2}}=\Big\{\Big((\mathrm{e}_1,\mathrm{e}_3),p^{\times_{1,3}}_{0.2}\Big),\Big((\mathrm{e}_1,\mathrm{e}_4),\overline{\overline{0}}\Big),\Big((\mathrm{e}_2,\mathrm{e}_3),\overline{\overline{0}}\Big),\Big((\mathrm{e}_2,\mathrm{e}_4),\overline{\overline{0}}\Big)\Big\}$

4-Binary soft point of type $sF^{\times_i}$ for some point $\times_i\in\mu$ with fuzzy point type p_α, for $\overline{\overline{0}}<\alpha<\overline{\overline{1}}$, binary soft fuzzy point of $sfF_{F_\alpha}\times_i$ such that $\forall(\mathrm{e}_i,\mathrm{e}_j)\in\pounds_i\times\pounds_j, sfF((\mathrm{e}_i,\mathrm{e}_j)=p^{\times i}_{\alpha}$ in this figure 3 in MATLAB depended ons $fF^{(\mathrm{e}_1,\mathrm{e}_3)}_{p^{\times 1,3}_{0.2}}$ of example above

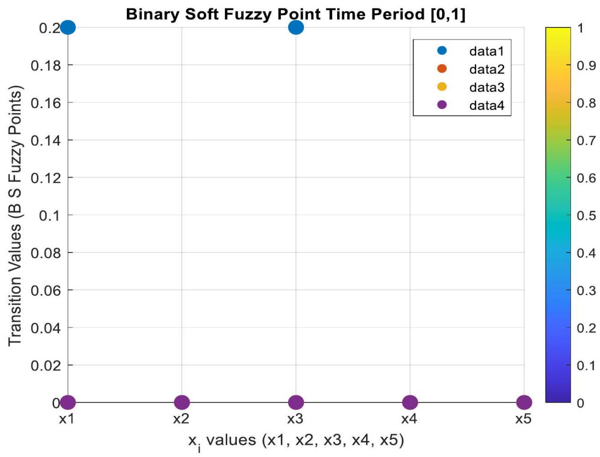

Figure 3: (The third binary soft fuzzy point)

Example 4.4

$$\mu = \{\times_1, \times_2, \times_3, \times_4, \times_5\}, \pounds = \{e_1, e_2, e_3, e_4\} \text{ such that } \pounds_1 = \{e_1, e_2\}, e_2 = \{e_3, e_4\}$$
$$sF^{\times_{1,2}} = \{(e_1, e_3), \{\times_1, \times_2\}), ((e_1, e_4), \{\times_1, \times_2\}), ((e_2, e_3), \{\times_1, \times_2\}), ((e_2, e_4), \{\times_1, \times_2\})\}$$
$$p_{0.2} = \{(\times_1, 0.2), (\times_2, 0.2), (\times_3, 0.2), (\times_4, 0.2), (\times_5, 0.2)\}$$
$$\mathrm{sfF}(e_i, e_j) = f\{\times_i\}(S) = \begin{cases} \alpha & \text{if } S = \times_i \\ \bar{0} & \text{if } S \neq \times_i \end{cases};$$
$$\mathrm{sfF}(e_1, e_3) = f\{\times_1, \times_2\} = \{(\times_1, 0.2), (\times_2, 0.2), (\times_3, 0), (\times_4, 0), (\times_5, 0)\}$$
$$\mathrm{sfF}((e_1, e_4) = f\{\times_1, \times_2\} = \{(\times_1, 0.2), (\times_2, 0.2), (\times_3, 0), (\times_4, 0), (\times_5, 0)\}$$
$$\mathrm{sfF}(e_2, e_3) = \{(\times_1, 0.2), (\times_2, 0.2), (\times_3, 0), (\times_4, 0), (\times_5, 0),$$
$$\mathrm{sfF}((e_2, e_4) = \{(\times_1, 0.2), (\times_2, 0.2), (\times_3, 0), (\times_4, 0), (\times_5, 0)\}$$
$$\mathrm{sfF}\, p_\alpha^{\times_{1,2}} = \left\{(e_1, e_3), p_{0.2}^{\times_{1,2}}\right), \left((e_1, e_4), p_{0.2}^{\times_{1,2}}\right), \left((e_2, e_3), p_{0.2}^{\times_{1,2}}\right), \left((e_2, e_4), p_{0.2}^{\times_{1,2}}\right)\right\}$$

fuzzy point in MATLAB of example above

0	0	0	0.2000	0.2000
0	0	0	0.2000	0.2000
0	0	0	0.2000	0.2000
0	0	0	0.2000	0.2000

Binary soft fuzzy with probability

0	0	0	0.0400	0.0400
0	0	0	0.0400	0.0400
0	0	0	0.0400	0.0400
0	0	0	0.0400	0.0400

In this figure 4 represented in MATLAB by depend on $sfF_{p_\alpha^{x_{1,2}}}$ *of example above binary soft fuzzy distribution with probability of fourth of binary soft fuzzy point)*

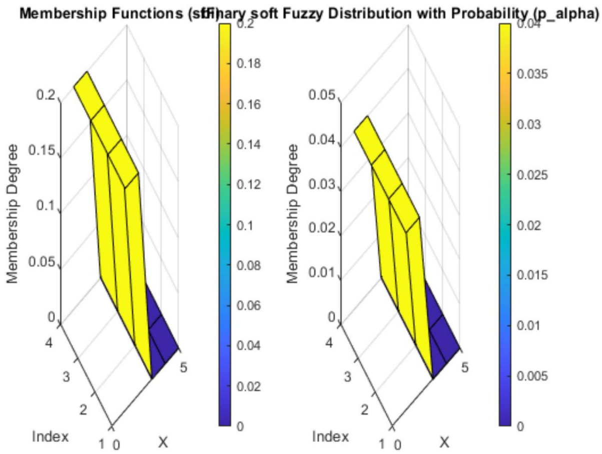

Figure 4: (binary soft fuzzy distribution with probability of fourth of binary soft fuzzy point)

5-Binary soft fuzzy point sF_{e_i,e_j} *for some* $(e_i, e_j) \in \pounds_i \times \pounds_j$ *with fuzzy point of type* p_α *for* $\bar{\bar{0}} < \alpha \leq \bar{\bar{1}}$,

$$\underset{p_\alpha^{F(e_i,e_j)}}{sf\,(e_i, e_j)}\,(c_i \times c_j) = \begin{cases} sp^{F(e_i,e_j)} & \text{if } c_i \times c_j = (e_i, e_j) \\ \bar{\bar{0}} & \text{if } c_i \times c_j \neq (e_i, e_j) \end{cases}$$

$$\text{Where } \underset{p_\alpha^{F(e_i e_j)}}{\text{sf }(e_i e_j)}\,(\times_i) = \begin{cases} \alpha & \text{if } \times_i \in F(e_i, e_j) \\ \bar{\bar{0}} & \text{if } \times_i \notin F(e_i, e_j) \end{cases}$$

Example 4.5

$$\mu = \{\times_1, \times_2, \times_3, \times_4\}, \pounds = \{e_1, e_2, e_3, e_4\} \text{ such that } \pounds_1 = \{e_1, e_2\}, \pounds_2 = \{e_3, e_4\}$$

$p_\alpha = \{(\times_i, \alpha)\, \times_i \in \mu\}$
$sF(e_1, e_3) = \{\{(e_1, e_3), \{X_1, X_3\}), ((e_1, e_4), \{\varnothing\}\}$
$\text{sfF}(e_1, e_3) = f\{\times_1, \times_3\} = \{(\times_1, \alpha), (\times_2, 0), (\times_3, \alpha), (\times_4, \alpha)\}$
$sfF\Big((e_1, e_4) = \{(\times_1, 0), (\times_2, 0), (\times_3, 0), (\times_4, 0)\} = \bar{\bar{0}}$
$\text{sfF}(e_2, e_3) = \{(\times_1, 0), (\times_2, 0), (\times_3, 0), (\times_4, 0)\} = \bar{\bar{0}},$
$\underset{p_\alpha^{F(e_1,e_3)}}{sf(e_1, e_3)} = \Big\{\Big((e_1, e_3), p_\alpha^{F(e_1,e_3)}\Big), \Big((e_1, e_4), \bar{\bar{0}}\Big), \Big((e_2, e_3), \bar{\bar{0}}\Big), \Big((e_2, e_4), \bar{\bar{0}}\Big)\Big\}.$

In this figure 5 represented in MATLAB by depend on $sfF^{(e_1,e_3)}_{p_\alpha^{F(e_1,e_3)}}$ *of example above*

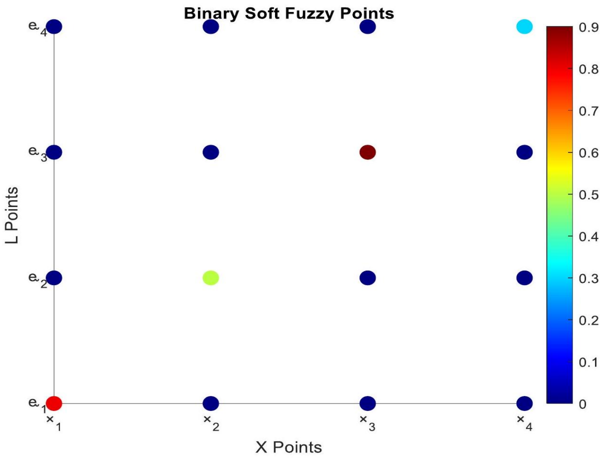

Figure 5: (Binary soft fuzzy point sF_{e_i,e_j})

6-combination Binary soft point $sF^{\times i,j}_{(e_i,e_j)}$ *with fuzzy point* p^A_α *for* $\bar{\bar{0}} < \alpha \leq \bar{\bar{1}}$

1- $\times_i \in A$ *we get binary soft fuzzy point of* $sfF^{(e_i,e_j)}_{p_\alpha^{(e_{i,j})}}$,

2- if $\times_i \notin A$ *we get null binary soft fuzzy set.*

Example 4.6 $\mu = \{\times_1, \times_2, \times_3, \times_4\}, \pounds = \{e_1, e_2, e_3, e_4\}$ *such that* $\pounds_1 = \{e_1, e_2\}, \pounds_2 = \{e_3, e_4\}, A = \{x_1, x_2, x_3\}$
$sF^{\times_1,\times_3}_{(e_1,e_3)} = \{\{(e_1, e_3), \{\times_1, \times_3\}), ((e_1, e_4), \{\varnothing\}), ((e_2, e_3), \{\varnothing\}), ((e_2, e_4), \{\varnothing\})\}$
$p^A_{0.2} = \{(\times_1, 0.2), (\times_2, 0.2), (\times_3, 0.2), (\times_4, 0)\}$, *the combination of* $sF^{\times_1,\times_3}_{(e_1,e_3)}$,
$p^A_{0.2}$ *we get* $sfF^{(e_1,e_3)}_{p_{0.2}^{x_1,x_3}} = \{$
$((e_1, e_3), \{(\times_1, 0.2), (\times_2, 0), (\times_3, 0.2), (\times_4, 0)\},), \left((e_1, e_4), \bar{\bar{0}}\right),$
$\left((e_2, e_3), \bar{\bar{0}}\right), \left((e_2, e_4), \bar{\bar{0}}\right)\Big\}$. *In this figure 6 represented in MATLAB by depend on point* $sfF^{(e_1,e_3)}_{p_{0.2}^{\times 1}\times 3}$ *of example above*

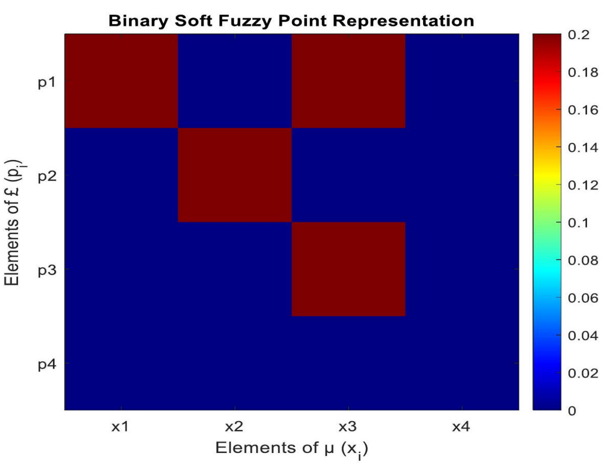

Figure 6: (combination Binary soft point $sF^{\times i,j}_{(e_i,e_j)}$ with fuzzy point p^A_α)

7- Combination Binary soft point $sF^{\times_{i,j}}$ *with fuzzy point* p^A_α *for* $\bar{\bar{0}} < \alpha \leq \bar{\bar{1}}$

1- $\times_i \in A$ *the binary soft fuzzy point of* $sfFp_{\alpha}^{\times_i}$,

2- if $\mathrm{x}_i \notin A$ *the null binary soft fuzzy set.*

Example 4.7 $\mu = \{x_1, x_2, x_3, x_4, x_5\}, \pounds = \{\mathrm{e}_1, \mathrm{e}_2, \mathrm{e}_3, \mathrm{e}_4\}$ *such that* $\pounds_1 = \{\mathrm{e}_1, \mathrm{e}_2\}, \pounds_2 = \{\mathrm{e}_3, \mathrm{e}_4\}, A = \{\times_1, \times_2, \times_3\}, B = \{\times_4, \times_5\}$

$$sF^{\times_1,\times_3} =$$
$$\{(\mathrm{e}_1, \mathrm{e}_3), \{\times_1, \times_3\}), ((\mathrm{e}_1, \mathrm{e}_4), \{\times_1, \times_3\}), ((\mathrm{e}_2, \mathrm{e}_3), \{\times_1, \times_3\}), ((\mathrm{e}_2, \mathrm{e}_4), \{\times_1, \times_3\})\}$$
$$p_{0.2}^{A} = \{(\times_1, 0.2), (\times_2, 0.2), (\times_3, 0.2), (\times_4, 0), (\times_5, 0)\}$$
$$sfF(\mathrm{e}_1, \mathrm{e}_3) = \{(\times_1, 0.2), (\times_2, 0), (\times_3, 0.2), (\times_4, 0), (\times_5, 0)\} = p_{0.2}^{\times_1,\times_3}$$
$$sfF((\mathrm{e}_1, \mathrm{e}_4) = \{(\times_1, 0.2), (\times_2, 0), (\times_3, 0.2), (\times_4, 0), (\times_5, 0)\}$$
$$sfF(\mathrm{e}_2, \mathrm{e}_3) = \{(\times_1, 0.2), (\times_2, 0), (\times_3, 0.2), (\times_4, 0), (\times_5, 0)\}, \text{ since } \times_1, \times_3 \in A$$
$$\text{but } \times_1, \times_3 \notin B$$
$$sfF^{\times_1,\times_3} = fF(\mathrm{e}_1, \mathrm{e}_3) = BfF\Big((\mathrm{e}_1, \mathrm{e}_4) = BfF(\mathrm{e}_2, \mathrm{e}_3) = BfF(\mathrm{e}_2, \mathrm{e}_4) = \overline{\overline{0}}$$

In this figure 7, 8 represented in MATLAB by depend on $sfFp_{\alpha}^{\times_i}$ *of example above*

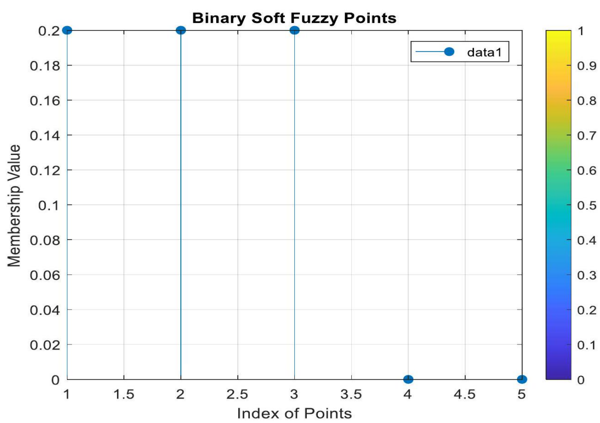

Figure 7: (Combination Binary soft point $sF^{\times}{}_{i,j}$ with fuzzy point p_{α}^{A})

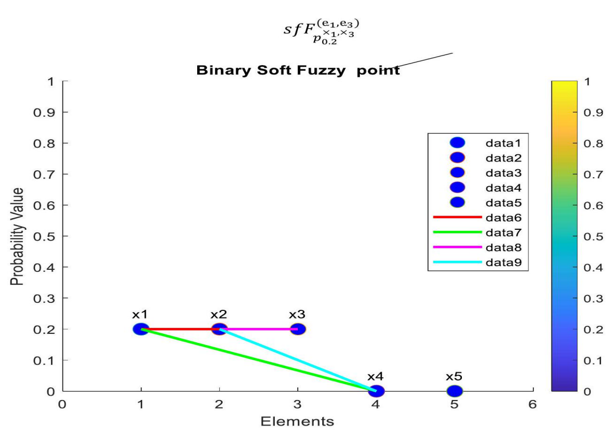

Figure 8: (Combination Binary soft point $sF^{\times}{}_{i,j}$ with fuzzy point p_{α}^{A})

8- combination Binary soft point $sF(\mathrm{e}_i, \mathrm{e}_j)$ *with fuzzy point* p_{α}^{A} *for* $\overline{\overline{0}} < \alpha \leq \overline{\overline{1}}$

1- if $sF(\mathrm{e}_i,\mathrm{e}_j)\sqcap A=\bar{\bar{0}}$ *the combination is binary null soft fuzzy set.*

2- if $sF(\mathrm{e}_i,\mathrm{e}_j)\sqcap A\neq\bar{\bar{0}}$ *the combination binary soft fuzzy point* $sf_{p_\alpha^{F(e_ie_j)\cap A}}(\mathrm{e}_i,\mathrm{e}_j)$

Example 4.8 $\mu=\{x_1,x_2,x_3,x_4,x_5\}$, $\pounds=\{\mathrm{e}_1,\mathrm{e}_2,\mathrm{e}_3,\mathrm{e}_4\}$ *such that* $\pounds_1=\{\mathrm{e}_1,\mathrm{e}_2\}$, $\pounds_2=\{\mathrm{e}_3,\mathrm{e}_4\}$, $A=\{\times_1,\times_2,\times_3\}$
$p_{0.2}^{A}=\{(\times_1,0.2),(\times_2,0.2),(\times_3,0.2),(\times_4,0),(\times_5,0)\}$,
$sF(\mathrm{e}_1,\mathrm{e}_3)=\{\{(\mathrm{e}_1,\mathrm{e}_3),\{\mathrm{X}_1,\mathrm{X}_2\}),((\mathrm{e}_1,\mathrm{e}_4),\{\varnothing\}),\}),((\mathrm{e}_2,\mathrm{e}_3),\{\varnothing\}),\}),((\mathrm{e}_2,\mathrm{e}_4),\{\varnothing\})\}$,
1- Combination $p_{0.2}^{A}, sF(\mathrm{e}_1,\mathrm{e}_3)$ *such that* $sF(\mathrm{e}_1,\mathrm{e}_3)\sqcap A=\{\mathrm{x}_1,\mathrm{x}_2\}$
To find $sF_{p_\alpha^{x_1,x_2}}^{(\mathrm{e}_1,\mathrm{e}_3)}$
$\mathrm{sfF}(\mathrm{e}_1,\mathrm{e}_3)=\{(\times_1,0.2),(\times_2,0.2),(\times_3,0),(\times_4,0),(\times_5,0)\}=p_{0.2}^{\times_1,\times_2}$

$$\mathrm{sfF}\Big((\mathrm{e}_1,\mathrm{e}_4)=\{(\times_1,0),(\times_2,0),(\times_3,0),(\times_4,0),(\times_5,0)\}=\bar{\bar{0}}$$
$$\mathrm{sfF}(\mathrm{e}_2,\mathrm{e}_3)=\{(\times_1,0),(\times_2,0),(\times_3,0),(\times_4,0),(\times_5,0)\}$$

$sfF(\mathrm{e}_2,\mathrm{e}_4)=\{(\times_1,0),(\times_2,0),(\times_3,0),(\times_4,0),(\times_5,0)\}$ *In this figure 9,10 represented in MATLAB by depend on* $sF_{p_\alpha^{1,1,2}}^{(\mathrm{e}_1,\mathrm{e}_3)}$ *of example above*

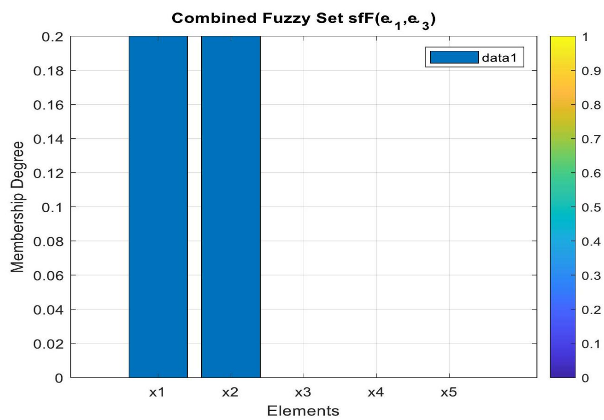

Figure 9: (combination Binary soft point $sF(\mathrm{e}_i,\mathrm{e}_j)$ with fuzzy point p_α^A)

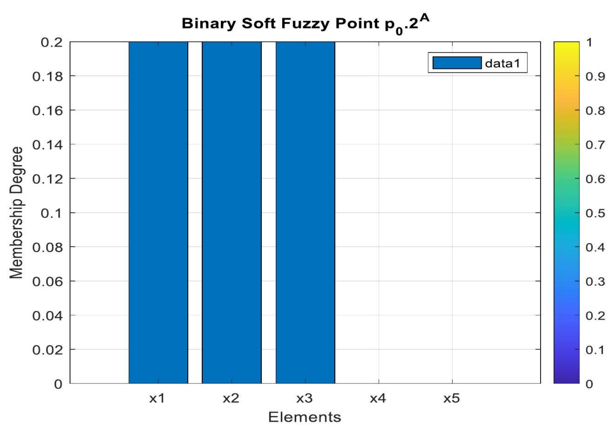

Figure 10: (combination Binary soft point $sF(\mathrm{e}_i,\mathrm{e}_j)$ with fuzzy point $p_{0.2}^{A}$

2- Combination fuzzy point $p_{0.2}^{A}$*, binary soft point* $sF(\mathrm{e}_1, \mathrm{e}_4)$ *such that* $sF(\mathrm{e}_1, \mathrm{e}_3) \sqcap A = \{\times_1, \times_2\}$

$sF(\mathrm{e}_1, \mathrm{e}_4) = \{\{(\mathrm{e}_1, \mathrm{e}_3), \{\varnothing\}), ((\mathrm{e}_1, \mathrm{e}_4), \{\mathrm{x}_4, \mathrm{x}_5\}),\}), ((\mathrm{e}_2, \mathrm{e}_3), \{\varnothing\}),\}), ((\mathrm{e}_2, \mathrm{e}_4), \{\varnothing\})\}$,

To find $sfF_{p_{0.2}^{x_1,x_2}}^{(\mathrm{e}_1,\mathrm{e}_4)}$

$\mathrm{sfF}(\mathrm{e}_1, \mathrm{e}_3) = \{(\times_1, 0), (\times_2, 0), (\times_3, 0), (\times_4, 0), (\times_5, 0)\} = \overline{\overline{0}}$

$\mathrm{sfF}\Big((\mathrm{e}_1, \mathrm{e}_4) = \{(\times_1, 0), (\times_2, 0), (\times_3, 0), (\times_4, 0), (\times_5, 0)\} = \overline{\overline{0}}$

$sfF(\mathrm{e}_2, \mathrm{e}_3) = \{(\times_1, 0), (\times_2, 0), (\times_3, 0), (\times_4, 0), (\times_5, 0)\}$

$sfF(\mathrm{e}_2, \mathrm{e}_4) = \{(\times_1, 0), (\times_2, 0), (\times_3, 0), (\times_4, 0), (\times_5, 0)\}$

Since $sF(\mathrm{e}_1, \mathrm{e}_4) \sqcap A = 0$

so $sfF_{p_{0.2}^{x_1,x_2}}^{(\mathrm{e}_1,\mathrm{e}_3)} = sfF_{p_{0.2}^{f_1,e_3)\sqcap A}}^{(\mathrm{e}_1,\mathrm{e}_3)}$.

5. CONCLUSIONS:

The binary Soft Fuzzy point is a new and promising domain which can lead to the development of new mathematical models that will significantly contribute to the applications in natural sciences such as information systems, decision making problems, biomathematics and others. Application binary soft fuzzy point in MATLAB. The novel structure of binary soft fuzzy-point is initiated in this paper. Some basic notions and concepts have been studied. the purpose of this paper is just to initiate the concept and some basic notions, and there is a lot of scope for the researchers to make their investigations in this field, i.e. this is a beginning of some new binary soft fuzzy structure and the concepts like other kinds of open and closed sets, topology, separation axioms, continuous functions and another main concepts can be studied in this new kind of soft fuzzy point.

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