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Ricci-Bourguignon Soliton on Perfect Fluid Spacetime

Mahima A., Madan M Desai, Somashekhara G., Shivaprasanna G. S., Savithri S. and Deepak A. S.

ABSTRACT: This paper examines perfect fluid spacetimes in the context of Ricci-Bourguignon solitons and η-Ricci-Bourguignon solitons, emphasizing the role of torse-forming vector fields. We derive the conditions for a Ricci-Borguignon soliton to be shrinking, steady, or expanding, by analyzing the associated scalar parameters. Additionally, we investigate perfect fluid spacetimes satisfying the condition $(\xi, \cdot)C \cdot Z = 0$, $(\xi, \cdot)H \cdot Z = 0$.

Keywords: Ricci-Bourguignon soliton, η-Ricci-Bourguignon soliton, quasi conformal curvature tensor, conharmonic curvature tensor.

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1. Introduction

Ricci-Bourguignon Soliton (RBS) was first discovered by Jean Pierre Bourguignon in 1981 [10] and is given by

$$\frac{\partial}{\partial t} g = -2(S - \rho r g), g(0) = g_0, \tag{1.1}$$

where g is the Riemannian metric, S is the Ricci curvature tensor, r is the scalar curvature, $\rho \in \mathbb{R}$.
The self-similar solutions for this flow are called Ricci-Bourguignon solitons. An n-dimensional Riemannian manifold (M, g) is said to admit RBS if

$$(\mathcal{L}_V g)(X, Y) + 2S(X, Y) + 2(\mu + \rho r)g(X, Y) = 0, \tag{1.2}$$

where $X, Y \in \chi(M)$, $\mathcal{L}_V$ is the Lie derivative operator in the direction of vector field V, μ and ρ are constants. An n-dimensional Riemannian manifold is said to admit η-Ricci-Bourguignon soliton (η-RBS) if

$$(\mathcal{L}_V g)(X, Y) + 2S(X, Y) + 2(\mu + \rho r)g(X, Y) + 2\omega\eta(X)\eta(Y) = 0, \tag{1.3}$$

where ω is a constant.
If $\omega = 0$, η-RBS reduces to RBS.
RBS is said to be shrinking, steady or expanding according to $\mu < 0$, $\mu = 0$ or $\mu > 0$ respectively.
The stress-energy tensor or the energy-momentum tensor plays a vital role in defining the matter content of the spacetime. It describes the density, energy flux and momentum in spacetime. A Perfect Fluid

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Spacetime (P.F.S) behaves like a perfect fluid or an ideal fluid that can be completely characterized by its rest frame mass density σ and isotropic pressure p_i.
Stress-energy tensor is given by [12]

$$T(X,Y) = p_i g(X,Y) + (\sigma + p_i)\eta(X)\eta(Y), \tag{1.4}$$

for any $X,Y \in \chi(M)$, $g(X,\xi) = \eta(X)$ is the 1-form and $g(\xi,\xi) = -1$.
Einstein's Field Equation describes how the curvature of the spacetime changes by the influence of matter and energy. It is given by [12]

$$S(X,Y) + \left(\lambda - \frac{r}{2}\right) g(X,Y) = kT(X,Y), \tag{1.5}$$

where λ is the cosmological constant, $k = 8\pi G$, G being the universal gravitational constant.
Substituting (1.4) into (1.5), we obtain

$$S(X,Y) = \left(kp_i - \lambda + \frac{r}{2}\right) g(X,Y) + k(\sigma + p_i)\eta(X)\eta(Y). \tag{1.6}$$

2. Preliminares

The (P.F.S) (M,g) satisfies (1.6). On contraction of (1.6), we get scalar curvature as

$$r = 4\lambda + k(\sigma - 3p_i). \tag{2.1}$$

Therefore, (1.6) becomes

$$S(X,Y) = \left[\lambda + \frac{k(\sigma - p_i)}{2}\right] g(X,Y) + k(\sigma + p_i)\eta(X)\eta(Y), \tag{2.2}$$

$$QX = \left[\lambda + \frac{k(\sigma - p_i)}{2}\right] X + k(\sigma + p_i)\eta(X)\xi, \tag{2.3}$$

where Q is a Ricci operator defined as $g(QX,Y) = S(X,Y)$.
If the unit timelike vector field ξ of the (P.F.S) satisfies

$$\nabla_X \xi = X + \eta(X)\xi, \tag{2.4}$$

then ξ is called torse-forming vector field.
From (2.4), for $X,Y,Z \in \chi(M)$ we can deduce the following

$$\nabla_\xi \xi = 0, \tag{2.5}$$

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y, \tag{2.6}$$

$$\eta(R(X,Y)Z) = \eta(X)g(Y,Z) - \eta(Y)g(X,Z), \tag{2.7}$$

where R is the curvature tensor of the manifold.

3. RBS on (P.F.S) with torse-forming vector field

On replacing V by ξ and substituting (2.2) in (1.2), we get

$$\left[1 + \lambda + \frac{k(\sigma + p_i)}{2} + \mu + \rho r\right] g(X,Y) + [1 + k(\sigma + p_i)]\,\eta(X)\eta(Y) = 0. \tag{3.1}$$

Taking $X = Y = \xi$ in the above equation, we get

$$\mu = k\left[\frac{1}{2}(\sigma + p_i) - \rho(\sigma - p_i)\right] - \lambda(1 + 4\rho). \tag{3.2}$$

Therefore, we can state the following theorem.

Theorem 3.1 *Let (P.F.S) admitting the RBS with the torse-forming vector field ξ. Then, the soliton is said to be shrinking or steady or expanding as* $k\left[\frac{1}{2}(\sigma+p_i)-\rho(\sigma-p_i)-\lambda(1+4\rho)\right] < 0$ *or* $k\left[\frac{1}{2}(\sigma+p_i)-\rho(\sigma-p_i)-\lambda(1+4\rho)\right] = 0$ *or* $k\left[\frac{1}{2}(\sigma+p_i)-\rho(\sigma-p_i)-\lambda(1+4\rho)\right] > 0$.

If

$$(\mathcal{L}_V g)(X,Y) = 2\Phi g(X,Y), \tag{3.3}$$

then V is called the conformal Killing vector field.
V is called proper if Φ is a non-constant. V is called a homothetic vector field if $\Phi \neq 0$ is a constant.
Consider V to be the conformal Killing vector field in (1.2). Then from (1.2) and (3.3) we get

$$S(X,Y) = -[\Phi+\mu+\rho r]g(X,Y). \tag{3.4}$$

Therefore, the spacetime is Einstein.
Conversely, if the spacetime is Einstein (i.e.,$S(X,Y)=\theta g(X,Y)$) then (1.2) is reduced to

$$(\mathcal{L}_V g)(X,Y) = 2\Psi g(X,Y), \tag{3.5}$$

where $\Psi = -(\theta-\mu-\rho r)$.
As a result, we can state that

Theorem 3.2 *The potential vector field V in (P.F.S) admitting RBS is conformal Killing vector field if and only if the spacetime is Einstein.*

By virtue of (2.2) and (3.4) we obtain

$$\left[\Phi+\mu+\rho r+\lambda+\frac{k}{2}(\sigma-p_i)\right]g(X,Y)+k(\sigma+p_i)\eta(X)\eta(Y)=0. \tag{3.6}$$

Taking $Y=\xi$, we get

$$\left[\Phi+\mu+\rho r+\lambda+\frac{k}{2}(\sigma-p_i)-k(\sigma+p_i)\right]\eta(X)=0. \tag{3.7}$$

Since, 1-form $\eta(X)\neq 0$, we have

$$\Phi = kp_i(1+2\rho). \tag{3.8}$$

We obtain Φ as a constant. Hence, we can state the theorem as follows:

Theorem 3.3 *If (PFS) admits RBS with the torse-forming vector field and the potential vector field V be the conformal Killing vector field, then V is the homothetic vector field.*

4. η-RBS on (P.F.S)

Let (M^4,g) be (P.F.S) and (g,μ,ω,ξ) be the $\eta-RBS$ on M^4. Then by the equations (2.1),(2.2) and (1.3) we have

$$\frac{1}{2}(\mathcal{L}_V g)(X,Y)+\left\{\lambda+\frac{k(\sigma-p_i)}{2}+\mu+\rho[4\lambda+k(\sigma-3p_i)]\right\}g(X,Y)+[k(\sigma+p_i)+\omega]\eta(X)\eta(Y)=0. \tag{4.1}$$

Let $e_i (1\leq i\leq 4)$ be an orthonormal frame field. Therefore, $\xi=\sum_{i=1}^{4}\xi^i e_i$ and from [2], $\sum_{i=1}^{4}\epsilon_{ii}(\xi^i)^2 = -1$ and $\eta(e_i)=\epsilon_{ii}\xi^i$. Multiply (4.1) by ϵ_{ii} and sum over i for $X=Y=e_i$ to get

$$4\mu-\omega = (-4\lambda-k\sigma+3kp_i)(4\rho+1)-div(\xi). \tag{4.2}$$

Lie derivative in the direction of V is given by

$$(\mathcal{L}_V g)(X,Y) = \frac{1}{2}[g(\nabla_X V,Y)+g(X,\nabla_Y V)]. \tag{4.3}$$

Using (1.3),(4.1) and (2.2), we obtain

$$\mu - \omega = \lambda(1+4\rho) + k\sigma(1-\rho) + 3kp_i\left(\frac{1}{2}+\rho\right). \tag{4.4}$$

Solve (4.2) and (4.4) simultaneously to get

$$\mu = \frac{1}{4}(-4\lambda - k\sigma + 3kp_i)(4\rho+1) + \frac{k}{12}(5\sigma+3p_i) - \frac{div(\xi)}{12}, \tag{4.5}$$

$$\omega = -\frac{k}{3}(5\sigma+3p_i) - \frac{div(\xi)}{3}. \tag{4.6}$$

Hence, we conclude that

Theorem 4.1 *Let (M^4, g) be a Riemannian manifold and η be the g-dual 1-form of the gradient vector field $\xi = grad(f)$, with $g(\xi,\xi) = -1$, where f is a smooth function. If (g, μ, ω, ξ) is the η-RBS on M^4, then Laplacian equation satisfied by f in $\eta-RBS$ becomes $\Delta(f) = 12\mu - 3(-4\lambda - k\sigma + 3kp_i)(4\rho+1) + k(5\sigma+3p_i)$.*

Definition 1 *[5] studied the Z-tensor which is given as*

$$Z(X,Y) = S(X,Y) + \phi g(X,Y), \tag{4.7}$$

where ϕ is arbitrary scalar function.

5. (P.F.S) satisfying $(\xi, \cdot)C \cdot Z = 0$

A Quasi Conformal curvature tensor C on a differentiable manifold (M^4, g) with Riemannian metric g is given by

$$C(X,Y)Z = aR(X,Y)Z + b\left[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\right] - \frac{r}{4}\left(\frac{a}{3}+2b\right)[g(Y,Z)X - g(X,Z)Y], \tag{5.1}$$

where $a, b \in \mathbb{R}$. [6], In (P.F.S) the condition $(\xi, \cdot)C \cdot Z = 0$ is equivalent to

$$Z(C(\xi,X)Y,\xi) + Z(Y, C(\xi,X)\xi) = 0. \tag{5.2}$$

By equations (4.7) and (5.2), we get

$$S(C(\xi,X)Y,\xi) + S(X, C(\xi,X)\xi) + \phi[\eta(C(\xi,X)Y) + g(Y, C(\xi,X)\xi)] = 0. \tag{5.3}$$

By virtue of (2.2),(2.6) and (2.7), we obtain

$$k(\sigma+p_i)\left\{a + 2b\left[\lambda + \frac{k(\sigma-p_i)}{2}\right] - b[k(\sigma+p_i)] - \frac{r}{4}\left(\frac{a}{3}+2b\right)\right\}[g(X,Y) + \eta(X)\eta(Y)] = 0. \tag{5.4}$$

Since $[g(X,Y) + \eta(X)\eta(Y)] \neq 0$, we have

$$p_i = \frac{\sigma(a+6b)}{(a-18b)} - \frac{4a(3-\lambda)}{k(a-18b)}. \tag{5.5}$$

The theorem can be stated as

Theorem 5.1 *Let (M^4, g) be a (P.F.S) which satisfies $(\xi, \cdot)C \cdot Z = 0$ where C is Quasi conformal curvature tensor and ξ is torse-forming vector field. Then*

$$p_i = \frac{\sigma(a+6b)}{(a-18b)} - \frac{4a(3-\lambda)}{k(a-18b)}.$$

6. (P.F.S) satisfying $(\xi,\cdot)H\cdot Z=0$

A Conharmonic curvature tensor H on a differentiable manifold (M^4,g) with Riemannian metric g is given by

$$H(X,Y)Z=R(X,Y)Z-\frac{1}{2}\left[S(Y,Z)X-S(X,Z)Y+g(Y,Z)QX-g(X,Z)QY\right]. \tag{6.1}$$

In (P.F.S) the condition $(\xi,\cdot)C\cdot Z=0$ is equivalent to

$$Z(H(\xi,X)Y,\xi)+Z(Y,H(\xi,X)\xi)=0. \tag{6.2}$$

By equations (4.7) and (6.2), we get

$$S(H(\xi,X)Y,\xi)+S(X,H(\xi,X)\xi)+\phi[\eta(H(\xi,X)Y)+g(Y,H(\xi,X)\xi)]=0. \tag{6.3}$$

By virtue of (2.2),(2.6) and (2.7), we have

$$k(\sigma+p_i)\left\{1-\frac{1}{2}\left[\lambda+\frac{k(\sigma-p_i)}{2}\right]+\frac{k(\sigma+p_i)}{4}\right\}[g(X,Y)+\eta(X)\eta(Y)]=0. \tag{6.4}$$

Since $[g(X,Y)+\eta(X)\eta(Y)]\neq 0$, we obtain

$$p_i=\frac{\lambda-2}{k}. \tag{6.5}$$

The theorem can be stated as

Theorem 6.1 *Let (M^4,g) be (P.F.S) which satisfies $(\xi,\cdot)H\cdot Z=0$ where H is Conharmonic curvature tensor and ξ is torse-forming vector field. Then*

$$p_i=\frac{\lambda-2}{k}.$$

7. (P.F.S) satisfying $(\xi,\cdot)Z\cdot C=0$

In (P.F.S), the condition $(\xi,\cdot)Z\cdot C=0$ is equivalent to

$$\begin{aligned}&Z(X_1,C(X_2,X_3)X_4)\xi-Z(\xi,C(X_2,X_3)X_4)X_1\\&+Z(X_1,X_2)C(\xi,X_3)X_4-Z(\xi,X_2)C(X_1,X_3)X_4\\&+Z(X_1,X_3)C(X_2,\xi)X_4-Z(\xi,X_3)C(X_2,X_1)X_4\\&+Z(X_1,X_4)C(X_2,X_3)\xi-Z(\xi,X_4)C(X_2,X_3)X_1=0,\end{aligned} \tag{7.1}$$

where $X_1,X_2,X_3,X_4\in\chi(M)$.
Contracting the above equation to ξ, we have

$$\begin{aligned}&-Z(X_1,C(X_2,X_3)X_4)-Z(\xi,C(X_2,X_3)X_4)\eta(X_1)\\&+Z(X_1,X_2)\eta(C(\xi,X_3)X_4)-Z(\xi,X_2)\eta(C(X_1,X_3)X_4)\\&+Z(X_1,X_3)\eta(C(X_2,\xi)X_4)-Z(\xi,X_3)\eta(C(X_2,X_1)X_4)\\&+Z(X_1,X_4)\eta(C(X_2,X_3)\xi)-Z(\xi,X_4)\eta(C(X_2,X_3)X_1)=0.\end{aligned} \tag{7.2}$$

Taking $X_3=X_4=\xi$ and by the virtue of (2.2) (2.3),(2.6),(2.7), (4.7),(5.1) and (7.2), we get

$$[g(X_1,X_2)+\eta(X_1)\eta(X_2)][-a-2bL+bK+R][-L-\phi]=0, \tag{7.3}$$

where $L=\left[\lambda+\frac{k(\sigma-p_i)}{2}\right]$, $K=k(\sigma+p_i)$ and $R=\frac{r}{4}\left(\frac{a}{3}+2b\right)$.
Since $[g(X_1,X_2)+\eta(X_1)\eta(X_2)]\neq 0$, we get

$$p_i=\frac{4a+4\lambda(2b-1)-k\sigma}{8b-3}.$$

Therefore, we can state the following theorem as

Theorem 7.1 *Let (M^4,g) be a (P.F.S) satisfying $(\xi,\cdot)Z\cdot C=0$ with the torse-forming vector field ξ. Then $p_i=\frac{4a+4\lambda(2b-1)-k\sigma}{8b-3}$.*

8. (P.F.S) satisfying $(\xi,\cdot)Z\cdot H=0$

In (P.F.S), the condition $(\xi,\cdot)Z\cdot H=0$ is equivalent to

$$\begin{aligned} &Z(X_1,H(X_2,X_3)X_4)\xi - Z(\xi,H(X_2,X_3)X_4)X_1\\ &+Z(X_1,X_2)H(\xi,X_3)X_4 - Z(\xi,X_2)H(X_1,X_3)X_4\\ &+Z(X_1,X_3)H(X_2,\xi)X_4 - Z(\xi,X_3)H(X_2,X_1)X_4\\ &+Z(X_1,X_4)H(X_2,X_3)\xi - Z(\xi,X_4)H(X_2,X_3)X_1=0, \end{aligned} \tag{8.1}$$

where $X_1,X_2,X_3,X_4\in\chi(M)$.
Contracting the above equation to ξ, we have

$$\begin{aligned} &-Z(X_1,H(X_2,X_3)X_4) - Z(\xi,H(X_2,X_3)X_4)\eta(X_1)\\ &+Z(X_1,X_2)\eta(H(\xi,X_3)X_4) - Z(\xi,X_2)\eta(H(X_1,X_3)X_4)\\ &+Z(X_1,X_3)\eta(H(X_2,\xi)X_4) - Z(\xi,X_3)\eta(H(X_2,X_1)X_4)\\ &+Z(X_1,X_4)\eta(H(X_2,X_3)\xi) - Z(\xi,X_4)\eta(H(X_2,X_3)X_1)=0. \end{aligned} \tag{8.2}$$

Taking $X_3=X_4=\xi$ and by the virtue of (2.2) (2.3),(2.6),(2.7), (4.7),(6.1) and (8.2), we get

$$[g(X_1,X_2)+\eta(X_1)\eta(X_2)]\left[-1-L+\frac{K}{2}\right][-L-\phi]=0, \tag{8.3}$$

where $L=\left[\lambda+\frac{k(\sigma-p_i)}{2}\right]$, $K=k(\sigma+p_i)$ and $R=\frac{r}{4}\left(\frac{a}{3}+2b\right)$.
Since $[g(X_1,X_2)+\eta(X_1)\eta(X_2)]\neq 0$, we get

$$p_i=4-k\sigma.$$

Therefore, we can state the following theorem as

Theorem 8.1 *Let (M^4,g) be a (P.F.S) satisfying $(\xi,\cdot)Z\cdot H=0$ with the torse-forming vector field ξ. Then $p_i=4-k\sigma$.*

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Mahima A., Somashekhara G. and Madan M Desai,

Department of Mathematics and Statistics,

M.S.Ramaiah University of Applied Sciences,

Bengaluru, India.

E-mail address: mahimamahi20012003@gmail.com, somashekhara.mt.ns@msruas.ac.in, madan.mp@msruas.ac.in

ORCID: http://orcid.org/0009-0001-9188-3405,

http://orcid.org/0000-0002-9069-8814,http://orcid.org/0009-0004-8661-0244

and

Shivaprasanna G. S.,

Department of Mathematics,

Dr.Ambedkar institute of technology,

Bengaluru, India.

E-mail address: shivaprasanna28@gmail.com

ORCID: http://orcid.org/0000-0002-9962-5216

and

Savithri S.,

Department of Mathematics,

R.V.College of Engineering,

Bengaluru, India.

E-mail address: savithrishashidhar@rvce.edu.in

ORCID: http://orcid.org/0000-0003-1501-4416

and

Deepak A. S.,

T A Pai Management Institute, Bengaluru,

Manipal Academy of Higher Education,

Manipal, India.

E-mail address: deepak.as@manipal.edu

ORCID: http://orcid.org/0000-0001-5890-1512