



Common Random Fixed Point of Set Valued Mapping on Convergence of Ishikawa Iteration on a Convex Metric Space

Amit Kumar Laha¹ and Ramakant Bhardwaj²

ABSTRACT: In this paper we have shown common random fixed point for a class of multivalued mapping over a convex metric space by the random Ishikawa iteration scheme has been used for the convergence of a sequence of random mapping to a common random fixed point.

Keywords: Set valued mapping, Ishikawa iteration, fixed point.

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1. Introduction

Recently, many researchers have successfully established the convergence of sequences generated by Mann iterates [2], [3].

To establish the strong convergence of a pseudo-contractive self-mapping on a convex and compact subset of a Hilbert space, the Ishikawa iteration scheme has been employed. In recent years, both the Mann and Ishikawa iteration schemes have been used to obtain strong convergence of the corresponding iterates for various contractive-type mappings in Hilbert spaces, and even in general normed linear spaces. In 2003 Ćirić et. al. had generalized the results of Naimpally and Singh [1] to a pair of self mappings S and T defined on a convex metric space (X, d) which satisfies the following condition

$$d(Sx, Ty) \leq h[d(x, y) + d(x, Tx) + d(y, Sx)] \tag{1.1}$$

where $0 < h < 1$. When $h = 1$, condition (1.1) always satisfies.

2. Preliminaries

Before we present our main results, we will introduce some basic preliminaries. Let (Ω, Σ) be a measurable space, (X, d) be a metric space, 2^X be the family of all subsets of X . $CB(X)$ be the family of all subsets of X . Let A be a subset of X .

Definition 2.1 A mapping $f : \Omega \rightarrow A$ is said to be measurable if $f^{-1}(B \cap A) \in \Sigma$ for every Borel subset B of X .

Definition 2.2 A mapping $f : \Omega \times X \rightarrow X$ is called a random operator if for any $x \in X$, $f(., x)$ is measurable.

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Definition 2.3 A measurable mapping $g : \Omega \rightarrow A$ is said to be a random fixed point of the random mapping $f : \Omega \times X \rightarrow X$ if $f(\omega, g(\omega)) = g(\omega)$ for all $\omega \in \Omega$.

Definition 2.4 A random mapping $f : \Omega \times X \rightarrow X$ is said to be continuous if for fixed $\omega \in \Omega$, $f(\omega, \cdot) : \Omega \times X \rightarrow X$ is continuous.

Definition 2.5 A mapping $T : \Omega \rightarrow 2^X$ is said to be measurable if for any open subset C of X ,

$$T^{-1}(C) = \{\omega \in \Omega : T(\omega) \cap C \neq \Phi\} \in \Sigma.$$

Definition 2.6 A mapping $\xi : \Omega \rightarrow X$ is said to be a measurable selector of T if ξ is measurable and for any $\omega \in \Omega$, $\xi(\omega) \in T(\omega)$.

Definition 2.7 A mapping $T : \Omega \times X \rightarrow CB(X)$ is called a random multivalued operator if for any $x \in X$, $T(\cdot, x)$ is measurable.

Definition 2.8 A measurable mapping $\xi : \Omega \rightarrow X$ is called a random fixed point of random multivalued operator $T : \Omega \times X \rightarrow CB(X)$ (or random single valued mapping $f : \Omega \times X \rightarrow X$) if for any $\omega \in \Omega$, $\xi(\omega) \in T(\omega, \xi(\omega))$ ($\xi(\omega) = f(\omega, \xi(\omega))$).

Definition 2.9 A random operator $T : \Omega \times X \rightarrow CB(X)$ is called Lipschitzian if $H(T(\omega, x), T(\omega, y)) \leq L(\omega)d(x, y)$, for any $x, y \in X$ and $\omega \in \Omega$ where, $L : \Omega \rightarrow [0, \infty)$ is a measurable map and H is the Hausdorff metric on $CB(X)$, induced by the metric d . When $L(\omega) < 1$ [$L(\omega) = 1$] for each $\omega \in \Omega$, then T is called contraction (non-expansive).

Definition 2.10 A random operator $T : \Omega \times X \rightarrow X$ is said to be a Banach operator if there exists a measurable map $\beta : \Omega \rightarrow [0, 1)$ and for each $x \in X$ and $\omega \in \Omega$

$$d(T(\omega, T(\omega, x)), T(\omega, x)) \leq \beta(\omega) \times d(T(\omega, x), x)$$

Definition 2.11 A continuous operator $W : X \times X \times I \rightarrow X$ is said to be a convex structure on X if for all $x, y, u \in X$, $\lambda \in I$,

$$d(u, W(x, y, \lambda)) \leq \lambda d(u, x) + (1 - \lambda)d(u, y)$$

where $I = [0, 1]$.

Definition 2.12 A metric space X with convex structure is called a convex metric space. Banach space and each of its convex subsets are simple examples of convex metric spaces.

A Banach space or any convex subset of it is a convex metric space with $W(x, y, \lambda) = \lambda x + (1 - \lambda)y$, $0 \leq \lambda \leq 1$.

More generally if X is a linear space with a translation invariant metric space satisfying

$$d(\lambda x + (1 - \lambda)y, 0) \leq \lambda d(x, 0) + (1 - \lambda)d(y, 0), 0 \leq \lambda \leq 1$$

then X is a convex metric space. It also follows that

$$\begin{aligned} d(x, W(x, y, \lambda)) &= (1 - \lambda)d(x, y) \\ d(y, W(x, y, \lambda)) &= \lambda d(x, y) \end{aligned} \tag{2.1}$$

In a metric space (X, d) , denote

$$\begin{aligned} P(X) &= \{A \in \mathcal{P}(X) : A \neq \Phi\}; P_b(X) = \{A \in P(X) : A \text{ is bounded}\}; \\ P_{cl}(X) &= \{A \in P(X) : A \text{ is closed}\}; P_{cp}(X) = \{A \in P(X) : A \text{ is compact}\}. \end{aligned}$$

Let $D : P(X) \times P(X) \rightarrow \mathbb{R}^+ \cup \{\infty\}$ defined as $D(A, B) = \inf\{d(a, b) : a \in A, b \in B\}$. The Pompeiu-Hausdorff ([6]) functional

$$H : P(X) \times P(X) \rightarrow \mathbb{R}^+ \cup \{\infty\},$$

is defined by

$$H(A, B) = \max\{\sup_{a \in A} D(a, B), \sup_{b \in B} D(A, b)\}.$$

The generalized diameter functional δ generated by d is defined by

$$\delta : P(X) \times P(X) \rightarrow \mathbb{R}^+ \cup \{\infty\},$$

$$\delta(A, B) = \sup\{d(a, b) : a \in A, b \in B\}.$$

In a metric space (X, d) and let $T : X \rightarrow P(X)$ be a multivalued operator, then $x \in X$ is said to a fixed point for T iff $x \in Tx$ and the set $Fix(T) = \{x \in X : x \in Tx\}$ is called fixed point set of T and $SFix(T) = \{x \in X : \{x\} = Tx\}$ is called strict fixed point set of T .

Here we will use S and T as random multivalued operator written as $S, T : \Omega \times X \rightarrow P_b(X)$ and defined as $S(\omega, x) = A$, where $\omega \in \Omega$, $x \in X$ and $A \subset P_b(X)$.

Definition 2.13 Let $S, T : \Omega \times C \rightarrow C$, where C is a non empty convex subset of a separable Banach space X , be two random operators. Let $g_0 : \Omega \rightarrow C$ be any measurable function. The random Ishikawa iteration scheme is defined as the following sequence of functions:

$$\begin{aligned} g_{n+1}(\omega) &= (1 - \alpha_n)g_n(\omega) + \alpha_n T(\omega, h_n(\omega)), \quad \text{where} \\ h_n(\omega) &= (1 - \beta_n)g_n(\omega) + \beta_n S(\omega, g_n(\omega)) \end{aligned} \tag{2.2}$$

Where $\alpha_n, \beta_n \in [\delta, 1 - \delta]$ for some $\delta \in (0, \frac{1}{2})$.

Let $g_1 : \Omega \rightarrow C$ be any arbitrary measurable map and $\{g_n\}$ and $\{h_n\}$ are sequences of maps from Ω to C .

3. Main Result

Theorem 3.1 Let (X, d) be linear metric space endowed with a convex structure and C be the non empty closed convex subset of X . Let $S, T : \Omega \times C \rightarrow C$ satisfies

$$d(S(\omega, x), T(\omega, y)) \leq h(\omega)[d(x, y) + d(x, T(\omega, y)) + d(S(\omega, x), y)]$$

for all $x, y \in C$, $\omega \in \Omega$, where $h(\omega)$ is a real valued random variable such that $0 < h(\omega) < 1$. Suppose that $\{g_n\}$ is random iteration scheme as defined in the Definition(2.13) is convergent to some point $p(\omega) \in C$, then $p(\omega)$ is the common random fixed point of S and T .

Proof: From the definition of convex structure of d we get,

$$\begin{aligned} d(x, W(x, y, \lambda)) &= (1 - \lambda)d(x, y) \\ d(y, W(x, y, \lambda)) &= \lambda d(x, y) \end{aligned} \tag{3.1}$$

So from (3.40) we get for $\omega \in \Omega$,

$$\begin{aligned} &d(g_n(\omega), g_{n+1}(\omega)) \\ &= d(g_n(\omega), (1 - \alpha_n)g_n(\omega) + \alpha_n T(\omega, h_n(\omega))) \\ &= \alpha_n d(g_n(\omega), T(\omega, h_n(\omega))). \end{aligned} \tag{3.2}$$

For $\omega \in \Omega$, since $\{g_n\} \rightarrow p(\omega)$ as $n \rightarrow \infty$, then we get $\lim_{n \rightarrow \infty} d(g_n(\omega), g_{n+1}(\omega)) = 0$, as α_n is bounded away from zero so we have for $\omega \in \Omega$,

$$\lim_{n \rightarrow \infty} d(g_n(\omega), T(\omega, h_n(\omega))) = 0. \tag{3.3}$$

Again,

$$\begin{aligned} &d(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) \\ &\leq h(\omega)[d(g_n(\omega), h_n(\omega)) + d(g_n(\omega), T(\omega, h_n(\omega))) + d(h_n(\omega), S(\omega, g_n(\omega)))]. \end{aligned} \tag{3.4}$$

From the Definition(2.13) we have,

$$\begin{aligned} & d(g_n(\omega), h_n(\omega)) \\ &= d(g_n(\omega), (1 - \beta_n)g_n(\omega) + \beta_n S(\omega, g_n(\omega))) \\ &= \beta_n d(g_n(\omega), S(\omega, g_n(\omega))). \end{aligned} \quad (3.5)$$

Again we have that,

$$\begin{aligned} & d(h_n(\omega), S(\omega, g_n(\omega))) \\ &= d((1 - \beta_n)g_n(\omega) + \beta_n S(\omega, g_n(\omega)), S(\omega, g_n(\omega))) \\ &= (1 - \beta_n)d(g_n(\omega), S(\omega, g_n(\omega))). \end{aligned} \quad (3.6)$$

Using (3.5) and (3.6) in (3.4) we obtain that,

$$\begin{aligned} & d(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) \\ &\leq h(\omega)[d(g_n(\omega), T(\omega, h_n(\omega))) + d(g_n(\omega), S(\omega, g_n(\omega)))]. \end{aligned} \quad (3.7)$$

Also,

$$\begin{aligned} & d(g_n(\omega), S(\omega, g_n(\omega))) \\ &\leq d(g_n(\omega), T(\omega, h_n(\omega))) + d(T(\omega, h_n(\omega)), S(\omega, g_n(\omega))) \end{aligned} \quad (3.8)$$

Hence by (3.8), we get from (3.7) that,

$$\begin{aligned} & d(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) \\ &\leq h(\omega)[d(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) + 2d(g_n(\omega), T(\omega, h_n(\omega)))]. \end{aligned} \quad (3.9)$$

So we get that,

$$\begin{aligned} & d(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) \\ &\leq \frac{2h(\omega)}{1 - h(\omega)} d(g_n(\omega), T(\omega, h_n(\omega))). \end{aligned} \quad (3.10)$$

Hence,

$$\lim_{n \rightarrow \infty} d(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) = 0, \quad (3.11)$$

Since, $\lim_{n \rightarrow \infty} d(g_n(\omega), T(\omega, h_n(\omega))) = 0$.

For $\omega \in \Omega$, and since $\lim_{n \rightarrow \infty} g_n(\omega) = p(\omega)$ we get,

$$\lim_{n \rightarrow \infty} T(\omega, h_n(\omega)) = p(\omega) \quad (3.12)$$

Since $\lim_{n \rightarrow \infty} T(\omega, h_n(\omega)) = p(\omega)$ so from (3.10) we see that, $\lim_{n \rightarrow \infty} S(\omega, g_n(\omega)) = p(\omega)$.

Again from (3.5) we have,

$$\begin{aligned} & \lim_{n \rightarrow \infty} d(g_n(\omega), h_n(\omega)) \\ &= \lim_{n \rightarrow \infty} \beta_n d(g_n(\omega), S(\omega, g_n(\omega))) = 0. \end{aligned} \quad (3.13)$$

So, $\lim_{n \rightarrow \infty} h_n(\omega) = p(\omega)$.

But,

$$\begin{aligned} & d(S(\omega, g_n(\omega)), T(\omega, p(\omega))) \\ &\leq h(\omega)[d(g_n(\omega), p(\omega)) + d(g_n(\omega), T(\omega, p(\omega))) + d(p(\omega), S(\omega, g_n(\omega)))]. \end{aligned} \quad (3.14)$$

Taking limit as $n \rightarrow \infty$ and since $0 < h(\omega) < 1$, we get for all $\omega \in \Omega$, $T(\omega, p(\omega)) = p(\omega)$. Similarly we get $S(\omega, p(\omega)) = p(\omega)$. So, $p(\omega) = S(\omega, p(\omega)) = T(\omega, p(\omega))$ i.e. S and T have common random fixed point $p(\omega)$. \square

We will show a supporting example against the Theorem(3.1).

Example 3.1 Let $X = (0, 1)$ be a subset of \mathbb{R} with usual metric of reals, define $S, T : \Omega \times X \rightarrow X$ by,

$$S(\omega, x) = \frac{x}{4} \quad (3.15)$$

and

$$T(\omega, x) = \frac{x}{3} \quad (3.16)$$

By routine verification we see all the condition that Theorem (3.1) are satisfied. Again random Ishikawa iteration scheme be defined for a sequence of functions $g_n(\omega) = 0$, $n \geq 1$, if we assume $g_n(\omega) = 0$ with constant $\alpha_n = \frac{1}{2}$ and $\beta_n = \frac{1}{2}$. Then we see that S and T have common random fixed point $p(\omega) = 0$.

3.1. Result on multivalued mapping

Let S and T are two random multivalued mapping.

Theorem 3.2 Let X be linear random convex metric space and C be the non empty closed convex subset of X . Let $S, T : \Omega \times C \rightarrow P_b(C)$ satisfies

$$\delta(S(\omega, x), T(\omega, y)) \leq h(\omega)[d(x, y) + \delta(x, T(\omega, y)) + \delta(S(\omega, x), y)]$$

for all $x, y \in C$, $\omega \in \Omega$, where $h(\omega)$ is a real valued random variable such that $0 < h(\omega) < 1$. Suppose that $\{g_n\}$ is random Ishikawa iteration scheme as defined in the Definition(2.13) is convergent to some point $p(\omega)$, then $Fix(S) = Fix(T) \neq \Phi$.

Proof: Clearly,

$$\begin{aligned} \delta(x, W(x, y, \lambda)) &= (1 - \lambda)d(x, y) \\ \delta(y, W(x, y, \lambda)) &= \lambda d(x, y) \end{aligned} \quad (3.17)$$

Hence for $\omega \in \Omega$,

$$\begin{aligned} d(g_n(\omega), g_{n+1}(\omega)) &= \delta(g_n(\omega), (1 - \alpha_n)g_n(\omega) + \alpha_n T(\omega, h_n(\omega))) \\ &= \alpha_n \delta(g_n(\omega), T(\omega, h_n(\omega))). \end{aligned} \quad (3.18)$$

For $\omega \in \Omega$, $\{g_n\} \rightarrow p(\omega)$ as $n \rightarrow \infty$, then we get $\lim_{n \rightarrow \infty} d(g_n(\omega), g_{n+1}(\omega)) = 0$, so we have for $\omega \in \Omega$,

$$\lim_{n \rightarrow \infty} \delta(g_n(\omega), T(\omega, h_n(\omega))) = 0, \quad (3.19)$$

as α_n is bounded away from zero.

Again,

$$\begin{aligned} &\delta(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) \\ &\leq h(\omega)[d(g_n(\omega), h_n(\omega)) + \delta(g_n(\omega), T(\omega, h_n(\omega))) + \delta(h_n(\omega), S(\omega, g_n(\omega)))]. \end{aligned} \quad (3.20)$$

So,

$$\begin{aligned} d(g_n(\omega), h_n(\omega)) &= \delta(g_n(\omega), (1 - \beta_n)g_n(\omega) + \beta_n S(\omega, g_n(\omega))) \\ &= \beta_n \delta(g_n(\omega), S(\omega, g_n(\omega))). \end{aligned} \quad (3.21)$$

Again,

$$\begin{aligned} & \delta(h_n(\omega), S(\omega, g_n(\omega))) \\ &= \delta((1 - \beta_n)g_n(\omega) + \beta_n S(\omega, g_n(\omega)), S(\omega, g_n(\omega))) \\ &= (1 - \beta_n)\delta(g_n(\omega), S(\omega, g_n(\omega))). \end{aligned} \quad (3.22)$$

Using (3.5) and (3.6) we obtain that,

$$\begin{aligned} & \delta(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) \\ & \leq h(\omega)[\delta(g_n(\omega), T(\omega, h_n(\omega))) + \delta(g_n(\omega), S(\omega, g_n(\omega)))]. \end{aligned} \quad (3.23)$$

and

$$\begin{aligned} & \delta(g_n(\omega), S(\omega, g_n(\omega))) \\ & \leq \delta(g_n(\omega), T(\omega, h_n(\omega))) + \delta(T(\omega, h_n(\omega)), S(\omega, g_n(\omega))) \end{aligned} \quad (3.24)$$

Thus by (3.8) we see that that,

$$\begin{aligned} & \delta(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) \\ & \leq h(\omega)[\delta(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) + 2\delta(g_n(\omega), T(\omega, h_n(\omega)))]. \end{aligned} \quad (3.25)$$

So,

$$\begin{aligned} & \delta(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) \\ & \leq \frac{2h(\omega)}{1 - h(\omega)}\delta(g_n(\omega), T(\omega, h_n(\omega))). \end{aligned} \quad (3.26)$$

Hence,

$$\lim_{n \rightarrow \infty} \delta(S(\omega, g_n(\omega)), T(\omega, h_n(\omega))) = 0, \quad (3.27)$$

as $\lim_{n \rightarrow \infty} \delta(g_n(\omega), T(\omega, h_n(\omega))) = 0$.

Also for $\omega \in \Omega$,

$$\begin{aligned} & \lim_{n \rightarrow \infty} \delta(g_n(\omega), T(\omega, h_n(\omega))) = 0 \\ \Rightarrow & \lim_{n \rightarrow \infty} T(\omega, h_n(\omega)) = \lim_{n \rightarrow \infty} g_n(\omega) = p(\omega) \end{aligned} \quad (3.28)$$

Now, $\lim_{n \rightarrow \infty} S(\omega, g_n(\omega)) = p(\omega)$.

Thus,

$$\begin{aligned} & \lim_{n \rightarrow \infty} \delta(g_n(\omega), h_n(\omega)) \\ &= \lim_{n \rightarrow \infty} \beta_n \delta(g_n(\omega), S(\omega, g_n(\omega))) = 0. \end{aligned} \quad (3.29)$$

So we can say that, $\lim_{n \rightarrow \infty} h_n(\omega) = p(\omega)$. Thus,

$$\begin{aligned} & \delta(S(\omega, g_n(\omega)), T(\omega, p(\omega))) \\ & \leq h(\omega)[\delta(g_n(\omega), p(\omega)) + \delta(g_n(\omega), T(\omega, p(\omega))) + \delta(p(\omega), S(\omega, g_n(\omega)))]. \end{aligned} \quad (3.30)$$

Taking limit as $n \rightarrow \infty$ in 3.30 we get for all $\omega \in \Omega$,

$$\begin{aligned} & \delta(p(\omega), T(\omega, p(\omega))) \\ & \leq h(\omega)\delta(p(\omega), T(\omega, p(\omega))). \end{aligned} \quad (3.31)$$

As, $0 < h(\omega) < 1$, $\delta(p(\omega), T(\omega, p(\omega))) = 0$ i.e. $p(\omega) = T(\omega, p(\omega))$.

Similarly, we get $p(\omega) = S(\omega, p(\omega))$.

So, $p(\omega) = S(\omega, p(\omega)) = T(\omega, p(\omega))$ i.e. $Fix(S) = Fix(T) \neq \Phi$. \square

Example 3.2 Let $X = (0, 1)$ in the usual metric of reals and $S, T : \Omega \times X \rightarrow P_b(X)$ where,

$$S(\omega, x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x \in [0, \frac{15}{32}) \cup (\frac{15}{32}, 1], \\ \{\frac{17}{96}, \frac{1}{4}\} & \text{if } x = \frac{15}{32} \end{cases} \quad (3.32)$$

and

$$T(\omega, x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x \in [0, \frac{13}{30}) \cup (\frac{13}{30}, 1], \\ \{\frac{19}{90}, \frac{1}{4}\} & \text{if } x = \frac{13}{30} \end{cases} \quad (3.33)$$

Here all the condition for Theorem (3.2) satisfies. Again random Ishikawa iteration scheme in 3.40 be defined on a sequence of functions $\{g_n(\omega)\}_{n \geq 1}$ by assuming $g_n(\omega) = 0$ with constant $\alpha_n = \frac{1}{2}$ and $\beta_n = \frac{1}{2}$. We now at once see that S and T have common random fixed point $p(\omega) = 0$.

Theorem 3.3 Let (X, d) be linear metric space endowed with a convex structure and C be the non empty closed convex subset of X . Let $S, T : \Omega \times C \rightarrow C$ satisfies

$$d(S(\omega, x), T(\omega, y)) \leq h(\omega)[d(x, y) + d(x, T(\omega, y)) + d(S(\omega, x), y)]$$

for all $x, y \in C$, $\omega \in \Omega$, where $h(\omega)$ is a real valued random variable such that $0 < h(\omega) < 1$. Suppose that $\{g_n\}$ is random iteration scheme as defined in the Definition(2.13). Let $p(\omega)$ be the common random fixed point of S and T then the random iteration converges to $p(\omega)$.

Proof: As, S and T have common random fixed point $p(\omega)$ we have, $S(\omega, p(\omega)) = T(\omega, p(\omega)) = p(\omega)$. From 3.40 it really follows that

$$\begin{aligned} d(p(\omega), h_n(\omega)) \\ = \beta_n d(p(\omega), S(\omega, p(\omega))). \end{aligned} \quad (3.34)$$

So, $\lim_{n \rightarrow \infty} h_n(\omega) = p(\omega)$. Hence $d(h_n(\omega), S(\omega, p(\omega)))$.

Now,

$$\begin{aligned} d(S(\omega, p(\omega)), T(\omega, h_n(\omega))) \\ \leq h(\omega)[d(p(\omega), h_n(\omega)) + d(p(\omega), T(\omega, h_n(\omega))) + d(h_n(\omega), S(\omega, p(\omega)))]. \end{aligned} \quad (3.35)$$

So,

$$\begin{aligned} d(p(\omega), T(\omega, h_n(\omega))) \\ \leq h(\omega)[d(p(\omega), h_n(\omega)) + d(p(\omega), T(\omega, h_n(\omega))) + d(h_n(\omega), S(\omega, p(\omega)))]. \end{aligned} \quad (3.36)$$

Hence,

$$\lim_{n \rightarrow \infty} d(p(\omega), T(\omega, h_n(\omega))) = 0.$$

Thus for $\omega \in \Omega$,

$$\begin{aligned} d(p(\omega), g_{n+1}(\omega)) \\ = \alpha_n d(p(\omega), T(\omega, h_n(\omega))). \end{aligned} \quad (3.37)$$

As α_n is bounded away from zero we get,

$$\lim_{n \rightarrow \infty} d(p(\omega), g_{n+1}(\omega)) = 0.$$

So, $\{g_{n+1}(\omega)\}$ converges to $p(\omega)$. \square

Example 3.3 Let $X = (0, 1)$ be equipped with usual metric of reals and $S, T : \Omega \times X \rightarrow X$ where,

$$S(\omega, x) = \begin{cases} \frac{1}{2} & \text{if } 0 < x \leq \frac{2}{3}, \\ 1 - \frac{1}{2}x & \text{if } \frac{2}{3} \leq x < 1 \end{cases} \quad (3.38)$$

and

$$T(\omega, x) = \begin{cases} \frac{1}{3} & \text{if } 0 < x \leq \frac{2}{3}, \\ \frac{4}{3} - x & \text{if } \frac{2}{3} \leq x < 1 \end{cases} \quad (3.39)$$

We see that all the condition for Theorem (3.3) satisfied. Let $g_0 : \Omega \rightarrow C$ be any measurable function. The random Ishikawa iteration scheme is defined as the following sequence of functions:

$$\begin{aligned} g_{n+1}(\omega) &= (1 - \alpha_n)g_n(\omega) + \alpha_n T(\omega, h_n(\omega)), \quad \text{where} \\ h_n(\omega) &= (1 - \beta_n)g_n(\omega) + \beta_n S(\omega, g_n(\omega)) \end{aligned} \quad (3.40)$$

where $\alpha_n = \frac{1}{2}$ and $\beta_n = \frac{1}{2}$. Then we see that both S and T have common fixed point $p(\omega) = \frac{2}{3}$ then we see that the random Ishikawa iteration $\{g_n(\omega)\}$ converges to $p(\omega) = \frac{2}{3}$.

4. Data Availability

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

5. Compliance with Ethical Standards

The authors declare that they have no conflict of interest.

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Amit Kumar Laha,
Amity University Kolkata
Major Arterial Road, AA II, Newtown
West Bengal 700135, India, <https://orcid.org/0000-0002-3927-5381>
E-mail address: amitlahaiit@gmail.com

and

Ramakant Bhardwaj,
Amity University Kolkata
Major Arterial Road, AA II, Newtown
West Bengal 700135, India, <https://orcid.org/0000-0002-1538-5615>
E-mail address: rkbhardwaj100@gmail.com