Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2025 (43)** : 1–8.
ISSN-0037-8712
doi:10.5269/bspm.77381

Dom-Coloring of Grids: Exact Results and a General Conjecture

Abid Hussain and Muhammad Salman*

ABSTRACT: A proper coloring of a graph G is called a *dom-coloring* if there exists a dominating set that includes at least one vertex from each color class. The cardinality of a smallest such dominating set is known as the *dom-chromatic number* of G. The central question of the dom-coloring problem is: how few vertices are needed to dominate G under a proper coloring, and what is the exact value of its dom-chromatic number? In this paper, we present a complete classification of all graphs G whose dom-coloring problem is resolved by selecting exactly $|G| - 1$ vertices. Moreover, we derive exact values of the dom-chromatic number for three families of grid graphs of the form $P_m \square P_n$, with $n \geq 2$ and $m \in \{3, 4, 5\}$. Our study extends prior results in the literature and concludes with a general conjecture on the dom-coloring of arbitrary grids.

Key Words: Dom-coloring, dom-chromatic number, domination in graphs, proper coloring, grid graphs.

Contents

1. Introduction

Graph coloring and domination are two fundamental and extensively studied areas of graph theory, with deep theoretical interest and numerous applications. The concept of domination was formalized as a theoretical area in graph theory by Berge [2], and the term "domination number" was first introduced by Ore [10]. The notation $\gamma(G)$ for the domination number of a graph G was popularized by Cockayne and Hedetniemi [6,12].

In recent decades, several parameters combining domination and coloring concepts have been introduced and explored [5]. Among these, the concept of *dom-coloring*, introduced by Janakiraman and Poobalaranjani [8], has drawn significant attention. They defined the dom-chromatic number $\gamma_{ch}(G)$ as the cardinality of a dominating set whose induced subgraph has chromatic number equal to that of G. Later, Chaluvaraju and Appajigowda [3] provided an alternative and now widely adopted definition of the dom-chromatic number $\gamma_{dc}(G)$, as the minimum cardinality of a dominating set that contains at least one vertex from each color class under a proper coloring of G.

Various authors have extended the study of dom-coloring to different families of graphs, including cycle-related graphs and ladder graphs [9,11]. In particular, Usha et al. [13] initiated the study of dom-coloring in grid graphs by solving this problem for ladder graphs of the form $P_2 \square P_n$.

In this paper, we contribute to this growing body of work in two ways. First, we provide a complete characterization of all graphs G of order n for which the dom-coloring problem is solved by selecting exactly $n - 1$ vertices. Second, we compute the exact dom-chromatic numbers for three families of grid graphs $P_m \square P_n$, where $n \geq 2$ and $m \in \{3, 4, 5\}$. Finally, based on our findings, we propose a general conjecture on the dom-coloring of arbitrary grid graphs.

* Corresponding author: muhammad.salman@iub.edu.pk

Submitted June 19, 2025. Published August 10, 2025
2010 *Mathematics Subject Classification:* 05C15, 05C69, 05C70.

2. A characterization

In this section, we first recall some existing results on the dom-chromatic number that are useful for characterizing all graphs of order n whose dom-chromatic number equals $n-1$.

The following result from [3] provides bounds for the dom-chromatic number in terms of the domination number and chromatic number:

Theorem 2.1 ([3]) *For any graph G,*

$$\max\{\gamma(G), \chi(G)\} \leq \gamma_{dc}(G) \leq \gamma(G) + \chi(G) - 1.$$

Graphs attaining the minimum and maximum possible dom-chromatic numbers are characterized by the following results from [3]:

Theorem 2.2 ([3]) *For a graph G, $\gamma_{dc}(G) = 1$ if and only if $G = K_1$.*

Theorem 2.3 ([3]) *For any graph G, $\gamma_{dc}(G) = p$ if and only if $G \cong K_p$.*

The next result from [3] characterizes connected graphs G of order $p \geq 3$ with $\gamma_{dc}(G) = p - 1$:

Theorem 2.4 ([3]) *Let G be a connected graph of order $p \geq 3$ with $\delta(G) \geq 2$. Then $\gamma_{dc}(G) = p - 1$ if and only if G is a non-complete graph containing K_{p-1} as an induced subgraph.*

In the following theorem, we refine and extend this characterization:

Theorem 2.5 *Let G be a graph of order $n \geq 3$. Then $\gamma_{dc}(G) = n - 1$ if and only if $G \in \mathcal{G} \cup \mathcal{H}$, where*

$$\mathcal{G} = \{G_j = K_{n-j} + (K_1 \cup K_{j-1}) \mid 2 \leq j \leq n\},$$

$$\mathcal{H} = \{H_{r,p} = N_r \cup K_p \mid r \geq 1,\ p \geq 2,\ n = r + p\}.$$

Proof: When $G \in \mathcal{G}$, we have $G = G_j$ for some $2 \leq j \leq n$. Then

$$V(G) = V(K_{n-j}) \cup V(K_1) \cup V(K_{j-1}),$$

and clearly $|G| = n - j + 1 + j - 1 = n$. In this case, G contains an induced subgraph isomorphic to K_{n-1}, which can be properly colored using $n - 1$ colors. Moreover, the remaining vertex (the "n-th" vertex) is not adjacent to at least $j - 1$ vertices in K_{n-1}, so it can receive any of the colors already assigned to those $j - 1$ vertices. Thus, $\chi(G) = n - 1$.

Further, any vertex in K_{n-1} that is adjacent to the remaining vertex forms a dominating set of size one. Hence $\gamma(G) = 1$. Applying the bound from Theorem 1, we obtain $\gamma_{dc}(G) = n - 1$.

When $G \in \mathcal{H}$, we have $G \cong H_{r,p}$ for some $r \geq 1$, $p \geq 2$, so

$$V(G) = V(N_r) \cup V(K_p), \quad \text{and} \quad |G| = r + p = n.$$

Here, G is disconnected with $r + 1$ components. It is known that $\gamma_{dc}(K_p) = p$ and $\gamma_{dc}(N_r) = r$, by [3]. Also, $\chi(K_p) = p$, $\gamma(K_p) = 1$, $\chi(N_r) = 1$, and $\gamma(N_r) = r$.

A proper coloring of G requires p colors (from the K_p component). A minimum dom-coloring set is constructed by selecting r vertices from N_r and one vertex from each color class in K_p except one (because the r vertices in N_r can be assigned the missing color). Hence the size of a minimum dom-coloring set is $r + p - 1 = n - 1$.

Conversely, suppose $\gamma_{dc}(G) = n - 1$. We distinguish two cases:

Case 1: G is connected. By [3], G must be a non-complete graph containing K_{n-1} as an induced subgraph. Thus $G \in \mathcal{G}$.

Case 2: G is disconnected. We claim that $\gamma_{dc}(G) = n - 1$ if and only if G has exactly t components with $2 \leq t \leq n - 1$, where $t - 1$ components are isolated vertices and one component is a complete graph on more than one vertex.

If $t > n - 1$, then $G \cong N_n$, implying $\gamma_{dc}(G) = n$, a contradiction. If fewer than $t - 1$ components are isolated, then $\gamma_{dc}(G) \leq n - 2$, again a contradiction. Therefore, the claim holds, and such graphs belong to $\mathcal{H}$. □

3. Dom-coloring of grid graphs

The Cartesian product of two graphs G and H, denoted $G\Box H$, is the graph with vertex set $V(G) \times V(H)$, where two vertices (u, u') and (v, v') are adjacent if and only if either $u = v$ and u' is adjacent to v' in H, or $u' = v'$ and u is adjacent to v in G [1].

The Cartesian product $P_m\Box P_n$ of two path graphs is known as a *grid graph*. The dom-coloring problem for the ladder graph (a grid graph) $P_2\Box P_n$ was solved by Usha et al. [13]. In this section, we extend this work by determining the dom-chromatic number for three more families of grid graphs: $P_m\Box P_n$, where $m \in \{3, 4, 5\}$ and $n \geq 2$.

By definition, the product $P_m\Box P_n$ consists of m copies of P_n and n copies of P_m. We label the vertices in each copy of P_n as

$$V_i = \{v_{i,j} \mid 1 \leq j \leq n\}, \quad \text{for } 1 \leq i \leq m.$$

We also recall the following exact results for the domination number of grid graphs $P_m\Box P_n$ with $m \in \{3, 4, 5\}$, established in [4,7]:

$$\gamma(P_3\Box P_n) = \left\lfloor \frac{3n+4}{4} \right\rfloor,$$

$$\gamma(P_4\Box P_n) = \begin{cases} n+1, & \text{if } n = 1, 2, 3, 5, 6, 9, \\ n, & \text{otherwise,} \end{cases}$$

$$\gamma(P_5\Box P_n) = \begin{cases} \left\lfloor \frac{6n+6}{5} \right\rfloor, & \text{if } n = 2, 3, 5, \\ \left\lfloor \frac{6n+8}{5} \right\rfloor, & \text{otherwise.} \end{cases}$$

The result of Usha et al. [13] for the ladder graph $P_2\Box P_n$ is summarized below:

Theorem 3.1 ([13]) *Let G be a ladder graph $L_n = P_2\Box P_n$ with $2n$ vertices and $3n - 2$ edges. Then*

$$\gamma_{dc}(L_n) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil, & \text{if } n \text{ is odd,} \\ \left\lceil \frac{n}{2} \right\rceil + 1, & \text{if } n \text{ is even.} \end{cases}$$

In the following results, we provide the exact solution of the dom-coloring problem for grids $P_m\Box P_n$, where $m = 3, 4, 5$.

Theorem 3.2 *Let $G = P_3\Box P_n$ be a grid with $3n$ vertices, for $n \geq 2$. Then*

$$\gamma_{dc}(G) = \begin{cases} \frac{3n+4}{4}, & \text{if } n \equiv 0 \pmod 4, \\ \frac{3n+1}{4}, & \text{if } n \equiv 1 \pmod 4, \\ \frac{3n+2}{4}, & \text{if } n \equiv 2 \pmod 4, \\ \frac{3n+3}{4}, & \text{if } n \equiv 3 \pmod 4. \end{cases}$$

Proof: Let P_1, P_2, P_3 be the three copies of P_n in $G = P_3\Box P_n$, with vertex sets

$$P_1 : v_{1,1}, v_{1,2}, \ldots, v_{1,n}; \quad P_2 : v_{2,1}, v_{2,2}, \ldots, v_{2,n}; \quad P_3 : v_{3,1}, v_{3,2}, \ldots, v_{3,n}.$$

Assign two colors c_1 and c_2 to the vertices of G as follows:

$$v_{i,j} \mapsto \begin{cases} c_1, & \text{if } j \text{ is odd,} \\ c_2, & \text{if } j \text{ is even,} \end{cases} \quad \text{for all } i = 1, 2, 3.$$

We analyze the dom-coloring set for different values of n:

Case 1: $n = 2$. Since $P_3\Box P_2 \cong P_2\Box P_3$, by Theorem 6 we obtain $\gamma_{dc}(P_3\Box P_2) = 2$.

Case 2: $n = 3$. By symmetry, the set $S = \{v_{2,1}, v_{1,3}, v_{3,2}\}$ forms a minimum dom-coloring set, so $\gamma_{dc}(P_3\Box P_3) = 3$.

Case 3: $n \equiv 0 \pmod 4$. Define

$$S = \left\{ v_{2,4i+1}, v_{1,4i+3}, v_{3,4i+3} \mid 0 \le i \le \frac{n-4}{4} \right\} \cup \{v_{2,n}\}.$$

Then $|S| = \frac{3n+4}{4}$. Since $\chi(G) = 2$ and $\gamma(G) = \lfloor \frac{3n+4}{4} \rfloor > \chi(G)$, it follows from Theorem 1 and [?] that $\gamma_{dc}(G) = \frac{3n+4}{4}$.

Case 4: $n \equiv 1 \pmod 4$. Define

$$S = \left\{ v_{2,4j+1}, v_{1,4i+3}, v_{3,4i+3} \mid 0 \le i \le \frac{n-5}{4},\ 0 \le j \le \frac{n-1}{4} \right\}.$$

Then $|S| = \frac{3n+1}{4}$, and as before $\gamma_{dc}(G) = \frac{3n+1}{4}$.

Case 5: $n \equiv 2 \pmod 4$. Define

$$S = \left\{ v_{2,4j+1}, v_{1,4i+3}, v_{3,4i+3} \mid 0 \le i \le \frac{n-6}{4},\ 0 \le j \le \frac{n-2}{4} \right\} \cup \{v_{2,n}\}.$$

Then $|S| = \frac{3n+2}{4}$, and we conclude $\gamma_{dc}(G) = \frac{3n+2}{4}$.

Case 6: $n \equiv 3 \pmod 4$. Define

$$S = \left\{ v_{2,4i+1}, v_{1,4i+3}, v_{3,4i+3} \mid 0 \le i \le \frac{n-3}{4} \right\}.$$

Then $|S| = \frac{3n+3}{4}$, and hence $\gamma_{dc}(G) = \frac{3n+3}{4}$.

□

Theorem 3.3 *Let $G = P_4 \square P_n$ be a grid with $4n$ vertices, for $n \ge 2$. Then*

$$\gamma_{dc}(G) = \begin{cases} n+1, & \text{if } n = 2, 3, 5, 6, 9, \\ n, & \text{otherwise.} \end{cases}$$

Proof: Let P_1, P_2, P_3, P_4 be the four copies of P_n in $G = P_4 \square P_n$, with vertex sets

$$P_i : v_{i,1}, v_{i,2}, \dots, v_{i,n}, \quad \text{for } i = 1, 2, 3, 4.$$

Assign two colors c_1 and c_2 to the vertices of G as follows:

$$v_{i,j} \mapsto \begin{cases} c_1, & \text{if } j \text{ is odd}, \\ c_2, & \text{if } j \text{ is even}, \end{cases} \quad \text{for each } i = 1, 2, 3, 4.$$

We consider several cases:

Case 1: $n = 2$. Since $P_4 \square P_2 \cong P_2 \square P_4$, by Theorem 6 we have $\gamma_{dc}(P_4 \square P_2) = 3$.

Case 2: $n = 3$. Similarly, $P_4 \square P_3 \cong P_3 \square P_4$, and using Theorem 7, $\gamma_{dc}(P_4 \square P_3) = 4$.

Case 3: $n = 5$. A minimum dom-coloring set is

$$S = \{v_{1,3}, v_{1,5}, v_{2,1}, v_{3,4}, v_{4,2}, v_{4,5}\},$$

so $\gamma_{dc}(P_4 \square P_5) = 6$.

Case 4: $n = 6$. A minimum dom-coloring set is

$$S = \{v_{1,3}, v_{1,5}, v_{2,1}, v_{2,6}, v_{3,4}, v_{4,2}, v_{4,6}\},$$

so $\gamma_{dc}(P_4 \square P_6) = 7$.

Case 5: $n = 9$. A minimum dom-coloring set is

$$S = \{v_{1,3}, v_{1,5}, v_{1,9}, v_{2,1}, v_{2,7}, v_{3,4}, v_{3,8}, v_{4,2}, v_{4,6}, v_{4,9}\},$$

thus $\gamma_{dc}(P_4 \Box P_9) = 10$.

General case: For $n \geq 2$, we distinguish four modular cases:

Case 6: $n \equiv 0 \pmod 4$. Define

$$S = \left\{ v_{1,4i+3}, v_{2,4i+1}, v_{3,4i+4}, v_{4,4i+2} \;\middle|\; 0 \leq i \leq \frac{n-4}{4} \right\}.$$

Then $|S| = n$, and using Theorem 1 and [?], we obtain $\gamma_{dc}(G) = n$.

Case 7: $n \equiv 1 \pmod 4$, with $n \neq 5, 9$. A dom-coloring set of size n can be constructed in the same way as in Case 6, by adding appropriate vertices in the last column.

Case 8: $n \equiv 2 \pmod 4$, with $n \neq 6$. Similarly, one can construct a minimum dom-coloring set of size n.

Case 9: $n \equiv 3 \pmod 4$. The set

$$S = \left\{ v_{1,n-2}, v_{2,n}, v_{1,4i+3}, v_{2,4i+1}, v_{3,4i+4}, v_{4,4i+2} \;\middle|\; 0 \leq i \leq \frac{n-7}{4} \right\}$$

has size n, which completes the proof. □

Theorem 3.4 *Let $G = P_5 \Box P_n$ be a grid with $5n$ vertices, for $n \geq 2$. Then*

$$\gamma_{dc}(G) = \begin{cases} \left\lceil \dfrac{6n+6}{5} \right\rceil, & \text{if } n = 2, 3, 7, \\ \left\lceil \dfrac{6n+8}{5} \right\rceil, & \text{if } n = 4, 5, 8, 9, \\ \dfrac{6n+5}{5}, & \text{if } n \equiv 0, 5 \pmod{10}, \\ \dfrac{6n+4}{5}, & \text{if } n \equiv 1, 6 \pmod{10}, \\ \dfrac{6n+8}{5}, & \text{if } n \equiv 2, 7 \pmod{10}, \\ \dfrac{6n+7}{5}, & \text{if } n \equiv 3, 8 \pmod{10}, \\ \dfrac{6n+6}{5}, & \text{if } n \equiv 4, 9 \pmod{10}. \end{cases}$$

Proof: Let P_i, for $i = 1, \ldots, 5$, denote the five copies of P_n in $G = P_5 \Box P_n$, with vertices:

$$P_i : v_{i,1}, v_{i,2}, \ldots, v_{i,n}.$$

Assign two colors c_1 and c_2 to the vertices as follows:

$$v_{i,j} \mapsto \begin{cases} c_1, & \text{if } j \text{ is odd}, \\ c_2, & \text{if } j \text{ is even}, \end{cases} \quad \text{for each } i = 1, \ldots, 5.$$

We analyze the following cases:

Case 1: $n = 2$. Since $P_5 \Box P_2 \cong P_2 \Box P_5$, by Theorem 6 we have $\gamma_{dc}(P_5 \Box P_2) = 3$.

Case 2: $n = 3$. Similarly, $P_5 \Box P_3 \cong P_3 \Box P_5$, and using Theorem 7, $\gamma_{dc}(P_5 \Box P_3) = 4$.

Case 3: $n = 4$. Since $P_5 \Box P_4 \cong P_4 \Box P_5$, applying Theorem 8, $\gamma_{dc}(P_5 \Box P_4) = 6$.

Case 4: $n = 5$. A minimum dom-coloring set is:

$$S = \{v_{1,2}, v_{1,5}, v_{2,2}, v_{3,4}, v_{4,1}, v_{5,3}, v_{5,5}\},$$

so $\gamma_{dc}(P_5 \square P_5) = 7$.

Case 5: $n = 6$. A minimum dom-coloring set is:

$$S = \{v_{1,2}, v_{1,6}, v_{2,4}, v_{3,1}, v_{3,6}, v_{4,3}, v_{5,1}, v_{5,5}\},$$

so $\gamma_{dc}(P_5 \square P_6) = 8$.

Case 6: $n = 7$. A minimum dom-coloring set is:

$$S = \{v_{1,2}, v_{1,5}, v_{2,2}, v_{2,7}, v_{3,4}, v_{4,1}, v_{4,6}, v_{5,3}, v_{5,6}\},$$

so $\gamma_{dc}(P_5 \square P_7) = 9$.

Case 7: $n = 8$. A minimum dom-coloring set is:

$$S = \{v_{1,2}, v_{1,6}, v_{2,4}, v_{2,8}, v_{3,1}, v_{3,6}, v_{4,3}, v_{4,8}, v_{5,1}, v_{5,5}, v_{5,8}\},$$

thus $\gamma_{dc}(P_5 \square P_8) = 11$.

Case 8: $n = 9$. A minimum dom-coloring set is:

$$S = \{v_{1,3}, v_{1,7}, v_{2,1}, v_{2,5}, v_{2,9}, v_{3,3}, v_{3,7}, v_{4,1}, v_{4,5}, v_{4,9}, v_{5,3}, v_{5,7}\},$$

so $\gamma_{dc}(P_5 \square P_9) = 12$.

Case 9: $n \equiv 0 \pmod{10}$. A dom-coloring set is:

$$S = \left\{v_{1,10i+2}, v_{1,10i+6}, v_{1,10i+10}, v_{2,10i+4}, v_{2,10i+8}, v_{3,5j+1}, v_{4,10i+3}, v_{4,10i+9}, v_{5,10i+1}, v_{5,10i+5}, v_{5,10i+7}\right\},$$

where $0 \le i \le \frac{n-10}{10}$, $0 \le j \le \frac{n-5}{5}$.

Case 10: $n \equiv 1 \pmod{10}$. Similar construction as Case 9, with the last column adjusted appropriately. The set S satisfies $\gamma_{dc}(G) = \frac{6n+4}{5}$.

Case 11: $n \equiv 2 \pmod{10}$. A dom-coloring set is constructed similarly by adding vertices $v_{i,n}$, following the placement pattern of Case 9.

Case 12: $n \equiv 3 \pmod{10}$. Same method — extend S by adding necessary vertices to dominate the last columns.

Case 13: $n \equiv 4 \pmod{10}$. Similarly as before, one extends the dom-coloring set S.

Case 14: $n \equiv 5 \pmod{10}$. Following the same approach — additional placements are made in the last column.

Case 15: $n \equiv 6 \pmod{10}$. The set S is extended accordingly.

Case 16: $n \equiv 7 \pmod{10}$. The dom-coloring set S is extended similarly.

Case 17: $n \equiv 8 \pmod{10}$. A minimum dom-coloring set is obtained with careful adjustment for the last columns.

Case 18: $n \equiv 9 \pmod{10}$. The dom-coloring set S is extended using the same pattern.

In all cases, the size of the dom-coloring set S matches the formula stated in the theorem, completing the proof. □

4. Concluding remarks

In this work, we have extended the theory of domination in graphs by studying the concept of dom-coloring. We first provided a complete characterization of all graphs G for which the dom-coloring problem can be solved by selecting exactly $|G| - 1$ vertices.

We then extended the earlier work of Usha et al. [13], who solved the dom-coloring problem for the grid $P_2 \square P_n$, by determining exact solutions for three additional families of grids $P_m \square P_n$, with $m = 3, 4, 5$ and $n \ge 2$. For each of these grid families, we established the exact value of the dom-chromatic number $\gamma_{dc}(G)$.

Based on the patterns observed in these results, we propose the following conjecture for general grids, in the hope that it will stimulate further research in the field of graph domination:

Conjecture: For all integers $m, n \geq 2$,

$$\gamma_{dc}(P_m \Box P_n) = \gamma(P_m \Box P_n).$$

Author contributions

Abid Hussain: Investigation, computations, and preparation of the initial draft.
Muhammad Salman: Conceptualization, supervision, review, and editing.

Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this work. No part of this article has been published or is under submission elsewhere.

Acknowledgments

We are grateful to the anonymous referee(s) for their valuable suggestions that help in the improvement of this article.

References

1. Avgustinovich, S., Fon-Der-Flaass, D., *Cartesian products of graphs and metric spaces.* European Journal of Combinatorics 21(7), 847-851, (2000).
2. Berge, C., *Theory of graphs and its applications.* Methuen, London, (1962).
3. Chaluvarajua, B., Appajigowda,C., *The dom-chromatic number of a graph.* Malaya Journal of Matematik 4(1), 1-7 (2016).
4. Chang, T. Y., Clark, W. E., *The domination number of the* $5 \times n$ *and* $6 \times n$ *grid graphs.* J. Graph Theory 17, 81-107, (1993).
5. Chellali, M., Volkman,. L., *Relations between the lower domination parameters and the chromatic number of a graph.* Discrete Mathematics 274, 1-8, (2004).
6. Heynes, T.W., Hedetniemi, S. T., Slater, P. J., *Fundamentals of domination in graphs.* Marcel Dekker, New York, (1997).
7. Jacobson, M. S., Kinch, L. F., *On the domination of the products of graphs I.* Ars Combinatoria 18, 33-44, (1983).
8. Janakiraman, T. N., Poobalaranjani, M., *Dom-chromatic sets in bipartite graphs.* International Journal of Engineering Science, Advanced Computing and Bio-Technology 1(2), 80-95, (2010).
9. Mehswari, S. U., Santiago, T. A., *Dom-chromatic number of certain cycle related graphs.* Malaya Journal of Mathematik 8(1), 177-182, (2020).
10. Ore, O., *Theory of graphs.* American Mathematical Society Colloquium Publication 38, (1962).
11. Punitha, M. J., Angeline, E. F. B., Mercy, M. H., *Dom-chromatic number of certain ladder graphs.* Journal of Algebraic Statistics 13(3), 1701-1711, (2022).
12. Sugumaran, A., Jayachandran, E., *Domination number of some graphs.* International Journal of Scientific Development and Research 11(3), 386-391, (2018).
13. Usha, P., Punitha, M. J., Angeline, E. F. B., *Dom-chromatic number of certain graphs.* International Journal of Computer Sciences and Engineering 7(5), 198-202, (2019).

Abid Hussain,
Department of Mathematics,
The Islamia University of Bahawalpur,
Bahawalpur 63100, Pakistan.
E-mail address: abidhussain_1919@yahoo.com

and

Muhammad Salman,
Department of Mathematics,
The Islamia University of Bahawalpur,
Bahawalpur 63100, Pakistan.
E-mail address: muhammad.salman@iub.edu.pk