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Exponential Degree Square Sum and Product Energy of Graph

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ABSTRACT: In this paper, we introduce and investigate Exponential degree square sum and product energy of graph. We give upper and lower bounds for $EDPE(G)$. The study also determines the EDP spectra and EDP energy for several key families of graphs.

Key Words: Exponential degree square sum and product matrix, spectra and energy.

Contents

1. Introduction

In algebraic graph theory, matrix representation of graph play an important role. Matrix representation of the graph is indeed a very old and highly beneficial topic of research in graph theory. In 1978, Ivan Gutman introduced the concept of energy of graph which is basically depends on the adjacency matrix of a molecular graph. The adjacency matrix carries only two values such as 0 if there is no edge between the vertices and 1, if two vertices are adjacent. Energy of graph [7] [8] is a popular graph invariant which is defined as the sum of the absolute values of the eigenvalues of the adjacency matrix.

Simple graphs without loops, sign and marks are considered in this study. In this paper we use $v_i \sim v_j$ to represent the adjacency between two vertices v_i and v_j.

Topological indices are often regarded as molecular descriptors which are indeed useful in the study of topology of graph structures. Many researchers investigated the molecular descriptors which are based on degree, distance, etc., For more on topological indices refer [3] [5], [6], [15]. K. N Prakasha et al. [11] proposed a new topological index, Exponential degree square sum and product index, which is defined as

$$EDP(G) = \sum_{uv \in E} e^{d_u^2 + d_v^2} + d_u d_v$$

where d_u and d_v represents the degree of the respective vertices u and v.

The matrix representation using the concept of topological indices are not new. Many matrices such as sum-connectivity matrix [10], Randić matrix [2], [4], Reduced reciprocal Randić matrix [12], atom bond connectivity matrix [9], Zagreb matrix [14], Edge Zagreb matrix [14] etc., are the matrices which are basically motivated by the respective topolological indices. Motivated by the concept of matrices of graphs, in this paper we introduce a new matrix which is known as Exponential degree square sum and product matrix, $EDP(G)$ as

$$EDP(G) = \begin{cases} e^{d_u^2 + d_v^2} + d_u d_v & uv \in E \\ 0 & otherwise \end{cases}$$

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Let ξ_i be the eigenvalues of Exponential degree square sum and product marices, then the Exponential degree square sum and product energy is given by

$$EDPE(G) = \sum_{i=1}^{n} |\xi_i|.$$

2. Properties of Exponential degree square sum and product energy of a graph

Proposition 2.1 *The first three coefficients of the polynomial $\phi_{EDP}(G, \xi)$ are given as follows:*

(i) $a_0 = 1,$
(ii) $a_1 = 0,$
(iii) $a_2 = -\sum_{i<j} \left[e^{d_u^2+d_v^2} + d_u d_v\right]^2.$

Proposition 2.2 *If $\xi_1, \xi_2, \ldots, \xi_n$ are the eigenvalues of $EDP(G)$, then*

$$\sum_{i=1}^{n} {\xi_i}^2 = 2\sum_{i<j} \left[e^{d_u^2+d_v^2} + d_u d_v\right]^2.$$

Theorem 2.1 *Let G be a graph with n vertices. Then the upper bound is given by*

$$EDPE(G) \leq \sqrt{2n\sum_{i<j} \left[e^{d_u^2+d_v^2} + d_u d_v\right]^2}$$

.

Proof: By Cauchy-Schwartz inequality, we have

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \leq \left(\sum_{i=1}^{n} {a_i}^2\right)\left(\sum_{i=1}^{n} {b_i}^2\right).$$

Let $\xi_1, \ \xi_2, \ \ldots, \ \xi_n$ be the eigenvalues of $EDP(G)$. On substitution of $a_i = 1$ and $b_i = \xi_i$, we have

$$\left(\sum_{i=1}^{n} |\xi_i|\right)^2 \leq \left(\sum_{i=1}^{n} 1\right)\left(\sum_{i=1}^{n} |\xi_i|^2\right)$$

Thus

$$[EDPE(G)]^2 \leq n(2\sum_{i<j} \left[e^{d_u^2+d_v^2} + d_u d_v\right]^2)$$

$$EDPE(G) \leq \sqrt{2n\sum_{i<j} \left[e^{d_u^2+d_v^2} + d_u d_v\right]^2}$$

□

Theorem 2.2 *Let G be a graph with n vertices and let $|EDP|=$ determinant of $EDP(G)$, then*

$$EDPE(G) \geq \sqrt{2\sum_{i<j} \left[e^{d_u^2+d_v^2} + d_u d_v\right]^2 + n(n-1)|EDP|^{\frac{2}{n}}}.$$

Theorem 2.3 *Let G be a regular graph of n vertices with regularity r, then*

$$EDPE(G) = (e^{2r^2} + r^2)E(G)$$

Theorem 2.4 *Let G be a semiregular graph of degrees $r \geq 1$ and $s \geq 1$. Then $EDPE(G) = (e^{r^2+s^2} + rs)E(G)$.*

Proof: Consider a semiregular graph of degrees $r \geq 1$ and $s \geq 1$, the $EDP-$matrix is given by

$$EDP(G) = \ (e^{r^2+s^2} + rs)\,(J - I)\ .$$

Let λ_i denotes the eigenvalue with respect to adjacency matrix of the corresponding graph.

$$\xi_i = (e^{r^2+s^2} + rs)\lambda_i.$$

Thus the proof follows. □

3. Exponential degree square sum and product energy of some standard graphs

Theorem 3.1 *Exponential degree square sum and product energy of complete graph K_n is*

$$EDPE(K_n) = 2(n-1)e^{2(n-1)^2} + (n-1)^2.$$

Proof:

For each and every vertex u in K_n, $d(u) = (n\text{–}1)$. Then every ij^{th}-entry of the Exponential degree square sum and product matrix will be

$$\begin{bmatrix} 0 & e^{2(n-1)^2}+(n-1)^2 & e^{2(n-1)^2}+(n-1)^2 & \dots & e^{2(n-1)^2}+(n-1)^2 & e^{2(n-1)^2}+(n-1)^2 \\ e^{2(n-1)^2}+(n-1)^2 & 0 & e^{2(n-1)^2}+(n-1)^2 & \dots & e^{2(n-1)^2}+(n-1)^2 & e^{2(n-1)^2}+(n-1)^2 \\ e^{2(n-1)^2}+(n-1)^2 & e^{2(n-1)^2}+(n-1)^2 & 0 & \dots & e^{2(n-1)^2}+(n-1)^2 & e^{2(n-1)^2}+(n-1)^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ e^{2(n-1)^2}+(n-1)^2 & e^{2(n-1)^2}+(n-1)^2 & e^{2(n-1)^2}+(n-1)^2 & \dots & 0 & e^{2(n-1)^2}+(n-1)^2 \\ e^{2(n-1)^2}+(n-1)^2 & e^{2(n-1)^2}+(n-1)^2 & e^{2(n-1)^2}+(n-1)^2 & \dots & e^{2(n-1)^2}+(n-1)^2 & 0 \end{bmatrix}$$

Hence the characteristic equation will be

$$\left(\xi - (e^{2(n-1)^2} + (n-1)^2)\right)^{n-1}\left(\xi - (n-1)(e^{2(n-1)^2} + (n-1)^2)\right) = 0$$

and therefore the spectrum becomes

$$Spec_{EDP}(K_n) = \begin{pmatrix} e^{2(n-1)^2} + (n-1)^2 & (n-1)(e^{2(n-1)^2} + (n-1)^2) \\ n-1 & 1 \end{pmatrix}.$$

Therefore,

$$EDPE(K_n) = 2(n-1)(e^{2(n-1)^2} + (n-1)^2).$$

□

Definition 3.1 [1] Crown graph is a graph with two sets of vertices $\{u_1, u_2, ...u_n\}$ and $\{v_1, v_2, ...v_n\}$ and with an edge from u_i to v_j for $i \neq j$.

Theorem 3.2 *The Exponential degree square sum and product energy of the crown graph S_n^0 is*

$$EDPE(S_n^0) = 4[(n-1)X_1].$$

(Where $X_1 = e^{2(n-1)^2} + (n-1)^2$.)

Proof: Let S_n^0 be the crown graph of order $2n$ with vertex set $\{u_1,\ u_2,\ \cdots,\ u_n,\ v_1,\ v_2,\ \cdots,\ v_n\}$. The Exponential degree square sum and product matrix is

$$\begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & X_1 & \dots & X_1 & X_1 \\ 0 & 0 & 0 & \dots & 0 & X_1 & 0 & \dots & X_1 & X_1 \\ 0 & 0 & 0 & \dots & 0 & X_1 & X_1 & \dots & 0 & X_1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & X_1 & X_1 & \dots & X_1 & 0 \\ 0 & X_1 & X_1 & \dots & X_1 & 0 & 0 & \dots & 0 & 0 \\ X_1 & 0 & X_1 & \dots & X_1 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ X_1 & X_1 & 0 & \dots & X_1 & 0 & 0 & \dots & 0 & 0 \\ X_1 & X_1 & X_1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}.$$

Where $X_1 = e^{2(n-1)^2} + (n-1)^2$. The characteristic equation is

$$(\xi - (X_1))^{n-1} (\xi + X_1)^{n-1} (\xi + (n-1)(X_1)) (\xi - (n-1)(X_1)) = 0$$

implying that the spectrum is

$$Spec_{EDP}(S_n^0) = \begin{pmatrix} -(n-1)(X_1) & (n-1)(X_1) & -e^{2(n-1)^2} + (n-1)^2 & e^{2(n-1)^2} + (n-1)^2 \\ 1 & 1 & n-1 & n-1 \end{pmatrix}.$$

Therefore,

$$EDPE(S_n^0) = 4(n-1)[e^{2(n-1)^2} + (n-1)^2].$$

□

Theorem 3.3 *The Exponential degree square sum and product energy of complete bipartite graph $K_{m\times n}$ is*

$$EDPE(K_{m,n}) = (2\sqrt{mn})(e^{m^2+n^2} + mn).$$

Proof: The Exponential degree square sum and product matrix of complete bipartite graph $K_{m\times n}$ is

$$(e^{m^2+n^2} + mn) \begin{bmatrix} 0_{m\times m} & J_{m\times n} \\ J_{n\times m} & 0_{n\times n} \end{bmatrix}.$$

$$Spec_{EDP}(K_{m,n}) = \begin{pmatrix} (\sqrt{mn})(e^{m^2+n^2} + mn) & 0 & -(\sqrt{mn})(e^{m^2+n^2} + mn) \\ 1 & m+n-2 & 1 \end{pmatrix}.$$

$$EDPE(K_{m,n}) = (2\sqrt{mn})(e^{m^2+n^2} + mn).$$

□

Definition 3.2 [1] The cocktail party graph, denoted by $K_{n\times 2}$, is graph having vertex set $V = \cup_{i=1}^{n}\{u_i, v_i\}$ and the edge set $E = \{u_iu_j, v_iv_j, u_iv_j, v_iu_j : 1 \le i < j \le n\}$

Theorem 3.4 *The Exponential degree square sum and product energy of Cocktail party graph $K_{n\times 2}$ is*

$$EDPE(K_{n\times 2}) = 2(n-1)(e^{8(n-1)^2} + 4(n-1)^2)$$

Proof: Let $K_{n\times 2}$ be a Cocktail party graph of order $2n$ with vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$. The Exponential degree square sum and product matrix is

$$EDPE(K_{n\times 2}) = (e^{8(n-1)^2} + 4(n-1)^2)\begin{pmatrix} (J-I)_{n\times n} & (J-I)_{n\times n} \\ (J-I)_{n\times n} & (J-I)_{n\times n} \end{pmatrix}.$$

Characteristic equation is

$$\xi^n(\xi + (e^{8(n-1)^2} + 4(n-1)^2))^{n-1}(\xi + (n-1)(e^{8(n-1)^2} + 4(n-1)^2) = 0$$

Hence, spectrum is
$$Spec_{EDP}(K_{n\times 2}) = \begin{pmatrix} -(e^{8(n-1)^2} + 4(n-1)^2)^{n-1} & 0 & (n-1)(e^{8(n-1)^2} + 4(n-1)^2) \\ n-1 & n & 1 \end{pmatrix}.$$
Therefore,
$$EDPE(K_{n\times 2}) = 2(n-1)(e^{8(n-1)^2} + 4(n-1)^2)$$

□

Theorem 3.5 *The Exponential degree square sum and product energy of star graph $K_{1,n-1}$ is*

$$EDPE(K_{1,n-1}) = 2(\sqrt{n-1})(e^{n^2-2n+2} + n - 1).$$

Proof: Let $K_{1,n-1}$ be the star graph with vertex set $V = \{v_0, v_1 ... v_{n-1}\}$. The Exponential degree square sum and product matrix is

$$EDP(K_{1,n-1}) = (e^{n^2-2n+2} + n - 1)\begin{pmatrix} 0_{1\times 1} & J_{1\times n-1} \\ J_{n-1\times 1} & 0_{n-1\times n-1} \end{pmatrix}.$$

Characteristic equation is

$$(\xi)^{n-2}\left(\xi^2 - (n-1)(e^{n^2-2n+2} + n - 1)^2\right)$$

spectrum is $Spec_{EDP}(K_{1,n-1}) = \begin{pmatrix} (\sqrt{n-1})(e^{n^2-2n+2} + n - 1) & 0 & -(\sqrt{n-1})(e^{n^2-2n+2} + n - 1) \\ 1 & n-2 & 1 \end{pmatrix}.$
Therefore, $EDPE(K_{1,n-1}) = 2(\sqrt{n-1})(e^{n^2-2n+2} + n - 1)$. □

Definition 3.3 [2] The friendship graph, denoted by F_3^n, is the graph obtained by taking n copies of the cycle graph C_3 with a vertex in common.
It is easy to see that $|V(F_3^n)| = 2n + 1$.

Theorem 3.6 *The Exponential degree square sum and product energy of the friendship graph F_3^n is*

$$EDPE(F_3^n) = (e^8 + 4)(2n - 1) + \sqrt{8ne^{8n^2+8} + e^{4n^2+4} + 128n^4 + 64n^3 + e^8(1 + e^8) + 16}.$$

Proof: Let F_3^n be the friendship graph with $2n + 1$ vertices. The Exponential degree square sum and product matrix is

$$\begin{bmatrix}
0 & e^{4n^2+4}+4n & e^{4n^2+4}+4n & e^{4n^2+4}+4n & e^{4n^2+4}+4n & \dots & e^{4n^2+4}+4n & e^{4n^2+4}+4n \\
e^{4n^2+4}+4n & 0 & e^8+4 & 0 & 0 & \dots & 0 & 0 \\
e^{4n^2+4}+4n & e^8+4 & 0 & 0 & 0 & \dots & 0 & 0 \\
e^{4n^2+4}+4n & 0 & 0 & 0 & e^8+4 & \dots & 0 & 0 \\
e^{4n^2+4}+4n & 0 & 0 & e^8+4 & 0 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
e^{4n^2+4}+4n & 0 & 0 & 0 & 0 & \dots & 0 & e^8+4 \\
e^{4n^2+4}+4n & 0 & 0 & 0 & 0 & \dots & e^8+4 & 0
\end{bmatrix}$$

Therefore the characteristic equation will be

$$\left(\xi^2-(e^8+4)\xi-2n(e^{4n^2+4}+4n)^2\right)\left(\xi-(e^8+4)\right)^{n-1}\left(\xi+(e^8+4)\right)^n=0.$$

Hence, the spectrum is

$$Spec_{EDP}(F_3^n)=\begin{pmatrix} -(e^8+4) & e^8+4 & \frac{e^8+4+D_1}{2} & \frac{e^8+4-D_1}{2} \\ n & n-1 & 1 & 1 \end{pmatrix}.$$

Where $D_1=\sqrt{8ne^{8n^2+8}+e^{4n^2+4}+128n^4+64n^3+e^8(1+e^8)+16}$.
Therefore, $EDPE(F_3^n)$ is

$$(e^8+4)(2n-1)+\sqrt{8ne^{8n^2+8}+e^{4n^2+4}+128n^4+64n^3+e^8(1+e^8)+16}.$$

□

Theorem 3.7 *The Exponential degree square sum and product energy of the double star graph $S_{n,n}$ is*

$$EDPE(S_{n,n})=2\sqrt{a^2+4b^2(n-1)}.$$

where $a=e^{n^2+1}+n$ and $b=e^{2n^2}+n^2$.

Proof: The Exponential degree square sum and product matrix is $EDP(S_{n,n})=$

$$\begin{bmatrix} 0 & e^{n^2+1}+n & e^{n^2+1}+n & \dots & e^{n^2+1}+n & e^{2n^2}+n^2 & 0 & 0 & \dots & 0 \\ e^{n^2+1}+n & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 \\ e^{n^2+1}+n & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ e^{n^2+1}+n & 0 & 0 & \dots & 0 & 0 & 0 & 0 & \dots & 0 \\ e^{2n^2}+n^2 & 0 & 0 & \dots & 0 & 0 & e^{n^2+1}+n & e^{n^2+1}+n & \dots & e^{n^2+1}+n \\ 0 & 0 & 0 & \dots & 0 & e^{n^2+1}+n & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & e^{n^2+1}+n & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & e^{n^2+1}+n & 0 & 0 & \dots & 0 \end{bmatrix}.$$

Hence the characteristic equation is

$$\xi^{2n-4}\left(\xi^2+e^{2n^2}+n^2\xi-\frac{n^3-1}{(2n+1)^2}\right)\left(\xi^2-e^{2n^2}+n^2\xi-\frac{n^3-1}{(2n+1)^2}\right)=0$$

implying that the spectrum is

$$\begin{pmatrix} 0 & \left(\frac{-a+\sqrt{a^2+4b^2(n-1)}}{2}\right) & \left(\frac{a+\sqrt{a^2+4b^2(n-1)}}{2}\right) & \left(\frac{-a-\sqrt{a^2+4b^2(n-1)}}{2}\right) & \left(\frac{a-\sqrt{a^2+4b^2(n-1)}}{2}\right) \\ 2n-4 & 1 & 1 & 1 & 1 \end{pmatrix},$$

where $a=e^{n^2+1}+n$ and $b=e^{2n^2}+n^2$.
Therefore,

$$EDPE(S_{n,n})=2\sqrt{a^2+4b^2(n-1)}.$$

□

4. Exponential degree square sum and product energy of graph complements

Definition 4.1 [13] The complement of a graph G is a graph is denoted by $\overline{G}$ and is a graph on the same vertices such that two distinct vertices of $\overline{G}$ are adjacent if and only if they are not adjacent in G.

Theorem 4.1 *The Exponential degree square sum and product energy of the complement $\overline{K_{1,n-1}}$ of the star graph is*

$$EDPE(\overline{K_{1,n-1}}) = 2(n-2)[e^{2(n-1)^2+(n-1)^2}].$$

Proof: Let $\overline{(K_{1,n-1})}$ be the complement of star graph with vertex set $V = \{v_0, v_1 ... v_{n-1}\}$. The Exponential degree square sum and product matrix is

$$EDP(\overline{K_{1,n-1}}) = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & e^{2(n-1)^2+(n-1)^2} & \dots & e^{2(n-1)^2+(n-1)^2} & e^{2(n-1)^2+(n-1)^2} \\ 0 & e^{2(n-1)^2+(n-1)^2} & 0 & \dots & e^{2(n-1)^2+(n-1)^2} & e^{2(n-1)^2+(n-1)^2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & e^{2(n-1)^2+(n-1)^2} & e^{2(n-1)^2+(n-1)^2} & \dots & 0 & e^{2(n-1)^2+(n-1)^2} \\ 0 & e^{2(n-1)^2+(n-1)^2} & e^{2(n-1)^2+(n-1)^2} & \dots & e^{2(n-1)^2+(n-1)^2} & 0 \end{bmatrix}.$$

Then the characteristic equation is

$$\xi^1 \left(\xi - e^{2(n-1)^2+(n-1)^2}\right)^{n-2} \left(\xi - (e^{2(n-1)^2+(n-1)^2})(n-2)\right) = 0$$

and therefore the spectrum is

$$Spec_{EDP}(\overline{K_{1,n-1}}) = \begin{pmatrix} e^{2(n-1)^2+(n-1)^2} & 0 & (n-2)[e^{2(n-1)^2+(n-1)^2}] \\ n-2 & 1 & 1 \end{pmatrix}.$$

Therefore,

$$EDPE(\overline{K_{1,n-1}}) = 2(n-2)[e^{2(n-1)^2+(n-1)^2}].$$

□

Theorem 4.2 *The Exponential degree square sum and product energy of the complement $\overline{K_{n\times 2}}$ of the cocktail party graph of order $2n$ is*

$$EDP(\overline{K_{n\times 2}}) = 2n(e^2+1).$$

Proof: Let $\overline{K_{n\times 2}}$ be the complement of the cocktail party graph with vertex set $\{u_1,\ u_2,\ \cdots,\ u_n,\ v_1,\ v_2,\ \cdots,\ v_n\}$. The Exponential degree square sum and product matrix is

$$EDP(\overline{K_{n\times 2}}) = \begin{bmatrix} 0 & 0 & 0 & 0 & \dots & e^2+1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & e^2+1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & e^2+1 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & e^2+1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ e^2+1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & e^2+1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & e^2+1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^2+1 & \dots & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Then the characteristic equation becomes $(\xi - (e^2+1))^n(\xi + (e^2+1))^n = 0$. Hence, the spectrum is obtained as

$$Spec_{EDP}(K_{n\times 2}) = \begin{pmatrix} (e^2+1) & -(e^2+1) \\ n & n \end{pmatrix}.$$

Therefore,

$$EDP(\overline{K_{n\times 2}}) = 2n(e^2+1).$$

□

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