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2Ailamujia Distribution for Sum Two Independent Random Variables

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ABSTRACT: This research presents two novel random variables generated from the Ailamujia probability distribution. Mathematically, the proposed model (termed 2Ailamujia) represents the summation of two independent and identically distributed Ailamujia variables with shared parameters. Key statistical properties are rigorously derived, encompassing moment-generating functions, raw/incomplete moments, and characteristic functions. For parameter inference, we employ maximum likelihood estimation (MLE) and assess its asymptotic efficiency via comprehensive Monte Carlo experiments.

Key Words: Ailamujia distribution, moment, Maximum likelihood method, characteristic function, incomplete moments.

Contents

1. Introduction

In applied sciences like engineering, medicine, and biomedical research, among others, real-life data analysis and modeling are essential. For this reason. Finding statistical distributions to deal with these actual model data became extremely urgent. The Ailamujia Distribution is one of these distributions; it was first presented by [10] for a number of engineering applications. Using a small sample size, [9] additionally, this distribution was examined hypothesis testing and interval estimation.

In the past few years, numerous authors have expanded, altered, and combined the Ailamujia Distribution with other distinct distributions. Referred to the new three parameters - Ailamujia Distribution [11], by [4] suggest the Generalized Ailamujia Distribution. Additionally, some interested researchers have introduced additional studies that have been applied to Ailamujia Distribution in an effort to enhance its performance. For instance, [3] introduced the weighted Ailamujia Distribution and its characteristics, [8] introduced power Ailamujia Distribution, [15] introduced the two-parameter Ailamujia distribution, Inverse Analogue of Ailamujia Distribution was introduced by [2], transmuted Ailamujia Distribution by [1].

Ailamujia Distribution has also been given special treatment in recent years. We can direct the reader to [9] for statistical developments regarding parameter estimation using statistical tests and confidence

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intervals, The Ailamujia Distribution's application to maintenance-decision-oriented modeling is demonstrated in Reference [6]. The parameter estimation using Bayesian techniques is referenced in [5], and The parameter's minima estimation under various loss functions is developed in reference [12]. This study will introduce a distribution of the sum for two independent variable [13] that are identical and have the same Ailamujia distribution and same parameters. "The cumulative distribution function (cdf)" , and "Probability Density Function (pdf)" for a 2Ailamujia distribution are supplied followed :

$$G^{*}(Z) = 1 - (1+\theta Z)\, e^{-\theta Z} \quad Z, \theta > 0 \tag{1.1}$$

$$g^{*}(Z) = \theta^{2} Z e^{-\theta Z} \quad Z, \theta > 0 \tag{1.2}$$

2. Methodolgy

In this This section, we determine the sum for each of the two independent random variables that follow the Ailamujia distribution. The discussion highlights the identical definitions of these distributions and examines specific characteristics associated with their sum.

Definition 2.1. *A review of the PDF provided by:*

$$f(x) = \frac{\theta^{4}}{6} x^{3} e^{-\theta x} \theta > 0, x > 0 \tag{2.1}$$

The following is this distribution's feature: Assume that $Z1$ and Z_2 are two independent (r.v) with parameter θ that follow the Ailamujia Distribution (AD). Then, In the pdf provided by (2.1), the random variable $X = Z_1 + Z_2$ has been applied (see Figure 1).

Proof: Given that Z_1 and Z_2 are (i.r.v),. We can find a (pdf) for the new distribution by following the steps below:

$$\text{"} f(x) = \int_{-\infty}^{+\infty} g^{*}(x-t) \times g^{*}(t) dt\text{"}$$

$$f\ (x) = \int_{0}^{x} \theta^{2}\,(\ x - t)\, e^{-\theta(x-t)} . \theta^{2} t e^{-\theta t}\ dt$$

$$f(x) = \frac{\theta^{4}}{6} x^{3} e^{-\theta x}\ \theta > 0\ ,\ x > 0$$

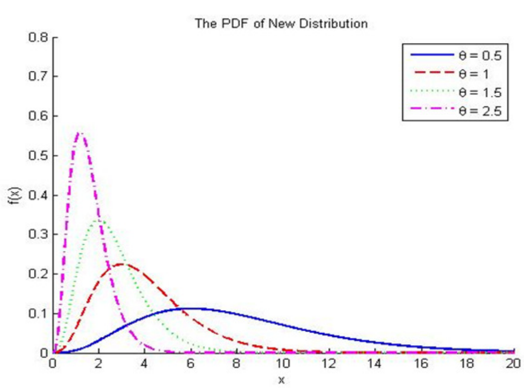


Figure 1: Plot the pdf for new distribution

The CDF of the 2Ailamujia Distribution is obtained by performing some algebraic operations (see Figure 2).

$$F(x) = \int_{0}^{x} f(t)\, dt$$

$$F(x) = \int_0^x \frac{\theta^4}{6} t^3 e^{-\theta t} dt$$

$$F(x) = 1 - \left(1 + \theta x + \frac{\theta^2 x^2}{2} + \frac{\theta^3 x^3}{6}\right) e^{-\theta x} \quad \theta > 0,\ x > 0$$

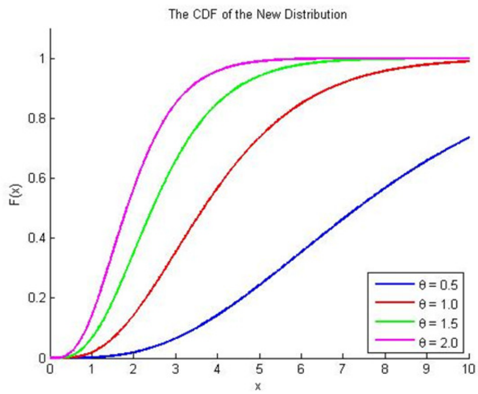


Figure 2: Plotting The CDF to the 2Ailamujia Distribution

Furthermore, the following is the survival function (See Figure 3).

$$S(x) = \left(1 + \theta x + \frac{\theta^2 x^2}{2} + \frac{\theta^3 x^3}{6}\right) e^{-\theta x}$$

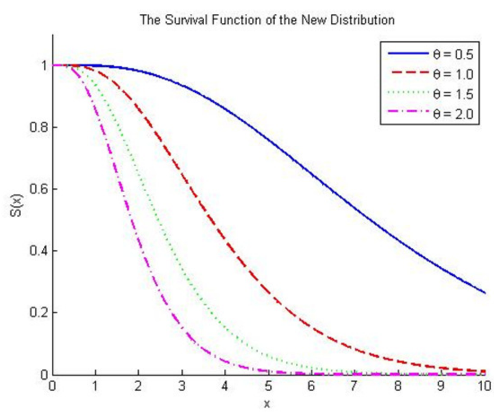


Figure 3: Plotting The Survival for the 2Ailamujia distribution

Here is the hezard rate funcation (hrf):

$$h(x) = \frac{f(x)}{S(x)}$$

$$h(x) = \frac{\frac{\theta^4}{6} x^3 e^{-\theta x}}{\left(1 + \theta x + \frac{\theta^2 x^2}{2} + \frac{\theta^3 x^3}{6}\right) e^{-\theta x}}$$

$$h(x) = \frac{\theta^4 x^3}{6 + 6\theta x + 3\theta^2 x^2 + \theta^3 x^3}$$

Additionally, the associated cumulative hazard rate function is provided by:

$$\Omega(x) = -ln(S(x))$$
$$\Omega(x) = -\left[ln\left(1 + \theta x + \frac{\theta^2 x^2}{2} + \frac{\theta^3 x^3}{6}\right) + ln\left(e^{-\theta x}\right)\right]$$
$$\Omega(x) = -\left[ln\left(1 + \theta x + \frac{\theta^2 x^2}{2} + \frac{\theta^3 x^3}{6}\right) - \theta x\right]$$
$$\Omega(x) = \theta x - ln\left(1 + \theta x + \frac{\theta^2 x^2}{2} + \frac{\theta^3 x^3}{6}\right)$$
$$\Omega(x) = \theta x - ln\left(\sum_{k=0}^{3} \frac{(\theta x)^k}{k!}\right)$$

By solving the following equation with respect to x, We determined the mode for the 2Ailamujia distribution. See Figure 4.

$$f(x) = \frac{\theta^4}{6} x^3 e^{-\theta x}$$
$$lnf(x) = 4ln\theta - ln6 + 3lnx - \theta x$$
$$\frac{\partial}{\partial x} lnf(x) = 0 - 0 + \frac{3}{x} - \theta = 0$$
$$x = \frac{3}{\theta}$$

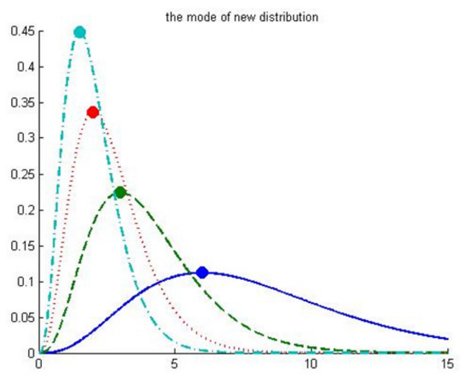


Figure 4: The Mode of New distribution

□

2.1. Moments

A moment in statistics represents a mathematical metric employed to characterize the properties of a statistical distribution concerning a dataset. The moment about the origin for the 2Ailamujia distribution has been computed as:

$$\mu_r^* = \frac{1}{6\theta^r}((r+4)$$

Proof: The definition of the gamma function is $\Gamma(x)=\int_0^\infty t^{x-1}e^{-t}dt,\ x>0$. By using Eq.(2.1), we get:

$$\mu_r^*=E\left(x^r\right)=\int_0^\infty x^r f\left(x\right)dx$$

$$\mu_r^*=E\left(x^r\right)=\int_0^\infty x^r\frac{\theta^4}{6}x^3e^{-\theta x}dx$$

$$\mu_r^*=\frac{\theta^4}{6}\int_0^\infty x^{r+3}e^{-\theta x}dx$$

$$z=\theta x\ \rightarrow x=\frac{z}{\theta}\ \rightarrow dx=\frac{1}{\theta}dz$$

Then

$$\mu_r^*=\frac{\theta^4}{6}\int_0^\infty \frac{z}{\theta}^{r+3}e^{-z}.\frac{1}{\theta}dz$$

$$\mu_r^*=\frac{\theta^4}{6}\int_0^\infty \left(\frac{z}{\theta}\right)^{r+3}e^{-z}.\frac{1}{\theta}dz$$

$$\mu_r^*=\frac{1}{6\theta^r}\int_0^\infty (z)^{r+4-1}e^{-z}dz$$

$$\mu_r^*=\frac{1}{6\theta^r}\Gamma\left(r+4\right)\ \theta>0,\ r>-4$$

$$\mu_1^*=\frac{1}{6\theta}\Gamma\left(5\right) \tag{2.2}$$

$$\mu_2^*=\frac{1}{6\theta^2}\Gamma\left(6\right),\mu_3^*=\frac{1}{6\theta^3}\Gamma\left(7\right),\mu_4^*=\frac{1}{6\theta^4}\Gamma\left(8\right)$$

Specifically, the $\bar{X}$ is given by $\mu=\mu_1^*$, and we gets $V(x)$ by:

$$V\left(x\right)=\sigma^2=\mu_2^*-\mu_1^*=\frac{1}{6\theta^2}\Gamma\left(6\right)-\frac{1}{6\theta}\Gamma\left(5\right)$$

We provide the Kurtosis and Skewnass coefficients of x, respectively:

$$C.S=\left(\frac{1}{\sigma^3}\sum_{k=0}^{3}\binom{3}{k}\mu_k^*\left(-\mu\right)^{3-k}\right)^2.$$

$$K.S=\frac{1}{\sigma^4}\sum_{k=0}^{4}\binom{4}{k}\mu_k^*\left(-\mu\right)^{4-k}.$$

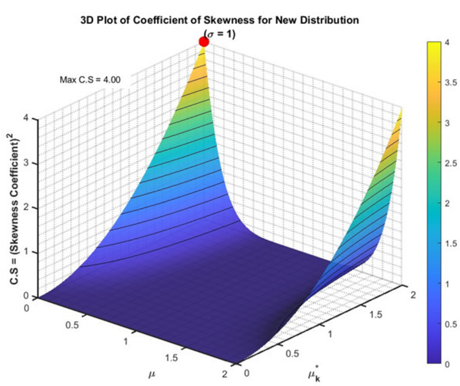


Figure 5: 3D plot of Coefficient of Skewedness for Now distribution

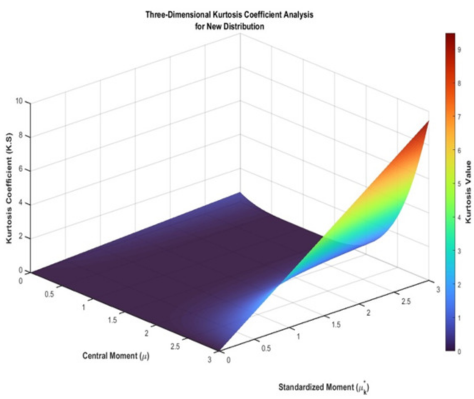


Figure 6: 3D plot of kurtosis coefficient for New distribution

2.2. Incompletes Moments

Let X be a (r.v) with parameter θ that follows the 2Ailamujia distribution, and let r be a positive integer. The lower gamma function, which is defined by

$$\Gamma\left(y, s\right) = \int_0^y t^{s-1} e^t dt \;\; s \; > 0, \;\; , t > 0$$

$$\mu_r^*\left(t\right) = E\left(x^r{}_{|\{x \leq t\}}\right)$$

$$\mu_r^*\left(t\right) = \int_0^t x^r f\left(x\right) dx$$

$$\mu_r^*\left(t\right) = \int_0^t x^r \frac{\theta^4}{6} x^3 e^{-\theta x} dx$$

$$\mu_r^*\left(t\right) = \frac{\theta^4}{6} \int_0^t x^{r+3} e^{-\theta x} dx$$

$$\mu_r^*\left(t\right) = \frac{\theta^4}{6} . \frac{\theta^2}{6} \frac{\Gamma\left(r+4, \;\theta t\right)}{\theta^{r+4}}$$

$$\mu_r^*\left(t\right) = \frac{1}{36 \; \theta^{r-2}} . \left(\left(r+4, \;\theta t\right)\right.$$

□

2.3. The Function of Moment Generating

The function that moment generating $\mu_x\left(t\right)$ for the random variable X∼2Ailamujia Distribution (θ) is provided by:

$$\mu_x\left(t\right) = \frac{\theta^4}{\left(\theta - t\right)^4}$$

Proof: With parameter (θ) and a (r.v) X that follows a 2Ailamujia Distribution, the (MGF) of X for any $t \in R$ is given by:

$$\mu_x\left(t\right) = \int_0^\infty e^{tx} f\left(x\right) dx$$

$$\mu_x\left(t\right) = \int_0^\infty e^{tx} \frac{\theta^4}{6} x^3 e^{-\theta x} dx$$

$$\mu_x\left(t\right) = \frac{\theta^4}{6} \int_0^\infty x^3 e^{-x(\theta - t)} dx$$

Let:

$$Z = x(\theta - t), dZ = (\theta - t)\,dx, dx = \frac{1}{\theta - t}dz$$
$$x^3 = \left(\frac{z}{\theta - t}\right)^3$$
$$\mu_x(t) = \frac{\theta^4}{6}\int_0^\infty \left(\frac{z}{\theta - t}\right)^3 e^{-z}\frac{1}{\theta - t}dz$$
$$\mu_x(t) = \frac{\theta^4}{6(\theta - t)^4}\Gamma(4)$$
$$\mu_x(t) = \frac{\theta^4}{(\theta - t)^4}$$

□

2.4. Characteristic Function

The expected value of the complex exponential function e^{itx}, where (i) is the imaginary unit, is known as the characteristic function $\Phi_x(t)$:

$$i = \sqrt{-1}$$

Then

$$\Phi_x(t) = \frac{\theta^4}{(\theta - it)^4}$$

2.5. Lorenz and Bonferreni Curves

Lorenz and Bonferreni curve suggested by [14]. They are employed to quantify how unequal a random variable X distribution is. It has applications in a wide range of domains, including reliability, economics, insurance, and demography. Here is the definition of the Lorenz and Bonferroni curves:

$$B(p) = \frac{1}{p\mu}\int_0^q xf(x)\,dx, \ \ L(p) = \frac{1}{\mu}\int_0^q xf(x)\,dx$$

Where

$$q = F^{-1}(p).$$

We can compute Bonferreni and Lorenz curves of x∼2 Ailamujia (θ)Distributions If x has the pdf in Eq.(2.1), as:

$$B(p) = \frac{1}{p\mu} * \mu_1^*(q), \ \ L(p) = \frac{1}{\mu} * \mu_1^*(q)$$

where: $\mu_1^*(q) = E\left(x/_{x\le q}\right)$ defined in eq. (2.2).

2.6. Order Statistic

Given a set of (iid) random variables $X_1, X_2, X_3, \ldots, X_n$, which are selected from a 2Ailamyjia distribution (θ), "the order statistics" are represented as "$X_{(1)}, X_{(2)}, X_{(3)}, \ldots, X_{(n)}$" represent the sorted values of data set. Where: $\{X_{(1)}\}$: min.value} and $\{X_{(n)}$: Max.value}. The pdf of the r-th order statistic $X(r)$ is given by:

$$f_{x(r)}(x, \theta) = \frac{n!}{(r-1)!\,(n-r)!}\frac{\theta^4}{6}x^3 e^{-\theta x}$$
$$\left[1 - \frac{-\theta^3 x^3 e^{-\theta x} - 3\left(\theta^2 x^2 e^{-\theta x} - 2\left(-\theta x e^{-\theta x} - e^{-\theta x}\right)\right) + 6}{6}\right]^{r-1}$$
$$\left[1 - 1 - \frac{-\theta^3 x^3 e^{-\theta x} - 3\left(\theta^2 x^2 e^{-\theta x} - 2\left(-\theta x e^{-\theta x} - e^{-\theta x}\right)\right) + 6}{6}\right]^{n-r}$$

Following that, the first- order $X_{(1)}$ 2Ailamujia distribution's pdf is provided by:

$$f_{x(1)}(x,\theta)=\frac{n!}{(n-1)!}\frac{\theta^4}{6}x^3e^{-\theta x}$$
$$\left[1-1-\frac{-\theta^3x^3e^{-\theta x}-3\left(\theta^2x^2e^{-\theta x}-2\left(-\theta xe^{-\theta x}-e^{-\theta x}\right)\right)+6}{6}\right]^{n-1}$$

And the pdf of nth order $X_{(n)}$ 2Ailamujia model is given as:

$$f_{x(r)}(x,\theta)=n\frac{\theta^4}{6}x^3e^{-\theta x}\left[1-\frac{-\theta^3x^3e^{-\theta x}-3\left(\theta^2x^2e^{-\theta x}-2\left(-\theta xe^{-\theta x}-e^{-\theta x}\right)\right)+6}{6}\right]^{n-1}$$

2.7. Estimation for Parameter

The estimation for parameter is a fundamental concept in statistics and data analysis, focusing on a process for used sample data to infer the properties of a population.

There are various methods for parameter estimation, including "method of moments" and "Maximum Likelihood Estimation (MLE)".

2.7.1. Method of Moments. The method of moments is a technique utilized for estimating paramaters in statistics. This method relies on the use of statistical moments, which are expected values of various functions of random variables.

$$\mu_1^*=\frac{\sum_{i=1}^n x_i}{n}$$
$$\bar{x}=\frac{1}{6\theta}\Gamma(5)=\frac{4}{\theta}\rightarrow\hat{\theta}=\frac{4}{\bar{x}}$$

2.7.2. Maximum Likelihood Estimation Method (MLE). Assume $(X_1,X_2,\ldots,X_n)$ be independent and identically distributed random variables representing a random sample of size n from the 2Ailamujia Distribution. Then, the likelihood function can be expressed as follows:

$$L(x,\theta)=\prod_{i=1}^{n}\left(\frac{\theta^4}{6}x^3e^{-\theta x}\right) \tag{2.3}$$

By taking the logarithm of Eq. (2.3), the log-likelihood function can be expressed as:

$$lnL(x,\theta)=4nln\theta-ln6+\left(3ln\sum_{i=1}^{n}x_i\right)-\theta x$$

We obtained the estimates $\hat{\theta}$ after maximizing $l(x,\theta)$ to θ.

$$\frac{\partial lnL(x,\theta)}{\partial\theta}=\frac{4n}{\hat{\theta}}-x=0$$
$$\frac{4n}{\hat{\theta}}=x$$
$$\hat{\theta}=\frac{4n}{x}$$

2.7.3. Cramer-Von Mises Method. Let $x_1,x_2,\ldots,x_n$ be a sample size (n) of sorted data, the formula for the method is:

$$C(\theta)=\frac{1}{12n}+\sum_{i=1}^{n}\left(F(x_i,\theta)-\frac{2i-1}{2n}\right)^2$$
$$C(\theta)=\frac{1}{12n}+\sum_{i=1}^{n}\left(1-\left(1+\theta x+\frac{\theta^2x^2}{2}+\frac{\theta^3x^3}{6}\right)e^{-\theta x}-\frac{2i-1}{2n}\right)^2$$

$$\frac{\partial C(\theta)}{\partial \theta} = \frac{\theta^3}{3} \sum_{i=1}^{n} \left(1 - \left(1 + \theta x_i + \frac{\theta^2 x_i^2}{2} + \frac{\theta^3 x_i^3}{6} \right) e^{-\theta x_i} - \frac{2i-1}{2n} \right) x_i^4 e^{-\theta x_i}$$

When

$$\frac{\partial C(\theta)}{\partial \theta} = 0,$$

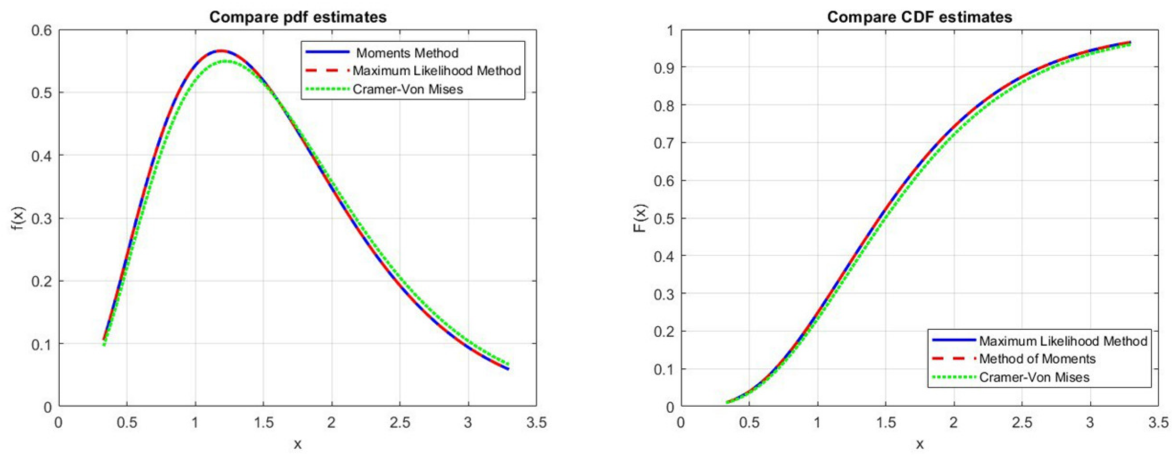
we get

$$\sum_{i=1}^{n} \left(1 - \left(1 + \theta x_i + \frac{\theta^2 x_i^2}{2} + \frac{\theta^3 x_i^3}{6} \right) e^{-\theta x_i} - \frac{2i-1}{2n} \right) x_i^4 e^{-\theta x_i} = 0$$

Because the **Eq. ( )** is nonlinear in θ than we cannot solved ananlytically. Therfor, we will solve by numerical methods by (Newton – Raphson Method).

$$\theta_{i+1} = \theta_i - \frac{G(\theta_i)}{G'(\theta_i)}$$

Where: θ_0 is initial value; $\theta_0 = 1$.



(a) Compare PDFs estimates

(b) Compare CDFs estimates

Figure 7: Compared for pdf and CDF of three Estmation methods

From the Figure 7a,b shows consistency of results, where we can use all estimation methods because they lead to the same result and the model fits data well.

3. Applications and Goodness of Fit

In this study, we utilize dataset to demonstrate the effectiveness for the suggested 2Ailamujia Distribution. Furthermore, we compare the performance of the suggested distribution with that of the original distribution, Weibull Distribution, and Exponential Distributions.

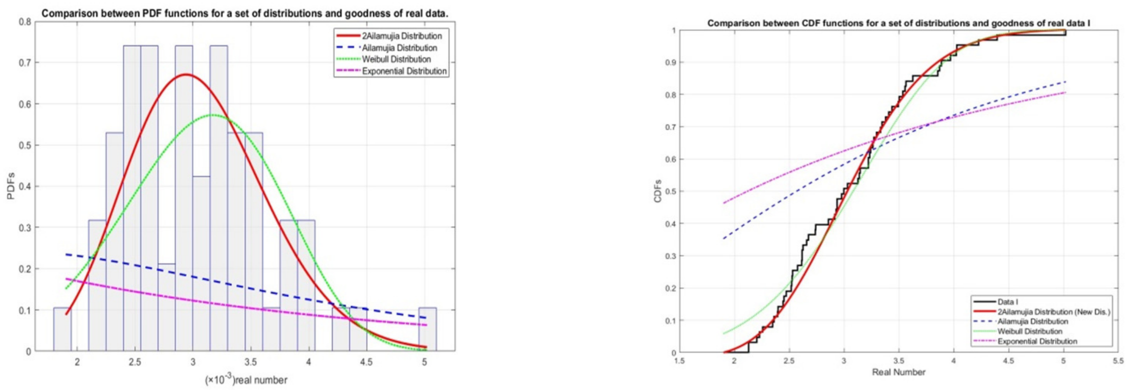
The real dataset pertains to the single fibers were tested under tension at gauge lengths of 10 mm", with a sample size of 63. This dataset comprises a series of recorded observations:

Table 1: The first real data set (*10^{-3}).

1901	2132	2203	2228	2257	2350	2361	2396	2397	2445	2454
2474	2518	2522	2525	2532	2575	2614	2616	2618	2624	2659
2675	2738	2740	2856	2917	2928	2937	2937	2977	2996	3030
3125	3139	3145	3220	3223	3235	3243	3264	3272	3294	3332
3346	3377	3408	3435	3493	3501	3537	3554	3562	3628	3852
3871	3886	3971	4024	4027	4225	4395	5020			

This data set is previously used by [7].

Figure 8a,b demonstrates that the Ailamujia distribution exhibits better flexibility in handling deviations at high values. In comparison, we observed that the Weibull and exponential distributions provided higher accuracy than other classical distributions.



(a) Comparison between pdfs functions for set of distriontionsand goodnessof real number

(b) Comparison between CDFs functions for set of distriontionsand goodnessof real number

Figure 8: Estimation pdf and Cdf Functions for real data

Table 2: Goodness-of-fit analysis for normal distribution: summary of statistical tests.

Test	Data I	
	Statistic	P-Value
Anderson-Darling	0.381	0.392
Baringhaus-Henze	0.428	0.193
Cramér-von Mises	0.053	0.410
Jarque-Bera ALM	2.107	0.349
Mardia Kurtosis	0.514	0.473
Mardia Skewness	1.593	0.207
Pearson Chi2	9.872	0.273
Shapiro-Wilk	0.981	0.275
Kolmogorov-Smirnov	0.072	0.511

From Table 2 The normality goodness-of-fit analyses indicate, show all statistical tests high p-values, therfor suggesting the dataset consistent with a normal distribution. And show consistency for results: like Kolmogorov-Smirnov tests.

Table 3: Descriptive statistics summary.

Coefficients	Data
Mean	3.059
Variance	0.386
Skewness	0.633
Kurtosis	3.286
Standard Deviation	0.621
Median	2.996
Max	5.020
Min	1.901

From Table 3 we estimated the unknown parameters using the Maximum Likelihood Method, employing the following metrics: the negative twice log-likelihood function ($-2l(\cdot)$), Akaike Information Criterion (AIC), Corrected Akaike Information Criterion (CAIC), Bayesian Information Criterion (BIC), and the Kolmogorov-Smirnov (K-S) test. These criteria are calculated as follows:

$$AIC = -2l(\cdot) + 2k, CAIC = AIC + [2k(k+1)]/(n-k-1), BIC = -2l(\cdot) + k \cdot ln(n)$$

The results demonstrated the clear superiority of the 2Ailamujia model compared to other distributions, recording the lowest values across all criteria ($-2\log(\cdot) = 113.76, AIC = 117.76, CAIC = 117.96$, and $BIC = 122.04$), while the Exponential distribution ranked last ($AIC = 268.89$).

Table 4: Descriptive statistics summary.

Distributions	-2log(.)	AIC	CAIA	BIS
2Ailamujia	113.76	117.76	117.96	122.04
Ailamujia	220.7	222.7	222.76	224.84
Weibull	123.91	127.91	128.11	132.2
Exponential	266.89	268.89	268.96	271.03

4. Conclusion

In this study, We obtained new distribution of the sum of two independent random variables are distrubition according to the same probability distriobution function. The new distribution called 2Ailumjia distributions. We explored the statistical properties of the 2Ailumjia distribution. including moments, incomplete moments, the characteristic function, and order statistics. Additionally, a detailed analysis of the statistical properties of the 2Ailumjia distribution was conducted.

The results that the statistical model fits the data well, with no statistical indications of inadequacy to evaluate and compare the goodness-of-fit between models, and the new distriobution showed high flexiblity in representing the real data.

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