Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2025 (43)** : 1–7.
ISSN-0037-8712
doi:10.5269/bspm.77813

Using a new complex integral transform for solving some scientific applications

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ABSTRACT: This research paper aims to provide solutions to some scientific problems using a new complex integral transformation, which provides easy-to-use and effective tools for solving a wide range of problems in various branches of science. This transformations was first introduced in 2022 and has proven highly effective in finding accurate solutions to differential and integral equations, in addition to its applications in applied sciences such as bacterial growth and chemistry.

Key Words: Complex integral transformation, bacterial growth, differential equations, scientific modeling, chemical applications.

Contents

1. Introduction

Integral transformations have made significant contributions to the scientific field. This was evident in the use of integral transformations in solving differential and integral equations as well as in solving equations in applied sciences. These transformations, the most important of which are the Laplace and Fourier transforms and other transformations that have recently appeared such as the Somodo transform and others, were able use simplified techniques to arrive at precise solution [7,6,1,9,10,8].

In 2022 a new complex integral transform was suggested to solve mathematical and some scientific problems which has proven its high efficiency in finding accurate solutions [11]. Therefore, in this research, we presented solutions to some scientific problems using this new transformation.

Definition 1.1. *Let $f(t)$ be a function that, for $t \geq 0$ is integrated and, $p(\mathfrak{s}) \neq 0$ is a real function that is positive, $im(q(s)) < 0, i \in \mathbb{C}$, while the general complex integral transform is referred by $T_g^c(\mathfrak{s})$ of $f(t)$ determined as formula:*

$$T_g^c\{f(t);\mathfrak{s}\} = F_g^c(\mathfrak{s}) = p(\mathfrak{s}) \int_{t=0}^{\infty} e^{-iq(\mathfrak{s})t} f(t)\, dt. \tag{1.1}$$

If, for any $q(s)$, the integral exists [11]

Some basic function's new complex integral transform are follows:

$$T_g^c\{t^n\} = (-i)^{n+1} \frac{n!\; p(s)}{[q(s)]^{n+1}},\; n \in \mathbb{N}. \tag{1.2}$$

$$T_g^c\{\, e^{\alpha t}\} = -p(s)\left[\frac{a}{a^2 + (q(s))^2} + i\frac{q(s)}{a^2 + (q(s))^2}\right],\; \alpha \in \mathbb{R}. \tag{1.3}$$

$$T_g^c\{sin(\alpha t)\} = \frac{-a\; p(s)}{(q(s))^2 - a^2}. \tag{1.4}$$

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2010 *Mathematics Subject Classification:* 62Pxx, 62P99.
Submitted July 12, 2025. Published September 01, 2025

$$T_g^c\{cos(\alpha t)\} = \frac{-i\ p(s)\ \ q(s)}{(q(s))^2 - a^2}. \tag{1.5}$$

$$T_g^c\{sinh(\alpha t)\} = \frac{-a\ p(s)}{(q(s))^2 + a^2} \tag{1.6}$$

$$T_g^c\{cosh(\alpha t)\} = \frac{-i\ p(s)\ \ q(s)}{(q(s))^2 + a^2} \tag{1.7}$$

The recently developed complex integral derivative transform:
Let $T_g^c\{f(t)\} = F_g^c(s)$, then

i. $T_g^c\{f'(t)\} = iq(s)\,F_g^c(s) - f(0)\,p(s)\,.$

ii. $T_g^c\{f''(t)\} = (iq(s))^2\,F_g^c(s) - p(s)\,f'(0) - iq(s)\,p(s)\,f(0)\,.$

iii. $T_g^c\left\{f^{(n)}(t)\right\} = (iq(s))^n\,F_g^c(s) - p(s)\left[\sum_{j=1}^{n-1}(iq(s))^{n-1-j}\,f^{(j)}(0)\right],\ n = 1, 2, 3,\ \ldots.$

2. The Bacteria Growth Model

To ascertain development of the bacteria in practical culture, have a look at Malthus model [2,4,3]. This model starts that the amount of bacteria present at any one moment is proportionate to pace at which they develop in practical culture. In terms of mathematics, it can be expressed as:

$$\frac{d\varpi}{dt} = K\varpi. \tag{2.1}$$

With initial condition

$$\varpi(t_0) = \varpi_{0.} \tag{2.2}$$

Where $K \in \mathbb{R}^+, \varpi$ is the number of bacteria at time t and ϖ_0 is the number of bacteria at initial time t_0.

Equation (2.1) is a first order linear ordinary differential equation. This equation together with Eq. (2.2) becomes an initial value problem.

2.1. Application

Application 2.1. *Estimate the initial number of bacteria in practical culture if, after two years, the number of bacteria has doubled and after three years became 20,000. Bacteria in practical culture grow at a rate proportional to number of bacteria currently living in a practical culture [5].*

$$\frac{d\varpi}{dt} = K\varpi. \tag{2.3}$$

Take the recently developed complex integral derivative transform for Eq. (2.3), we get

$$T_g^c\left\{\frac{d\varpi}{dt}\right\} = KT_g^c\{\varpi(t)\}\,. \tag{2.4}$$

Using new complex integral transform to derivative a function property on Eq. (2.4), we obtain:

$$iq(s)\,T_g^c\{\varpi(t)\} - p(s)\,\varpi(0) = KT_g^c\{\varpi(t)\}\,. \tag{2.5}$$

Since at $t = 0, \varpi = \varpi_0$, so using this in Eq. (2.5), we have:

$$T_g^c\{\varpi(t)\}\,[iq(s) - K] = p(s)\,\varpi_0.$$

Then,

$$T_g^c\{\varpi(t)\} = \frac{p(s)\,\varpi_0}{[iq(s) - K]}. \tag{2.6}$$

Operating inverse of $\left\{T_g^c\left\{\varpi\left(t\right)\right\}\right\}^{-1}$ on both sides of (2.6), to get a form:

$$\begin{aligned}\varpi\left(t\right) &= \varpi_0\ T_g^{c-1}\left\{\frac{p\left(s\right)}{\left[iq\left(s\right)-K\right]}\right\},\\ &= \varpi_0\ T_g^{c-1}\left\{\frac{p\left(s\right)}{\left[iq\left(s\right)-K\right]}\bullet\frac{K+iq\left(s\right)}{K+iq\left(s\right)}\right\},\\ &= \varpi_0\ T_g^{c-1}\left\{\frac{p\left(s\right)\left[K+iq\left(s\right)\right]}{K^2+\left(q\left(s\right)\right)^2}\right\},\end{aligned}$$

$$\varpi\left(t\right)=\varpi_0 e^{Kt}.\forall t\geq 0 \tag{2.7}$$

At $t=2,\ \varpi=2\varpi_0$, in order to use Eq. (2.7), to get:

$$2\varpi_0=\varpi_0 e^{2K}\Rightarrow e^{2K}=2$$

$$K=0.5\log_e 2=0.347 \tag{2.8}$$

Again at $t=3,\varpi=20,000$, in Eq. (2.7), to get:

$$20,000=\varpi_0 e^{3K}, \tag{2.9}$$

Substitute value of K to Eq. (2.7) in (2.8), to get

$$20,000=\varpi_0 e^{3\times 0.347},$$

$$\varpi_0\cong 7062$$

which gives the required number of bacteria initially in a certain culture.

Application 2.2. *A culture's bacteria population grows at a pace corresponds to its size. Calculate how many bacteria are in certain culture after 1.5 hours if number G rises from 1000 to 2000 in a single hour [5].*

The given problem given in mathematical form as:

$$\frac{d\varpi}{dt}=K\varpi. \tag{2.10}$$

Take the recently developed complex integral derivative transform for Eq. (2.10), we get

$$T_g^c\left\{\frac{d\varpi}{dt}\right\}=KT_g^c\left\{\varpi\left(t\right)\right\}. \tag{2.11}$$

Using new complex integral transform to derivative a function property on Eq. (2.11), we obtain:

$$iq\left(s\right)T_g^c\left\{\varpi\left(t\right)\right\}-p\left(s\right)\varpi\left(0\right)=KT_g^c\left\{\varpi\left(t\right)\right\}. \tag{2.12}$$

Since at time $t=0,\varpi=1000$, so using this in Eq. (2.12), we have

$$T_g^c\left\{\varpi\left(t\right)\right\}\left[iq\left(s\right)-K\right]=1000p\left(s\right).$$

$$T_g^c\left\{\varpi\left(t\right)\right\}=\frac{1000p\left(s\right)}{\left[iq\left(s\right)-K\right]}. \tag{2.13}$$

Operating inverse of the integral transform on both sides of (2.13), so we have:

$$\varpi\left(t\right)=1000\ T_g^{c-1}\left\{\frac{p\left(s\right)}{\left[iq\left(s\right)-K\right]}\right\},$$

$$\varpi(t) = 1000e^{Kt}. \ \ \forall t \geq 0 \tag{2.14}$$

Now at $t = 1, \varpi = 2000$, so using this in Eq. (2.14), we obtain

$$2000 = 1000e^{K} \Rightarrow e^{K} = 2$$

$$K = \log_e 2 = 0.693 \tag{2.15}$$

Now at $t = 1.5, \varpi$ in a certain culture is given by Eq. (2.14) as:

$$\begin{aligned} \varpi(1.5) &= 1000e^{Kt}, \\ &= 1000e^{1.5\times 0.693}, \end{aligned} \tag{2.16}$$

$$\varpi(1.5) \cong 2827.80$$

3. Application in Chemical Science

This part apply the new complex integral transform to various phenomenon in chemical science.

Application 3.1. *In chemical sciences, mixing problems are crucial. Think about its below. A tank with a set capacity that is filled with a completely mixed solution (such as salt) is a common example of a mixing problems. After being fully agitated, solution with a certain concentration enters the tank at a defined average and exits at fixed rate that may be different from incoming average. The average at which the substance in introduced less rate at which it is withdrawn is $y\prime(t)$, when $y(t)$ represents the sum of substance in the tank at t. This station's mathematical description frequently results in a differential equation of the first order. Twenty kilograms of salt are dissolved in five thousand liters of water in tank. At a rate of 25L/min, brine continuing 0.03kg of salt per liter of water enters the tank. The solution drains from tank at same pace and is kept well mixed. After thirty minutes, how much salt is still in tank? [12].*

At initial time $t = 0, y(t)$ denote the sum of salt. Therefore tank contains 20 kg salt, i.e. $y(t)$ and sum of salt remaining after 30 min., i.e. $y(30)$

$$y'(t) = \frac{dy}{dt} = \text{ average of sum of salt.}$$

$$\frac{dy}{dt} = average\ in\ -\ average\ out. \tag{3.1}$$

$$average\ in = (0.03kg/L)(25L/min) = \frac{0.75kg}{min}.$$

$$average\ out = \left(\frac{y(t)}{5000}\right)\left(\frac{kg}{L}\right)(25)\left(\frac{L}{min.}\right) = \frac{y(t)}{200}\frac{kg}{min.}[12].$$

From Eq. (3.1)

$$\frac{dy}{dt} = 0.75\frac{kg}{min.} - \frac{y(t)}{200}\frac{kg}{min.}$$

$$\frac{dy}{dt} = \frac{3}{4} - \frac{y(t)}{200},$$

$$\frac{dy}{dt} + \frac{y(t)}{200} = \frac{3}{4},\ y(0) = 20. \tag{3.2}$$

Eq. (3.2) can be generalized as

$$\frac{dy}{dt} + ky(t) = r. \tag{3.3}$$

Take the recently developed complex integral for Equation (3.3), we have

$$T_g^c\left\{\frac{dy}{dt}\right\} + kT_g^c\{y(t)\} = rT_g^c\{1\}. \tag{3.4}$$

Using recently complex integral transform to derivative a function property on Eq. (3.4), we obtain:

$$iq(s)\,T_g^c\{y(t)\} - 20p(s) + kT_g^c\{y(t)\} = r\left(\frac{-ip(s)}{q(s)}\right).$$

Then,

$$T_g^c\{y(t)\} = \frac{\frac{-irp(s)}{q(s)} + 20p(s)}{iq(s)+k},$$

$$T_g^c\{y(t)\} = \frac{rp(s) + 20iq(s)p(s)}{iq(s)\left[iq(s)+k\right]}.$$

By partial fractional,

$$\frac{rp(s) + 20iq(s)p(s)}{iq(s)\left[iq(s)+k\right]} = \frac{A}{iq(s)} + \frac{B}{iq(s)+k},$$

We get

$$A = \frac{r}{k},\ B = 20 - \frac{r}{k}.$$

$$T_g^c\{y(t)\} = \frac{\left(\frac{r}{k}\right)p(s)}{iq(s)} + \frac{\left(20-\frac{r}{k}\right)p(s)}{iq(s)+k}$$

Applying inverse of the new complex integral transform on the last equation, we have:

$$\begin{aligned} y(t) &= \frac{r}{k}\,T_g^{c-1}\left\{\frac{p(s)}{iq(s)}\right\} + \left(20-\frac{r}{k}\right)T_g^{c-1}\left\{\frac{p(s)}{iq(s)+k}\right\}, \\ &= \frac{r}{k}\,T_g^{c-1}\left\{\frac{-ip(s)}{q(s)}\right\} + \left(20-\frac{r}{k}\right)T_g^{c-1}\left\{\frac{-p(s)}{-k-iq(s)}\bullet\frac{-k+iq(s)}{-k+iq(s)}\right\}, \end{aligned}$$

$$y(t) = \frac{r}{k}(1) + \left(20-\frac{r}{k}\right)e^{-kt}. \tag{3.5}$$

For exact solution according to the conditions given in the problem put $k = \frac{1}{200}$ and $r = \frac{3}{4}$,

$$y(t) = 150 - 130e^{\frac{-t}{200}}.$$

One way to determine half-life duration is to replace (t). by $\frac{y(0)}{2}$ in above equation.

Application 3.2.

Application in Organic Chemistry: Saponification

The minimal concentration level for sodium chloride waste in any liquid that is released into the environment must not exceed 11.00g/L in order to create "homemade" soap, according to the municipal regulation. The main waste product of operation is liquid water that contains sodium chloride. The company's waste storage tank is merely 15L. When the waste tank was filled, it held 750 gram of sodium chloride and 15L liters of water. Fresh water should be pumped into tank at rate 2 liters per minute in order to maintain production and comply with local regulations, while waste salt water, which contains 25g of salt per liter, would be injected at rate 1.5L per minute.

Waste is released at a rate of 3.5L per minute in order to maintain a solution level to 15L. It is assumed in the flow sketch that when streams A and B enter the tank, the chloride concentration in the tank instantly changes to the exit concentration, x.

In this case, A stands for the process waste stream, B for the fresh water stream, and C for the discharge stream to the environment.

The equation accumulation=input-output+removal via reaction may be used to express the material (sodium chloride) balance on the tank system [12]. Given that there is no chemical reaction taking place in the storage tank, the equation above may be expressed as:

$$\frac{dy}{dt} = (25g/L)(1.5L/min) + (0g/L)(2L/min) - \left(\frac{xg}{L}\right)(3.5L/min) + 0,$$

$$\frac{dy}{dt} = \frac{37.5g}{min} + 0 - 3.5xg/min,$$

Therefore,

$$\frac{dy}{dt} + 3.5x = 37.5. \tag{3.6}$$

For the initial condition of the ordinary differential Eq. (3.6) at $t = 0$ the salt conc. In tank gives as 750g/L, i.e., $\frac{50g}{L}$ [12].

By using recently complex integral transform for Eq. (3.6), we have

$$T_g^c\left\{\frac{dy}{dt}\right\} + 3.5T_g^c\{x\} = 37.5T_g^c\{1\},$$

Applying the property, the integral transform of derivative of function, on above equation, to get:

$$iq(s)\,T_g^c\{y(t)\} - 50p(s) + 3.5T_g^c\{y(t)\} = 37.5\left(\frac{-ip(s)}{q(s)}\right).$$

Then,

$$T_g^c\{y(t)\} = \frac{\frac{-i(37.5)p(s)}{q(s)} + 50p(s)}{iq(s) + 3.5},$$

$$T_g^c\{y(t)\} = \frac{37.5p(s) + 50iq(s)p(s)}{iq(s)\,[iq(s) + 3.5]}.$$

By partial fractional,

$$\frac{37.5p(s) + 50iq(s)p(s)}{iq(s)\,[iq(s) + 3.5]} = \frac{A}{iq(s)} + \frac{B}{iq(s) + 3.5},$$

We get

$$A = \frac{37.5}{3.5} = \frac{75}{7},\ B = 50 - \frac{75}{7} = \frac{275}{7}.$$

$$T_g^c\{y(t)\} = \frac{\left(\frac{75}{7}\right)p(s)}{iq(s)} + \frac{\left(\frac{275}{7}\right)p(s)}{iq(s) + 3.5}$$

Operating inverse of $\left\{T_g^c\{\varpi(t)\}\right\}^{-1}$ on both sides of above equation, to get a form:

$$\begin{aligned} y(t) &= \frac{75}{7}\,T_g^{c-1}\left\{\frac{p(s)}{iq(s)}\right\} + \frac{275}{7}T_g^{c-1}\left\{\frac{p(s)}{iq(s) + 3.5}\right\}, \\ &= \frac{75}{7}\,T_g^{c-1}\left\{\frac{-ip(s)}{q(s)}\right\} + \frac{275}{7}T_g^{c-1}\left\{\frac{-p(s)}{-3.5 - iq(s)} \bullet \frac{-3.5 + iq(s)}{-3.5 + iq(s)}\right\}, \end{aligned}$$

4. Conclusion

We used the new complex integral transform of scientific problems in this paper. The applications presented here demonstrate the significance of the suggested transformation of these applications. The results show that this transformation provides the accurate solution without any tedious calculation work.

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