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Unit groups of some multiquadratic number fields

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Abstract: Let q, p and s be three different prime integers such that $p \equiv 1 \pmod 8$, $q \equiv 3 \pmod 8$ and $s \equiv 7 \pmod 8$ with $\left(\frac{p}{q}\right) = \left(\frac{p}{s}\right) = \left(\frac{q}{s}\right) = -1$. In this paper we investigate the unit groups of the number fields of the form $\mathbb{L}^+ = \mathbb{Q}(\sqrt{2}, \sqrt{pq}, \sqrt{ps})$.

Key Words: Unit group, multiquadratic number fields, unit index.

Contents

1. Introduction

One of the central problems in algebraic number theory particularly in the study of unit groups of number fields, which are deeply connected to many aspects of the theory is the explicit computation of a fundamental system of units. For quadratic fields, this problem is relatively straightforward. In the case of certain quartic bicyclic fields, Kubota [13] proposed a method for determining such a system. Wada [14] later generalized Kubota's approach by developing an algorithm for computing fundamental units in arbitrary multiquadratic fields. However, in general, computing the unit group of a number field especially when the degree exceeds 4 is a difficult task. In several recent works, Azizi, Chems-Eddin, El Hamam, and Zekhnini carried out explicit computations to determine the unit groups of some number fields of degrees 8 and 16 (see [3,4,5,6,7,8,9]). All the fields studied in those works are of the form $\mathbb{Q}(\sqrt{2}, \sqrt{p}, \sqrt{q})$ and $\mathbb{Q}(\sqrt{2}, \sqrt{p}, \sqrt{q}, \sqrt{-\ell})$, where p and q are two different prime integers and ℓ a positive odd square-free integer. In [10,11,12], El Hamam determined the unit groups of fields of the form $\mathbb{L}^+ = \mathbb{Q}(\sqrt{2}, \sqrt{pq}, \sqrt{ps})$ and $\mathbb{L} = \mathbb{Q}(\sqrt{2}, \sqrt{pq}, \sqrt{ps}, \sqrt{-\ell})$, where p, q, and s are distinct prime numbers satisfying certain conditions.

In the present work, shall determine the unit group of fields k of degree 8 and 16 of the form $\mathbb{Q}(\sqrt{2}, \sqrt{pq}, \sqrt{ps})$, where p, q, and s are distinct prime numbers satisfy the following conditions $p \equiv 1 \pmod 8$, $q \equiv 3 \pmod 8$ and $s \equiv 7 \pmod 8$ with $\left(\frac{p}{q}\right) = \left(\frac{p}{s}\right) = \left(\frac{q}{s}\right) = -1$.

We note that the computation of the unit group of these fields may be very important to deal with the problem of the 2-class field tower of biquadratic number fields (see for example [1]).

In the rest of this article we use the following notations: let ε_m denote the fundamental unit of the quadratic field $\mathbb{Q}(\sqrt{m})$, $\left(\frac{\cdot}{\cdot}\right)$ the Legendre Symbol and $h(k)$ (resp. $h_2(k)$) the class number (resp. 2-class number) of a number field k. Finally, let $h_2(d)$ denote the 2-class number of the quadratic field $\mathbb{Q}(\sqrt{d})$ and ζ_n denote a primitive n-th root of unity.

2. Preliminary results

Let us first recall the result stated by Wada [14] to determine a fundamental system of units of a multiquadratic number field. Let K_0 be a multiquadratic number field. Denote by σ and τ two different generators of the group $\mathrm{Gal}(K_0/\mathbb{Q})$, let then K_1, K_2 and K_3 be respectively the invariant subfields of K_0 by σ, τ and $\sigma\tau$, and E_{K_i} the unit group of K_i. Then the unit group E_{K_0} of K_0 is generated by the

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elements of each E_i and the square roots of elements of the product $E_{K_1}E_{K_2}E_{K_3}$ that are perfect squares in K_0.

We close this section by stating the following lemmas.

Lemma 2.1. *([2, Lemma 2.4]) Let $p \equiv 1 \pmod 8$ and $q \equiv 3 \pmod 8$ be two primes such that $\left(\frac{p}{q}\right) = -1$.*

1. *Let x and y be two integers such that $\varepsilon_{2pq} = x + y\sqrt{2pq}$. Then*
   a. *$(x-1)$ is a square in $\mathbb{N}$,*
   b. *$\sqrt{2\varepsilon_{2pq}} = y_1 + y_2\sqrt{2pq}$ and $2 = -y_1^2 + 2pqy_2^2$, for some integers y_1 and y_2.*
2. *There are two integers v and w such that $\varepsilon_{pq} = v + w\sqrt{pq}$. Then*
   a. *$(v-1)$ is a square in $\mathbb{N}$,*
   b. *$\sqrt{2\varepsilon_{pq}} = w_1 + w_2\sqrt{pq}$ and $2 = -w_1^2 + pqw_2^2$, for some integers w_1 and w_2.*

Lemma 2.2. *([7, Lemma 3.1]) Let $p \equiv 1 \pmod 8$ and $s \equiv 7 \pmod 8$ be two primes such that $\left(\frac{p}{s}\right) = -1$.*

1. *Let x and y be two integers such that $\varepsilon_{ps} = x + y\sqrt{ps}$. Then*
   a. *$x+1$ is a square in $\mathbb{N}$,*
   b. *$\sqrt{2\varepsilon_{ps}} = y_1 + y_2\sqrt{ps}$ and $2 = y_1^2 - psy_2^2$, for some integers y_1 and y_2.*
2. *There are two integers a and b such that $\varepsilon_{2ps} = a + b\sqrt{2ps}$. Then*
   a. *$a+1$ is a square in $\mathbb{N}$,*
   b. *$\sqrt{2\varepsilon_{2ps}} = b_1 + b_2\sqrt{2ps}$ and $2 = b_1^2 - 2psb_2^2$, for some integers b_1 and b_2.*

Lemma 2.3. *([5, Lemma 5]) Let $q \equiv 3 \pmod 8$ and $s \equiv 7 \pmod 8$ be two primes such that $\left(\frac{q}{s}\right) = -1$.*

1. *Let x and y be two integers such that $\varepsilon_{sq} = x + y\sqrt{sq}$. Then*
   a. *$2s(x+1)$ is a square in $\mathbb{N}$,*
   b. *$\sqrt{\varepsilon_{sq}} = y_1\sqrt{s} + y_2\sqrt{q}$ and $1 = sy_1^2 - qy_2^2$, for some integers y_1 and y_2 such that $y = 2y_1y_2$.*
2. *There are two integers a and b such that $\varepsilon_{2sq} = a + b\sqrt{2sq}$. Then*
   a. *$2s(a+1)$ is a square in $\mathbb{N}$,*
   b. *$\sqrt{2\varepsilon_{2sq}} = b_1\sqrt{2s} + b_2\sqrt{q}$ and $2 = 2sb_1^2 - qb_2^2$, for some integers b_1 and b_2 such that $b = b_1b_2$.*

In the rest of this paper, we compute the unit group of the fields $\mathbb{L}^+ = \mathbb{Q}(\sqrt{2}, \sqrt{pq}, \sqrt{ps})$, where p, q and s are three different prime integers such that $p \equiv 1 \pmod 8$, $q \equiv 3 \pmod 8$ and $s \equiv 7 \pmod 8$ with $\left(\frac{p}{q}\right) = \left(\frac{p}{s}\right) = \left(\frac{q}{s}\right) = -1$.

3. The Main Result

Now we state our first main Theorem.

Theorem 3.1. *Let q, p and s are three different prime integers such that $p \equiv 1 \pmod 8$, $q \equiv 3 \pmod 8$ and $s \equiv 7 \pmod 8$ with $\left(\frac{p}{q}\right) = \left(\frac{p}{s}\right) = \left(\frac{q}{s}\right) = -1$. Put $\mathbb{L}^+ = \mathbb{Q}(\sqrt{2}, \sqrt{pq}, \sqrt{ps})$. The group unit of $\mathbb{L}^+$ is one of the following:*

i. $E_{\mathbb{L}^+} = \left\langle -1, \varepsilon_2, \varepsilon_{sq}, \sqrt{\varepsilon_{pq}}, \sqrt{\varepsilon_{ps}}, \sqrt{\varepsilon_{2ps}}, \sqrt{\varepsilon_{2pq}}, \sqrt{\varepsilon_{sq}\varepsilon_{2sq}} \right\rangle$,

ii. $E_{\mathbb{L}^+} = \left\langle -1, \varepsilon_{sq}, \sqrt{\varepsilon_{pq}}, \sqrt{\varepsilon_{ps}}, \sqrt{\varepsilon_{2ps}}, \sqrt{\varepsilon_{2pq}}, \sqrt{\varepsilon_{sq}\varepsilon_{2sq}}, \sqrt{\varepsilon_2\sqrt{\varepsilon_{pq}}\sqrt{\varepsilon_{ps}}\sqrt{\varepsilon_{2pq}}\sqrt{\varepsilon_{2ps}}} \right\rangle$,

iii. $E_{\mathbb{L}^+}=\left\langle -1, \varepsilon_{sq}, \sqrt{\varepsilon_{pq}}, \sqrt{\varepsilon_{ps}}, \sqrt{\varepsilon_{2ps}}, \sqrt{\varepsilon_{2pq}}, \sqrt{\varepsilon_{sq}\varepsilon_{2sq}}, \sqrt{\varepsilon_2\varepsilon_{sq}\sqrt{\varepsilon_{pq}}\sqrt{\varepsilon_{ps}}\sqrt{\varepsilon_{2pq}}\sqrt{\varepsilon_{2ps}}}\right\rangle$.

Proof. Consider the following diagram of subfields of $\mathbb{L}^+/\mathbb{Q}(\sqrt{2})$

$$\begin{tikzcd}
 & \mathbb{L}^+=\mathbb{Q}(\sqrt{2},\sqrt{pq},\sqrt{ps}) & \\
L_1=\mathbb{Q}(\sqrt{2},\sqrt{pq}) \arrow[ur] & L_2=\mathbb{Q}(\sqrt{2},\sqrt{ps}) \arrow[u] & L_3=\mathbb{Q}(\sqrt{2},\sqrt{sq}) \arrow[ul] \\
 & \mathbb{Q}(\sqrt{2}) \arrow[ul] \arrow[u] \arrow[ur] &
\end{tikzcd}$$

Figure 1: Subfields of $\mathbb{L}^+/\mathbb{Q}(\sqrt{2})$

Put $\mathrm{Gal}(\mathbb{L}^+/\mathbb{Q})=\langle\sigma_1,\sigma_2,\sigma_3\rangle$, where

$$\begin{array}{lll}
\sigma_1(\sqrt{2})=-\sqrt{2}, & \sigma_1(\sqrt{pq})=\sqrt{pq}, & \sigma_1(\sqrt{ps})=\sqrt{ps},\\
\sigma_2(\sqrt{2})=\sqrt{2}, & \sigma_2(\sqrt{pq})=-\sqrt{pq}, & \sigma_2(\sqrt{ps})=\sqrt{ps},\\
\sigma_3(\sqrt{2})=\sqrt{2}, & \sigma_3(\sqrt{pq})=\sqrt{pq}, & \sigma_3(\sqrt{ps})=-\sqrt{ps}.
\end{array}$$

By Lemma 2.1, we have

$$E_{L_1}=\left\{\varepsilon_2, \sqrt{\varepsilon_{pq}}, \sqrt{\varepsilon_{2pq}}\right\}.$$

By Lemma 2.2, we have

$$E_{L_2}=\left\{\varepsilon_2, \sqrt{\varepsilon_{ps}}, \sqrt{\varepsilon_{2ps}}\right\}.$$

By Lemma 2.3, we have

$$E_{L_3}=\left\{\varepsilon_2, \varepsilon_{sq}, \sqrt{\varepsilon_{sq}\varepsilon_{2sq}}\right\}.$$

Thus

$$E_{L_1}E_{L_2}E_{L_3}=\left\langle -1, \varepsilon_2, \varepsilon_{sq}, \sqrt{\varepsilon_{pq}}, \sqrt{\varepsilon_{ps}}, \sqrt{\varepsilon_{2pq}}, \sqrt{\varepsilon_{2ps}}, \sqrt{\varepsilon_{sq}\varepsilon_{2sq}}\right\rangle.$$

To find a fundamental system of units of $\mathbb{L}^+$, it suffices, as we have said in the beginning of the section, to find elements ξ of $E_{L_1}E_{L_2}E_{L_3}$ which are squares in $\mathbb{L}^+$. Put

$$\xi^2=\varepsilon_2^a\cdot\varepsilon_{sq}^b\cdot\sqrt{\varepsilon_{pq}}^c\cdot\sqrt{\varepsilon_{ps}}^d\cdot\sqrt{\varepsilon_{2pq}}^e\cdot\sqrt{\varepsilon_{2ps}}^f\cdot\sqrt{\varepsilon_{sq}\varepsilon_{2sq}}^g,$$

with $a,b,c,d,e,f,g\in\{0,1\}$. We will use norm maps from $\mathbb{L}^+$ to its biquadratic subextensions. The computations of these norms are summarized in the following table (see Table 1). Note that the third line of Table 1, is constructed as follows (we similarly construct the rest of the table):
By Lemma 2.1, we have

$$\begin{cases}\sqrt{2\varepsilon_{pq}}=w_1+w_2\sqrt{pq} \quad\text{and}\quad 2=-w_1^2+pqw_2^2,\\ \sqrt{2\varepsilon_{2pq}}=y_1+y_2\sqrt{2pq} \quad\text{and}\quad 2=-y_1^2+2pqy_2^2.\end{cases}$$

Thus

$$\begin{aligned}
\sqrt{\varepsilon_{pq}}^{1+\sigma_1} &= \sqrt{\varepsilon_{pq}}\cdot\sigma_1(\sqrt{\varepsilon_{pq}})\\
&= -\varepsilon_{pq},\\[1ex]
\sqrt{\varepsilon_{pq}}^{1+\sigma_2} &= \sqrt{\varepsilon_{pq}}\cdot\sigma_2(\sqrt{\varepsilon_{pq}})\\
&= -1,\\[1ex]
\sqrt{\varepsilon_{pq}}^{1+\sigma_3} &= \sqrt{\varepsilon_{pq}}\cdot\sigma_2(\sqrt{\varepsilon_{pq}})\\
&= \varepsilon_{pq},
\end{aligned}$$

$$\begin{aligned}
\sqrt{\varepsilon_{pq}}^{1+\sigma_1\sigma_2} &= \sqrt{\varepsilon_{pq}}\cdot\sigma_1(\sigma_2(\sqrt{\varepsilon_{pq}}))\\
&= 1,\\
\sqrt{\varepsilon_{pq}}^{1+\sigma_1\sigma_3} &= \sqrt{\varepsilon_{pq}}\cdot\sigma_1(\sigma_3(\sqrt{\varepsilon_{pq}}))\\
&= -\varepsilon_{pq},\\
\sqrt{\varepsilon_{pq}}^{1+\sigma_2\sigma_3} &= \sqrt{\varepsilon_{pq}}\cdot\sigma_2(\sigma_3(\sqrt{\varepsilon_{pq}}))\\
&= -1,\\
\sqrt{\varepsilon_{2pq}}^{1+\sigma_1} &= \sqrt{\varepsilon_{2pq}}\cdot\sigma_1(\sqrt{\varepsilon_{2pq}})\\
&= 1.
\end{aligned}$$

By Lemmas 2.2 and 2.3 we have

$$\begin{cases}
\sqrt{2\varepsilon_{ps}} = y_1 + y_2\sqrt{ps} \quad \text{and} \quad 2 = y_1^2 - psy_2^2,\\
\sqrt{2\varepsilon_{2ps}} = b_1 + b_2\sqrt{2ps} \quad \text{and} \quad 2 = b_1^2 - 2psb_2^2,\\
\sqrt{\varepsilon_{sq}} = y_1\sqrt{s} + y_2\sqrt{q} \quad \text{and} \quad 1 = sy_1^2 - qy_2^2,\\
\sqrt{2\varepsilon_{2sq}} = b_1\sqrt{2s} + b_2\sqrt{q} \quad \text{and} \quad 2 = 2sb_1^2 - qb_2^2.
\end{cases}$$

With the same technique, we full fill the following table:

ε	$\varepsilon^{1+\sigma_1}$	$\varepsilon^{1+\sigma_2}$	$\varepsilon^{1+\sigma_3}$	$\varepsilon^{1+\sigma_1\sigma_2}$	$\varepsilon^{1+\sigma_1\sigma_3}$	$\varepsilon^{1+\sigma_2\sigma_3}$
ε_2	-1	ε_2^2	ε_2^2	-1	-1	ε_2^2
$\sqrt{\varepsilon_{pq}}$	$-\varepsilon_{pq}$	-1	ε_{pq}	1	$-\varepsilon_{pq}$	-1
$\sqrt{\varepsilon_{ps}}$	$-\varepsilon_{ps}$	ε_{ps}	1	$-\varepsilon_{ps}$	-1	1
$\sqrt{\varepsilon_{2pq}}$	1	-1	ε_{2pq}	$-\varepsilon_{2pq}$	1	-1
$\sqrt{\varepsilon_{2ps}}$	-1	ε_{2ps}	1	-1	$-\varepsilon_{2ps}$	1
$\sqrt{\varepsilon_{sq}\varepsilon_{2sq}}$	ε_{sq}	1	1	ε_{2sq}	ε_{2sq}	$\varepsilon_{sq}\varepsilon_{2sq}$

Table 1: Norms in $\mathbb{L}^+/\mathbb{Q}(\sqrt{2})$

Let us eliminate some forms of ξ^2 such that ξ can not be in $\mathbb{L}^+$.

➥ Let us start by applying the norm $N_{\mathbb{L}^+/L_2} = 1+\sigma_2$, where $L_2 = \mathbb{Q}(\sqrt{2},\sqrt{ps})$. We have

$$\begin{aligned}
N_{\mathbb{L}^+/L_2}(\xi^2) &= \varepsilon_2^{2a}\cdot 1\cdot(-1)^c\cdot\varepsilon_{ps}^d\cdot(-1)^e\cdot\varepsilon_{2ps}^f\cdot 1\\
&= (-1)^{c+e}\cdot\varepsilon_2^{2a}\cdot\varepsilon_{ps}^d\cdot\varepsilon_{2ps}^f.
\end{aligned}$$

Then $c = e$. Therefore, $\xi^2 = \varepsilon_2^a\cdot\varepsilon_{sq}^b\cdot\sqrt{\varepsilon_{pq}}^c\cdot\sqrt{\varepsilon_{ps}}^d\cdot\sqrt{\varepsilon_{2pq}}^c\cdot\sqrt{\varepsilon_{2ps}}^f\cdot\sqrt{\varepsilon_{sq}\varepsilon_{2sq}}^g$.

➥ Consider $L_4 = \mathbb{Q}(\sqrt{pq},\sqrt{ps})$, we will apply the norm $N_{\mathbb{L}^+/L_4} = 1+\sigma_1$. We have

$$\begin{aligned}
N_{\mathbb{L}^+/L_4}(\xi^2) &= (-1)^a\cdot\varepsilon_{sq}^{2b}\cdot(-\varepsilon_{pq})^c\cdot(-\varepsilon_{ps})^d\cdot 1\cdot(-1)^f\cdot(\varepsilon_{qs})^g\\
&= (-1)^{a+c+d+f}\cdot\varepsilon_{sq}^{2b}\cdot\varepsilon_{pq}^c\cdot\varepsilon_{ps}^d\cdot\varepsilon_{qs}^g.
\end{aligned}$$

So $a+c+d+f\equiv 0 \pmod 2$. By Lemmas 2.1, 2.2 and 2.3, we have $\varepsilon_{pq}^c\cdot\varepsilon_{ps}^d\cdot\varepsilon_{qs}$, ε_{pq} and ε_{ps} are not squares in L_4, then $g=0$ and $c=d$. Therefore, $a=f$ and $\xi^2=\varepsilon_2^a\cdot\varepsilon_{sq}^b\cdot\sqrt{\varepsilon_{pq}}^c\cdot\sqrt{\varepsilon_{ps}}^c\cdot\sqrt{\varepsilon_{2pq}}^c\cdot\sqrt{\varepsilon_{2ps}}^a$.

➥ $L_6=\mathbb{Q}(\sqrt{pq},\sqrt{2ps})$, we will apply the norm $N_{\mathbb{L}^+/L_6}=1+\sigma_1\sigma_3$.

$$\begin{aligned} N_{\mathbb{L}^+/L_2}(\xi^2) &= (-1)^a\cdot 1\cdot(-\varepsilon_{pq})^c\cdot(-1)^c\cdot 1\cdot(-\varepsilon_{2ps})^a\\ &= \varepsilon_{pq}^c\cdot\varepsilon_{2ps}^a. \end{aligned}$$

By Lemmas 2.1 and 2.2, we have ε_{pq} and ε_{2ps} are not squares in L_6, then $c=a$. Therefore, $\xi^2=\varepsilon_2^a\cdot\varepsilon_{sq}^b\cdot\sqrt{\varepsilon_{pq}}^a\cdot\sqrt{\varepsilon_{ps}}^a\cdot\sqrt{\varepsilon_{2pq}}^a\cdot\sqrt{\varepsilon_{2ps}}^a$.

As ε_{sq} is not a square in $\mathbb{L}^+$, then: If $a=0$, so $b=0$. Hence ξ^2 can take one of the following forms:

* $\xi^2=\varepsilon_2\cdot\sqrt{\varepsilon_{pq}}\cdot\sqrt{\varepsilon_{ps}}\cdot\sqrt{\varepsilon_{2pq}}\cdot\sqrt{\varepsilon_{2ps}}$,
* $\xi^2=\varepsilon_2\cdot\varepsilon_{sq}\cdot\sqrt{\varepsilon_{pq}}\cdot\sqrt{\varepsilon_{ps}}\cdot\sqrt{\varepsilon_{2pq}}\cdot\sqrt{\varepsilon_{2ps}}$.

□

References

1. A. Azizi, M. M. Chems-Eddin and A. Zekhnini, Note on the Hilbert 2-class field tower, Math. Bohem., 147 (2022), 513-524.
2. M. M. Chems-Eddin, Arithmetic of some real triquadratic fields: Units and 2-class groups. Moroccan Journal of Algebra and Geometry with Applications, vol. 0(0):1-7, (2021).
3. M. M. Chems-Eddin, Unit groups of some multiquadratic number fields and 2-class groups, Period. Math. Hung. 84, 235-249 (2022).
4. M. M. Chems-Eddin, On units of some fields of the form $\mathbb{Q}(\sqrt{2},\sqrt{p},\sqrt{q},\sqrt{-\ell})$, Math. Bohem., 148 (2023), 237-242.
5. M. M. Chems-Eddin, A. Azizi and A. Zekhnini, Unit groups and Iwasawa Lambda invariants of some multiquadratic number fields, Bol. Soc. Mat. Mex. 27, 24 (2021). https://doi.org/10.1007/s40590-021-00329-z
6. M. M. Chems-Eddin, A. Zekhnini and A. Azizi, Units and 2-class field towers of some multiquadratic number fields, Turk. J. Math., 44 (2020), 1466-1483.
7. M. M. Chems-Eddin, M. B. T. El Hamam and M. A. Hajjami, On the unit group and the 2-class number of $\mathbb{Q}(\sqrt{2},\sqrt{p},\sqrt{q})$, Ramanujan J., (2024) DOI: 10.1007/s11139-024-00947-x
8. M.B.T. El Hamam, The unit group of some fields of the form $\mathbb{Q}(\sqrt{2},\sqrt{p},\sqrt{q},\sqrt{-\ell})$, Math. Bohem., (2023), doi: 10.21136/MB.2023.0077-22
9. M. B. T. El Hamam, On units of some fields of the form $\mathbb{Q}(\sqrt{2},\sqrt{p},\sqrt{q},\sqrt{-\ell})$, Asian-European Journal of Mathematics (2025) https://doi.org/10.1142/S1793557125500780
10. M. B. T. El Hamam, The unit group of some fields of the form $\mathbb{Q}(\sqrt{2},\sqrt{pq},\sqrt{ps})$ and $\mathbb{Q}(\sqrt{2},\sqrt{pq},\sqrt{ps},\sqrt{-l})$. Period Math Hung 91, 88–100 (2025). https://doi.org/10.1007/s10998-025-00655-z.
11. M. B. T. El Hamam, Fundamental systems of units of some multiquadratic fields of degree 8 and degree 16. Ricerche di Matematica (2025). https://doi.org/10.1007/s11587-025-01015-2.
12. M. B. T. El Hamam, The unit group and the 2-class number of some fields of the form $\mathbb{Q}(\sqrt{2},\sqrt{pq},\sqrt{ps})$ and $\mathbb{Q}(\sqrt{2},\sqrt{pq},\sqrt{ps},\sqrt{-l})$. arXiv:2505.07092, https://doi.org/10.48550/arXiv.2505.07092
13. T. Kubota, Über den bizyklischen biquadratischen Zahlkörper. Nagoya. Math. J., 10: 65-85 (1956) (in German).
14. H. Wada, On the class number and the unit group of certain algebraic number fields. J. Fac. Univ. Tokyo. 13 (1966), 201-209.

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