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An Algorithm Based on Cost Distribution to Find the Optimal Solution for Transportation Problems

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ABSTRACT: Within the realm of operations research, solving the transportation problem has become essential to improving methods of getting the product from the original source to the customer in the quickest amount of time or at the lowest cost. This importance arises from the economic aspects of the transportation problem, its classification as a specific instance of linear programming that focuses on finding the most efficient distribution of goods from supply centers to customers, and the increasing relevance of globalization and rapid development. Using an ordered tree, we provide a more straightforward, reliable, and cost-effective method for businesses to handle transportation challenges. Using an ordered tree, we accomplish this by developing a new proposed algorithm that relies on the distribution of costs among cells. Thus, the total cost of the transportation problem is equal to the sum of the costs for those cells represented in a connected acyclic directed graph, which also addresses the logistical problems related to supplying goods and their arrival at their destination.

Key Words: Transportation problems, Optimal solution, Graph theory, Distribution cost (D.C).

Contents

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1. Introduction

Issue with transportation one of the first and most outstanding solutions to the linear programming problems (LLP) is TP. When it comes to everyday effectiveness in our lives, transportation problems are a unique type of linear programming [16,5]. Transportation models primarily concentrate on the best way, which is the transfer of the product from multiple plants or factories (it is supply assets) to multiple storehouses or clients (it is demand destinations). This is because in the nature of public life, a designated quantity of homogeneous goods is available in a number of provenances, and a stationary quantity is desired to satisfy the request in every consumption venue [2,3]. Finding the best shipping schedule for the product while meeting demand at each location is the primary goal of this problem. In 1941, Hitchcock presented the transportation dilemma for the first time. F. L. [11]. Tjalling C. Koopmans presented his article in 1947 in an attempt to address the transportation issue [12]. The twotioned pieces of research represent the primary advancements in the various methods for resolving the transportation model. The TP can also be solved by using the simplex methodology, which was developed by Dantzig G. B. in 1951, to represent it as an LP model. However, this method involves a significant number of variables and constraints, and solving them takes a lot of time and effort. To find an IBFS that accounts for transportation expenses, numerous researchers have extended substitution techniques. One of the three traditional methods the least cost method (LCM), Vogel's approximation approach (VAM), or the northwest corner method (NWCM) can be used to obtain an IBFS for the TP. Vogel's Approximation Method (VAM) is the best methodology among those three, according to the studies. One of two approaches the Modified Distribution Method (MODI) or the Stepping Stone Method allows us to improve the results of the initial solution to obtain the optimal solution to the TP. In essence, the primary distinction between these methods is which strategy yields the best initial answer; a good initial solution will yield a higher objective value. There are two types of transportation problems: balanced transportation problems and unbalanced transportation problems. A transportation problem is considered balanced if the number of demands and the number of sources are equal. If not, we refer to the issue of uneven transportation [15]. To determine IBFS for the transportation model, a number of approaches have been put out in recent years. According to Md. Ashraful Babu and colleagues (2014), the Implied Cost Method (ICM) is superior to or comparable to VAM. Mollah Mesbahuddin Ahmed and associates presented a novel method in 2017 for obtaining a (IBFS) for the (TP) [1]. In 2020, a novel approach is put forth to solve various kinds of (TP) for a minimization-type objective function. Which Hussein Ali Hussein Al-Saeedi has proposed [13]. Noor suggested a method for determining the best or almost best course of action [17]. Using an ordered tree, we proposed an algorithm that depends on distributing costs to cells. Thus, the total cost of the transportation problem equals the sum of the costs for those cells through a connected acyclic directed graph [4]. When comparing the new solution with previous solutions (NWC, LCM, and VAM) and the Saeedi 1 technique in order to present the best results that demonstrated the effectiveness of the new method. The ideal solution is provided by the Matlab 2013 application, which we used to compute it.

2. Some Basic Concepts of Transportation problems (TP) and Graph Theory (GT)

2.1. Transportation Model

There are m supply origins in (TP) $S_1, S_2, \ldots, S_m$ and n demand destinations $D_1, D_2, \ldots, D_n$ each of which is uniquely represented. The brackets represent the tracks that link sources and destinations. Bracket (i, j) links between origin $i (i = 1, 2, 3, \ldots, m)$ and destination $j (j = 1, 2, 3, \ldots, n)$, where $z_{ij} = c_{ij} x_{ij}$ for each unit of items. The units that are available at supply in origin i are denoted by a_i, while the units that are available at request in destination j are denoted by b_j.

Minimize

$$Z = z_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\ x_{ij} \text{ (gross cost)} \tag{2.1}$$

Subject to

$$\sum_{i=1}^{m} x_{ij} = a_i,\ i = 1, 2, \ldots, m \text{ (supply restrictions)} \tag{2.2}$$

$$\sum_{j=1}^{n} x_{ij} = b_j,\ j = 1, 2, \ldots, n \text{ (demand restrictions)} \tag{2.3}$$

2.2. Representation of Transportation Problems

One type of linear programming problem (LPP) is represented by transportation problems (TP). Shipping varied amounts of uniform items from multiple origins (like factories) to different destinations (like warehouses) in order to minimize the overall cost (or time) of transportation is a transportation problem.

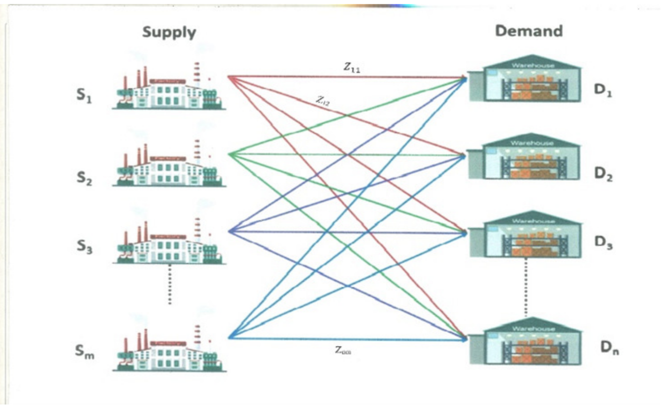

Figure 1: The graph structure of TP

2.3. Schedule of Transportation

The TP can be seen using the mathematical model for linear programming of the TP in a specific table (Table 1 Schedule of TP). The TP model, with all pertinent parameters, can be shown in tabular form. Within the transportation schedule, the destination requirement (b_j) is shown in the bottom row, while the supply availability (a_{ij}) is shown in the rightmost column for each source. Each cell shows one route, z_{ij} represents the product of the cost of one shipment multiplied by the quantity of the shipped item. z_{ij} a ppears in the upper right corner of each cell in the table [6].

Table 1: Schedule of (TP) Depending on z_{ij}.

Sources (i)	Destinations (j)				Supply (a_i)
	D_1	D_2	$\ldots$	D_n	
S_1	z_{11}	z_{12}	$\ldots$	z_{1n}	a_1
S_2	z_{21}	z_{22}	$\ldots$	z_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$	z_{ij}	$\vdots$	$\vdots$
S_m	z_{m1}	z_{m2}	$\ldots$	z_{mn}	a_m
Demand (b_j)	b_1	b_2	$\ldots$	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

3. Some Definition

Definition 3.1. *[Feasible Solution (FS)] A set of non-negative assigned values $x_{ij} \geq 0$ that account for constraints (in this case, supply and demand constraints) is a workable solution to transportation problems [8].*

Definition 3.2. *[Basic Feasible Solution (BFS)] A fundamental feasible solution (BFS) is a workable solution to transportation issues that does not involve any of the most from $(m+n-1)$ positive assigned quantities, where m is the number of rows and n is the number of columns of TP [8].*

Definition 3.3. *[The initial Basic Feasible Solution (IBFS)] It is the starting point from which the problem-solving process starts, and after further refinements to that answer, the optimal solution is reached.*

4. General Procedure to solve a Transportation Problem

We follow the following steps [14]:

1. The table of TP is expressed mathematically.
2. Determine the Basic Feasible Solution (BFS) at first.
3. To determine the best option, the initial basic feasible solution (IBFS) acquired in Phase 2 was modified.

4.1. Types of Transportation Problems

4.1.1. Balanced (TP). The TP is said to be balanced when the quantity of items in the apportionment pivot matches the quantity of products that the request pivot wants, i. e, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

4.1.2. Unbalanced (TP). When the quantity of items needed by the request pivot is less than the quantity of products available in the apportionment pivot, the TP is referred to as imbalanced, $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$.

4.2. Methods for Finding IBFS of TP

The following characteristics must be present in the first solution that we find using either the new or classic solution approaches:

1. It must be possible.
2. The non-negativity must be satisfied.
3. It should be basic.

The following are some traditional techniques for obtaining the initial solution for TP:

1. North –West Corner Method (NWCM).
2. Least Cost Method (LCM).
3. Vogel's Approximation Method (VAM).

4.3. Methods of Finding Optimal Solution

The optimality test will be conducted once the initial basic viable solution has been determined in order to determine whether or not the acquired feasible solution is optimal. The following techniques can be used to carry out this test:

1. Stepping Stone Method (SSM).
2. Modified Distribution (MODI) Method.

4.4. Graph Theory

A subfield of discrete mathematics called graph theory examines mathematical structures called graphs. An ordered pair $G = (V, E)$ is the formal definition of a graph, where V is the set of vertices and E is the set of edges joining vertex pairs. Modeling binary relationships and resolving network and distributional optimization issues are two applications of graph theory [4].

Definition 4.1. *[Tree] A tree is an undirected network that is linked and free of cycles, which means that there is only one path connecting any two nodes. The number of edges $|E|$ in a tree $G = (V, E)$ with n nodes is hence $n - 1$. Because they can depict hierarchical structures, trees are a useful tool in many applications involving algorithmic analysis and optimization [10].*

Definition 4.2. *[Rooted Tree] A tree that has one node designated as the root and forces the other nodes into a hierarchical order is said to be rooted. There is a unique path to the root for each node except the root. Its "parent" is the node that comes before it, and its "children" are the nodes that immediately follow it. Several optimization techniques and mathematical models are predicated on this hierarchical structure [7].*

Definition 4.3. *[Tree in Graph] According to graph theory, a "tree in a graph" is a sub graph made up of all of the original graph's connected edges and vertices that are acyclic-free. Stated differently, if a tree* $T = (V', E')$ *and a graph* $G = (V, E)$ *such that* $V' \subseteq V$ *and* $E' \subseteq E$*, then* T *is a tree in* G *if the following criteria are satisfied.*

1. *Connected: Every pair of vertices in* T *has a path connecting them.*

2. *Cycle-free: No path that starts and ends at the same vertex can have edges that repeat.*

A sub graph is referred to as a universal tree when T *contains every vertex* V *in the original graph [4,9].*

Definition 4.4. *[M- ary Tree][7]*

1. *A m- ary tree* ($m \geq 2$) *is a rooted tree with* m *or less children at each vertex.*

2. *A complete m- ary tree is one in which all of the leaves have the same depth and each internal vertex has exactly* m *children.*

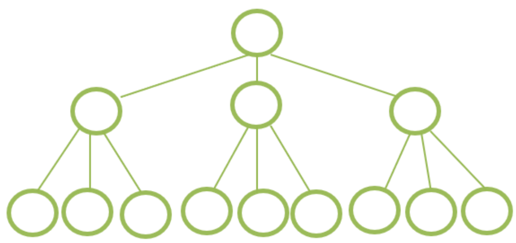

Figure 2: Complete 3-ary tree

5. The Proposed Algorithm

We will talk about the technique we suggested to find the optimal solution (OS) directly, which uses the rooted tree.

6. Algorithm

Below are the steps of the proposed algorithm:

Note: After comparing the suggested algorithm's step 6 with SSM and MODI, we obtain the lowest cost that corresponds to the optimal solution (OS).

7. Numerical Examples

We will give some examples that demonstrate the suggested algorithm.

Example 7.1. *Examine the transportation issues table that follows.*

1: Step (1): We begin by allocating the lowest value between supply and demand to the first cell in the northwest corner. Then we calculate the cost for this cell, which is the result of multiplying the allocation to it by its value in the cost matrix.
2: Step (2): We start from the cell we chose in the first step by creating a branch extension for the tree by taking the cells in the second row, starting from the first cell to the last cell in the second row, and allocating each cell's share of supply and demand, and then we calculate the cost of each cell.
3: Step (3): The tree extends to the remaining cells by taking the branches of the cells in the rows following the second row until the end of the rows of the transportation problem, assigning each cell and calculating its cost.
4: Step (4): We repeat the above steps until we complete calculating the cost of all cells in the first row.

5: Step (5): We calculate the total cost for each branch of the tree to meet the supply and demand for the transportation problem.
6: Step (6): We repeat the previous procedures for each cell of the transportation problem, calculating the total cost of the transportation problem.
7: Step (7): We choose the least total cost transportation problem with the least total cost allocation matrix.

Table 2: Initial Data of Example (1).

Source	Destination			Supply
	D_1	D_2	D_3	
S_1	1	2	2	15
S_2	6	4	5	35
S_3	3	2	9	25
Demand	23	33	19	75

1. We begin with the first row and allocate the cell (northwest) until all of the first row's items have been allocated.

Table 3: Transportation Problem table based on Z_{11}.

Source	Destination		
	D_1	D_2	D_3
S_1	15	0	0
S_2	48	108	0
S_3	0	12	171

We take the tree branches for Table 3.

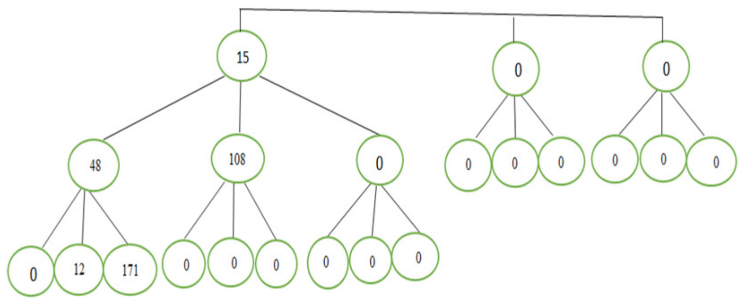

Figure 3: The Tree of the first row rooted tree by z_{11}

Based on the first cell, we calculate the cost. Is

$$Z = \sum_{i=1}^{3}\sum_{j=1}^{3} z_{ij}$$

$$Z = (15) + (48) + (108) + (12) + (171) = 354 \text{ units.}$$

2. We allocate the second cell and repeat the procedure for the first row.

Table 4: Transportation Problem table based on Z_{12}.

Source	Destination		
	D_1	D_2	D_3
S_1	0	30	0
S_2	138	48	0
S_3	0	12	171

We take the tree branches for Table 4.

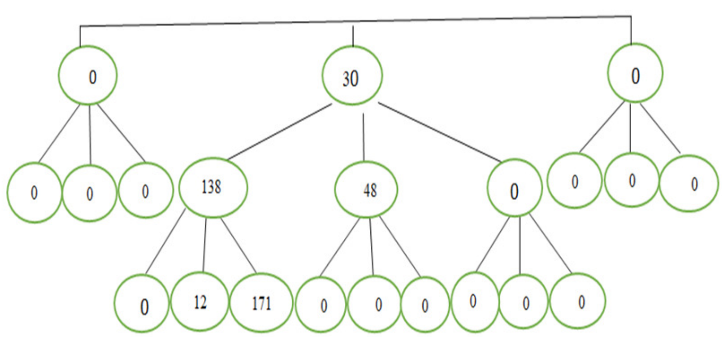

Figure 4: The tree of the first row rooted tree by z_{12}

Based on the second cell, we calculate the cost is

$$Z = \sum_{i=1}^{3}\sum_{j=1}^{3} z_{ij}$$

$$Z = (30) + (138) + (48) + (12) + (171) = 399 \text{ units.}$$

3. By assigning the third cell, we carry out the process once more for the first row.

Table 5: Transportation Problem table based on Z_{13}.

Source	Destination		
	D_1	D_2	D_3
S_1	0	0	30
S_2	138	48	0
S_3	0	42	36

We take the tree branches for Table 5.

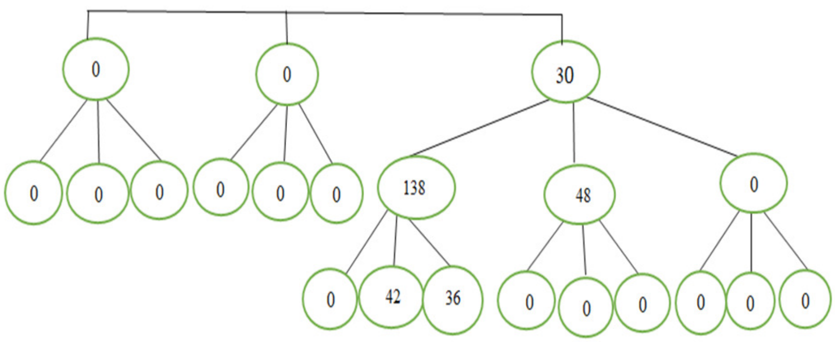

Figure 5: The tree of the first row rooted tree by z_{13}

Based on the third cell, we calculate the cost is

$$Z = \sum_{i=1}^{3} \sum_{j=1}^{3} z_{ij}$$

$$Z = (30) + (138) + (48) + (42) + (36) = 294 \text{ units.}$$

We swapped the second and third rows while keeping the allocation for each column and row the same. We can determine the lowest cost of the cost matrix that corresponds to the OS-based optimal solution by running the suggested method again on each cell, as shown in the accompanying table.

Table 6: Optimal arrangement of the initial table for (TP) to find the OS.

Source	Destination			Supply
	D_1	D_2	D_3	
S_1	1	2	2	15
S_3	3	2	9	25
S_2	6	4	5	35
Demand	23	33	19	75

The optimal solution is obtained using the MATLAB 2013 software when we repeat the algorithm's application on each element of the cost matrix, as shown in the accompanying table:

Table 7: Transportation Problem table based on Z_{11}.

Source	Destination		
	D_1	D_2	D_3
S_1	15	0	0
S_2	24	34	0
S_3	0	64	95

We take the tree branches for Table.

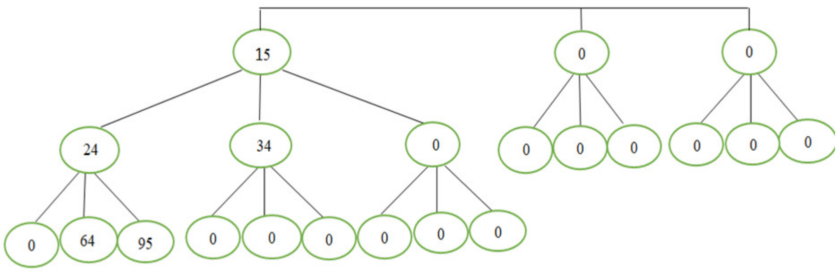

Figure 6: The tree of the first row rooted tree by z_{11}

Then,

$$Z = (15) + (24) + (34) + (64) + (95) = 232 \text{ units},$$

which is the optimal solution.

Example 7.2. *Examine the transportation issues table that follows.*

Table 8: Initial Data of Example (2).

Source	Destination				Supply
	D_1	D_2	D_3	D_4	
S_1	1	2	3	4	30
S_2	7	6	2	5	50
S_3	4	3	2	7	35
Demand	15	30	25	45	115

Rooted Tree, the ideal answer, was obtained by using the suggested approach, which was coded in MATLAB 2013. While keeping the allocation for each column and row the same, we shift the fourth column to the second. The following table is the outcome.

Table 9: Optimal arrangement of the initial table for (TP) to find the (OS).

Source	Destination				Supply
	D_1	D_4	D_3	D_2	
S_1	1	4	3	2	30
S_2	7	5	2	6	50
S_3	4	7	2	3	35
Demand	15	45	25	30	115

Using MATLAB 2013, we determine the lowest feasible cost from the above table by repeating the algorithm's application to each element of the cost matrix.

Table 10: Transportation Problem table based on Z_{11}.

Source	Destination			
	D_1	D_4	D_3	D_2
S_1	15	0	0	30
S_2	0	225	10	0
S_3	0	40	45	90

We take the tree branches for Table.

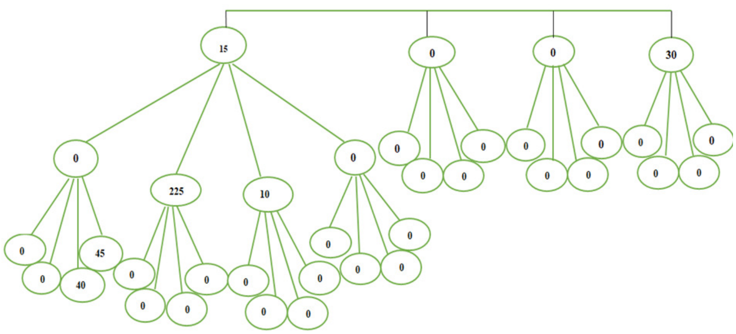

Figure 7: The tree of the first row rooted tree

Then,

$$Z = (15) + (30) + (225) + (10) + (40) + (45) = 365 \text{ units.}$$

8. Comparison

The outcomes of the case-solving sessions conducted in the current study were compared in order to illustrate the viability of the unique approach. The conventional methods of problem-solving (NWCM, LCM, VAM, and methodology Al-Saeedi1) have been contrasted. In terms of IBFS, the proposed method performs better than the traditional methods. This algorithm and the IBFS optimized by the currently suggested algorithm are in close or optimal places. The results produced using Vogel's Approximation approaches are demonstrated to be inferior to those achieved with this procedure. Observe how the methods.

Table 11: Comparing the results.

Name of method	NWCM	LCM	VAM	Saeedi1	The proposed algorithm	OS
Example1	354	240	247	247	232	232
Example2	480	405	375	405	365	365

Can be represented Table 11 by the following chart:

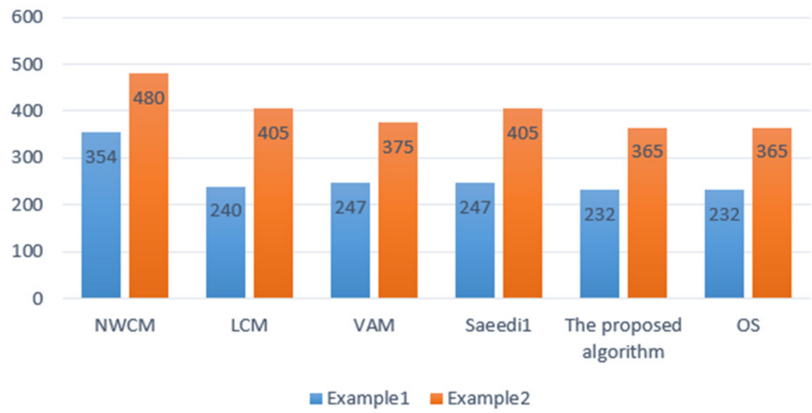

Figure 8: Illustration of Table 11

9. Conclusion

When comparing the outcomes of solving the examples in this work using the Saeedi 1 technique and the three traditional approaches (NWC, LCM, and VAM), the tree approach yields superior results to the Saeedi (1) method and the three traditional methods. Based on the previously supplied information, it can be inferred that the new method has produced suitable and desirable results. As a result, the (TP) was solved using an objective function of the minimization kind.The new algorithm's ease of understanding and process applications is what defines it. The optimal solution (OS) was obtained with less time and effort as a result. It allows us to make an effective selection based on the best response. As a result, the algorithm of our approach is enhanced by possible extension.

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