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Multi-Objective Inventory Model Incorporating Shortage Cost and Delayed Replenishment With Diverse Fuzzy Number Applications Using C++ Programming Language

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ABSTRACT: Today's supply chains face many problems, especially when it comes to managing inventory. Delays in getting supplies, sudden changes in customer demand, and the high cost of running out of stock make planning difficult. Most traditional models use fixed values, which don't work well when things are uncertain. Although fuzzy logic has been used to handle uncertainty, many studies only use simple fuzzy numbers, which don't fully capture the complexity of real-life situations. This study introduces an improved inventory model that considers both shortage costs and delays in restocking. It uses four types of fuzzy numbers trapezoidal, pentagonal, hexagonal, and decagonal to show different levels of uncertainty more accurately. The model is programmed in C++ and uses a special method called graded mean integration to turn fuzzy numbers into useful values. Results show that while trapezoidal numbers produce basic results, pentagonal and hexagonal numbers are better especially hexagonal, which provides the lowest overall cost. The model also deals with uncertainty in how long restocking takes. The article examines a fuzzy inventory model using trapezoidal, pentagonal, and hexagonal fuzzy numbers. It finds trapezoidal fuzzy numbers ineffective for cost efficiency, while pentagonal and hexagonal fuzzy numbers outperform crisp models, enhancing inventory management decision-making.

Key Words: Fuzzification; Economic order quantity; Shortage cost; Delayed replenishment; Defuzzification; Graded mean integration Method.

Contents

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1. Introduction

In modern production environments, effective inventory control plays a critical role in ensuring operational efficiency and customer satisfaction. One of the central challenges in inventory management is achieving an optimal balance. Holding excessive inventory increases storage and handling costs, while insufficient inventory can lead to shortages, production delays, and reduced customer service levels. Particularly in discrete production systems, the overarching objective is to minimize the long-term average inventory cost by making well-informed, data-driven decisions.

Fuzzy set theory has increasingly been recognized as a valuable approach for improving inventory modeling, particularly in situations where uncertainty is a defining characteristic. Unlike traditional crisp models, fuzzy approaches allow for a more flexible and realistic representation of the vagueness inherent in real-world supply chain systems. Several studies have built upon this foundation to enhance the reliability and efficiency of inventory-related decision-making. For example, Abdullah Ali et al. [1] developed a multi-objective optimization model aimed at promoting sustainability in green supply chains, specifically within the domains of inventory control and production planning. Adrian Ban and Lucian Coroianu [2] studied the basic math behind fuzzy logic by looking at how some operators behave when using trapezoidal fuzzy numbers, which adds to our understanding of fuzzy modeling. In a real-world example, Ata Allah Taleizadeh et al. [3] created models for sustainable economic production quantities that take into account shortage costs, showing how to handle challenges in inventory management. These contributions underscore the growing relevance of fuzzy set theory in creating more adaptable and responsive inventory systems. Chen and Wang [4] introduced a fuzzy inventory system that includes backordering, using the function principle to manage uncertainty more effectively. Building on this, Chang [5] modified the traditional EOQ model by integrating fuzzy set theory to handle situations involving imperfect-quality items. Later, Chang et al. [6] extended this concept further by developing a fuzzy reorder point model that also incorporates backorder quantities. Cheng [7] contributed by formulating an EOQ model where production costs depend on demand levels and account for flaws in the production process. In another important study, Chinmay Saha et al. [8] applied fuzzy optimization techniques, along with differential evolution algorithms, to enhance inventory control for defective items. Further contributions to this area include the work of Chakraborty et al. [9], who developed a two-warehouse inventory model considering partial backlogging and deterioration under inflationary pressure. Lee and Yao [10] proposed a fuzzy EOQ model suitable for systems without backordering. Similarly, Hsu and Hsu [11] developed an EOQ framework that considers a wide range of real-world issues, including imperfect items, inspection errors, shortages, and returns. Kazemi et al. [12] included fuzzy logic in both the factors and choices of an inventory model that allows backorders, while Aslam et al. [13] created a nonlinear fuzzy supply chain model that takes into account uncertain lead times using the Takagi–Sugeno method. Multi-objective optimization has also gained attention in recent studies. Mishra et al. [14] explored inventory modeling using a variety of fuzzy techniques to address multiple conflicting goals. Chaudhary et al. [15] introduced a sustainable inventory model tailored to defective items under fuzzy conditions. Salameh and Jaber [16] presented a production quantity model that accounts for imperfect-quality goods. Suvetha et al. [17] proposed a three-stage sustainable inventory system incorporating trapezoidal demand patterns and time-sensitive holding costs. Meanwhile, Sanni and Chukwu [18] addressed the deterioration of inventory items through an EOQ model that considers Weibull distribution and ramp-type demand. The mathematical treatment of fuzzy numbers has also played a vital role in refining such models. Mandal and Mandal [19] explored the structural properties of pentagonal fuzzy numbers and demonstrated their applicability in solving fuzzy equations. Sangeetha and Parimala [20] applied hexagonal fuzzy numbers in solving fuzzy game problems, highlighting their flexibility. Tinarelli [21] provided helpful details about the ongoing challenges in inventory control theory, while Yao and Wu [22] presented methods for ranking fuzzy numbers using the decomposition principle and signed distance techniques.

Building on the strengths of these previous works, the present study proposes a comprehensive multi-objective production-inventory model that takes into account both shortage-related costs and delayed replenishment under uncertain conditions. To capture the nuances of uncertainty more precisely, this model employs trapezoidal, pentagonal, and hexagonal fuzzy numbers to represent key parameters. The graded mean integration method is used to simplify fuzzy numbers, creating a decision-support system that is both precise and flexible for complicated inventory situations.

RELATED WORK

There are several real-world issues related to optimizing a ratio of two functions. There are several examples in the fields of economics, medicine, manufacturing, finance, inventories, corporate planning, and water resource management. Fractional programming problems are the name given to these optimization issues. Furthermore, it is known as a Linear Fractional Programming Problem (LFPP) if the functions involved are linear. Optimizing output/employee, nurse/patient, cost/student, profit/cost, debt/equity, etc. are a few examples of LFPP. Numerous methods have been developed over time to identify the best LFPP solution. The variable transformation approach was developed by Charnes and Cooper [23], who took into account an additional variable to solve LFPP. LFPP was resolved by Tantawy [24] by applying the conjugate gradient method. The parametric approach is examined by Almogy and Levin [25] to solve a sum of fractional objectives. Since Dantzig developed the simplex method [26], numerous researchers have proposed various approaches to solve LFPP [27–30]. For fractional programming problems involving absolute value functions, the solution strategy has been covered by Chadha [31]. Das et al. [32] introduced a novel approach to solving LFPP with absolute value functions. Borza et al. [33] used Charnes and Cooper's [23] variable transformation method to solve LFPP with the objective coefficients as intervals.

Early on in the decision-making process, prediction is crucial. One of the prerequisites for sound decision-making is accurate prediction, which also plays a crucial guiding role in the process. Decision-making errors frequently have a more severe impact on the industrial sector because of its quick, electric, violent, destructive, far-reaching, and other real-life features. Making a reasonable decision is essential to the practical problem because the success or failure of the current and future practical problems is directly correlated with the correctness of the decision. To better advise the practical situation and create the appropriate practical strategy, fuzzy numbers are crucial for describing confusing facts. A powerful mathematical tool that effectively models such circumstances is the Fuzzy Linear Fractional Programming Problem (FLFPP), which combines LFPP and fuzzy numbers. The concept of fuzzy sets was first presented by Zadeh [34] in 1965, and it has since been widely applied in many other fields. Bellman and Zadeh's [35] incorporation of fuzziness into decision-making issues led to the increasing popularity of fuzzy sets in the field of mathematical optimization. Tanaka et al. [36] created the first Linear Programming Problem (LPP) using fuzzy parameters. Buckley and Feuring [37] introduced a type of fully fuzzy LPP where all variables and parameters take the form of fuzzy numbers. Ganesan and Veeramani [38] created a solution to the fuzzy LPP using symmetric trapezoidal fuzzy numbers. A lexicographic method for solving fully fuzzy LPP with trapezoidal fuzzy numbers was presented by Das et al. [39]. A ranking function approach was presented by Das et al. [40] for the solution of fuzzy LPP with Triangular Fuzzy Numbers (TFNs). Safaei [41] used the decomposition method to suggest an FLFPP solution. Using a simplex method and a denominator objective limitation technique, Pandian and Jayalakshmi [42] resolved LFPP. Luhandjula [43] used fuzzy techniques to tackle the Multi-Objective Linear Fractional Programming Problem (MOLFPP). A fully fuzzy LFPP solution was put forth by Pop and Stancu-Minasian [44], who accounted for the variables and parameters as TFNs. Das et al. [45] provided a method for solving FLFPP in which the upper, middle, and lower limits were found by converting the FLFPP into an analogous tri-objective LFPP. The ranking method established for TFNs served as the foundation for the comparison of the outcomes. Das et al. [46] introduced a unique method for solving FLFPP using multi-objective LPP. Stanojevic and Stanojevic [47] developed a method for solving LFPP with fuzzy parameters in the objective function. Deb and De [48] solved the completely fuzzy LFPP using the graded means integration representation technique.

One important computational technique that aids in identifying the best solution for multi-objective programming problems which typically contain conflicting objectives is goal programming (GP) [49]. GP applied to an optimization problem with fuzzy parameters is known as fuzzy goal programming, or FGP. GP is frequently used to discover the answer to several optimization issues. Pal et al. [50] developed a GP-based approach for multi-objective FLFPP. The FLFPP solution was derived by Veeramani and Sumathi [51] using fuzzy mathematical programming. The FLFPP is solved using fuzzy mathematical programming and transformed into an analogous deterministic MOLFPP. An FGP approach was created by Pramanik et al. [52] to solve multiobjective linear plus LFPP problems. To get the best solution, Veeramani and Sumathi [53] transformed FLFPP into a triobjective LFPP and then used FGP. Using the Taylor series approximation, Pramanik et al. [54] used FGP to solve a linear fractional bi-level decentralized programming problem. To address linear fractional bi-level multi-objective programming problems, Dey et al. [55] used the TOPSIS technique with FGP. Pramanik and Roy [56] applied the FGP approach to a multi-objective fuzzy transportation problem. Pramanik and Roy [57] employed the FGP approach to tackle the multi-level programming challenge. Multi-objective linear and LFPP linearization strategies have been researched by Borza et al. [58]. Edalatpanah has tackled a multi-dimensional solution to the fuzzy LPP [59]. Sheikhi and Ebadi [60] have devised a new approach to solve linear interval fractional transportation issues. Veeramani et al. [61] utilized the neutrosophic GP strategy to solve the multi-objective fractional transportation issue. Khalifa [62] proposed a signed distance for interval-valued fuzzy numbers. Shirneshan et al. [63] proposed a stochastic programming method to solve the shift scheduling issue that caretakers encounter. A novel model known as LR-type totally Pythagorean fuzzy LPPs with equality constraints was introduced by Akram et al. [64]. In order to establish an integrated supply chain, Arabzad et al. [65] created an FGP model to address challenges in allocating linear facility sites. A novel approach to simulating zero-base budgeting in a fuzzy environment has been presented by El-Morsy [66]. In order to solve cost-efficiency issues, Khalifa and Yousif [67] ordered piecewise quadratic fuzzy quotients linearly.

Assuming that the parameters are TFNs, this study proposes an FGP-based approach for solving FLFPP. First, the FLFPP is transformed into an equivalent deterministic LFPP with three objectives by applying Zadeh's [68] extension principle. The TFN and centroid of triangle technique attributes are used to eliminate the fuzziness in the constraints. The variable transformation approach determines the maximum and minimum values of each objective function by solving it separately. The objectives' hazy aims are then created using these values. Each objective goal's associated membership function is then developed; because these functions are nonlinear, they are then linearized to prevent computational issues. Lastly, FGP is used to minimize the negative deviational variables in order to achieve the highest value of the membership goals. Two real-world examples illustrate the usefulness and effectiveness of the suggested approach, and the outcomes are contrasted with those of other approaches.

MOTIVATION OF THIS STUDY

The paper investigates a fuzzy inventory model using trapezoidal, pentagonal, and hexagonal fuzzy numbers to represent costs. It finds that trapezoidal fuzzy numbers are ineffective in minimizing expenses, while pentagonal and hexagonal fuzzy numbers outperform the crisp model in cost efficiency. The study emphasizes the benefits of hexagonal fuzzy numbers for improved inventory management decision-making.

2. Preliminaries

Definition 2.1 Give χ a collection of all the values. Then the fuzzy set $\widetilde{A} = \{(x, \mu_A(x))/x \in \chi\}$ of χ is established by its function within the membership $\mu_A : \chi \to [0,1]$.

2.1. Fuzziness

When an observed variable becomes fuzzy in the actual world, there are two common situations. Technical measurement conditions make it impossible to measure the response variable precisely in the first case. Consequently, in order to show the required tolerance to measurement mistakes, data can only be expressed in linguistic terms rather than directly using accurate (non-fuzzy) statistics. In the second case, the variable's answer is provided in linguistic forms that are not numerical, such a farmer's report about his produce or an expert's linguistic report. The data in both cases can be represented as a country of fuzzy sets for the purpose of experiment analysis. Since the values derived from experiment results are frequently fuzzy, fuzzy sets theory must be employed to model and manage the results from experiments in many applicable fields. Since its introduction to the scientific community by Zadeh (1965), the fuzzy sets theory has been applied by several individuals in a wide range of scientific fields.

2.2. Fuzzy Numbers

A fuzzy number $\widetilde{A}$ is a fuzzy set on a real line R such that

1. $\mu_A(x_0)$ is piecewise continuous
2. There exists at least one $x_0 \in$ R with $\mu_{\widetilde{A}}(x_0) = 1$
3. A must be normal and convex

2.3. Trapezoidal Fuzzy Numbers

$\widetilde{A}$ = {a, b, c, d} is defined by trapezoidal fuzzy numbers, where a, b, c, and d are real numbers. The membership function $\mu_{\widetilde{A}}(x)$ of these numbers is as follows:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & ; for\ x < a \\ \frac{(x-a)}{(b-c)} & ; for\ a \le x \le b \\ 1 & ; for\ b \le x \le c \\ \frac{(d-x)}{(d-c)} & ; for\ c \le x \le d \\ 0 & ; for\ x > d \end{cases}$$

2.4. Pentagonal Fuzzy Numbers

The pentagonal fuzzy number is defined as $\widetilde{A}$ = {a, b, c, d, e}, where all a, b, c, d, e are real numbers, and its membership function $\mu_{\widetilde{A}}(x)$ is given below:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & ; for\ x < a \\ \frac{(x-a)}{(b-a)} & ; for\ a \le x \le b \\ \frac{(x-b)}{(c-b)} & ; for\ b \le x \le c \\ 1 & ; for\ x = c \\ \frac{(d-x)}{(d-c)} & ; for\ c \le x \le d \\ \frac{(e-x)}{(e-d)} & ; for\ d \le x \le e \\ 0 & ; for\ x > e \end{cases}$$

2.5. Hexagonal Fuzzy Numbers

Hexagonal fuzzy numbers are defined as $\widetilde{A}$= { a_1, a_2, a_3, a_4, a_5, a_6}, where all a_1, a_2, a_3, a_4, a_5, a_6 are real numbers, and their membership function $\mu_{\widetilde{A}}(x)$ is given below:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & ; x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & ; for\ a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & ; for\ a_2 \le x \le a_3 \\ 1 & ; for\ a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{a_5-x}{a_5-a_4}\right) & ; for\ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & ; for\ a_5 \le x \le a_6 \\ 0 & ; for\ x > a_6 \end{cases}$$

3. Model Description

3.1. Notations

p = purchase cost
k = storage cost for one unit per day
b = backorder cost for one unit per day
c = cost of placing as order
d = total demand over the planning time period [0, T]
T = scheduling time period
q = order quantity per cycle

s = shortage quantity per cycle
α = screening cost per unit
β = reworking cost per unit
θ = percentage of defective items
TC = total annual cost
Z = order level
$\widetilde{p}$ = fuzzy purchase cost
$\widetilde{k}$ = storage fuzzy cost for one unit per day
$\widetilde{b}$ = backorder fuzzy cost for one unit per day
$\widetilde{c}$ = fuzzy cost of placing as order
$\widetilde{d}$ = total fuzzy demand over the planning time period [0, T]
$\widetilde{T}$ = fuzzy scheduling time period
$\widetilde{q}$ = fuzzy order quantity per cycle
$\widetilde{s}$ = fuzzy shortage quantity per cycle
$\widetilde{\alpha}$ = fuzzy screening cost per unit
$\widetilde{\beta}$ = fuzzy reworking cost per unit
$\widetilde{TC}$ = fuzzy total annual cost
$\widetilde{z}$ = fuzzy order level

3.2. Assumptions

- Lead time is allowed.
- Shortages are allowed.
- The function of the form R(L) = aL^{-b}, where a and b are real constants, ties the lead time crash cost to the lead time. a > 0, 0 < b ≤ 0.5,
- The storage cost is considered.
- The back ordering cost is considered.
- The ordering cost is considered.
- The only delay in the screening procedure is time; the buyer getsthe additional expense.
- The storage cost (k), backordering cost (b), ordering cost (c), demand (d), shortage cost (s), screening cost (α) and reworking cost (β) are taken as trapezoidal, pentagonal and hexagonal fuzzy numbers.
- The screening cost and reworking cost are constants.
- q is the order quality per cycle.
- The length of the plan, expressed in days, is represented by L.
- tq is the cycle length or scheduling period.
- At the start of each cycle, only one order is produced, and the entire lot is delivered in a single batch.
- The goals are to reduce the overall yearly cost that is pertinent.

4. Model Formulation

4.1. Proposed Inventory Model in Crisp Sense

From the above assumptions and notations, we can determine the total annual cost for the inventory model, which includes shortages and lead time in a crisp environment. Over the planning period [0, T], the total crisp cost is determined by
TC(T) = Storage cost + Shortage cost + Ordering cost + Lead time crash cost + Screening cost + Reworking cost.

$$TC(T) = \left[\frac{kz^2}{2dT}\right] + \frac{c}{T} + s(dT - z)^2 + \frac{R(L)}{T} + \alpha dT + \beta\theta dT \tag{4.1}$$

$$TC(T) = \frac{ksdT}{2(k+s)} + \frac{c}{T} + \frac{aL^{-b}}{T} + \alpha dT + \beta\theta dT \tag{4.2}$$

From (4.1),

$$TC(q) = \frac{kz^2}{2q} + \frac{cd}{q} + s(q-z)^2 + \frac{d}{qaL^{-b}} + \alpha q + \beta\theta q \tag{4.3}$$

Where z = $\frac{sdT}{k+s}$ and q = dT
From (4.2),

$$TC(q) = \frac{ksq}{2(k+s)} + \frac{cd}{q} + \frac{d}{q}aL^{-b} + \alpha q + \beta\theta q \tag{4.4}$$

Partially differentiating (4.2) with relation to 'T', yields

$$\frac{\partial TC}{\partial T} = \frac{ksq}{2(k+s)} - \frac{C}{T^2} - \frac{aL^{-b}}{T} + \alpha q + \beta\theta q \tag{4.5}$$

Finding the first-order partial derivative with regard to 'T' and equating it to zero will yield the ideal values of Q and TC.
(i.e.,) $\frac{\partial TC}{\partial T} = 0$ gives
Optimum Time Period

$$T = \sqrt{\frac{2\left(c + aL^{-b}\right)(k+s)}{ksd + 2\alpha d\,(k+s) + 2\beta\theta d(k+s)}} \tag{4.6}$$

Optimum order quantity

$$q = dT = \sqrt{\frac{2\left(c + aL^{-b}\right)(k+s)}{ks + 2\alpha\,(k+s) + 2\beta\theta(k+s)}} \tag{4.7}$$

Optimal Total Annual Cost

$$TC = \sqrt{2d(c + aL^{-b})\left[\frac{ks}{k+s} + 2\alpha + 2\beta\theta\right]} \tag{4.8}$$

4.2. Proposed Model in the Fuzzy Sense

From the above notations and assumptions, we obtain the total annual cost for the inventory model with shortages and lead time in a fuzzy environment. The planning period [0, T] fuzzy total cost is determined by
$\widetilde{TC}$ ($\widetilde{T}$) = Storage fuzzy cost + Shortage cost + Ordering cost + Lead time crashing cost + Screeningcost + Reworking cost.

$$\widetilde{TC}(\widetilde{T}) = \left[\frac{\widetilde{k}\widetilde{z}^2}{2\widetilde{d}\widetilde{T}}\right] + \frac{\widetilde{C}}{\widetilde{T}} + \widetilde{S}(\widetilde{d}\widetilde{T} - \widetilde{Z})^2 + \widetilde{\alpha}\widetilde{d}\widetilde{T} + \widetilde{\beta}\theta\widetilde{d}\widetilde{T} \tag{4.9}$$

$$\widetilde{TC}(\widetilde{T}) = \frac{\widetilde{k}\widetilde{s}\widetilde{d}\widetilde{T}}{2(\widetilde{k} + \widetilde{s})} + \frac{\widetilde{C}}{\widetilde{T}} + \frac{\widetilde{a}L^{-b}}{L} + \widetilde{\alpha}\widetilde{d}\widetilde{T} + \widetilde{\beta}\theta\widetilde{T} \tag{4.10}$$

From (4.9),

$$\widetilde{TC}(\widetilde{q}) = \frac{\widetilde{k}\widetilde{z}^2}{2\widetilde{q}} + \frac{\widetilde{c}\widetilde{d}}{\widetilde{q}} + \widetilde{s}(\widetilde{q}-\widetilde{z})^2 + \frac{\widetilde{d}}{\widetilde{q}}aL^{-b} + \widetilde{\alpha}\widetilde{q} + \widetilde{\beta}\theta\widetilde{q} \tag{4.11}$$

where $\widetilde{z} = \frac{\widetilde{s}\widetilde{d}\widetilde{T}}{\widetilde{k}+\widetilde{s}} and \widetilde{q} = \widetilde{d}\widetilde{T}$

From (4.10),

$$\widetilde{TC}(\widetilde{q}) = \frac{\widetilde{k}\widetilde{s}\widetilde{q}}{2(\widetilde{k}+\widetilde{s})} + \frac{\widetilde{c}\widetilde{d}}{\widetilde{q}} + \frac{\widetilde{d}}{\widetilde{q}}aL^{-b} + \widetilde{\widetilde{\alpha}q} + \widetilde{\beta}\theta\widetilde{q} \tag{4.12}$$

By partially differentiating (10) with respect to $\widetilde{T}$, we obtain

$$\frac{\partial\widetilde{TC}}{\partial\widetilde{T}} = \frac{\widetilde{k}\widetilde{s}\widetilde{q}}{2(\widetilde{k}+\widetilde{s})} - \frac{\widetilde{c}}{\widetilde{T}^2} - \frac{aL^{-b}}{\widetilde{T}} + \widetilde{\alpha}\widetilde{q} + \widetilde{\beta}\theta\widetilde{q} \tag{4.13}$$

The first-order partial derivative with respect '$\widetilde{T}$' can be found and equated to zero in order to get the optimal $\widetilde{q}$ and $\widetilde{T}\widetilde{c}$.

(i.e.,) $\frac{\partial\widetilde{T}\widetilde{c}}{\partial\widetilde{T}} = 0$ gives

Fuzzy Optimum Time Period

$$\widetilde{T} = \sqrt{\frac{2\left(\widetilde{C}+aL^{-b}\right)(\widetilde{K}+\widetilde{S})}{\widetilde{K}\widetilde{S}\widetilde{d} + 2\widetilde{\alpha}\widetilde{d}\left(\widetilde{k}+\widetilde{s}\right) + 2\widetilde{\beta}\theta\widetilde{d}\widetilde{(\widetilde{k}+\widetilde{s})}}} \tag{4.14}$$

Fuzzy Optimum order quantity

$$\widetilde{q} = \sqrt{\frac{2\widetilde{d}\left(\widetilde{C}+aL^{-b}\right)(\widetilde{K}+\widetilde{S})}{\widetilde{K}\widetilde{S} + 2\widetilde{\alpha}\left(\widetilde{k}+\widetilde{s}\right) + 2\widetilde{\beta}\theta\widetilde{(\widetilde{k}+\widetilde{s})}}} \tag{4.15}$$

Fuzzy Optimal Total Annual Cost

$$\widetilde{TC} = \sqrt{2\widetilde{d}\left(\widetilde{C}+aL^{-b}\right)\left[\frac{\widetilde{k}\widetilde{s}}{\widetilde{k}+\widetilde{s}} + 2\widetilde{\alpha} + 2\widetilde{\beta}\theta\right]} \tag{4.16}$$

5. Numerical Example

5.1. Numerical Example in Crisp Sense

An item's yearly demand Rs. 500 units/year, its annual inventory holding cost is Rs.25 per unit, and making an order costs Rs.50/unit, and the shortage cost is Rs.20/unit/year. If there are 10% defective items, then the duplicate cost for the defective items is Rs.5/unit, and the screening cost is Rs.2/unit. L=0.5, a=1, b=0.1 and the optimum time period, optimal economic order quantity and total annual cost are determined.

Let d = Rs.500 units/year
k = Rs.25 units/year
c = Rs.50 units/year
s = Rs.20 units/year
θ = 10%
α= Rs.5 units
β = Rs.2units
L = 0.5
a = 1

b = 0.1

Optimum Time Period

$$T=\sqrt{\frac{2\left(c+aL^{-b}\right)(k+s)}{ksd+2\alpha d\,(k+s)+2\beta\theta d(k+s)}}=0.1 \tag{5.1}$$

Optimum order quantity

$$q=\sqrt{\frac{2\left(c+aL^{-b}\right)(k+s)}{ks+2\alpha\,(k+s)+2\beta\theta(k+s)}}=48.7258 \tag{5.2}$$

Optimal Total Annual Cost

$$TC=\sqrt{2d\left(c+aL^{-b}\right)\left[\frac{ks}{k+s}+2\alpha+2\beta\theta\right]}=1048.1463 \tag{5.3}$$

5.2. Numerical Example in the Fuzzy Sense

5.2.1. Trapezoidal Fuzzy Numbers. Let $\widetilde{d}$ = (480, 500, 540, 560) units / year

$\widetilde{k}$ = (20, 25, 35, 40) units / year

$\widetilde{c}$ = (40, 50, 70, 80) units / year

$\widetilde{s}$ = (15, 20, 30, 35) units / year

θ = 10%

$\widetilde{\alpha}$ = (4, 5, 7, 8)/unit

$\widetilde{\beta}$ = (1, 2, 4, 5) / unit

L= 0.5

a = 1

b = 0.1

Trapezoidal fuzzy optimum time period

$$\widetilde{T}=\sqrt{\frac{2\left(\widetilde{C}+aL^{-b}\right)(\widetilde{K}+\widetilde{S})}{\widetilde{K}\widetilde{S}\widetilde{d}+2\widetilde{\alpha}\widetilde{d}\left(\widetilde{k}+\widetilde{s}\right)+2\widetilde{\beta}\theta\widetilde{d}\widetilde{(\widetilde{k}+\widetilde{s})}}}=0.0946 \tag{5.4}$$

Trapezoidal fuzzy optimum order quantity

$$\widetilde{q}=\sqrt{\frac{2\widetilde{d}\left(\widetilde{C}+aL^{-b}\right)(\widetilde{K}+\widetilde{S})}{\widetilde{K}\widetilde{S}+2\widetilde{\alpha}\left(\widetilde{k}+\widetilde{s}\right)+2\widetilde{\beta}\theta\widetilde{(\widetilde{k}+\widetilde{s})}}}=49.20 \tag{5.5}$$

Trapezoidal fuzzy optimal total annual cost

$$\widetilde{TC}=\sqrt{2\widetilde{d}\left(\widetilde{C}+aL^{-b}\right)\left[\frac{\widetilde{k}\widetilde{s}}{\widetilde{k}+\widetilde{s}}+2\widetilde{\alpha}+2\widetilde{\beta}\theta\right]}=1290.88 \tag{5.6}$$

5.2.2. Pentagonal Fuzzy Numbers. Let $\widetilde{d}$ = (480, 490, 500, 510,520)

$\widetilde{k}$ = (23, 25, 27,29,31) units / year

$\widetilde{c}$ = (40, 45, 50, 55, 60) units / year

$\widetilde{s}$ = (18, 20, 22, 24, 26) units / year

θ = 10%

$\widetilde{\alpha}$ = (4, 5, 6, 7, 8) / unit

$\widetilde{\beta}$ = (1, 2, 3, 4, 5) / unit

L= 0.5
a = 1
b = 0.1

Pentagonal fuzzy Optimal Time period

$$\widetilde{T} = \sqrt{\frac{2\left(\widetilde{C} + aL^{-b}\right)(\widetilde{K} + \widetilde{S})}{\widetilde{K}\widetilde{S}\widetilde{d} + 2\widetilde{\alpha}\widetilde{d}\left(\widetilde{k} + \widetilde{s}\right) + 2\widetilde{\beta}\theta\widetilde{d}\widetilde{(\widetilde{k} + \widetilde{s})}}} = 0.0909 \tag{5.7}$$

Pentagonal fuzzy optimum order quantity

$$\widetilde{q} = \sqrt{\frac{2\widetilde{d}\left(\widetilde{C} + aL^{-b}\right)(\widetilde{K} + \widetilde{S})}{\widetilde{K}\widetilde{S} + 2\widetilde{\alpha}\left(\widetilde{k} + \widetilde{s}\right) + 2\widetilde{\beta}\theta\widetilde{(\widetilde{k} + \widetilde{s})}}} = 45.45 \tag{5.8}$$

Pentagonal fuzzy optimal total annual cost

$$\widetilde{TC} = \sqrt{2\widetilde{d}\left(\widetilde{C} + aL^{-b}\right)\left[\frac{\widetilde{k}\check{s}}{\widetilde{k} + \widetilde{s}} + 2\widetilde{\alpha} + 2\widetilde{\beta}\theta\right]} = 1123.66 \tag{5.9}$$

5.2.3. Hexagonal Fuzzy Numbers. Let $\widetilde{d}$ = (470, 480, 490, 500,510,520)
$\widetilde{k}$ = (21, 23, 25, 27, 29, 31) units/year
$\widetilde{c}$ = (35, 40, 45, 50, 55, 60) units / year
$\widetilde{s}$ = (16, 18, 20, 22, 24, 26) units / year
θ = 10%
$\widetilde{\alpha}$ = (3, 4, 5, 6, 7, 8) / unit
$\widetilde{\beta}$ = (1, 2, 3, 4, 5, 6) / unit
L = 0.5
a = 1
b = 0.1

Hexagonal fuzzy Optimal Time period

$$\widetilde{T} = \sqrt{\frac{2\left(\widetilde{C} + aL^{-b}\right)(\widetilde{K} + \widetilde{S})}{\widetilde{K}\widetilde{S}\widetilde{d} + 2\widetilde{\alpha}\widetilde{d}\left(\widetilde{k} + \widetilde{s}\right) + 2\widetilde{\beta}\theta\widetilde{d}\widetilde{(\widetilde{k} + \widetilde{s})}}} = 0.0917 \tag{5.10}$$

Hexagonal fuzzy optimum order quantity

$$\widetilde{q} = \sqrt{\frac{2\widetilde{d}\left(\widetilde{C} + aL^{-b}\right)(\widetilde{K} + \widetilde{S})}{\widetilde{K}\widetilde{S} + 2\widetilde{\alpha}\left(\widetilde{k} + \widetilde{s}\right) + 2\widetilde{\beta}\theta\widetilde{(\widetilde{k} + \widetilde{s})}}} = 45.41 \tag{5.11}$$

Hexagonal fuzzy optimal total annual cost

$$\widetilde{TC} = \sqrt{2\widetilde{d}\left(\widetilde{C} + aL^{-b}\right)\left[\frac{\widetilde{k}\widetilde{s}}{\widetilde{k} + \widetilde{s}} + 2\widetilde{\alpha} + 2\widetilde{\beta}\theta\right]} = 1058.87 \tag{5.12}$$

6. Sensitivity Analysis

The following tables are shows all values

6.1. Sensitivity Analysis for Crisp Value

D	T	q	TC
500	0.0974	48.70	1052.48
600	0.0889	53.34	1146.87
700	0.0824	57.68	1238.25
800	0.0770	61.60	1325.01
900	0.0726	65.34	1409.21
1000	0.0689	68.90	1482.46

Table 1:

6.2. Sensitivity Analysis for Trapezoidal Fuzzy Numbers

$\tilde{d}$	$\tilde{T}$	$\tilde{q}$	$\tilde{TC}$
(480, 500, 540, 560)	0.0946	49.20	1290.88
(580, 600, 640, 660)	0.0867	53.73	1409.56
(680, 700, 740, 760)	0.0804	57.90	1518.98
(780, 800, 840, 860)	0.0735	61.179	1621.04
(880, 900, 940, 960)	0.0711	65.45	1717.04
(980, 1000, 1040,1060)	0.0646	66.70	1799.78

Table 2:

6.3. Sensitivity Analysis for Pentagonal Fuzzy Numbers

$\tilde{d}$	$\tilde{T}$	$\tilde{q}$	$\tilde{TC}$
(480, 490, 500, 510, 520)	0.0909	45.45	1290.88
(580, 590, 600, 610, 620)	0.0830	49.79	1230.91
(680, 690, 700, 710, 720)	0.0768	53.78	1329.54
(780, 790, 800, 810, 820)	0.0719	57.49	1421.33
(880, 890, 900, 910,920)	0.0678	60.98	1507.55
(980, 990, 1000, 1010, 1020)	0.0615	61.54	1574.93

Table 3:

6.4. Sensitivity Analysis for Hexagonal Fuzzy Numbers

$\tilde{d}$	$\tilde{T}$	$\tilde{q}$	$\tilde{TC}$
(470, 480, 490, 500, 510, 520)	0.0917	45.41	1058.87
(570, 580, 590, 600, 610, 620)	0.0837	49.79	1160.92
(670, 680, 690, 700, 710, 720)	0.0777	54.01	1249.10
(770, 780, 790, 800, 810, 820)	0.0724	57.55	1341.92
(870, 880, 890, 900, 910, 920)	0.0682	61.06	1423.82
(970, 980, 990, 1000, 1010,1020)	0.0619	61.98	1484.41

Table 4:

Program 6.1: "Defuzzification of Hexagonal Fuzzy Numbers Using GMI for Calculating Optimum Time, Order Quantity, and Total Cost in Inventory Models"

```
#include ¡iostream.h¿
    #include ¡conio.h¿
    #include ¡math.h¿
```

```
// Function to defuzzify hexagonal fuzzy numbers
float defuzzify (float a1, float a2, float a3, float a4, float a5, float a6)
{
    return (a1 + 2*a2 + 2*a3 + 2*a4 + 2*a5 + a6) / 10.0;
}
void main()
{
clrscr();
     // Given hexagonal fuzzy numbers
    float d = defuzzify(470, 480, 490, 500, 510, 520);
    float k = defuzzify(21, 23, 25, 27, 29, 31);
    float c = defuzzify(35, 40, 45, 50, 55, 60);
    float s = defuzzify(16, 18, 20, 22, 24, 26);
    float alpha = defuzzify(3, 4, 5, 6, 7, 8);
    float beta = defuzzify(1, 2, 3, 4, 5, 6);
     // Constants
    float theta = 0.10;
    float a = 1.0;
    float b = 0.1;
    float L = 0.5;
// Lead time crash cost R(L)
    float R = a * pow(L, -b);
// Optimum Time Period T
    float numerator_T = 2 * (c + R) * (k + s);
    float denominator_T = k * s * d + 2 * alpha * d * (k + s) + 2 * beta * theta * d * (k + s);
    float T = sqrt(numerator_T / denominator_T);
// Optimum Order Quantity q = d * T
    float numerator_q = 2 * (c + R) * (k + s);
    float denominator_q = k * s + 2 * alpha * (k + s) + 2 * beta * theta * (k + s);
    float q = sqrt(numerator_q / denominator_q);
    q = d * T;
// Optimum Total Cost TC
    float TC = sqrt(2 * d * (c + R) * (k * s / (k + s) + 2 * alpha + 2 * beta * theta));
cout ¡¡ ”Defuzzified Demand (d):” ¡¡ d ¡¡ ”units” ¡¡ endl;
cout<< ”Defuzzified Storage Cost (k):” << k << ”units/year” <<endl;
cout<< ”Defuzzified Ordering Cost (c):” << c << ”units/year” <<endl;
cout<< ”Defuzzified Shortage Cost (s):” << s << ”units/year” <<endl;
cout<< ”Defuzzified Screening Cost (alpha):” << alpha <<endl;
cout<< ”Defuzzified Reworking Cost (beta):” << beta <<endl;
cout<< ”Lead Time Crash Cost R(L):” << R <<endl;
cout<< ”Optimum Time Period (T):” << T <<endl;
cout<< ”Optimum Order Quantity (q):” << q <<endl;
cout<< ”Total Annual Cost (TC):” << TC <<endl;
getch();
}
```

OUTPUT:

Defuzzified Demand (d): 495 units
Defuzzified Storage Cost (k): 26 units / year
Defuzzified Ordering Cost (c): 47.5 units / year
Defuzzified Shortage Cost (s): 21 units / year
Defuzzified Screening Cost (alpha): 5.5
Defuzzified Reworking Cost (beta): 3.5
Lead Time Crash Cost R(L): 1.0717

Optimum Time Period (T): 0.0917
Optimum Order Quantity (q): 45.41
Total Annual Cost (TC): 1058.87

Program 6.2: Comparative Analysis of Trapezoidal, Pentagonal, and Hexagonal Fuzzy Numbers in Inventory Optimization Using Graded Mean Integration defuzzification-Based Evaluation of Optimum Time, Order Quantity, and Total Cost.

```
#include <iostream.h>
#include <conio.h>
#include <math.h>
// GMI Functions
float gmiTrapezoidal(float a1, float a2, float a3, float a4)
{
    return (a1 + 2*a2 + 2*a3 + a4) / 6.0;
}
float gmiPentagonal(float a1, float a2, float a3, float a4, float a5)
{
    return (a1 + 2*a2 + 3*a3 + 2*a4 + a5) / 9.0;
}
float gmiHexagonal(float a1, float a2, float a3, float a4, float a5, float a6)
{
    return (a1 + 2*a2 + 2*a3 + 2*a4 + 2*a5 + a6) / 10.0;
}
// Function to compute T, q, TC
void computeAndDisplay(char* label, float d, float k, float c, float s, float alpha, float beta, float theta,
float a, float b, float L)
{
    float R = a * pow(L, -b);
    float numerator_T = 2 * (c + R) * (k + s);
    float denominator_T = k * s * d + 2 * alpha * d * (k + s) + 2 * beta * theta * d * (k + s);
    float T = sqrt(numerator_T / denominator_T);
    float numerator_q = 2 * (c + R) * (k + s);
    float denominator_q = k * s + 2 * alpha * (k + s) + 2 * beta * theta * (k + s);
    float q = d * T;
    float TC = sqrt(2 * d * (c + R) * (k * s / (k + s) + 2 * alpha + 2 * beta * theta));
cout<< "\n---" << label << "Fuzzy Number ---";
cout<< "\nOptimum Time Period (T): "<< T;
cout<< "\nOptimum Order Quantity (q):" << q;
cout<< "\nTotal Annual Cost (TC):" << TC << "\n";
}
void main()
{
clrscr();
    float theta = 0.10;
    float a = 1.0;
    float b = 0.1;
    float L = 0.5;
    // Trapezoidal inputs (4 values each)
    float d_t = gmiTrapezoidal(480, 500, 540, 560);
    float k_t = gmiTrapezoidal(20, 25, 35, 40);
    float c_t = gmiTrapezoidal(40, 50, 70, 80);
    float s_t = gmiTrapezoidal(15, 20, 30, 35);
    float alpha_t = gmiTrapezoidal(4, 5, 7, 8);
    float beta_t = gmiTrapezoidal(1, 2, 4, 5);
```

```
// Pentagonal inputs (5 values each)
float d_p = gmiPentagonal(480, 490, 500, 510, 520);
float k_p = gmiPentagonal(23, 25, 27, 29, 31);
float c_p = gmiPentagonal(40, 45, 50, 55, 60);
float s_p = gmiPentagonal(18, 20, 22, 24, 26);
float alpha_p = gmiPentagonal(4, 5, 6, 7, 8);
float beta_p = gmiPentagonal(1, 2, 3, 4, 5);
// Hexagonal inputs (6 values each)
float d_h = gmiHexagonal(470, 480, 490, 500, 510, 520);
float k_h = gmiHexagonal(21, 23, 25, 27, 29, 31);
float c_h = gmiHexagonal(35, 40, 45, 50, 55, 60);
float s_h = gmiHexagonal(16, 18, 20, 22, 24, 26);
float alpha_h = gmiHexagonal(3, 4, 5, 6, 7, 8);
float beta_h = gmiHexagonal(1, 2, 3, 4, 5, 6);
// Compute and display results for each
compute And Display("Trapezoidal", d_t, k_t, c_t, s_t, alpha_t, beta_t, theta, a, b, L);
compute And Display("Pentagonal", d_p, k_p, c_p, s_p, alpha_p, beta_p, theta, a,b, L);
compute And Display("Hexagonal", d_h, k_h, c_h, s_h, alpha_h, beta_h, theta, a, b, L);
getch();
}
```

OUTPUT:

-- Trapezoidal Fuzzy Number ---

Optimum Time Period (T): 0.0946
Optimum Order Quantity (q): 49.20
Total Annual Cost (TC): 1290.88

--- Pentagonal Fuzzy Number ---

Optimum Time Period (T): 0.0909
Optimum Order Quantity (q): 45.45
Total Annual Cost (TC): 1123.66

--- Hexagonal Fuzzy Number ---

Optimum Time Period (T): 0.0917
Optimum Order Quantity (q): 45.41
Total Annual Cost (TC): 1058.87

7. Conclusion

This paper explores a fuzzy inventory model accounting for shortages and constant demand, utilizing trapezoidal, pentagonal and hexagonal fuzzy numbers to represent carrying, backordering, and ordering costs. The results of the investigation show that trapezoidal fuzzy numbers are ineffective at minimizing overall expenses. In contrast, both pentagonal and hexagonal fuzzy numbers outperform the crisp model in cost efficiency, with hexagonal fuzzy numbers yielding the most favorable results. The numerical examples substantiate these findings, highlighting the advantages of adopting hexagonal fuzzy numbers for improved decision-making in inventory management.

DATA AVAILABILITY

This work is based on non-empirical analysis.

DECLARATION OF COMPETING INTEREST

The authors state that none of the cited in this study publication may have been impacted by any known conflicting financial interests or personal connections.

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