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Commutativity Criteria for 3-Prime Near-Rings via Zero-Power Valued Homoderivations

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ABSTRACT: In this work, we discuss the commutativity of a 3-prime near-ring $\mathcal{N}$ that satisfies some algebraic identities under the action of a homoderivation. Additionally, we provide examples to illustrate that the 3-primeness and zero-power valued conditions imposed in our theorems are essential and cannot be disregarded.

Key Words: Prime near-rings, homoderivation, commutativity.

Contents

1. Introduction

Throughout this paper, $\mathcal{N}$ represents a left zero-symmetric near-ring. The symbol $\mathcal{Z}(\mathcal{N})$ will denote the multiplicative center of $\mathcal{N}$, that is $\mathcal{Z}(\mathcal{N}) = \{x \in \mathcal{N} \mid xy = yx \text{ for all } y \in \mathcal{N}\}$. Note that $\mathcal{N}$ is a zero-symmetric if $0.x = 0$ for all $x \in \mathcal{N}$, (recall that left distributive yields $x.0 = 0$). A near-ring $\mathcal{N}$ is said to be 3-prime if, for $x, y \in \mathcal{N}$, the condition $x\mathcal{N}y = \{0\}$ implies $x = 0$ or $y = 0$, and we say that $\mathcal{N}$ is 2-torsion-free if $(\mathcal{N}, +)$ has no elements of order 2. For any pair of elements $x, y \in \mathcal{N}$, $[x, y] = xy - yx$ and $x \circ y = xy + yx$ will denote the well-known Lie product and Jordan product, respectively. A derivation on $\mathcal{N}$ is an additive mapping $d : \mathcal{N} \to \mathcal{N}$ such that $d(xy) = d(x)y + xd(y)$ for all $x, y \in \mathcal{N}$. Over the past decades, several authors have studied the commutativity of prime rings and 3-prime near-rings that admit certain additive mappings, such as derivations, semi-derivations, and generalized derivations, which satisfy appropriate algebraic conditions. For more details, see for example, [4,7,8,9,11,12,13] and [14].

The concept of a homoderivation was initially defined by El Sofy [10] on prime ring $\mathcal{R}$ as an additive mapping $h : \mathcal{R} \to \mathcal{R}$ such that $h(xy) = h(x)h(y) + h(x)y + xh(y)$ for all $x, y \in \mathcal{R}$. He proved that the prime ring $\mathcal{R}$ is commutative whenever h satisfies the condition $h([x, y]) = [x, y]$ for all $x, y \in \mathcal{U}$, where $\mathcal{U}$ is a nonzero ideal of $\mathcal{R}$. Note that a homoderivation h is a derivation on $\mathcal{R}$ if and only if $h(x)h(y) = 0$ for all $x, y \in \mathcal{R}$. This result leads us to $h(x)\mathcal{R}h(y) = \{0\}$ for all $x, y \in \mathcal{R}$. Consequently, if $\mathcal{R}$ is prime, the only additive map that is both a derivation and a homoderivation is the zero map. Let S be a subset of $\mathcal{R}$. A mapping $f : \mathcal{R} \to \mathcal{R}$ is said to be zero-power valued on S if it preserves S and, for any element $x \in S$, there exists a positive integer $k(x) > 1$ such that $f^{k(x)}(x) = 0$.

Recently, significant work has been conducted on the commutativity of prime and semi-prime rings with homoderivations satisfying certain differential identities (see, for example, [1], [5] and [10]). In [6], A. Boua extended some of these results to 3-prime near-rings. Specifically, he proved notable results concerning Jordan ideals that meet specific conditions under the action of homoderivations. These studies show that the presence of a suitably constrained homoderivation on a 3-prime near-ring compels the near-ring to be commutative. In this paper, we continue this line of research by examining 3-prime near-rings with homoderivations.

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2. Preliminary results

In this section, we present some well-known lemmas that will be used to develop the proof of our main results.

Lemma 2.1 *Let $\mathcal{N}$ be a 3-prime near-ring and $\mathcal{I}$ be a nonzero semigroup ideal of $\mathcal{N}$.*

(i) [3, Lemma 1.4(i)] *If $x, y \in \mathcal{N}$ and $x\mathcal{I}y = \{0\}$, then $x = 0$ or $y = 0$.*

(ii) [3, Lemma 1.3(i)] *If $x \in \mathcal{N}$ and $x\mathcal{I} = \{0\}$ or $\mathcal{I}x = \{0\}$, then $x = 0$.*

Lemma 2.2 *Let $\mathcal{N}$ be a 3-prime near-ring.*

(i) [3, Lemma 1.2(iii)] *If $z \in \mathcal{Z}(\mathcal{N}) \setminus \{0\}$ and $xz \in \mathcal{Z}(\mathcal{N})$, then $x \in \mathcal{Z}(\mathcal{N})$.*

(ii) [3, Lemma 1.5] *If $\mathcal{N} \subseteq \mathcal{Z}(\mathcal{N})$, then $\mathcal{N}$ is a commutative ring.*

(iii) [4, Lemma 3(ii)] *If $\mathcal{Z}(\mathcal{N})$ contains a nonzero element z for which $z + z \in \mathcal{Z}(\mathcal{N})$, then $(\mathcal{N}, +)$ is abelian.*

Lemma 2.3 [6, Lemma 2.4] *Let $\mathcal{N}$ be a 3-prime near-ring and h be a nonzero homoderivation of $\mathcal{N}$. If $ah(\mathcal{N}) = \{0\}$, then $a = 0$.*

Lemma 2.4 [6, Theorem 3.7(i)] *Let $\mathcal{N}$ be a 2-torsion free 3-prime near-ring. If $\mathcal{N}$ admits a nonzero homoderivation h, then the following assertions are equivalent:*

(i) *$h(\mathcal{N}) \subseteq \mathcal{Z}(\mathcal{N})$,*

(ii) *$\mathcal{N}$ is a commutative ring.*

3. Main results

M. Ashraf and N. Ur-Rehman [2] proved the commutativity of prime ring $\mathcal{R}$ admitting a derivation d that satisfies any one of the properties $d(xy) - xy \in \mathcal{Z}(\mathcal{R})$ or $d(xy) + xy \in \mathcal{Z}(\mathcal{R})$, for all $x, y \in \mathcal{I}$, where $\mathcal{I}$ is a nonzero ideal of $\mathcal{R}$. Additionally, using the same notations, E. F. Al harfie et al. [1] proved the commutativity of $\mathcal{R}$ admitting a homoderivation h such that $h(xy) + xy \in \mathcal{Z}(\mathcal{R})$ or $h(xy) - xy \in \mathcal{Z}(\mathcal{R})$ for all $x, y \in \mathcal{I}$. Motivated by these results, we prove new statements regarding homoderivations by considering the case where $\mathcal{N}$ is a near-ring instead of $\mathcal{R}$, admitting a homoderivation h that is zero-power valued on $\mathcal{N}$. We begin by proving the first theorem, which is crucial for proving the subsequent theorems.

Theorem 3.1 *Let $\mathcal{N}$ be a 2-torsion free 3-prime near-ring. If $\mathcal{N}$ admits a nonzero homderivation h which is zero-power valued on $\mathcal{N}$, then the following conditions are equivalent:*

(i) *$[xh(y) + [x, y], t] \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$,*

(ii) *$[(xh(y) + [x, y]) \circ t, r] \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t, r \in \mathcal{N}$,*

(iii) *$[xh(y) + [x, y], t] \circ r \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t, r \in \mathcal{N}$,*

(iv) *$\mathcal{N}$ is a commutative ring.*

Proof: Condition (iv) readily implies conditions (i), (ii), and (iii).
$(i) \Rightarrow (iv)$. Assume that $\mathcal{N}$ admits a homoderivation h such that

$$[xh(y) + [x, y], t] \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y, t \in \mathcal{N}. \tag{3.1}$$

Substituting $(xh(y)+[x, y])t$ for t in (3.1), we get $(xh(y)+[x, y])[xh(y)+[x, y], t] \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$. In virtue of Lemma 2.2(i), the latter relation shows that $[xh(y) + [x, y], t] = 0$ or $xh(y) + [x, y] \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$ which implies that

$$xh(y) + [x, y] \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y \in \mathcal{N}. \tag{3.2}$$

Since h is zero-power valued on $\mathcal{N}$ and is a nonzero mapping, there exists an element $y_0 \in \mathcal{N}$ for which a positive integer $k = k(y_0) > 1$ exists such that $h^k(y_0) = 0$ and $z = h^{k-1}(y_0) \neq 0$. In particular, replace y by z in (3.2), we infer that $[x, z] = xh(z) + [x, z] \in \mathcal{Z}(\mathcal{N})$ for all $x \in \mathcal{N}$. Taking $x = zx$ in the last relation, we obtain $z[x, z] \in \mathcal{Z}(\mathcal{N})$ for all $x \in \mathcal{N}$ which, because of Lemma 2.2(i), implies that $[x, z] = 0$ or $z \in \mathcal{Z}(\mathcal{N})$ for all $x \in \mathcal{N}$, so that $z \in \mathcal{Z}(\mathcal{N})$. Now, replacing x by z in (3.2), we get $zh(y) \in \mathcal{Z}(\mathcal{N})$ for all $y \in \mathcal{N}$. As $0 \neq z \in \mathcal{Z}(\mathcal{N})$ and in view of Lemma 2.2(i), the previous expression gives $h(y) \in \mathcal{Z}(\mathcal{N})$ for all $y \in \mathcal{N}$ and hence $\mathcal{N}$ is a commutative ring by Lemma 2.4.
$(i) \Rightarrow (iv)$. Assume that

$$[(xh(y) + [x, y]) \circ t, r] \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y, t, r \in \mathcal{N}. \tag{3.3}$$

It follows that $\big[[(xh(y)+[x, y]) \circ t, r], m\big] = 0$ for all $x, y, t, r, m \in \mathcal{N}$. Replacing r by $((xh(y)+[x, y]) \circ t)r$ in the last relation, we get $\Big[\big((xh(y) + [x, y]) \circ t\big)\big[(xh(y) + [x, y]) \circ t, r\big], m\Big] = 0$ for all $x, y, t, r, m \in \mathcal{N}$, which means that $\big[(xh(y) + [x, y]) \circ t, r\big]\big[(xh(y) + [x, y]) \circ t, m\big] = 0$ for all $x, y, t, r, m \in \mathcal{N}$. According to (3.3), we can see that $\big[(xh(y) + [x, y]) \circ t, r\big]\mathcal{N}\big[(xh(y) + [x, y]) \circ t, m\big] = \{0\}$ for all $x, y, t, r, m \in \mathcal{N}$. The 3-primeness of $\mathcal{N}$ implies that either $[(xh(y) + [x, y]) \circ t, r] = 0$ or $[(xh(y) + [x, y]) \circ t, m] = 0$ for all $x, y, t, r, m \in \mathcal{N}$. Thus in every case, we obtain

$$(xh(y) + [x, y]) \circ t \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y, t \in \mathcal{N}. \tag{3.4}$$

Now, our goal is to show that $\mathcal{Z}(\mathcal{N}) \neq \{0\}$. Indeed, suppose that $\mathcal{Z}(\mathcal{N}) = \{0\}$ and substituting $xh(y) + [x, y]$ for t in (3.4), we find that $2(xh(y) + [x, y])^2 = 0$ for all $x, y \in \mathcal{N}$. Thus, 2-torsion freeness of $\mathcal{N}$ implies that $(xh(y) + [x, y])^2 = 0$ for all $x, y \in \mathcal{N}$. On the other hand, (3.4) yields $(xh(y) + [x, y])t + t(xh(y) + [x, y]) = 0$ for all $x, y, t \in \mathcal{N}$. Left multiplying the last result by $(xh(y) + [x, y])$, we get $(xh(y)+[x, y])t(xh(y)+[x, y]) = 0$ for all $x, y, t \in \mathcal{N}$ hence it follows that $\big(xh(y)+[x, y]\big)\mathcal{N}\big(xh(y)+[x, y]\big) = 0$ for all $x, y \in \mathcal{N}$. In the light of the 3-primeness of $\mathcal{N}$, we conclude that

$$xh(y) + [x, y] = 0 \text{ for all } x, y \in \mathcal{N}. \tag{3.5}$$

So, by induction, we can see that

$$xh^n(y) + [x, h^{n-1}(y)] = 0 \text{ for all } x, y \in \mathcal{N}, n \in \mathbb{N}^*. \tag{3.6}$$

Let $y_0 \in \mathcal{N}$ such that $h(y_0) \neq 0$. Since h is zero-power valued on $\mathcal{N}$, there exists a positive integer $k = k(y_0) > 1$ such that $h^k(y_0) = 0$ and $z = h^{k-1}(y_0) \neq 0$. From (3.6), we have $xh(z) + [x, z] = 0 = [x, z]$ for all $x \in \mathcal{N}$ which gives $z \in \mathcal{Z}(\mathcal{N})$. Replacing x by z in (3.5), we obtain $zh(y) = 0$ for all $y \in \mathcal{N}$ which means that $zh(\mathcal{N}) = \{0\}$ and thus $z = 0$ by Lemma 2.3; a contradiction. Consequently, $\mathcal{Z}(\mathcal{N}) \neq \{0\}$.
Choosing $0 \neq z_0 \in \mathcal{Z}(\mathcal{N})$ and substituting z_0 for t in (3.4), we get $z_0\big(xh(y)+[x, y]+xh(y)+[x, y]\big) \in \mathcal{Z}(\mathcal{N})$ for all $x, y \in \mathcal{N}$. Invoking Lemma 2.2(i), we find that $2(xh(y)+[x, y]) \in \mathcal{Z}(\mathcal{N})$ for all $x, y \in \mathcal{N}$. Replacing t by $xh(y) + [x, y]$ in (3.4), we obtain $(xh(y) + [x, y])(2(xh(y) + [x, y])) \in \mathcal{Z}(\mathcal{N})$ for all $x, y, \in \mathcal{N}$. Once again, using Lemma 2.2(i) together with the 2-torsion freeness of $\mathcal{N}$, we infer that $xh(y) + [x, y] \in \mathcal{Z}(\mathcal{N})$ for all $x, y \in \mathcal{N}$. Since the last result is the same as (3.2), we conclude that $\mathcal{N}$ is a commutative ring.
$(iii) \Rightarrow (iv)$ Suppose that

$$\big[xh(y) + [x, y], t\big] \circ r \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y, t, r \in \mathcal{N}. \tag{3.7}$$

Accordingly, $\big[\big[xh(y) + [x, y], t\big] \circ r, m\big] = 0$ for all $x, y, t, r, m \in \mathcal{N}$. Replacing r by $\big[xh(y) + [x, y], t\big]r$ in the last relation, we get $\Big[\big[xh(y)+[x, y], t\big]\Big(\big[xh(y)+[x, y], t\big] \circ r\Big), m\Big] = 0$ for all $x, y, t, r, m \in \mathcal{N}$ which, in virtue of (3.7), implies that $\Big(\big[xh(y) + [x, y], t\big] \circ r\Big)\big[[xh(y) + [x, y], t], m\big] = 0$ for all $x, y, t, r, m \in \mathcal{N}$. Left multiplying by k, where $k \in \mathcal{N}$, and using (3.7), we find that $\big(\big[xh(y)+[x, y], t\big]\circ r\big)\, k\, \big[[xh(y)+[x, y], t], m\big] = 0$. This can be rewritten as $\big(\big[xh(y) + [x, y], t\big] \circ r\big)\mathcal{N}\big[[xh(y) + [x, y], t], m\big] = \{0\}$ for all $x, y, t, r, m \in \mathcal{N}$. Since $\mathcal{N}$ is 3-prime, the above relation shows that either

$$\big[xh(y) + [x, y], t\big] \circ r = 0 \text{ or } \big[[xh(y) + [x, y], t], m\big] = 0 \text{ for all } x, y, t, r, m \in \mathcal{N} \tag{3.8}$$

• Suppose that $[x\circ h(y)+x\circ y,t]\circ r=0$ for all $x,y,t,r\in\mathcal{N}$, thus $[xh(y)+[x,y],t]r=r(-[xh(y)+[x,y],t])$ for all $x,y,t,r\in\mathcal{N}$. For any $n\in\mathcal{N}$, setting $r=nr$ in the latter equation and applying it again, we deduce that $n(-[xh(y)+[x,y],t])r=nr(-[xh(y)+[x,y],t])$ for all $x,y,r,t,n\in\mathcal{N}$, which leads to $\mathcal{N}[-[xh(y)+[x,y],t],r]=\{0\}$ for all $x,y,t,r\in\mathcal{N}$. By Lemma 2.1(ii), the preceding result yields to $-[xh(y)+[x,y],t]\in\mathcal{Z}(\mathcal{N})$ for all $x,y,t\in\mathcal{N}$. Now, substituting $(xh(y)+[x,y])t$ for t and using Lemma 2.2(i), we arrive at $xh(y)+[x,y]\in\mathcal{Z}(\mathcal{N})$ for all $x,y\in\mathcal{N}$. Since this expression corresponds to the relation (3.2) in the proof of $(i)\Rightarrow(iv)$, we deduce that $\mathcal{N}$ is a commutative ring.
• Assume that there exist $x_0,y_0,t_0,r_0\in\mathcal{N}$ such that $[x_0\circ h(y_0)+x_0\circ y_0,t_0]\circ r_0\neq 0$. An immediate consequence of this condition is that $[x_0h(y_0)+[x_0,y_0],t_0]\neq 0$; also from (3.8), it follows that $\big[[x_0h(y_0)+[x_0,y_0],t_0],m\big]=0$ for all $m\in\mathcal{N}$ and thus $[x_0h(y_0)+[x_0,y_0],t_0]\in\mathcal{Z}(\mathcal{N})$. For these elements, (3.7) leads to $[x_0h(y_0)+[x_0,y_0],t_0](2r)\in\mathcal{Z}(\mathcal{N})$ for all $r\in\mathcal{N}$. Applying Lemma 2.2(i) and using the fact that $[x_0h(y_0)+[x_0,y_0],t_0]\neq 0$, we find that $2r\in\mathcal{Z}(\mathcal{N})$ for all $r\in\mathcal{N}$. Replacing r by r^2 in the last result, we obtain $r(2r)\in\mathcal{Z}(\mathcal{N})$ for all $r\in\mathcal{N}$. By Lemma 2.2(i) and considering the 2-torsion freeness of $\mathcal{N}$, we conclude that $\mathcal{N}\subseteq\mathcal{Z}(\mathcal{N})$ and hence $\mathcal{N}$ is a commutative ring by Lemma 2.2(ii). □

Theorem 3.2 *Let $\mathcal{N}$ be a 3-prime near-ring. If $\mathcal{N}$ admits a nonzero homderivation h which is zero-valued power of $\mathcal{N}$, then the following conditions are equivalent:*

(i) $[x\circ h(y)+xy,t]\circ r\in\mathcal{Z}(\mathcal{N})$ *for all* $x,y,t\in\mathcal{N}$,

(ii) $[(x\circ h(y)+xy)\circ t,r]\in\mathcal{Z}(\mathcal{N})$ *for all* $x,y,t,r\in\mathcal{N}$,

(iii) $\mathcal{N}$ *is a commutative ring.*

Proof: $(i)\Rightarrow(iii)$. By hypothesis, we have

$$[x\circ h(y)+xy,t]\circ r\in\mathcal{Z}(\mathcal{N}) \text{ for all } x,y,t,r\in\mathcal{N}. \tag{3.9}$$

Suppose that $\mathcal{Z}(\mathcal{N})=\{0\}$. It follows that $[x\circ h(y)+xy,t]\circ r=0$ for all $x,y,t,r\in\mathcal{N}$, that is $[x\circ h(y)+xy,t]r=r\big(-[x\circ h(y)+xy,t]\big)$ for all $x,y,t,r\in\mathcal{N}$. Taking $r=sr$, where $s\in\mathcal{N}$, in the previous equation and using it again, we get $r\big[-[x\circ h(y)+xy,t],s\big]=0$ for all $x,y,r,t,s\in\mathcal{N}$, which can be rewritten as $\mathcal{N}\big[-[x\circ h(y)+xy,t],s\big]=\{0\}$ for all $x,y,t,s\in\mathcal{N}$. Applying Lemma 2.1(ii) and using our hypothesis that $\mathcal{Z}(\mathcal{N})=\{0\}$, we arrive at $x\circ h(y)+xy=0$ for all $x,y\in\mathcal{N}$. Since h is zero-power valued on $\mathcal{N}$ and is nonzero mapping, there exist $y_0\in\mathcal{N}$ and a positive integer $k=k(y_0)>1$ such that $h^k(y_0)=0$ and $z=h^{k-1}(y_0)\neq 0$. It follows that $xz=x\circ h(z)+xz=0$ for all $x\in\mathcal{N}$. By Lemma 2.1(ii), we obtain $z=0$ which is a contradiction, and hence $\mathcal{Z}(\mathcal{N})\neq\{0\}$. Now, choosing $z_0\in\mathcal{Z}(\mathcal{N})\setminus\{0\}$ and substituting z_0 for r in (3.9), we get $z_0\big([x\circ h(y)+xy,t]+[x\circ h(y)+xy,t]\big)\in\mathcal{Z}(\mathcal{N})$ for all $x,y,t\in\mathcal{N}$. Using Lemma 2.2(i), we find that $2[x\circ h(y)+xy,t]\in\mathcal{Z}(\mathcal{N})$ for all $x,y,t\in\mathcal{N}$. Again, taking $r=[x\circ h(y)+xy,t]r$ in (3.9) and invoking Lemma 2.2(i), we find that

$$2[x\circ h(y)+xy,t]=0 \text{ or } [x\circ h(y)+xy,t]\in\mathcal{Z}(\mathcal{N}) \text{ for all } x,y,t\in\mathcal{N}. \tag{3.10}$$

Let $x_0,y_0,t_0\in\mathcal{N}$ such that $2[x_0\circ h(y_0)+x_0y_0,t_0]=0$, then $[x_0\circ h(y_0)+x_0y_0,t_0]=-[x_0\circ h(y_0)+x_0y_0,t_0]$. Replacing x,y,t,r by x_0,y_0,t_0 and $[x_0\circ h(y_0)+x_0y_0,t_0]r$, respectively, in (3.9), we obtain $\big[x_0\circ h(y_0)+x_0y_0,t_0\big]\Big(\big[x_0\circ h(y_0)+x_0y_0,t_0\big]\circ r\Big)\in\mathcal{Z}(\mathcal{N})$ for all $r\in\mathcal{N}$. It follows that

$$\big[x_0\circ h(y_0)+x_0y_0,t_0\big]\in\mathcal{Z}(\mathcal{N}) \text{ or } \big[x_0\circ h(y_0)+x_0y_0,t_0\big]\circ r=0 \text{ for all } r\in\mathcal{N}. \tag{3.11}$$

Assume that there exists $r_0\in\mathcal{N}$ such that $[x_0\circ h(y_0)+x_0y_0,t_0]\circ r_0\neq 0$, then (3.11) shows that $[x_0\circ h(y_0)+x_0y_0,t_0]\in\mathcal{Z}(\mathcal{N})$. If $[x_0\circ h(y_0)+x_0y_0,t_0]\circ r=0$ for all $r\in\mathcal{N}$, expanding this condition, we find that

$$\begin{aligned}[x_0\circ h(y_0)+x_0y_0,t_0]r &= -r[x_0\circ h(y_0)+x_0y_0,t_0]\\ &= r(-[x_0\circ h(y_0)+x_0y_0,t_0])\\ &= r[x_0\circ h(y_0)+x_0y_0,t_0]\end{aligned}$$

for all $r \in \mathcal{N}$, which means that $[x_0 \circ h(y_0) + x_0y_0, t_0] \in \mathcal{Z}(\mathcal{N})$. Consequently, (3.10) reduces to

$$[x \circ h(y) + xy, t] \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y, t \in \mathcal{N}. \tag{3.12}$$

Accordingly, $\big[[x \circ h(y) + xy, t], m\big] = 0$ for all $x, y, t, m \in \mathcal{N}$. In particular, for $t = (x \circ h(y) + xy)t$, and in view of (3.12), we get

$[x \circ h(y) + xy, t][x \circ h(y) + xy, m] = 0$ for all $x, y, t, m \in \mathcal{N}$. Taking $m = t$ and multiplying by n, where $n \in \mathcal{N}$, we infer that $[x \circ h(y) + xy, t]n[x \circ h(y) + xy, t] = 0$ for all $x, y, t, n \in \mathcal{N}$, which mean that $[x \circ h(y) + xy, t]\mathcal{N}[x \circ h(y) + xy, t] = \{0\}$ for all $x, y, t \in \mathcal{N}$. Thus, the 3-primeness of $\mathcal{N}$, leads to

$$x \circ h(y) + xy \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y \in \mathcal{N}. \tag{3.13}$$

Since h is zero-power valued on $\mathcal{N}$ and h is nonzero mapping, it necessarily follows that for some nonzero element $y \in \mathcal{N}$, there exists a positive integer $k = k(y) > 1$ such that $h^k(y) = 0$ and $z = h^{k-1}(y) \neq 0$. From (3.13), we deduce that

$$xz = x \circ h(z) + xz \in \mathcal{Z}(\mathcal{N}) \text{ for all } x \in \mathcal{N}. \tag{3.14}$$

Putting $x = zx$ in (3.14), we find that $zxz \in \mathcal{Z}(\mathcal{N})$ for all $x \in \mathcal{N}$. Applying Lemma 2.2(i) together with (3.14), we obtain $xz = 0$ for all $x \in \mathcal{N}$ or $z \in \mathcal{Z}(\mathcal{N})$. According to Lemma 2.1(ii), if the first condition holds, we infer that $z = 0$ which is a contradiction, and hence $0 \neq z \in \mathcal{Z}(\mathcal{N})$. Consequently, (3.14) leads to $x \in \mathcal{Z}(\mathcal{N})$ for all $x \in \mathcal{N}$ and therefore $\mathcal{N}$ is a commutative ring by Lemma 2.2(ii).
(ii) $\Rightarrow$ (iii). Assume that

$$[(x \circ h(y) + xy) \circ t, r] \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y, t, r \in \mathcal{N}. \tag{3.15}$$

By replacing r with $\big((x \circ h(y) + xy) \circ t\big)r$ in (3.15) and using Lemma 2.1(i), we arrive at $\big[(x \circ h(y) + xy) \circ t, r\big] = 0$ or $(x \circ h(y) + xy) \circ t \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t, r \in \mathcal{N}$. Both cases lead us to

$$(x \circ h(y) + xy) \circ t \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y, t \in \mathcal{N}. \tag{3.16}$$

Assume that $\mathcal{Z}(\mathcal{N}) = \{0\}$. Thus, (3.16) gives

$$(x \circ h(y) + xy)t = t(-(x \circ h(y) + xy)) \text{ for all } x, y, t \in \mathcal{N}. \tag{3.17}$$

Putting mt instead of t in (3.17) and using it again, we find that $m[-(x \circ h(y) + xy), t] = 0$ for all $x, y, m, t \in \mathcal{N}$, which means that $\mathcal{N}[-(x \circ h(y) + xy), t] = \{0\}$ for all $x, y, t \in \mathcal{N}$. Using Lemma 2.1(ii) together with our statement hypothesis, we obtain $x \circ h(y) + xy = 0$ for all $x, y \in \mathcal{N}$. Let y_0 a nonzero element of $\mathcal{N}$ for which there exists a positive integer $k = k(y_0) > 1$ such that $h^k(y_0) = 0$ and $h^{k-1}(y_0) \neq 0$. By taking $y = h^{k-1}(y_0)$ in the previous relation, we get $x \circ h^k(y_0) + xh^{k-1}(y_0) = xh^{k-1}(y_0) = 0$ for all $x \in \mathcal{N}$ which, in virtue of Lemma 2.1(ii), implies that $h^{k-1}(y_0) = 0$, a contradiction; and therefore $\mathcal{Z}(\mathcal{N}) \neq \{0\}$. Choosing $z_0 \in \mathcal{Z}(\mathcal{N}) \setminus \{0\}$ and replacing t by z_0 in (3.16), we get $z_0\big(2(x \circ h(y) + xy)\big) \in \mathcal{Z}(\mathcal{N})$ for all $x, y \in \mathcal{N}$. Account of Lemma 2.2(ii) and $z_0 \neq 0$, we obtain $2\big(x \circ h(y) + xy\big) \in \mathcal{Z}(\mathcal{N})$ for all $x, y \in \mathcal{N}$. Once again, replacing t by $x \circ h(y) + xy$ in (3.16), we get $(x \circ h(y) + xy)(2(x \circ h(y) + xy)) \in \mathcal{Z}(\mathcal{N})$ for all $x, y \in \mathcal{N}$. In view of Lemma 2.2(ii), the last relation yields

$$2(x \circ h(y) + xy) = 0 \text{ or } x \circ h(y) + xy \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y \in \mathcal{N}. \tag{3.18}$$

Suppose that there exist $x_0, y_0 \in \mathcal{N}$ such that $2(x_0 \circ h(y_0) + x_0y_0) = 0$, it follows that $x_0 \circ h(y_0) + x_0y_0 = -(x_0 \circ h(y_0) + x_0y_0)$. Also, taking $x = x_0, y = y_0$ and $t = (x_0 \circ h(y_0) + x_0y_0)t$ in (3.16), we obtain $(x_0 \circ h(y_0) + x_0y_0)\Big((x_0 \circ h(y_0) + x_0y_0) \circ t\Big) \in \mathcal{Z}(\mathcal{N})$ for all $t \in \mathcal{N}$ which, because of Lemma 2.2(i), leads to

$$(x_0 \circ h(y_0) + x_0y_0) \circ t = 0 \text{ or } x_0 \circ h(y_0) + x_0y_0 \in \mathcal{Z}(\mathcal{N}) \text{ for all } t \in \mathcal{N}. \tag{3.19}$$

But, if the first condition holds, we can easily verified that $x_0 \circ h(y_0) + x_0y_0 \in \mathcal{Z}(\mathcal{N})$ and therefore from (3.18) and (3.19), we conclude that $x \circ h(y) + xy \in \mathcal{Z}(\mathcal{N})$ for all $x, y \in \mathcal{N}$ which implies that $\mathcal{N}$. As this expression is the same as (3.13), then we can use the same arguments as those used in the first part, we get the desired result.
For (iii) implies both conditions (i) and (ii) is obvious. □

Theorem 3.3 *Let $\mathcal{N}$ be a 2-torsion free 3-prime near-ring. If $\mathcal{N}$ admits a nonzero homoderivation h which is zero-valued power of $\mathcal{N}$, then the following assertions are equivalent:*

(i) $[x \circ h(y) + x \circ y, t] \in \mathcal{Z}(\mathcal{N})$ *for all* $x, y, t \in \mathcal{N}$,

(ii) $[x \circ h(y) + x \circ y, t] \circ r \in \mathcal{Z}(\mathcal{N})$ *for all* $x, y, t, r \in \mathcal{N}$,

(iii) $[(x \circ h(y) + x \circ y) \circ t, r] \in \mathcal{Z}(\mathcal{N})$ *for all* $x, y, t, r \in \mathcal{N}$,

(iv) $\mathcal{N}$ *is a commutative ring.*

Proof: It is straightforward to verify that condition (iv) implies conditions (i), (ii), and (iii).
$(i) \Rightarrow (iv)$. We are assuming that $[x \circ h(y) + x \circ y, t] \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$. Replacing t by $(x \circ h(y) + x \circ y)t$ in the above relation, we obtain $(x \circ h(y) + x \circ y)[x \circ h(y) + x \circ y, t] \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$, by Lemma 2.2(i), this implies that either $[x \circ h(y) + x \circ y, t] = 0$ or $x \circ h(y) + x \circ y \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$. Hence, in both the cases we have

$$x \circ h(y) + x \circ y \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y \in \mathcal{N}. \tag{3.20}$$

Suppose that $\mathcal{Z}(\mathcal{N}) = \{0\}$. In this case, (3.20) gives $x \circ h(y) = -(x \circ y)$ for all $x, y \in \mathcal{N}$. Proceeding by induction, we obtain $x \circ h^n(y) = (-1)^n(x \circ y)$ for all $x, y \in \mathcal{N}$, where $n \in \mathbb{N} \setminus \{0\}$. Since h is zero-power valued on $\mathcal{N}$, for any $y \in \mathcal{N}$, there exists a positive integer $k = k(y) > 1$ such that $h^k(y) = 0$, and thus $x \circ y = 0$ for all $x, y \in \mathcal{N}$. It follows that $x^2 = 0$ for all $x \in \mathcal{N}$ and thus we can see that $xyx = 0$ for all $x, y \in \mathcal{N}$. The 3-primeness of $\mathcal{N}$ forces that $\mathcal{N} = \{0\}$, a contradiction. Consequently, $\mathcal{Z}(\mathcal{N}) \neq \{0\}$. Letting $z_0 \in \mathcal{Z}(\mathcal{N}) \setminus \{0\}$ and taking $x = z_0$ in (3.20), we get $z_0(h(y) + h(y) + y + y) \in \mathcal{Z}(\mathcal{N})$ for all $y \in \mathcal{N}$. In virtue of Lemma 2.2(i) and $z_0 \neq 0$, we obtain

$$h(y) + h(y) + y + y \in \mathcal{Z}(\mathcal{N}) \text{ for all } y \in \mathcal{N}. \tag{3.21}$$

Let y_0 be a nonzero element of $\mathcal{N}$ such that $h(y_0) \neq 0$. As h is zero-power valued on $\mathcal{N}$, there exists a positive integer $k = k(y_0) > 1$ such that $h^k(y_0) = 0$ and $z = h^{k-1}(y_0) \neq 0$. Thus, $2z = h(z)+h(z)+z+z \in \mathcal{Z}(\mathcal{N}) \setminus \{0\}$. Replacing y by $2z$ in (3.21), we get $h(2z)+h(2z)+2z+2z = 2z+2z \in \mathcal{Z}(\mathcal{N})$. Accordingly, $(\mathcal{N}, +)$ is abelian by Lemma 2.2(iii). Consequently, (3.21) becomes $2(h(y) + y) \in \mathcal{Z}(\mathcal{N})$ for all $y \in \mathcal{N}$. Again, substituting $y - h(y) + h^2(y) + ... + (-1)^{k(y)-1}h^{k(y)-1}(y)$ for y in the last relation, we find that $2y \in \mathcal{Z}(\mathcal{N})$ for all $y \in \mathcal{N}$. Putting y^2 instead of y, we obtain $y(2y) \in \mathcal{Z}(\mathcal{N})$ which, because of the 2-torsion freeness of $\mathcal{N}$ together with Lemma 2.2(i), implies that $y \in \mathcal{Z}(\mathcal{N})$ for all $y \in \mathcal{N}$ and hence $\mathcal{N}$ is a commutative ring by Lemma 2.2(ii).
$(ii) \Rightarrow (iv)$. We are assuming that

$$[x \circ h(y) + x \circ y, t] \circ r \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y, t, r \in \mathcal{N}. \tag{3.22}$$

Then $\big[[x \circ h(y) + x \circ y, t] \circ r, m\big] = 0$ for all $x, y, t, r, m \in \mathcal{N}$. For $r = [x \circ h(y) + y \circ x, t]r$, and using (3.22), we get $\big([x \circ h(y) + x \circ y, t] \circ r\big)\big[[x \circ h(y) + x \circ y, t], m\big] = 0$ for all $x, y, t, r, m \in \mathcal{N}$. Once again, according to (3.22), the preceding result assures that $\big([x \circ h(y) + x \circ y, t] \circ r\big)\mathcal{N}\big[[x \circ h(y) + x \circ y, t], m\big] = \{0\}$ for all $x, y, t, r, m \in \mathcal{N}$. In the light of the 3-primeness of $\mathcal{N}$, we find that

$$[x \circ h(y) + x \circ y, t] \circ r = 0 \text{ or } \big[[x \circ h(y) + x \circ y, t], m\big] = 0 \text{ for all } x, y, t, r, m \in \mathcal{N} \tag{3.23}$$

For $x, y, t \in \mathcal{N}$, if the second condition of (3.23) holds for all $m \in \mathcal{N}$, then $[x \circ h(y) + x \circ y, t] \in \mathcal{Z}(\mathcal{N})$; else the first condition of (3.23) will be satisfy for all $r \in \mathcal{N}$; i.e, $[x \circ h(y) + x \circ y, t]r = r(-([x \circ h(y) + x \circ y, t]))$ for all $r \in \mathcal{N}$. Taking $r = rn$ in the preceding relation and using it again, we get $r\Big[-[x \circ h(y) + x \circ y, t], n\Big] = 0$ for all $r, n \in \mathcal{N}$ which reduces to $\mathcal{N}\big[-[x \circ h(y) + x \circ y, t], n\big] = \{0\}$ for all $n \in \mathcal{N}$. According to Lemma 2.1(ii), we obtain $\big[-[x \circ h(y) + x \circ y, t], n\big] = 0$ for all $n \in \mathcal{N}$, that is $-[x \circ h(y) + x \circ y, t] \in \mathcal{Z}(\mathcal{N})$. Accordingly, for all $x, y, t \in \mathcal{N}$, we have either

$$[x \circ h(y) + x \circ y, t] \in \mathcal{Z}(\mathcal{N}) \text{ or } -[x \circ h(y) + x \circ y, t] \in \mathcal{Z}(\mathcal{N}). \tag{3.24}$$

Suppose that there exist $x_0, y_0, t_0 \in \mathcal{N}$ such that $[x_0 \circ h(y_0) + x_0 \circ y_0, t_0] \notin \mathcal{Z}(\mathcal{N})$. From (3.24), it follows that $0 \neq k_0 = -[x_0 \circ h(y_0) + x_0 \circ y_0, t_0] \in \mathcal{Z}(\mathcal{N})$. Now, return to (3.22) and replacing r by k_0, we obtain $k_0\Big([x \circ h(y) + x \circ y, t] + [x \circ h(y) + x \circ y, t]\Big) \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$. Thereby, in view of Lemma 2.2(i) and the fact that $k_0 \neq 0$, $2[x \circ h(y) + x \circ y, t] \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$. On the other hand, taking $r = [x \circ h(y) + x \circ y, t]r$ in (3.22), we get $[x \circ h(y) + x \circ y, t]\Big(2[x \circ h(y) + x \circ y, t]\Big) \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$. Using the fact that $\mathcal{N}$ is 2-torsion free and in virtue of Lemma 2.2(i), we find that $[x \circ h(y) + x \circ y, t] \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$. This contradicts our assumption that $[x_0 \circ h(y_0) + x_0 \circ y_0, t_0] \notin \mathcal{Z}(\mathcal{N})$, which means that this case cannot occur. Hence, (3.24) reduces to $[x \circ h(y) + x \circ y, t] \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$. So, replacing t by $(x \circ h(y) + x \circ y)t$ and invoking Lemma 2.2(i), we find that $[x \circ h(y) + x \circ y, t] = 0$ or $x \circ h(y) + x \circ y \in \mathcal{Z}(\mathcal{N})$ for all $x, y, t \in \mathcal{N}$. We can see that the two conditions give $x \circ h(y) + x \circ y \in \mathcal{Z}(\mathcal{N})$ for all $x, y \in \mathcal{N}$. In this case, all that remains is to follow the same steps as those used after relation (3.20), we deduce that $\mathcal{N}$ is a commutative ring.
$(iii) \Rightarrow (iv)$. Suppose that

$$[(x \circ h(y) + x \circ y) \circ t, r] \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y, t, r \in \mathcal{N}. \tag{3.25}$$

It follows that, $\big[[(x \circ h(y) + x \circ y) \circ t, r], m\big] = 0$ for all $x, y, t, r, m \in \mathcal{N}$. Putting $((x \circ h(y) + x \circ y) \circ t)r$ instead of r, we get $\Big[\big((x \circ h(y) + x \circ y) \circ t\big)\big[(x \circ h(y) + x \circ y) \circ t, r\big], m\Big] = 0$ for all $x, y, t, r, m \in \mathcal{N}$ which, because of (3.25), means that $\big[(x \circ h(y) + x \circ y) \circ t, r\big]\big[(x \circ h(y) + x \circ y) \circ t, m\big] = 0$ for all $x, y, t, r, m \in \mathcal{N}$. Once again, in the light of (3.25), the previous expression shows that $\big[(x \circ h(y) + x \circ y) \circ t, r\big]\,\mathcal{N}\,\big[(x \circ h(y) + x \circ y) \circ t, m\big] = \{0\}$ for all $x, y, t, r, m \in \mathcal{N}$. In view of the 3-primeness of $\mathcal{N}$, we infer that

$$\big[(x \circ h(y) + x \circ y) \circ t, r\big] = 0 \text{ or } \big[(x \circ h(y) + x \circ y) \circ t, m\big] = 0 \text{ for all } x, y, t, r, m \in \mathcal{N}.$$

Accordingly,

$$(x \circ h(y) + x \circ y) \circ t \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y \in \mathcal{N}. \tag{3.26}$$

In the following, our goal is to prove that $\mathcal{Z}(\mathcal{N}) \neq \{0\}$. In fact, suppose that $\mathcal{Z}(\mathcal{N}) = \{0\}$ and replacing t by $x \circ h(y) + x \circ y$ in (3.26), we get $2(x \circ h(y) + x \circ y)^2 = 0$ for all $x, y \in \mathcal{N}$. By the 2-torsion freeness of $\mathcal{N}$, we obtain $(x \circ h(y) + x \circ y)^2 = 0$ for all $x, y \in \mathcal{N}$. On the other hand, (3.26) yields $(x \circ h(y) + x \circ y)t + t(x \circ h(y) + x \circ y) = 0$ for all $x, y, t \in \mathcal{N}$. Left multiplying by $(x \circ h(y) + x \circ y)$ and using the 3-primeness of $\mathcal{N}$, we get $x \circ h(y) + x \circ y = 0$ for all $x, y \in \mathcal{N}$. Following the same steps used after (3.20), we conclude that $\mathcal{N} = \{0\}$, which leads to a contradiction. Therefore, $\mathcal{Z}(\mathcal{N}) \neq \{0\}$. Choosing $0 \neq z_0 \in \mathcal{Z}(\mathcal{N})$ and replacing t by z_0 in (3.26), we get $z_0\big(x \circ h(y) + x \circ y + x \circ h(y) + x \circ y\big) \in \mathcal{Z}(\mathcal{N})$ for all $x, y \in \mathcal{N}$. By Lemma 2.2(i), we find that $2(x \circ h(y) + x \circ y) \in \mathcal{Z}(\mathcal{N})$ for all $x, y \in \mathcal{N}$. On the other hand, taking $x \circ h(y) + x \circ y$ instead of t in (3.26), we obtain $\big(x \circ h(y) + x \circ y\big)\big(2(x \circ h(y) + x \circ y)\big) \in \mathcal{Z}(\mathcal{N})$ for all $x, y, \in \mathcal{N}$. By Lemma 2.2(i), the previous relation shows that

$$2(x \circ h(y) + x \circ y) = 0 \text{ or } x \circ h(y) + x \circ y \in \mathcal{Z}(\mathcal{N}) \text{ for all } x, y \in \mathcal{N}. \tag{3.27}$$

In the light of the 2-torsion freeness of $\mathcal{N}$, the first condition of (3.27) gives $x \circ h(y) + x \circ y = 0$ and therefore (3.27) yields to $x \circ h(y) + x \circ y \in \mathcal{Z}(\mathcal{N})$ for all $x, y \in \mathcal{N}$. Since this relation is the same as (3.20), we conclude that $\mathcal{N}$ is a commutative ring. □

In the following, we present two illustrative examples. The first demonstrates the construction of a homoderivation defined on a 3-prime near-ring that satisfies all the identities established in the third theorem and in not zero-power valued. The second explores the significance of the 3-prime assumption as it appears throughout our theorems.

Example 3.1 *Let $\mathcal{N} = M_2(\mathbb{Z})$ the ring of all 2 ×2 matrices over the integers $\mathbb{Z}$, with usual matrix addition and multiplication. Then $(\mathcal{N}, +, .)$ is 2-torsion-free and 3-prime near-ring. Consider the map $h = -id_{\mathcal{N}}$, it is clear that h is not zero-power valued homoderivation on $\mathcal{N}$, which satisfies the following conditions:*

1. $[A \circ h(B) + A \circ B, C] \in \mathcal{Z}(\mathcal{N})$,

2. $[A \circ h(B) + A \circ B, C] \circ D \in \mathcal{Z}(\mathcal{N})$,

3. $[(A \circ h(B) + A \circ B) \circ C, D] \in \mathcal{Z}(\mathcal{N})$

for all $A, B, C, D \in \mathcal{N}$. *However, the near-ring* $\mathcal{N}$ *fails to be commutative.*

Example 3.2 *Let* $\mathcal{S}$ *be a zero-symmetric, 2-torsion free, noncommutative left near-ring, and let*

$$\mathcal{N} = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} \mid 0, x, y \in \mathcal{S} \right\}.$$

Then, $\mathcal{N}$ *is a 2-torsion free near-ring which is not 3-prime. Define a nonzero mapping* $h : \mathcal{N} \to \mathcal{N}$ *by:*

$$h \begin{pmatrix} 0 & 0 & 0 \\ x & 0 & y \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ y & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Then, h *is a nonzero zero-power valued homoderivation on* $\mathcal{N}$ *that satisfies all identities of our theorems. Furthermore,* $\mathcal{N}$ *is a noncommutative near-ring.*

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