Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2025 (43)** : 1–9.
ISSN-0037-8712
doi:10.5269/bspm.77981

Degree Based Root Connectivity Energy of Graph

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ABSTRACT: The purpose of this paper is to introduce and investigate Degree based root connectivity matrix and the corresponding energy of graph structures. The upper and lower bounds have been established. Further we generalize the Degree based root connectivity energy to various families of graphs including some regular and semi regular graphs.

Key Words: Degree based root connectivity matrix, spectra and energy.

Contents

1. Introduction

One of the fast growing topic of research in Graph theory is energy and spectral analysis. The adjacency matrix plays a vital role in understanding the algebraic properties of graphs. Ivan Gutman [2] introduced the concept of energy of graph which is basically depends on the eigen values of the adjacency matrix of graphs. He defined the energy of graph as the sum of the absolute values of the adjacency matrix of graph.

A chemical compound can be represented by a graph and it is known as molecular graph. Thereby many researchers discussed the energies of not only graph structures but also for chemical compounds. Energy of graph attracted researchers a lot. Many graph theorists/chemists introduced and investicated different sort of matrix representations and also corresponding energies of the graphs as well as chemical compounds. In this paper we introduce a new matrix for a graph G as follows.

Here d_u and d_v represents the degree of the respective vertices u and v.

$$DBRC(G) = \begin{cases} \sqrt{d_u} + \sqrt{d_v} + (d_u d_v)^{\frac{1}{3}} & uv \in E \\ 0 & otherwise \end{cases}$$

Let $\digamma_i$ be the eigenvalues of degree based root connectivity marices, then the degree based root connectivity energy is given by

$$DBRCE(G) = \sum_{i=1}^{n} |\digamma_i|.$$

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2010 *Mathematics Subject Classification:* 05C50.

Submitted July 22, 2025. Published October 30, 2025

2. Upper and lower bounds of degree based root connectivity energy of a graph

The following result gives an upper bound for the DBRC energy of a graph G.

Theorem 2.1 *Let G be a graph with n vertices. Then*

$$DBRCE(G) \leq \sqrt{2n\left[\sum_{i<j}\sqrt{d_u}+\sqrt{d_v}+(d_ud_v)^{\frac{1}{3}}\right]^2}$$

.

Proof: Let $\digamma_1,\ \digamma_2,\ \ldots,\ \digamma_n$ be the eigenvalues of $DBRC(G)$. Now by the Cauchy-Schwartz inequality we have

$$\left(\sum_{i=1}^{n}a_ib_i\right)^2 \leq \left(\sum_{i=1}^{n}{a_i}^2\right)\left(\sum_{i=1}^{n}{b_i}^2\right).$$

We let $a_i=1$ and $b_i=\digamma_i$. Then

$$\left(\sum_{i=1}^{n}|\digamma_i|\right)^2 \leq \left(\sum_{i=1}^{n}1\right)\left(\sum_{i=1}^{n}|\digamma_i|^2\right)$$

which implies that

$$[DBRCE(G)]^2 \leq n(2\sum_{i<j}\left[\sqrt{d_u}+\sqrt{d_v}+(d_ud_v)^{\frac{1}{3}}\right]^2)$$

and finally

$$DBRCE(G) \leq \sqrt{2n\sum_{i<j}\left[\sqrt{d_u}+\sqrt{d_v}+(d_ud_v)^{\frac{1}{3}}\right]^2}$$

which is an upper bound. □

Theorem 2.2 *Let G be a graph with n vertices and let R= determinant of $DBRC(G)$, then*

$$DBRCE(G) \geq \sqrt{2\sum_{i<j}\left[\sqrt{d_u}+\sqrt{d_v}+(d_ud_v)^{\frac{1}{3}}\right]^2+n(n-1)R^{\frac{2}{n}}}.$$

Proof: By definition,

$$\begin{aligned}(DBRCE(G))^2 &= \left(\sum_{i=1}^{n}\mid\digamma_i\mid\right)^2\\ &= \sum_{i=1}^{n}\mid\digamma_i\mid\sum_{j=1}^{n}\mid\digamma_j\mid\\ &= \left(\sum_{i=1}^{n}\mid\digamma_i\mid^2\right)+\sum_{i\neq j}\mid\digamma_i\mid\mid\digamma_j\mid.\end{aligned}$$

Using arithmetic mean and geometric mean inequality, we have

$$\frac{1}{n(n-1)}\sum_{i\neq j}\mid\digamma_i\mid\mid\digamma_j\mid \quad\geq\quad \left(\prod_{i\neq j}\mid\digamma_i\mid\mid\digamma_j\mid\right)^{\frac{1}{n(n-1)}}.$$

Therefore,

$$\begin{aligned}
(DBRCE(G))^2 &\geq \sum_{i=1}^{n} \mid \digamma_i \mid^2 + n(n-1)\left(\prod_{i\neq j} \mid \digamma_i \mid\mid \digamma_j \mid\right)^{\frac{1}{n(n-1)}} \\
&\geq \sum_{i=1}^{n} \mid \digamma_i \mid^2 + n(n-1)\left(\prod_{i=1}^{n} \mid \digamma_i \mid^{2(n-1)}\right)^{\frac{1}{n(n-1)}} \\
&= \sum_{i=1}^{n} \mid \digamma_i \mid^2 + n(n-1)R^{\frac{2}{n}} \\
&= 2\sum_{i<j}\left[\sqrt{d_u}+\sqrt{d_v}+(d_u d_v)^{\frac{1}{3}}\right]^2 + n(n-1)R^{\frac{2}{n}}.
\end{aligned}$$

Thus,

$$DBRCE(G) \geq \sqrt{2\sum_{i<j}\left[\sqrt{d_u}+\sqrt{d_v}+(d_u d_v)^{\frac{1}{3}}\right]^2 + n(n-1)R^{\frac{2}{n}}}.$$

□

3. Properties of degree based root connectivity energy of a graph

Proposition 3.1 *The first three coefficients of the polynomial $\phi_{DBRC}(G,\digamma)$ are given as follows:*

(i) $a_0 = 1$,
(ii) $a_1 = 0$,
(iii) $a_2 = -\sum_{i<j}\left[\sqrt{d_u}+\sqrt{d_v}+(d_u d_v)^{\frac{1}{3}}\right]^2$.

Proof: **(i)** From the definition, $\phi_{DBRC}(G,\digamma) = det[\digamma I - DBRC(G)]$ and then we get $a_0 = 1$ after easy calculations.

(ii) The sum of the determinants of all 1×1 principal submatrices of $DBRC(G)$ is equal to the trace of $DBRC(G)$. Therefore

$$a_1 = (-1)^1 \cdot \text{trace of } [DBRC(G)] = 0.$$

(iii) Similarly we have

$$\begin{aligned}
(-1)^2 a_2 &= \sum_{1\leq i<j\leq n} \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix} \\
&= \sum_{1\leq i<j\leq n} a_{ii}a_{jj} - a_{ji}a_{ij} \\
&= \sum_{1\leq i<j\leq n} a_{ii}a_{jj} - \sum_{1\leq i<j\leq n} a_{ji}a_{ij} \\
&= -\sum_{i<j}\left[\sqrt{d_u}+\sqrt{d_v}+(d_u d_v)^{\frac{1}{3}}\right]^2.
\end{aligned}$$

□

Proposition 3.2 *If $\digamma_1, \digamma_2, \ldots, \digamma_n$ are the eigenvalues of $DBRC(G)$, then*

$$\sum_{i=1}^{n} {\digamma_i}^2 = 2\sum_{i<j}\left[\sqrt{d_u}+\sqrt{d_v}+(d_u d_v)^{\frac{1}{3}}\right]^2.$$

Proof: We know that

$$\begin{aligned}\sum_{i=1}^{n} \digamma_i^2 &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}a_{ji} \\ &= 2\sum_{i<j} a_{ij}^2 + \sum_{i=1}^{n} a_{ii}^2 \\ &= 2\sum_{i<j} a_{ij}^2 \\ &= 2\sum_{i<j}\left[\sqrt{d_u}+\sqrt{d_v}+(d_u d_v)^{\frac{1}{3}}\right]^2 .\end{aligned}$$

□

Theorem 3.1 *Let G be a regular graph of n vertices with regularity r, then*

$$DBRCE(G) = (2\sqrt{r} + {}^{2/3}\!\sqrt{r})E(G)$$

Theorem 3.2 *Let G be a semiregular graph of degrees $r_1 \geq 1$ and $r_2 \geq 1$.*
Then $\sqrt{r_1}+\sqrt{r_2}+\sqrt[3]{r_1 r_2}$

Proof: Consider a semiregular graph of degrees $r_1 \geq 1$ and $r_2 \geq 1$, the $FL-$matrix is given by

$$DBRC(G) = \left(\sqrt{r_1}+\sqrt{r_2}+\sqrt[3]{r_1 r_2}\right)(J-I) \; .$$

$$\digamma_i = (\sqrt{r_1}+\sqrt{r_2}+\sqrt[3]{r_1 r_2})\lambda_i.$$

(Here λ_i represents the eigenvalue with respect to adjacency matrix of the corresponding graph.)
Thus the proof follows. □

4. Degree based root connectivity energy of some standard graphs

Theorem 4.1 *degree based root connectivity energy of complete graph K_n is*

$$DBRCE(K_n) = 2(n-1)\left(\frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}}\right).$$

Proof: For each and every vertex u in K_n, $d(u) = (n–1)$. Then every ij^{th}-entry of the degree based root connectivity matrix will be

$$\begin{bmatrix} 0 & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \cdots & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} \\ \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & 0 & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \cdots & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} \\ \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & 0 & \cdots & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \cdots & 0 & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} \\ \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & \cdots & \frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}} & 0 \end{bmatrix}.$$

Hence the characteristic equation will be

$$\left(\digamma - \left(\frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}}\right)\right)^{n-1}\left(\digamma - (n-1)\left(\frac{2}{(n-1)^{\frac{3}{2}}}+\frac{1}{(n-1)^{\frac{4}{3}}}\right)\right) = 0$$

and therefore the spectrum becomes

$$Spec_{DBRC}(K_n) = \begin{pmatrix} \frac{2}{(n-1)^{\frac{3}{2}}} + \frac{1}{(n-1)^{\frac{4}{3}}} & (n-1)\left(\frac{2}{(n-1)^{\frac{3}{2}}} + \frac{1}{(n-1)^{\frac{4}{3}}}\right) \\ n-1 & 1 \end{pmatrix}.$$

Therefore,

$$DBRCE(K_n) = 2(n-1)\left(\frac{2}{(n-1)^{\frac{3}{2}}} + \frac{1}{(n-1)^{\frac{4}{3}}}\right).$$

□

Theorem 4.2 *The degree based root connectivity energy of the crown graph S_n^0 is*

$$DBRCE(S_n^0) = 4(n-1)\left[\frac{2}{(n-1)^{\frac{3}{2}}} + \frac{1}{(n-1)^{\frac{4}{3}}}\right].$$

Proof: Let S_n^0 be the crown graph of order $2n$ with vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$. The degree based root connectivity matrix is

$$\begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & \alpha_1 & \dots & \alpha_1 & \alpha_1 \\ 0 & 0 & 0 & \dots & 0 & \alpha_1 & 0 & \dots & \alpha_1 & \alpha_1 \\ 0 & 0 & 0 & \dots & 0 & \alpha_1 & \alpha_1 & \dots & 0 & \alpha_1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & \alpha_1 & \alpha_1 & \dots & \alpha_1 & 0 \\ 0 & \alpha_1 & \alpha_1 & \dots & \alpha_1 & 0 & 0 & \dots & 0 & 0 \\ \alpha_1 & 0 & \alpha_1 & \dots & \alpha_1 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_1 & \alpha_1 & 0 & \dots & \alpha_1 & 0 & 0 & \dots & 0 & 0 \\ \alpha_1 & \alpha_1 & \alpha_1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}.$$

Where $\alpha_1 = \frac{2}{(n-1)^{\frac{3}{2}}} + \frac{1}{(n-1)^{\frac{4}{3}}}$. In that case the characteristic equation is

$$(F - (\alpha_1))^{n-1}(F + \alpha_1)^{n-1}(F + (n-1)(\alpha_1))(F - (n-1)(\alpha_1)) = 0$$

implying that the spectrum is

$$Spec_{DBRC}(S_n^0) = \begin{pmatrix} -(n-1)(\alpha_1) & (n-1)(\alpha_1) & -\alpha_1 & \alpha_1 \\ 1 & 1 & n-1 & n-1 \end{pmatrix}.$$

Therefore,

$$DBRCE(S_n^0) = 4(n-1)\left[\frac{2}{(n-1)^{\frac{3}{2}}} + \frac{1}{(n-1)^{\frac{4}{3}}}\right].$$

□

Theorem 4.3 *The degree based root connectivity energy of complete bipartite graph $K_{m\times n}$ is*

$$DBRCE(K_{m,n}) = 2\left(\frac{\sqrt{m} + \sqrt{n} + (mn)^{\frac{1}{3}}}{\sqrt{mn}}\right).$$

Proof: The degree based root connectivity matrix of complete bipartite graph $K_{m\times n}$ is

$$\left(\frac{\sqrt{m} + \sqrt{n} + (mn)^{\frac{1}{3}}}{mn}\right)\begin{bmatrix} 0_{m\times m} & J_{m\times n} \\ J_{n\times m} & 0_{n\times n} \end{bmatrix}.$$

$$Spec_{DBRC}(K_{m,n}) = \begin{pmatrix} (\sqrt{mn})\left(\frac{\sqrt{m}+\sqrt{n}+(mn)^{\frac{1}{3}}}{mn}\right) & 0 & -(\sqrt{mn})\left(\frac{\sqrt{m}+\sqrt{n}+(mn)^{\frac{1}{3}}}{mn}\right) \\ 1 & m+n-2 & 1 \end{pmatrix}.$$

$$DBRCE(K_{m,n}) = 2\left(\frac{\sqrt{m}+\sqrt{n}+(mn)^{\frac{1}{3}}}{\sqrt{mn}}\right).$$

□

Theorem 4.4 *The degree based root connectivity energy of Cocktail party graph $K_{n\times 2}$ is*

$$DBRCE(K_{n\times 2}) = 2(n-1)(2\sqrt{2n-2}+(2n-2)^{\frac{2}{3}})$$

Proof: Let $K_{n\times 2}$ be a Cocktail party graph of order $2n$ with vertex set $\{u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$. The degree based root connectivity matrix is $2\sqrt{2n-2}+(2n-2)^{\frac{2}{3}}$

$$DBRCE(K_{n\times 2}) = (2\sqrt{2n-2}+(2n-2)^{\frac{2}{3}})\begin{pmatrix} (J-I)_{n\times n} & (J-I)_{n\times n} \\ (J-I)_{n\times n} & (J-I)_{n\times n} \end{pmatrix}.$$

Characteristic equation is

$$F^n(F+(2\sqrt{2n-2}+(2n-2)^{\frac{2}{3}}))^{n-1}(F+(n-1)(2\sqrt{2n-2}+(2n-2)^{\frac{2}{3}}) = 0$$

Hence, spectrum is

$$Spec_{DBRC}(K_{n\times 2}) = \begin{pmatrix} -(2\sqrt{2n-2}+(2n-2)^{\frac{2}{3}})^{n-1} & 0 & (n-1)(2\sqrt{2n-2}+(2n-2)^{\frac{2}{3}}) \\ n-1 & n & 1 \end{pmatrix}.$$

Therefore,

$$DBRCE(K_{n\times 2}) = 2(n-1)(2\sqrt{2n-2}+(2n-2)^{\frac{2}{3}})$$

□

Theorem 4.5 *The degree based root connectivity energy of star graph $K_{1,n-1}$ is*

$$DBRCE(K_{1,n-1}) = \frac{2(n-1)^{\frac{1}{2}}+2(n-1)^{\frac{1}{3}}+2}{\sqrt{n-1}}.$$

Proof: Let $K_{1,n-1}$ be the star graph with vertex set $V = \{v_0, v_1 ... v_{n-1}\}$. The degree based root connectivity matrix is

$$DBRC(K_{1,n-1}) = \frac{(n-1)^{\frac{1}{2}}+(n-1)^{\frac{1}{3}}+1}{n-1}\begin{pmatrix} 0_{1\times 1} & J_{1\times n-1} \\ J_{n-1\times 1} & 0_{n-1\times n-1} \end{pmatrix}.$$

Characteristic equation is

$$(F)^{n-2}\left(F^2-(n-1)\left(\frac{(n-1)^{\frac{1}{2}}+(n-1)^{\frac{1}{3}}+1}{n-1}\right)^2\right)$$

spectrum is $Spec_{DBRC}(K_{1,n-1}) = \begin{pmatrix} (\sqrt{n-1})\frac{(n-1)^{\frac{1}{2}}+(n-1)^{\frac{1}{3}}+1}{n-1} & 0 & -(\sqrt{n-1})\frac{(n-1)^{\frac{1}{2}}+(n-1)^{\frac{1}{3}}+1}{n-1} \\ 1 & n-2 & 1 \end{pmatrix}.$

Therefore, $DBRCE(K_{1,n-1}) = \frac{2(n-1)^{\frac{1}{2}}+2(n-1)^{\frac{1}{3}}+2}{\sqrt{n-1}}$. □

It is easy to see that $|V(F_3^n)| = 2n+1$.

Theorem 4.6 *The degree based root connectivity energy of the friendship graph F_3^n is*

$$DBRCE(F_3^n) = (5.3482)(2n-1) + \sqrt{(5.3482)^2 + 8n(3.4142\sqrt{n}+\sqrt{2})^2}.$$

Proof: Let F_3^n be the friendship graph with $2n+1$ vertices. The degree based root connectivity matrix is

$$\begin{bmatrix} 0 & L_1 & L_1 & L_1 & L_1 & \dots & L_1 & L_1 \\ L_1 & 0 & 5.3482 & 0 & 0 & \dots & 0 & 0 \\ L_1 & 5.3482 & 0 & 0 & 0 & \dots & 0 & 0 \\ L_1 & 0 & 0 & 0 & 5.3482 & \dots & 0 & 0 \\ L_1 & 0 & 0 & 5.3482 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ L_1 & 0 & 0 & 0 & 0 & \dots & 0 & 5.3482 \\ L_1 & 0 & 0 & 0 & 0 & \dots & 5.3482 & 0 \end{bmatrix}.$$

Here $L_1 = 3.4142\sqrt{n}+\sqrt{2}$. Therefore the characteristic equation will be

$$\left(\digamma^2 - (5.3482)\digamma - 2n(L_1)^2\right)\left(\digamma - (5.3482)\right)^{n-1}\left(\digamma + (5.3482)\right)^n = 0.$$

Hence, the spectrum is

$$Spec_{DBRC}(F_3^n) = \begin{pmatrix} -(5.3482) & 5.3482 & \frac{5.3482+B}{2} & \frac{5.3482-B}{2} \\ n & n-1 & 1 & 1 \end{pmatrix}.$$

Where $B = \sqrt{(5.3482)^2 + 8n(3.4142\sqrt{n}+\sqrt{2})^2}$, therefore, $DBRCE(F_3^n)$ is

$$(5.3482)(2n-1) + \sqrt{(5.3482)^2 + 8n(3.4142\sqrt{n}+\sqrt{2})^2}.$$

□

5. Degree based root connectivity energy of cubic graphs of order 10

There are 21 cubic graphs of order 10. They are represented in the figure [1]. The eigen values and the inverse sum indeg energy of cubic graphs of order 10 are given in the following table.

Graph	Eigen values	ISI Energy
G_1	-11.0882, -11.0882, -8.6574, -5.5441, -5.5441, -0.0000, 5.5441, 5.5441, 14.2015, 16.6323	83.8440
G_2	-13.6174, -11.0882, -8.8998, -5.5441, -2.0419, -0.0000, 5.5441, 7.1138, 11.9012, 16.6323	82.3829
G_3	-13.7500 -9.9901 -8.5102 -5.5441 -2.4674 - 0.8233 4.0222 6.9134 13.5172 16.6323	82.1701
G_4	-11.0882, -5.5441, -2.9346, -2.2964, -0.0000, 0.0000, 7.4453, 13.3846, 16.6323	74.9245
G_5	-11.0882, -11.0882, -9.6027, -5.5441, -2.2964, -0.0000, 0.0000, 9.6027, 13.3846, 16.6323	79.2392
G_6	-16.6323, -8.9705, -8.9705, -3.4264, -3.4264, 3.4264, 3.4264, 8.9705, 8.9705, 16.6323	82.8525
G_7	-14.5146, -12.7668, -8.9705, -3.4264, -2.1177, 3.4264, 5.5441, 7.2227, 8.9705, 16.6323	83.5922

G_8	-14.2015, -11.0882, -11.0882, -5.5441, 0.0000, 5.5441, 5.5441, 5.5441, 8.6574, 16.6323	83.8440
G_9	-14.3933 -11.0882, -8.4941, -6.5567, -1.9254, 2.8592, 5.5441, 7.0026, 10.4195, 16.6323	84.9155
G_{10}	-14.5146, -14.5146, -3.4264, -3.4264, -2.1177, -2.1177, 5.5441, 8.9705, 8.9705, 16.6323	80.2350
G_{11}	-14.5146, -10.3165, -8.9705, -3.4264, -2.1177, -1.4088, 3.4264, 8.9705, 11.7253, 16.6323	81.5091
G_{12}	-11.0882, -11.0882, -11.0882, -5.5441, -5.5441, 5.5441, 5.5441, 5.5441, 11.0882, 16.6323	88.7056
G_{13}	-14.7858, -12.4575, -7.1746, -3.0767, -2.3619, 0.0000, 4.4460, 7.2595, 11.5187, 16.6323	79.7131
G_{14}	10.4004, 10.4004,10.4004,10.4004,10.4004,10.40	23.2762
G_{15}	-15.0205, -9.9901, -9.9901, -2.4674, -2.4674, -1.0752, 6.9134, 6.9134, 10.5516, 16.6323	82.0213
G_{16}	-16.6323 -11.0882 -5.5441 -5.5441 -0.0000 0.0000 5.5441 5.5441 11.0882 16.6323	77.6174
G_{17}	-11.0882, -11.0882, -11.0882, -11.0882, 5.5441, 5.5441, 5.5441, 5.5441, 5.5441, 16.6323,	88.7056
G_{18}	-13.8009, -11.0882, -5.5441, -5.5441, -1.6032, 0.0000, 0.0000, 5.5441, 15.4040, 16.6323	75.1609
G_{19}	-13.7096, -8.9705, -8.9705, -8.1088, -3.4264, 3.4264, 3.4264, 8.9705, 10.7302, 16.6323	86.3719
G_{20}	-11.0882, -11.0882, -5.5441, -5.5441, -5.5441, -0.0000, 0.0000, 5.5441, 16.6323, 16.6323	77.6174
G_{21}	16.6323, -11.0882, -5.5441, 0.0000, 0.0000, -0.0000, -5.5441, 16.6323, -5.5441, -5.5441	66.5292

Acknowledgments

We the authors would like to thank the anonymous reviewers for your suggestions.

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