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Construction of Balanced Incomplete Block Designs (BIBDs) Through Variable Constrained Neighborhood Search (VCNS) Algorithm

Pratibha Karki and Samrat Hore*

ABSTRACT: The issue of comparing two or more treatments in the presence of single or multiple covariates is a pivotal concern across various research domains. While various studies have explored optimal treatment allocation under completely randomized designs with regard to D- and A- optimality criteria but only few have addressed this challenge within a blocked framework. Addressing this problem, authors have developed an efficient, computationally tractable iterative search algorithm under the block design setup. This article extends the work by adapting the algorithm for the construction of Balanced Incomplete Block Designs (BIBDs). The primary objective of this study is to construct the Balanced Incomplete Block Designs (BIBDs) by the proposed Variable Constrained Neighborhood Search (VCNS) algorithm and assess the efficacy of the proposed VCNS algorithm with existing algorithms. The key contribution of this research lies in the extension of the previously developed VCNS algorithm to the domain of BIBD construction by offering a new computational tool that maintains competitive performance while reducing computational demands. This contribution adds value to the existing literature by bridging gaps in efficient design methodologies for treatment comparisons.

Key Words: Incomplete block design, iterative search, near-optimal allocation design, mathematical programming, computational method.

Contents

1. Introduction

Balanced Incomplete Block Designs (BIBDs) are an essential combinatorial structure in experimental design, statistical analysis, and optimization. A BIBD consists of v treatments arranged into b blocks, each containing k treatments, such that every treatment appears in exactly r blocks, and every pair of treatments co-occurs in precisely λ blocks [1,2]. These designs play a fundamental role in various fields, including agricultural experiments [3], clinical trials [4], cryptography [5], and network security [6]. The balanced nature of BIBDs ensures equal treatment comparisons, reducing biases and improving experimental efficiency [1].

The construction of BIBDs has been a significant research focus, with various approaches developed over the years. Classical methods rely on algebraic techniques, such as finite projective geometry [7], difference sets [8], and combinatorial structures [9]. While these methods provide theoretical frameworks

* Corresponding author: sam.stat724@gmail.com
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for BIBD generation, they often lack generalizability and practical applicability to arbitrary parameter sets. Yasmin [10] proposed the method of cyclic shifts, which constructs infinite series of BIBDs through systematic transformations. Although this method simplifies the construction process and maintains the required balance properties, it remains restricted to specific configurations and does not offer a generalized solution for all possible BIBD parameters.With advancements in computational techniques, mathematical programming has emerged as an alternative method for BIBD construction. Yokoya and Yamada [11] introduced a nonlinear mixed-integer programming (MIP) approach, incorporating branch-and-bound techniques and tabu search to generate BIBDs. These computational methods provide flexibility by enabling the construction of BIBDs for various parameter sets without relying on predefined algebraic structures. However, they suffer from high computational costs, particularly when dealing with large-scale designs. The complexity of solving large instances often results in excessive processing times, limiting their practical applicability. Houghten [12] and Bilous [13] demonstrated the computational challenges associated with exhaustive searches, where verifying the existence of certain BIBDs required extensive computing resources over prolonged periods.

To address these limitations, heuristic and metaheuristic algorithms have been explored as efficient alternatives for BIBD construction. Techniques such as genetic algorithms [14], simulated annealing [15], and tabu search [16] have been applied with varying degrees of success. These methods leverage probabilistic search strategies to navigate large solution spaces efficiently. However, local search-based heuristics often suffer from premature convergence, causing them to become trapped in local optima and limiting their effectiveness in finding optimal BIBDs across diverse parameter settings.

In BIBD construction, Variable Neighborhood Search (VNS), a metaheuristic first presented by [17], is a promising but underutilized method. VNS systematically explores different neighborhoods of a solution, allowing for both intensification (local search refinement) and diversification (global search through structural perturbations). Unlike conventional local search methods, VNS avoids stagnation in local optima by dynamically adjusting its search strategy, making it particularly suitable for combinatorial optimization problems like BIBD generation [17]. Given its effectiveness in solving complex combinatorial problems, VNS presents a promising alternative to existing BIBD construction techniques.

This study aims to develop an efficient Variable Neighborhood Search (VNS)-based algorithm for the construction of BIBDs, addressing the limitations of existing algebraic and computational methods. The specific objectives of the research are:

- To design and implement a novel VNS-based algorithm tailored for BIBD construction, integrating adaptive search strategies to enhance efficiency and scalability.

- To compare the performance of the proposed VNS approach with existing methods.

By achieving these objectives, this research seeks to advance the field of combinatorial optimization by introducing a robust and scalable heuristic approach for BIBD construction. The findings will provide valuable insights into the applicability of VNS in combinatorial design problems, offering an efficient alternative to traditional and computational methods.

The article has been organized in the following sections. In Section 2, the problem has been formulated. In Section 3, the initial design of experimental units has been described. Section 4 describes about the proposed Variable Neighborhood search (VCNS) algorithm and Section 5 describes with the working of the algorithm. In Section 6 the comparison of the proposed neighborhood search algorithm with the existing algorithms has been carried out. The article ends with concluding remarks and scope for the future in Section 7.

2. Problem Formulation

In experimental design, especially when comparing multiple treatments under the influence of covariates, the use of Balanced Incomplete Block Designs (BIBDs) offers an efficient framework to ensure balanced treatment comparisons while reducing variability. A BIBD is characterized by parameters (v, b, r, k, λ), representing the number of treatments, blocks, replications, block size, and pairwise concurrence, respectively. These designs must satisfy the following combinatorial conditions via an incidence matrix $N=(n_{ij})_{v\times b}$, where n_{ij} takes either 0 or 1, according to the absence or presence of the i^{th} treatment in the j^{th} block.

1. Each treatment occurs in exactly r blocks i.e.,

$$\sum_{j=1}^{b} n_{ij} = r \quad \forall i = 1, 2, ..., v.$$

2. Each block contains exactly k treatments: i.e.,

$$\sum_{i=1}^{v} n_{ij} = k \quad \forall j = 1, 2, ..., b.$$

3. Each pair of treatments appears together in exactly λ blocks: i.e.,

$$\sum_{j=1}^{b} n_{ij} n_{i'j} = \lambda \quad \forall i < i'$$

A BIBD is called **Symmetric BIBD** if $b = v$, i.e., the number of treatments equals the number of blocks. Under this condition, and using the standard identity for BIBDs, $bk = vr$, it follows that k = r. This implies that each treatment appears in r blocks and each block contains r treatments. For symmetric BIBDs, the matrix product NN' satisfies:

$$NN' = (r - \lambda)I + \lambda J,$$

where I is the identity matrix and J is a matrix of all ones.

The determinant of NN' is given by:

$$|NN'| = rk(r - \lambda)^{v-1}.$$

Since the incidence matrix N contains only integer entries, when v is even, the quantity $(r - \lambda)$ must be a perfect square to ensure that the determinant is also an integer. Additionally, due to the incomplete nature of BIBDs, the following conditions must hold to avoid degenerating into a complete (randomized) block design:

$$v > k, \quad r > \lambda.$$

Due to the combinatorial complexity of this problem, particularly with large values of v and b, traditional algebraic and exact optimization techniques become computationally infeasible or overly rigid for general use. Additionally, heuristic methods frequently encounter challenges such as convergence to suboptimal solutions or inability to scale effectively across varying parameter sets. To address these challenges, this study formulates the BIBD construction task as a constrained combinatorial optimization problem. The objective is to identify a valid incidence matrix N satisfying all BIBD properties while enabling efficient allocation of experimental units to treatments within blocks.

The solution approach involves using a Variable Constrained Neighborhood Search (VCNS) algorithm, which systematically explores the space of feasible incidence matrices through iterative neighborhood generation, evaluation, and selection. The objective of this study is to find an optimal design that meets all incidence constraints and supports accurate and stable parameter estimation in the presence of covariates. This formulation enables a flexible, computationally viable framework for constructing BIBDs across a wide range of experimental settings, thereby addressing the limitations of conventional and existing heuristic approaches.

3. Initial Allocation of Experimental Units

Selecting an effective initial allocation design is crucial, as it facilitates faster convergence toward an optimal or near-optimal solution. In contrast, a poor initial design may lead to poor final outcomes. To address this issue, we implement the re-randomization strategy proposed by [18], which involves generating $t = 10$ randomized initial designs and selecting the one with the most favorable preliminary criterion value. This method, as utilized in [19] and [20], improves the robustness and reliability of the allocation procedure.

4. Proposed Variable Constrained Neighborhood Search (VCNS) Algorithm

The approach of finding the optimal or near-optimal under block design set up has been discussed by [20] and it has been extended for unequal allocation of experimental units by the authors. In this study, the VCNS algorithm proposed in [20] has been extended to assess its efficiency while constructing the Balanced Incomplete Block Designs (BIBDs). The steps of the algorithm have been given as follows:
Step 1 (**Initial Allocation**): Start with an initial allocation design, denoted by $\alpha^{(0)}$ and the corresponding objective function $V(\alpha^{(0)})$.
Step 2 (**Constrained Approach**): For the allocation design the incidence matrix N=(n_{ij}) where i= 1,2,..., v and j=1,2,3,...,b, denotes the number of times the ith treatment occurs in the jth block. The allocation design $\alpha^{(0)}$, subject to satisfy the condition that :

$$\sum_{j=1}^{b} n_{ij} = r \quad \forall i = 1, 2, ..., v.$$

then move forward otherwise go to step 1.
Step 3 (**Neighborhood Search**): Construct the neighborhood $N(\alpha^{(0)})$ of the allocation design $\alpha^{(0)}$.
Step 4 (**Better Initial Selection**):Compute $V(\alpha^{'})$,for all $\alpha' \in N(\alpha^{(0)})$,if max $\{V(\alpha^{'}), \alpha^{'} \in N(\alpha^{(0)}) > V(\alpha^{(0)})\}$ then choose the next improved allocation to be $\alpha^{(0)'}$ such that $\alpha^{(0)'}$= arg max $\{ V(\alpha^{'}),\alpha' \in N(\alpha^{(0)})\}$ otherwise choose $\alpha^{(0)'}$ randomly among the allocations.
Step 5 (**Best Initial Selection**): Replace $\alpha^{(0)}$ by $\alpha^{(0)'}$ and repeat steps 2 and 3 until it convergences with some desired accuracy.
Step 6 (**Stopping Rule**): The algorithm stops when the following two conditions are satisfied

$$\sum_{j=1}^{b} n_{ij} n_{i'j} = \lambda \quad \forall i < i'$$

and

$$|NN'| = rk(r - \lambda)^{v-1}.$$

5. Working of the Algorithm

To illustrate the working of the proposed Variable Constrained Neighborhood Search (VCNS) algorithm, we consider the construction of a symmetric Balanced Incomplete Block Design (BIBD) with the parameter set: $v = 7$,$b = 7$, $r = 3$, $k = 3$, $\lambda = 1$. These parameters satisfy the two fundamental combinatorial conditions required for a valid BIBD:

$$bk = vr \quad \text{and} \quad \lambda(v - 1) = r(k - 1),$$

which ensure that the total number of treatment positions is balanced across the design and that every pair of treatments appears together in exactly one block.

For computational implementation, the treatments $T_1, T_2, \ldots, T_7$ are represented by the integers $1, 2, \ldots, 7$. An initial block design is generated using simple random sampling without replacement (SRSWOR), where each of the $b = 7$ blocks is independently assigned a subset of $k = 3$ distinct treatments. This SRSWOR method is repeated t=10 times and the best one (maximum time pre-fixed r=3 appears) has been selected. The resulting initial allocation of treatments to blocks is as follows: Block 1 includes treatments (2, 4, 7), Block 2 includes (3, 5, 2), Block 3 includes (1, 6, 3), Block 4 includes (7, 1, 4), Block 5 includes (5, 3, 4), Block 6 includes (6, 1, 5), and Block 7 includes (3, 7, 6).While this initial configuration satisfies the basic requirement of block size, it does not necessarily meet the replication condition $r = 3$ for each treatment, nor the pairwise concurrence condition $\lambda = 1$. Some treatments may be over- or under-represented, and not all treatment pairs may co-occur exactly once. This initial design serves as the starting point for the VCNS algorithm, which iteratively explores the neighborhood of the current solution, making refinements to treatment allocations until all BIBD conditions are satisfied. We now calculate the corresponding Incidence matrix N. The incidence matrix calculated is given below.

Treatments	**[1]**	**[2]**	**[3]**	**[4]**	**[5]**	**[6]**	**[7]**	**r**
[1]	0	0	1	1	0	1	0	3
[2]	1	1	0	0	0	0	0	2
[3]	0	1	1	0	1	0	1	4
[4]	1	0	0	1	1	0	0	3
[5]	0	1	0	0	1	1	0	3
[6]	0	0	1	0	0	1	1	3
[7]	1	0	0	1	0	0	1	3
k	3	3	3	3	3	3	3	

It is observed that the replication values r for treatments T_2 and T_3 deviate from the required value of 3, which is a necessary condition for a Balanced Incomplete Block Design (BIBD) with parameters $v = 7, b = 7, r = 3, k = 3, \lambda = 1$. Specifically, treatment T_2 appears in only 2 blocks, while treatment T_3 appears in 4 blocks. This imbalance in replication violates the uniform treatment replication condition of a BIBD, thereby rendering the current design invalid.To correct this discrepancy, one occurrence of treatment T_3 must be replaced with treatment T_2. By doing so, the replication count of T_2 increases to 3, and that of T_3 decreases to 3, thereby restoring the uniform replication condition $r = 3$ for all treatments.

5.1. First Iteration

In the first iteration of the neighborhood search, the VCNS algorithm identifies a violation of the replication condition required for a valid Balanced Incomplete Block Design (BIBD) with parameters (7,7,3,3,1). Specifically, treatment T_3 was found to be over-represented with four replications, while T_2 was under-represented with only two replications, deviating from the required replication number $r = 3$. To correct this imbalance, the algorithm examines the neighborhood of the current design and identifies Block 3 (B3), which initially contains treatments (1, 6, 3), as a suitable candidate for modification. Within this block, treatment T_3 is replaced by T_2, resulting in the updated configuration (1, 6, 2). This targeted adjustment simultaneously reduces the replication count of T_3 and increases that of T_2, thereby aligning both with the specified replication requirement. The modified design now satisfies the replication condition across all treatments and is used as the basis for the next iteration, where further adjustments are evaluated to satisfy the pairwise concurrence condition $\lambda = 1$.

Treatments	**[1]**	**[2]**	**[3]**	**[4]**	**[5]**	**[6]**	**[7]**	**r**
[1]	0	0	1	1	0	1	0	3
[2]	1	1	1	0	0	0	0	3
[3]	0	1	0	0	1	0	1	3
[4]	1	0	0	1	1	0	0	3
[5]	0	1	0	0	1	1	0	3
[6]	0	0	1	0	0	1	1	3
[7]	1	0	0	1	0	0	1	3
k	3	3	3	3	3	3	3	

The updated incidence matrix N does not satisfy the conditions of a Balanced Incomplete Block Design (BIBD), as the treatment pairs (T_1, T_3), (T_4, T_6), and (T_5, T_7) are absent. Therefore, the pairwise concurrence condition $\lambda = 1$ is not met, and the design cannot be considered a valid BIBD.

5.2. Second Iteration

In the second iteration of the neighborhood search, the focus shifts to satisfying the pairwise concurrence condition $\lambda = 1$, which remained incomplete after the previous adjustment. Based on the incidence matrix from the first iteration, certain treatment pairs, specifically (T_1, T_3), (T_4, T_6), and (T_5, T_7) were identified as missing, indicating that these pairs had not co-occurred in any block. To address this, the VCNS algorithm searches for suitable modifications by analyzing the blocks that already include one member of each missing pair. The goal is to identify feasible swap opportunities that introduce the missing combinations while preserving the replication number $r = 3$ for each treatment and maintaining the block size $k = 3$.

Accordingly, several replacements are made. In Block 4 (B4), treatment T_7 is replaced with T_3, allowing T_1 and T_3 to co-occur. In Block 5 (B5), treatment T_5 is replaced with T_6, introducing the T_4, T_6 pair. Finally, in Block 6 (B6), treatment T_6 is replaced with T_7, enabling the inclusion of the T_5, T_7 pair. These targeted adjustments ensure that all missing treatment combinations are now present in the design.This updated configuration maintains the uniformity of replication and block sizes while also satisfying the pairwise concurrence requirement for a valid Balanced Incomplete Block Design (BIBD). The design is now significantly closer to the ideal structure specified by the parameters $v = 7$, $b = 7$, $r = 3$, $k = 3$, and $\lambda = 1$.

Treatments	**[1]**	**[2]**	**[3]**	**[4]**	**[5]**	**[6]**	**[7]**	**r**
[1]	0	0	1	1	0	1	0	3
[2]	1	1	1	0	0	0	0	3
[3]	0	1	0	1	1	0	1	4
[4]	1	0	0	0	1	0	1	3
[5]	0	1	0	0	1	0	0	2
[6]	0	0	1	0	0	1	1	3
[7]	1	0	0	1	0	1	0	3
k	3	3	3	3	3	3	3	

5.3. Third Iteration

In the third iteration of the VCNS algorithm, the focus remains on correcting the replication imbalance detected in the previous step. Specifically, treatment T_3 was found to be over-replicated, appearing in four blocks, while treatment T_5 was under-replicated, appearing in only two blocks. To resolve this inconsistency and ensure that the replication condition $r = 3$ is satisfied for all treatments, the algorithm explores the neighborhood of the current design. Block 5 (B5), which initially contains treatments $(6, 3, 4)$, is identified as a suitable candidate for modification. Since both T_3 and T_4 already have the correct replication count and T_5 requires one additional occurrence, the algorithm replaces treatment T_3 with T_5 in B5. After this adjustment, B5 is updated to $(6, 5, 4)$, reducing the replication count of T_3 to 3 and increasing that of T_5 to 3.

This targeted modification ensures that all treatments now meet the required replication number $r = 3$, setting the stage for the next iteration, which will focus on verifying and satisfying the pairwise concurrence condition $\lambda = 1$.

Treatments	**[1]**	**[2]**	**[3]**	**[4]**	**[5]**	**[6]**	**[7]**	**r**
[1]	0	0	1	1	0	1	0	3
[2]	1	1	1	0	0	0	0	3
[3]	0	1	0	1	1	0	0	3
[4]	1	0	0	0	1	0	1	3
[5]	0	1	0	0	1	0	1	3
[6]	0	0	1	0	0	1	1	3
[7]	1	0	0	1	0	0	1	3
k	3	3	3	3	3	3	3	

From the final incidence matrix, we observe that the replication number $r = 3$ and block size $k = 3$ are satisfied for all treatments and blocks, respectively. Furthermore, all unordered treatment pairs appear exactly once across the blocks, which confirms that the pairwise concurrence condition $\lambda = 1$ is also satisfied. This indicates that the design meets all fundamental conditions for a Balanced Incomplete Block Design (BIBD).

To validate the design algebraically, we construct the matrix NN', where N is the incidence matrix. The matrix NN' is shown below:

$$NN' = \begin{bmatrix} 3 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 3 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 3 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 3 \end{bmatrix}$$

We compute the determinant of the matrix NN' and obtain $|NN'| = 576$. To verify the theoretical condition for a Balanced Incomplete Block Design (BIBD), we check whether the identity

$$|NN'| = rk(r - \lambda)^{v-1}$$

holds for the given parameters. Substituting $r = 3$, $k = 3$, $\lambda = 1$, and $v = 7$, we have:

$$|NN'| = 3 \times 3 \times (3 - 1)^{7-1} = 9 \times 2^6 = 9 \times 64 = 576.$$

Since the computed value of the determinant matches the theoretical expression, the algebraic condition is satisfied. The final block configuration for the design is as follows: Block 1 contains treatments (2, 4, 7), Block 2 contains (3, 5, 2), Block 3 includes (1, 6, 2), Block 4 holds (3, 1, 4), Block 5 contains (6, 5, 4), Block 6 includes (7, 1, 5), and Block 7 comprises (3, 7, 6). This design satisfies all the necessary combinatorial constrained: each treatment appears in exactly three blocks ($r = 3$), each block contains exactly three treatments ($k = 3$), and every unordered pair of treatments appears together in exactly one block ($\lambda = 1$). Therefore, this configuration is confirmed to be a valid BIBD with parameters $v = 7, b = 7, r = 3, k = 3, \lambda = 1$.

6. Comparison of the Proposed Algorithm Based on Existing Algorithms

To evaluate the performance of the proposed algorithm, we attempted to construct 11 benchmark Symmetric Balanced Incomplete Block Designs (BIBDs) out of the 86 listed by [21] . The performance of the proposed VCNS algorithm is compared against several existing algorithms, including the Linear Integer Programming-based algorithm (LIP), Neural Network by [22] and Simulated Annealing (NN-SA) by [23] , Local Search (LS) algorithm by [21] , Branch and Bound (BB) algorithm , and Tabu Search (TS) algorithm by [11]. Table 1 summarizes the solution status for the 11 symmetric instances across these algorithms, where "Yes" denotes that the solution was found using the algorithm and "No" indicates that the solution was not found for the respective instances.

Table 1: Comparison of proposed algorithm with different algorithms for solution status of 11 symmetric instances by Prestwich (2001).

Sr No.	v	b	r	k	λ	NN-SA	LS	BB	TS	LIP	VCNS
1	11	11	5	5	2	Yes	Yes	Yes	Yes	Yes	Yes
2	13	13	4	4	1	Yes	Yes	Yes	Yes	Yes	Yes
3	15	15	7	7	3	Yes	Yes	Yes	Yes	Yes	Yes
4	16	16	6	6	2	Yes	Yes	Yes	No	Yes	Yes
5	19	19	9	9	4	No	Yes	Yes	Yes	Yes	Yes
6	21	21	5	5	1	Yes	Yes	Yes	Yes	Yes	Yes
7	23	23	11	11	5	No	No	No	Yes	Yes	No
8	27	27	13	13	6	No	No	No	Yes	Yes	Yes
9	31	31	6	6	1	No	Yes	Yes	Yes	Yes	Yes
10	31	31	10	10	3	No	No	No	Yes	No	Yes
11	31	31	15	15	7	No	No	No	Yes	No	Yes

(Source: Computed by Authors,2025)

Table 2: Summary performance of existing algorithms with the proposed algorithm

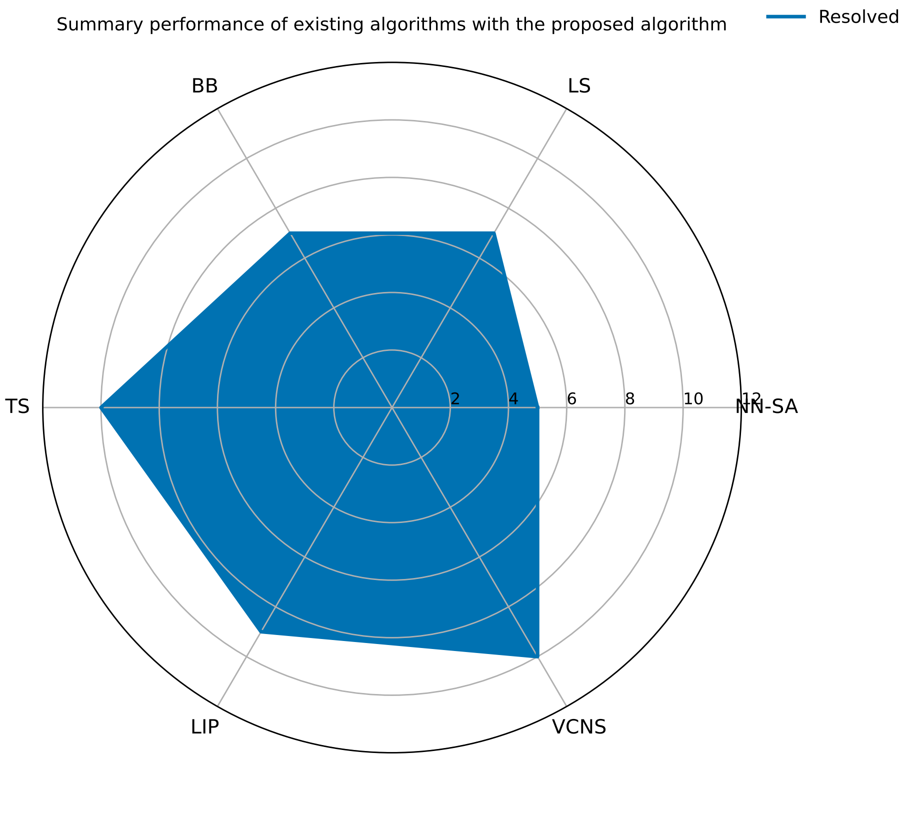
Algorithm	Resolved	Unresolved
NN-SA	5	6
LS	7	4
BB	7	4
TS	10	1
LIP	9	2
VCNS	10	1

(Source: Computed by Authors,2025)

The analysis reveals that the proposed VCNS algorithm successfully solved 10 out of the 11 instances, achieving a 91% success rate. This performance is on par with the Tabu Search (TS) algorithm, which also solved 10 instances. The Linear Integer Programming (LIP) approach solved 9 instances, while Branch and Bound (BB) and Local Search (LS) each solved 7. The Neural Network with Simulated Annealing (NN-SA) algorithm demonstrated the least effectiveness, solving only 5 out of the 11 instances. Thus, VCNS outperformed NN-SA, LS, BB, and LIP in terms of the number of problems successfully solved. Although both VCNS and TS achieved the same number of successful outcomes, a closer examination of Table 1 indicates that the single unsolved instance by VCNS differs from the one unsolved by TS. This suggests that the two algorithms may be updated to strengthen and increase overall solution coverage. Such insights demonstrate the value of algorithmic diversity when addressing complex combinatorial design problems like BIBDs. Fig. 1 shows the radar chart of the solved instances of different algorithms.

The proposed VCNS algorithm demonstrates strong performance in solving symmetric BIBD instances. It consistently performs better than most existing algorithms and matches the best-performing method in this study. These findings provide strong evidence for the effectiveness of the VCNS approach and highlight its potential for broader application in combinatorial design generation.

Fig. 1: Single-layer radar chart showing the number of solved instances out of 11 for each algorithm



7. Concluding Remarks

This study introduced a Variable Constrained Neighborhood Search (VCNS) algorithm for the construction of Balanced Incomplete Block Designs (BIBDs), with a primary focus on symmetric configurations. By formulating the design generation problem as a constrained combinatorial optimization task, the VCNS algorithm systematically explores the solution space while adhering to the strict combinatorial requirements of BIBDs. The algorithm demonstrated high success rates and efficient convergence across various initialization distributions, and outperformed or matched several established methods in solving benchmark instances.The results affirm the efficacy and robustness of the VCNS approach in constructing valid BIBDs, particularly in settings where classical algebraic methods or exact optimization become computationally prohibitive. The consistent satisfaction of replication and pairwise concurrence conditions, along with competitive performance metrics, positions VCNS as a valuable heuristic tool in experimental design.

Future work may consider extending this framework to the construction of asymmetric BIBD configurations, where the number of treatments differs from the number of blocks. Additionally, the algorithm can be adapted to generate other combinatorial and statistical designs such as partially balanced incomplete block designs (PBIBDs), resolvable designs, or t-designs, thereby broadening its applicability across a wider range of experimental and optimization problems.

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Pratibha Karki,
Department of Statistics,
Tripura University,
Agartala-799022, India.
E-mail address: pratibha.karki02@gmail.com

and

Samrat Hore,
Department of Statistics,
Tripura University,
Agartala-799022, India.
E-mail address: sam.stat724@gmail.com