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Weak convergence of sequences defined by Orlicz function in $n-$normed space

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ABSTRACT: In this article, we investigate the weak convergence of sequences defined by Orlicz functions within the framework of n-normed spaces. The concept of Orlicz sequence spaces, which generalize classical ℓ^p spaces, provides a flexible structure for analyzing sequence behavior through convex functions. On the other hand, n-normed spaces, originally introduced by Gähler, extend the notion of norm by considering n-tuples of vectors, offering a rich setting for functional analysis. We introduce and study a new classes of weakly convergent sequences in Orlicz sequence spaces under the n-norm We have explained its different algebraic and topological properties, We have also proved the geometric properties of these sequence spaces.

Key Words: Convergence, Orlicz function, $n-$norm, weakly solid, symmetric weakly convex, monotone, sequence algebra, balanced.

Contents

1. Introduction

The concept of 2-normed spaces was initially introduced by Diminnie and Gähler[12] in the mid-1960s, while Misiak[1] later extended the idea by defining n-normed spaces. Since then, many researchers have investigated these spaces and obtained various significant results see, for instance, the studies by Gunawan[11] and Khan[17].

Let ω denote the set of all sequences of real or complex numbers. Let $\mathbb{N}, \mathbb{R}$, and $\mathbb{C}$ represent the sets of natural numbers, real numbers, and complex numbers, respectively. Define ℓ_∞ and c as the Banach spaces consisting of all bounded and convergent sequences $\eta = (\eta_k)$, respectively, equipped with the standard norm $\|\eta\| = \sup |\eta_k|$.

An Orlicz function $\mu : [0, \infty) \to [0, \infty)$ is a continuous, non-decreasing, and convex function such that $\mu(0) = 0$, $\mu(\eta) > 0$, for all $\eta > 0$, and $\mu(\eta) \to \infty$, as $\eta \to \infty$.

Using the concept of an Orlicz function, Lindenstrauss and Tzafriri[14] defined the Orlicz sequence space ℓ_μ as follows:

$$\ell_\mu = \left\{ \eta = (\eta_k) \in \omega : \sum_{k=1}^{\infty} \mu\left(\frac{|\eta_k|}{\rho}\right) < \infty, \text{ for some } \rho > 0 \right\}.$$

The space ℓ_μ becomes a Banach space when equipped with the Luxemburg norm defined by:

$$\|\eta\| = \inf\left\{ \rho > 0 : \sum_{k=1}^{\infty} \mu\left(\frac{|\eta_k|}{\rho}\right) \le 1 \right\}.$$

It was shown in Lindenstrauss and Tzafriri[14] that every Orlicz sequence space ℓ_μ possesses the structure of a Banach space under this norm.

In this paper we will generalized the notion of weak convergence of sequences defined by Orlicz function in n-normed space.

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2. Preliminaries and Definition

Definition 2.1 Assume that X is a real vector space and $n \in \mathbb{N}$. A real valued function $||(.,.,...,.)||$ on X^n satisfies the following four properties :

N(1). $||(\xi_1, \xi_2, ..., \xi_{n-1}, \xi_n)|| = 0$ iff $\xi_1, \xi_2, ..., \xi_{n-1}, \xi_n$ are linearly dependent vectors in X.

N(2). $||(\xi_1, \xi_2, ..., \xi_{n-1}, \xi_n)||$ is invariant under permutation.

N(3). $||(\xi_1, \xi_2, ..., \xi_{n-1}, \alpha\xi_n)|| = |\alpha| ||(\xi_1, \xi_2, ..., \xi_{n-1}, \xi_n)||$, for any $\alpha \in \mathbb{R}$.

N(4). $||(\xi_1, \xi_2, ..., \xi_{n-1}, \eta + \zeta)|| \leq ||(\xi_1, \xi_2, ..., \xi_{n-1}, \eta)|| + ||(\xi_1, \xi_2, ..., \xi_{n-1}, \zeta)||$

such a function is called an n-norm on X and the pair $(X, ||(.,.,...,.)||_n)$ is called an n-normed sequence space.

Accordingly, the class of all n-normed sequence space is denoted by $(w, ||(.,.,...,.)||_n)$.

Definition 2.2 A sequence (η_k) is said to be convergent to L in n-normed space, if for a given $\varepsilon \geq 0$, one can find n_0 such that

$$||(\xi_1, \xi_2, ..., \xi_{n-1}, \eta_k - L)|| < \varepsilon, \text{ for all } n \geq n_0,$$

where $\xi_1, \xi_2, ..., \xi_{n-1}$ are linearly independent elements in X.

Definition 2.3 A sequence (η_k) is called a Cauchy in n-normed space, if for a given $\varepsilon \geq 0$, one can find n_0 such that

$$||(\xi_1, \xi_2, ..., \xi_{n-1}, \eta_m - \eta_n)|| < \varepsilon, \quad \text{for all } n, m \geq n_0,$$

where $\xi_1, \xi_2, ..., \xi_{n-1}$ are linearly independent elements in X.

Definition 2.4 A sequence (η_k) is called weakly converge to L in n-normed space on $\mathbb{R}^n$ if, for each $h \in X'$,

$$||(h(\xi_1), h(\xi_2), ..., h(\xi_{n-1}), h(\eta_k - L))|| \to 0,$$

as $k \to \infty$.

Definition 2.5 A sequence (η_k) is called weakly Cauchy in n-normed space on $\mathbb{R}^n$ if, for each $h \in X'$,

$$||(h(\xi_1), h(\xi_2), ..., h(\xi_{n-1}), h(\eta_m - \eta_n))|| \to 0,$$

as $m, n \to \infty$.

Definition 2.6 A subset E of sequences in $n-$normed space is called solid if $(\zeta_k) \in E$, whenever $(\eta_k) \in E$ and

$$||(\xi_1, \xi_2, ..., \xi_{n-1}, \zeta_k)|| \leq ||(\xi_1, \xi_2, ..., \xi_{n-1}, \eta_k)||, \text{ for all } k \in \mathbb{N}.$$

Definition 2.7 A subset E of sequences in ω is said to be convex if, for any two sequences $(\eta_k), (\zeta_k) \in E$ implies $\lambda\eta_k + (1 - \lambda)\zeta_k \in E$, where $0 < \lambda < 1$.

Definition 2.8 A seqence space E is said to be symmetric if $(\eta_k) \in E$ implies that $(\eta_{\pi(n)}) \in E$ for every permutation π of $\mathbb{N}$.

Definition 2.9 A subset E of sequences in w is said to be be balanced if $\alpha E \subset E$ for all α with $|\alpha| \leq 1$.

Definition 2.10 Let $K = \{k_1 < k_2 < k_3 < ......k_n < ....\} \subset \mathbb{N}$ (set of natural numbers). Let $(\eta_k) \in \omega$. Then the $K-$step space of the sequence space E is defined by

$$\lambda_K^E = \{(\eta_k) \in \omega : (\eta_k) \in E\}.$$

Definition 2.11 A canonical pre-image (ζ_k) of a sequence $(\eta_k) \in E$, where $K-$step space is considered, is defined by

$$\zeta_k = \begin{cases} \eta_k, & k \in \mathbb{N}; \\ 0 & otherwise. \end{cases}$$

Definition 2.12 A sequence space E is said to be monotone, if it contains all pre-images of its step spaces.

3. Main results

Here we defined the classes of weakly convergent sequences defind by Orlicz function as follows,
$c^w(X^n, ||(.,.,...,.)||, \mu)$
$= \{\eta = (\eta_k) \in \omega : \sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\eta_k - L)|}{\rho}\right|\right|\right) < \infty$, as $k \to \infty$, for some $\rho > 0\}$,
This is the class of all weakly convergent sequences defined by Orlicz function in $n-$ normed space.

In the above definition taking $L = \theta$, the zero element of X , we get the class of weakly null sequences defined by Orlicz function in $n-$ normed space., denoted by $c_0^w(X^n, ||(.,.,...,.)||, \mu)$
$\ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$
$= \left\{\eta = (\eta_k) \in \omega : \sup_{\rho>0} \sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\eta_k)|}{\rho}\right|\right|\right) < \infty, \text{ as } k \to \infty, \text{ for some } \rho > 0\right\}$,
the class of all weakly bounded sequences in n-normed space.

Theorem 3.1 *The spaces* $c^w(X^n, ||(.,.,...,.)||, \mu)$, $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ *and* $\ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$ *are linear spaces.*

Proof: We established the proof for $c_0^w(X^n, ||(.,.,...,.)||, \mu)$. Let $\eta, \zeta \in c_0^w(X^n, ||(.,.,...,.)||, \mu)$, where $\eta = (\eta_k), \zeta = (\zeta_k), \eta_k, \zeta_k \in X^n$, for all $k \in \mathbb{N}$.
So, we have

$$\sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\eta_k)|}{\rho_1}\right|\right|\right) \to 0, \text{as } k \to \infty, \rho_1 > 0$$

and

$$\sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\zeta_k)|}{\rho_2}\right|\right|\right) \to 0, \text{as } k \to \infty, \rho_2 > 0. \tag{3.1}$$

Now, choose $\rho_3 = \max\{|\alpha\rho_1|, |\beta\rho_2|\}$. Then we have

$$\begin{aligned}\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\alpha\eta_k+\beta\zeta_k)|}{\rho_3}\right|\right|\right) &= \sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\alpha\eta_k)+f(\beta\zeta_k)|}{\rho_3}\right|\right|\right)\\ &= \sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\alpha\eta_k)|+|f(\beta\zeta_k)|}{\rho_3}\right|\right|\right)\\ &\leq \sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\alpha\eta_k)|}{\rho_3}\right|\right|\right)\\ &+ \sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\beta\zeta_k)|}{\rho_3}\right|\right|\right)\\ &= |\alpha|\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\eta_k)|}{\rho_3}\right|\right|\right)\\ &+ |\beta|\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\zeta_k)|}{\rho_3}\right|\right|\right)\\ &\to 0.\ \text{`}by(3.1) \qquad (3.2)\end{aligned}$$

Therefore, $\alpha\eta_k + \beta\zeta_k \in c_0^w(X^n, ||(.,.,...,.)||, \mu)$. Hence, $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ is a linear space.

Theorem 3.2 *The spaces* $c^w(X^n, ||(.,.,...,.)||, \mu)$, $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ *and* $\ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$ *are normed linear spaces, with respect to the normed defined by,*

$$g(x) = \inf_{\rho}\ \{\ \rho > 0 : e\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\eta_k)|}{\rho}\right|\right|\right) < \infty.\}$$

Proof: Clearly $g(\eta) = g(-\eta)$ and $g(\theta) = 0$, where $\theta = (0, 0, ..., 0)$, the zero sequence. By using theorem 1 for $\alpha = \beta = 1$, we get $g(\eta + \zeta) \leq g(\eta) + g(\zeta)$.

Now, by using the definition of g we get,

$$\begin{aligned}g(\alpha\eta) &= \inf_{\rho}\ \{\ \rho > 0 : \sum_{k=1}^{\infty}||(f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \mu(\frac{|f(\alpha\eta_k)|}{\rho}))|| < \infty\}\\ &\leq \inf_{\rho}\ \{\ \frac{\rho}{\alpha} > 0 : \sum_{k=1}^{\infty}||(f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \mu(\frac{|f(\eta_k)|}{\rho}))|| < \infty\}\end{aligned}$$

where, $\sigma = \frac{\rho}{\alpha}$. since

$$\begin{aligned}&\leq \max(1, |\alpha|) \times \inf_{\sigma\to 0}\ \{\ \frac{\rho}{\alpha} : \sum_{k=1}^{\infty}||(f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \mu(\frac{|f(\eta_k)|}{\rho}))|| < \infty\}\\ &= (\max(1, |\alpha|)) \times g(\eta).\end{aligned}$$

So, the function satisfies all the conditions of a norm. As the spaces are alredy proved to belinear space. Hence, they are normed linear space.

Theorem 3.3 *The spaces* $c^w(X^n, ||(.,.,...,.)||, \mu)$, $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ *and* $\ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$ *are solid.*

Proof: Let $\eta = (\eta_k) \in c_0^w(X^n, ||(.,.,...,.)||, \mu)$, and let $\alpha = (\alpha_k)$ be any sequence of scalars such that $|\alpha_k| \leq 1$, for all $k \in \mathbb{N}$. So, we have

$$\begin{aligned}\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\alpha\eta_k)|}{\rho}\right|\right|\right) < \infty\} &\leq \sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\eta_k)|}{\rho}\right|\right|\right) < \infty\}\\ &\to 0, \text{as } n \to \infty\end{aligned}$$

Hence, $\alpha\eta \in c_0^w(X^n, ||(.,.,...,.)||, \mu)$, whenever $x \in c_0^w(X^n, ||(.,.,...,.)||, \mu)$.
Hence, the space $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ is solid. The other cases can be establish similarly.
In view of Lemma 1, we formulate the following theorem without proof.

Theorem 3.4 *The spaces* $c^w(X^n, ||(.,.,...,.)||, \mu)$, $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ *and* $\ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$ *are monotone.*

The following Remarkcan be proved by usingstandard techniques.

Remark 3.1 The spaces $c^w(X^n, ||(.,.,...,.)||, \mu)$, $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ and $\ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$ are symmetric.

Theorem 3.5 *The spaces* $c^w(X^n, ||(.,.,...,.)||, \mu)$, $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ *and* $\ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$ *are weakly convex.*

Proof: We will establish the proof for $\ell_\infty(X^n, ||(.,.,...,.)||, \mu)$. So the other cases can be establish similarly. Let $(\eta_k), (\zeta_k) \in \ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$. So, we have

$$\sup_{\rho>0} \sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\eta_k)|}{\rho_1}\right|\right|\right) < \infty, as\ k \to \infty, \text{for some } \ \rho_1 > 0. \tag{3.3}$$

$$\sup_{\rho>0} \sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\zeta_k)|}{\rho_2}\right|\right|\right) < \infty, as\ \ k \to \infty, \text{for some } \ \rho_2 > 0. \tag{3.4}$$

Now, choose $\rho_3 = \max\{\rho_1, \rho_2\}$. Then we have

$$\sup_{\rho>0} \sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\lambda\eta_k + (1-\lambda)\zeta_k)|}{\rho_3}\right|\right|\right) \tag{3.5}$$
$$\leq \sup_{\rho>0} \sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\lambda\eta_k)| + |f(1-\lambda)\zeta_k)|}{\rho_3}\right|\right|\right)$$
$$\leq \sup_{\rho>0} \sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\lambda\eta_k)|}{\rho_3}\right|\right|\right) + \sup_{\rho>0} \sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(1-\lambda)\zeta_k)|}{\rho_3}\right|\right|\right)$$
$$= |\lambda| \sup_{\rho>0} \sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\eta_k)|}{\rho_3}\right|\right|\right) + |1-\lambda| \sup_{\rho>0} \sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\zeta_k)|}{\rho_3}\right|\right|\right)$$
$$< \infty. \text{ by (1) and (2)}$$

Hence, $\lambda\eta_k + (1-\lambda)\zeta_k \in \ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$. Therefore, $\ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$ is convex.

Theorem 3.6 *The spaces* $c^w(X^n, ||(.,.,...,.)||, \mu)$, $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ *and* $\ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$ *are weakly balanced.*

Proof: We will establish the proof for the case $c_0^w(X^n, ||(.,.,...,.)||, \mu)$. The other cases can be established following similar techniques. Let $x \in c_0^w(X^n, ||(.,.,...,.)||, \mu)$, where $\eta = (\eta_k), \eta_k \in X^n$, for all $k \in \mathbb{N}$. So we have,

$$\sum_{k=1}^{\infty} \mu\left(\left|\left|f(\xi_1), f(\xi_2), ..., f(\xi_{n-1}), \frac{|f(\eta_k)|}{\rho}\right|\right|\right) \to 0, \text{as} k \to \infty, \rho > 0, f \in X'. \tag{3.6}$$

Consider a scalar α with $|\alpha| \leq 1$, then we need to show that $\alpha\eta_k \in c_0^w(X^n, ||(.,.,...,.)||, \mu)$.
Now, we have

$$\begin{aligned}\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1),f(\xi_2),...,f(\xi_{n-1}),\frac{|f(\alpha\eta_k)|}{\rho}\right|\right|\right) &= \sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1),f(\xi_2),...,f(\xi_{n-1}),\frac{|\alpha f(\eta_k)|}{\rho}\right|\right|\right)\\ &= \sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1),f(\xi_2),...,f(\xi_{n-1}),\frac{|\alpha||f(\eta_k)|}{\rho}\right|\right|\right)\\ &\leq \ |\alpha|\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1),f(\xi_2),...,f(\xi_{n-1}),\frac{|f(\eta_k)|}{\rho}\right|\right|\right)\\ &\to 0, \text{by } 3.7\end{aligned}$$

So, $(\alpha x_k) \in c_0^w(X^n, ||(.,.,...,.)||, \mu)$. Hence, $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ is balanced.
The other cases will follow similarly.

Theorem 3.7 *The spaces* $c^w(X^n, ||(.,.,...,.)||, \mu)$, $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ *and* $\ell_\infty^w(X^n, ||(.,.,...,.)||, \mu)$ *are sequence algebra.*

Proof: We will establish the proof for $c_0^w(X^n, ||(.,.,...,.)||, \mu)$. Let $(\eta_k), (\zeta_k) \in c_0^w(X^n, ||(.,.,...,.)||, \mu)$. So, we have

$$\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1),f(\xi_2),...,f(\xi_{n-1}),\frac{|f(\eta_k)|}{\rho_1)}\right|\right|\right) \to 0, \text{as} k\to\infty, \rho_1>0, f\in X^{'}$$

$$\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1),f(\xi_2),...,f(\xi_{n-1}),\frac{|f(\zeta_k)|}{\rho_2}\right|\right|\right) \to 0, \text{as} \to\infty, \rho_2>0, f\in X^{'}$$

Now, let $\rho_3 = \rho_1\rho_2 > 0$, then we can easily prove

$$\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1),f(\xi_2),...,f(\xi_{n-1}),\frac{|f(\eta_k\zeta_k)|}{\rho_3}\right|\right|\right) \to 0, \text{as} k\to\infty, \rho_3>0, f\in X^{'}.$$

$$\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1),f(\xi_2),...,f(\xi_{n-1}),\frac{|f(\eta_k\zeta_k)|}{\rho_3}\right|\right|\right) \to 0, \text{as} k\to\infty, \rho_3>0, f\in X^{'}.$$

Together imply thefollowing

$$\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1),f(\xi_2),...,f(\xi_{n-1}),\frac{|f(\eta_k)|}{\rho_1}\right|\right|\right)\sum_{k=1}^{\infty}\mu\left(\left|\left|f(\xi_1),f(\xi_2),...,f(\xi_{n-1}),\frac{|f(\zeta_k)|}{\rho_2}\right|\right|\right) \to 0,$$

as $k \to \infty, \rho_1, \rho_2 > 0, f \in X^{'}$.

Hence, $(\eta_k\zeta_k) \in c_0^w(X^n, ||(.,.,...,.)||, \mu)$. Hence, $c_0^w(X^n, ||(.,.,...,.)||, \mu)$ is sequence algebra.
The other cases will follow similarly.

4. Conclusion

In this investigation, we have introduced new classes of weakly convergent sequences defined by Orlicz functions within the framework of n-normed spaces. By combining the structural richness of Orlicz sequence spaces with the generality of n-norms, we have developed a refined approach to weak convergence in a broader functional-analytic setting. We have explored various topological and geometrical properties of these newly defined sequence classes, including criteria for convergence, linearity, solidity, monotonocity, convexity, balanced, symmetric and structural behaviors under modular and norm-based frameworks. These findings extend and generalize existing results in classical sequence spaces and provide deeper insights into the interaction between Orlicz functionals and n-normed topologies. The results presented here not only contribute to the theoretical advancement of sequence space theory but also pave the way for future research on operator theory, duality, and fixed point problems in generalized normed spaces defined by Orlicz functions.

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