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Total Secure Domination in Graphs

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ABSTRACT: In this paper, we introduce and study the concept of total secure domination in graphs. A set $D \subseteq V(G)$ is called a *total secure dominating set* of a graph G if it is a total dominating set (i.e., every vertex of G is adjacent to some vertex in D) and for every vertex $u \notin D$, there exists a vertex $v \in D$ adjacent to u such that the set $(D \setminus \{v\}) \cup \{u\}$ remains a dominating set in G (it need not be total dominating). We denote the minimum size of such a set by $\gamma_{ts}(G)$, the *total secure domination number*. We establish fundamental properties, derive bounds, and characterize $\gamma_{ts}(G)$ for standard graph classes. We also propose a greedy algorithm for trees. Finally, we discuss applications and directions for future research.

Key Words: Domination, total domination, secure domination, total secure domination.

Contents

1. Introduction

By a graph $G = (V, E)$ we mean a finite, undirected graph with neither loops nor multiple edges. The order $|V|$ and the size $|E|$ are denoted by n and m respectively. For graph theoretic terminology we refer to Chartrand and Lesniak [9].

Domination parameters have been central to graph theory with widespread applications in network design, security, and optimization. A dominating set of a graph $G = (V, E)$ is a subset $D \subseteq V$ such that every vertex in $V \setminus D$ has at least one neighbor in D. Total domination requires every vertex in G to be adjacent to at least one vertex in D [5], [20]. Secure domination extends these concepts by imposing an additional robustness condition: if a vertex outside D were to replace a vertex in D, the resulting set should still dominate G.

For an excellent treatment of the fundamentals of domination we refer to Haynes et al. [2], [6]. A survey of several advanced topics in domination is given in Haynes et al. [12].

Strategies for protection of a graph $G = (V, E)$ by placing one or more guards at every vertex of a subset S of V, where a guard at a vertex can protect all vertices in its closed neighborhood have resulted in the study of several concepts such as Roman domination, weak Roman domination and secure domination. The concept of secure domination is motivated by the following situation and was introduced by Cockayne et al. [11], studied by several authors [3], [8], [10], [13], [17]. Given a graph $G = (V, E)$, we wish to place one guard at each vertex of a subset S of V in such a way that S is a dominating set of G and if a guard at v moves along an edge to protect an unguarded vertex u, then the resulting configuration of guards also forms a dominating set. This leads to the concept of secure domination.

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In this work, we introduce the concept of *total secure domination* by combining the concepts of total domination and secure domination which arises naturally and present several results. Total secure domination is different from secure total domination which is introduced by William F. Klostermeyer and Christina M. Mynhardt [4] and studied by several authors [7], [14], [15], [16], [18], [19]. Specifically, a set D is a total secure dominating set(TSDS) of G if:

1. D is a total dominating set, and
2. For every vertex $u \notin D$, there exists a vertex $v \in D$ adjacent to u such that replacing v with u, i.e., forming $(D \setminus \{v\}) \cup \{u\}$, yields a dominating set of G (note: the replacement set is required only to be dominating, not necessarily total dominating).

We denote the minimum size of such a set by $\gamma_{ts}(G)$.

This new parameter has important applications in areas where both connectivity and resilience against node failures are crucial. In what follows, we establish several theorems concerning $\gamma_{ts}(G)$, discuss algorithmic aspects, and explore potential applications and open research directions.

2. Preliminaries and Definitions

Let $G = (V, E)$ be a simple, undirected graph.

Definition 2.1 A set $D \subseteq V$ is a *dominating set* of G, if every vertex in $V \setminus D$ is adjacent to at least one vertex in D. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set.

Definition 2.2 [1] A set $D \subseteq V$ is a *total dominating set* (TDS) of G, if every vertex in V is adjacent to at least one vertex in D. The *total domination number* $\gamma_t(G)$ is the minimum cardinality of a TDS.

Definition 2.3 [11] Let $G = (V, E)$ be a graph. A set $D \subseteq V$ is called a *secure dominating set* if:

1. D is a dominating set of G.
2. For every vertex $u \in V \setminus D$, there exists a vertex $v \in D$ (with $uv \in E$) such that the set $(D \setminus \{v\}) \cup \{u\}$ is also a dominating set of G.

Definition 2.4 The corona of two graphs G_1 and G_2, denoted by $G_1 \circ G_2$, is the graph obtained by taking $|V(G_1)|$ copies of G_2 and joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2.

3. Results

Since a total dominating set is defined only for graphs without isolated vertices, we consider only graphs with no isolated vertices in this study.

Definition 3.1 *A set $D \subseteq V$ is a total secure dominating set (TSDS) of G if:*

1. *D is a total dominating set, and*
2. *For every vertex $u \notin D$, there exists a vertex $v \in D$ adjacent to u such that the set $(D \setminus \{v\}) \cup \{u\}$ is a dominating set of G.*

The total secure domination number $\gamma_{ts}(G)$ is the minimum size of a TSDS in G.

Theorem 3.1 *For any graph G,*

$$\gamma_{ts}(G) \geq \gamma(G).$$

Proof: Since any total secure dominating set D after replacement yields a dominating set, it follows that D is, in particular, a dominating set. Therefore, its size cannot be smaller than the minimum size required to dominate G, i.e., $\gamma(G)$. □

Theorem 3.2 *Let $G = (V, E)$ be a graph of order n with minimum degree $\delta \geq 1$. Then*

$$\gamma_{ts}(G) \leq n - \delta.$$

Proof: We proceed by constructing a total secure dominating set of size $n - \delta$. Since every vertex in G has degree at least δ, we can choose a subset of $V(G)$ with at most $n - \delta$ vertices for a total secure dominating set. Let $X \subseteq V(G)$ be an independent set of δ vertices and each vertex in X has distinct neighborhoods in $V(G) \setminus X$. This is possible due to the minimum degree assumption . Let $D = V(G) \setminus X$. Then $|D| = n - \delta$. We claim that D is a total secure dominating set: Since each vertex $v \in X$ has at least δ neighbors in $V(G) \setminus X = D$, it is dominated by some vertex in D. For any vertex $u \in D$, since u is not in X, it must be adjacent to some other vertex in D (as $\delta \geq 1$, and X is an independent set, so u's neighbors are not limited only to X). Thus, every vertex in $V(G)$ has a neighbor in D, and hence D is a total dominating set. For each vertex $x \in X = V(G) \setminus D$, there exists a neighbor $v \in D$ such that replacing v by x results in a new set $D' = (D \setminus \{v\}) \cup \{x\}$ that is still a dominating set. Since $x \in X$ has δ neighbors in D, and because $\delta \geq 1$, we can choose such a neighbor $v \in D$. Now since x continues to dominate all its neighbors (in D) and neighbors of v are still dominated because they are adjacent to other vertices in $D \setminus \{v\}$ or to x (if x was adjacent to them), we have $D' = (D \setminus \{v\}) \cup \{x\}$ is a dominating set of G. Therefore, $\gamma_{ts}(G) \leq |D| = n - \delta$. □

Theorem 3.3 *For any complete graph K_n with $n \geq 2$,*

$$\gamma_{ts}(K_n) = 2.$$

Proof: Consider any two distinct vertices $v_1, v_2 \in V(K_n)$ and let $D = \{v_1, v_2\}$. Since K_n is complete, every vertex is adjacent to every other vertex. In particular, every vertex $u \in V(K_n)$ is adjacent to a vertex in D. Hence, D is a total dominating set. Now, let u be any vertex in $V(K_n) \setminus D$. Since K_n is complete, u is adjacent to both v_1 and v_2. Consider the replacement of one vertex from D with u. Without loss of generality, suppose we replace v_1 with u then the set $D' = \{u, v_2\}$ is a dominating set of K_n. Hence $\gamma_{ts}(K_n) \leq 2$. Now to prove , $\gamma_{ts}(K_n) \geq 2$. Suppose, that there exists a total secure dominating set D with $|D| = 1$, say $D = \{v\}$. Then the vertex v does not have any neighbor in D violating the definition of total domination. Hence $\gamma_{ts}(K_n) \geq 2$. Therefore, $\gamma_{ts}(K_n) = 2$. □

Theorem 3.4 *Let S_n be a star graph of order n . Then*

$$\gamma_{ts}(S_n) = n - 1.$$

Proof: Let S_n have vertex set $V = \{c, \ell_1, \ell_2, \ldots, \ell_{n-1}\}$, where c is the central vertex and $\ell_1, \ldots, \ell_{n-1}$ are the leaves. Consider the subset $D = \{c, \ell_2, \ell_3, \ldots, \ell_{n-1}\}$, of $V(S_n)$ omitting exactly the single leaf ℓ_1. Clearly D is a TSDS of S_n with $|D| = n - 1$. Hence $\gamma_{ts}(S_n) \leq n - 1$. Now consider any TSDS D of S_n. Since D is total-dominating, every vertex of S_n, including those in D, has a neighbor in D. The leaves each have degree one, so each leaf's only neighbor is c. Thus c must belong to D. If two or more leaves are not in D, say $\ell_i, \ell_j \notin D$, then in the required swap for ℓ_i we must remove its only neighbor $c \in D$, leaving c out and replacing it by ℓ_i. But then ℓ_j would have no neighbor in the new set, violating domination. Hence at most one leaf can lie outside D. Therefore D contains c and at least $n - 2$ of the $n - 1$ leaves, giving $|D| \geq 1 + (n - 2) = n - 1$. Thus we have $\gamma_{ts}(S_n) = n - 1$. □

Theorem 3.5 *Let C_n be a cycle graph on $n \geq 3$ vertices. Then*

$$\gamma_{ts}(C_n) = \begin{cases} \frac{n}{2}, & \text{if } n \equiv 0 \pmod 4, \\ \frac{n}{2} + 1, & \text{if } n \equiv 2 \pmod 4, \\ \lceil \frac{n}{2} \rceil, & \text{if } n \text{ is odd.} \end{cases}$$

Proof: Label the vertices of the cycle C_n in order as $v_1, v_2, \ldots, v_n$, with $v_{n+1} = v_1$. Then we have the following three cases.

Case (i): $n \equiv 0 \pmod 4$**.** Let $n = 4k$. Consider the set $D = \{v_2, v_3, v_6, v_7, \ldots, v_{4k-2}, v_{4k-1}\}$. Clearly D is a TSDS of C_n. We have $|D| = 2k = \frac{n}{2}$ and hence $\gamma_{ts}(C_{4k}) \leq 2k$. Now suppose D is any TSDS with $|D| < 2k$. Then $|V \setminus D| > 2k$. But each swap can introduce at most one new vertex outside D, so all n vertices cannot be covered under the security condition, which means at least $2k$ vertices are needed. Hence $\gamma_{ts}(C_{4k}) \geq 2k$.

Case (ii): $n \equiv 2 \pmod 4$**.** Let $n = 4k + 2$. Choose $D = \{\, v_2, v_3,\ v_6, v_7,\ \ldots,\ v_{4k-2}, v_{4k-1},\ v_{4k+2}\}$, so $|D| = 2k+1 = (n/2)+1$ and it can be easily verified that D is a TSDS of C_n. Hence $\gamma_{ts}(C_{4k+2}) \leq 2k+1$. Also any TSDS must include two consecutive vertices in each block of four to avoid isolating a chosen vertex; with $4k + 2$ vertices, that forces at least $2k + 1$. Thus $\gamma_{ts}(C_{4k+2}) \geq 2k + 1$.

Case (iii): n **is odd.** Let $n = 2k + 1$. Take $D = \{\, v_2, v_3,\ v_6, v_7,\ \ldots\} \cup \{v_{2k}\}$, choosing $k + 1 = \lceil n/2 \rceil$ vertices. Clearly D is a TSDS of C_n which gives $\gamma_{ts}(C_n) \leq (k + 1)$. At the same time, any TSDS must cover all vertices and cannot leave more than one out of every two consecutive vertices without isolating a vertex. Hence $|D| \geq k + 1$.

This completes the proof. □

Theorem 3.6 *Let P_n be the path on $n \geq 3$ vertices. Then*

$$\gamma_{ts}(P_n) = \begin{cases} \frac{n}{2}, & n \equiv 0 \pmod 4, \\ \frac{n}{2} + 1, & n \equiv 2 \pmod 4, \\ \lceil \frac{n}{2} \rceil, & n \text{ odd.} \end{cases}$$

Proof: Label vertices $v_1, v_2, \ldots, v_n$ in order. Then we have the following three cases.

Case (i): $n = 4k$. Choose $D = \{v_2, v_3,\ v_6, v_7,\ \ldots,\ v_{4k-2}, v_{4k-1}\}$, so $|D| = 2k = n/2$. Clearly D is a TSDS of P_n. Now suppose D is any TSDS with $|D| < 2k$. Then at least $2k + 1$ vertices lie outside D. But each swap can introduce only one new outside vertex into domination, and the endpoints require adjacency within D. It shows $|D| \geq 2k$. Thus $\gamma_{ts}(P_{4k}) = 2k$.

Case (ii): $n = 4k + 2$. Take $D = \{v_2, v_3,\ v_6, v_7,\ \ldots,\ v_{4k-2}, v_{4k-1}\} \cup \{v_{4k+1,4k+2}\}$, so $|D| = 2k + 1 = (n/2) + 1$. Clearly D is a TSDS of P_n. Any TSDS must cover each block of four with two chosen vertices to avoid isolating a member, and the extra two vertices force at least $2k + 1$. Hence $\gamma_{ts}(P_{4k+2}) = 2k + 1$.

Case (iii): $n = 2k+1$. The set $D = \{\, v_2, v_3,\ v_6, v_7,\ \ldots\} \cup \{v_{2k}\}$, is a TSDS of P_n with $|D| = k+1$. Hence $\gamma_{ts}(P_{2k+1}) \leq k + 1$. If $|D| < k + 1$, then two consecutive non-chosen vertices would isolate a member of D or fail the security condition. Thus $\gamma_{ts}(P_{2k+1}) \geq k + 1$. Therefore $\gamma_{ts}(P_{2k+1}) = k + 1$.

This completes the proof. □

Theorem 3.7 *Let $K_{m,n}$ be a complete bipartite graph with partite sets U and V, where $|U| = m$, $|V| = n$ and $2 \leq m \leq n$. Then the total secure domination number $\gamma_{ts}(K_{m,n})$ is given by*

$$\gamma_{ts}(K_{m,n}) = \begin{cases} 2, & \text{if } m = n = 2, \\ 3, & \text{if } m = 2 \text{ and } n > 2, \\ 4, & \text{if } m, n \geq 3. \end{cases}$$

Proof: Let $U = \{u_1, u_2, ..., u_m\}$ and $V = \{v_1, v_2, ..., v_n\}$. Clearly if S is any minimum total dominating set of $K_{m,n}$, then $\{S \cap U\} \neq \phi$ and $\{S \cap V\} \neq \phi$. We consider the following three cases .

Case (i): $m = n = 2$ Clearly $D = \{u_1, v_1\}$ is a TSDS of $K_{2,2}$. Also $\gamma_{ts}(K_{2,2}) \geq 2$. Hence $\gamma_{ts}(K_{2,2}) = 2$.

Case (ii): $m = 2$ **and** $n > 2$. The set $D = U \cup \{v_1\}$ is a TSDS of $K_{2,n}$ implying $\gamma_{ts}(K_{2,n}) \leq 3$. It can be easily verified that $D = \{x, y/x \in U, y \in V$ and $x \neq y\}$ is not a TSDS of $K_{2,n}$ so that $\gamma_{ts}(K_{2,n}) \geq 3$. Hence, $\gamma_{ts}(K_{m,n}) = 3$.

Case (iii): $m, n \geq 3$. Clearly $D = \{u_1, u_2, v_1, v_2\}$ is a TSDS of $K_{m,n}$. It can be easily verified that $D = \{x, y, z/x, y, z \in V(K_{m,n})$ and $x \neq y\}$ is not a TSDS of $K_{m,n}$ so that $\gamma_{ts}(K_{m,n}) \geq 4$. Thus, $\gamma_{ts}(K_{m,n}) = 4$. □

Theorem 3.8 *Let G be a non-empty graph. Then,*

$$\gamma_{ts}(G + K_n) = 2.$$

Proof: Let $H = G + K_n$. Consider the set $D = \{x, y\}$, where $x, y \in V(K_n)$. Every vertex in H is adjacent to at least one vertex in D. For any vertex $u \notin D$, the sets $\{x, u\}$ and $\{y, u\}$ are dominating sets of H. It follows that D is a TSDS of H. By the definition of total dominating set, any TSDS set has cardinality at least two, we have $\gamma_{ts}(H) = 2$. □

Theorem 3.9 *Let G be a connected graph of order n and H be a graph with no isolated vertices. Then the total secure domination number of the corona $G \circ H$ is*

$$\gamma_{ts}(G \circ H) = n.$$

Proof: Consider any total secure dominating set D of $G \circ H$. For each vertex $v \in V(G)$, note that in $G \circ H$, the only vertices that can dominate v are either vertices in the copy H_v attached to v or vertices in $V(G)$ adjacent to v in the graph G. However, if we choose one vertex from each copy H_v, then the vertices chosen from different copies are not adjacent to each other (since copies H_v and H_w have no edges between them for $v \neq w$). Thus, to obtain a total dominating set in $G \circ H$ we must include *all* vertices of $V(G)$ (or an equivalent set of size at least n) to ensure that every vertex in $V(G)$ is dominated by a vertex in the set. Hence, $\gamma_{ts}(G \circ H) \geq n$.

Conversely, in $G \circ H$, every vertex $v \in V(G)$ is adjacent to every vertex in the corresponding copy H_v. If we take $D = V(G)$, then for any vertex $v \in V(G)$, since v is in D, we must have a neighbor in D. If G itself has edges, then there exists at least one edge between vertices of $V(G)$, and so every vertex in $V(G)$ has a neighbor in $V(G)$ ensuring that $V(G)$ is a total dominating set. Now consider any vertex $u \notin V(G)$. Then $u \in H_v$ for some $v \in V(G)$. Since by the corona construction u is adjacent to v. Consider the set $D' = (V(G) \setminus \{v\}) \cup \{u\}$. Because every vertex in $V(G) \setminus \{v\}$ is adjacent to vertices in the copies H_w (for $w \neq v$) and u is adjacent to v, D' dominates $G \circ H$.

Thus, the set $D = V(G)$ is a total secure dominating set for $G \circ H$. Hence, $\gamma_{ts}(G \circ H) \leq n$. Combining, we obtain: $\gamma_{ts}(G \circ H) = n$. □

4. Algorithmic Aspects for Trees

We present a greedy algorithm that uses a level-order traversal followed by a reverse level-order processing to construct a total secure dominating set in trees.

Greedy Algorithm for Trees

Explanation

- **Phase 1 (Total Domination):** We perform a level-order traversal of the tree to organize the vertices by their distance from the root. Processing the levels in reverse order (from leaves upward), we check each vertex v to see if it is already dominated by the current set D. If not, we add its parent p to D. This step ensures that every vertex becomes adjacent to at least one vertex in D and that many vertices in D will have a neighbor from the same higher level (thus promoting internal connectivity within D).

Algorithm 1 Compute a Total Secure Dominating Set in a Rooted Tree

```
Require: A rooted tree T = (V, E) with root r
Ensure: A total secure dominating set D ⊆ V(T)
D ← ∅
Level-order traverse T to build levels L_0, ..., L_k with L_0 = {r}    ▷ Phase 1: ensure total domination
for i ← k downto 1 do
    for each vertex v ∈ L_i do
        if v is not dominated by D then
            let p be the parent of v
            D ← D ∪ {p}
        end if
    end for
end for                                                                ▷ Phase 2: enforce security
for each vertex u ∉ D do
    if no neighbor v ∈ D satisfies that swapping keeps domination then
        add an appropriate neighbor of u to D
    end if
end for
return D
```

- **Phase 2 (Security Check):** After obtaining a candidate set D that is a total dominating set, we then verify the security condition. For every vertex $u \notin D$, we ensure that there exists a vertex $v \in D$ adjacent to u such that replacing v with u yields a dominating set. If a vertex u fails this check, we adjust D (by adding an appropriate neighbor) to enforce the swap condition.
- **Result:** The final set D output by the algorithm is a total secure dominating set of the tree T. The algorithm leverages the hierarchical structure of trees to make local decisions that guarantee both total domination and the security condition.

This greedy algorithm provides a systematic way to construct a total secure dominating set in trees. By first ensuring total domination via a reverse level-order traversal and then performing a security adjustment, the algorithm produces a candidate set that meets both required conditions. Future work may analyze the approximation quality and time complexity of this approach in more detail.

5. Applications of Total Secure Domination

Total Secure Dominating Sets (TSDS) have practical significance in several domains, including:

- **Network Security and Resilience:** TSDS can be used to design robust and secure communication networks. In such networks, each node is guaranteed to be monitored by a secure subset of nodes, ensuring that even if a node is compromised, the overall network remains functional. TSDS provide a model for redundancy and rapid reconfiguration in response to failures or attacks.
- **Wireless Sensor Networks:** In sensor networks, TSDS help in the optimal placement of sensors to ensure full coverage and fault tolerance.
- **Social Networks and Influence Spreading:** TSDS can be applied to social networks to identify robust sets of influential individuals. These sets ensure that, even if some nodes (individuals) become inactive, the overall influence or information flow is maintained.
- **Distributed Control Systems:** In distributed systems such as multi-agent robotics or networked control systems, TSDS ensure that control nodes are arranged so that every node is both covered and can substitute a neighboring controller if needed, thereby increasing system robustness.
- **Biological Networks:** TSDS can model resilient structures in biological networks such as neural networks or metabolic networks, where maintaining functionality despite localized failures is critical.

6. Future Research

The concept of total secure domination opens several avenues for further investigation. Some potential research directions include:

- Develop and analyze efficient algorithms (both exact and approximation) for computing TSDS in graphs.
- Characterize TSDS for various graph families and study how structural properties such as connectivity, degree distribution, and diameter influence the total secure domination number.
- Investigate the behavior of TSDS in dynamic networks where vertices or edges are added or removed over time.
- Explore the relationships between TSDS and other domination parameters such as the total domination number, secure domination number, and connected domination number.
- Apply TSDS in practical network design problems to develop new strategies for enhancing security and robustness.
- Exploring relationships between $\gamma_{ts}(G)$ and other invariants such as independence number, chromatic number, and connectivity.

7. Conclusion

We have introduced the total secure domination number $\gamma_{ts}(G)$, defined as the minimum size of a total dominating set D in G such that for every vertex $u \notin D$ there exists $v \in D$ with the property that $(D \setminus \{v\}) \cup \{u\}$ remains a dominating set in G. We established a series of fundamental theorems providing lower and upper bounds and exact values for standard graph classes. Furthermore, we presented a greedy algorithm for trees and discussed potential applications and future research directions. This work lays a foundation for further exploration of robust domination concepts in graphs. We believe the study of Total Secure Dominating Sets is a promising research area with significant theoretical and practical implications. Future work in these directions can contribute to the development of more resilient and secure networked systems.

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