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Exploring Fuzzy Geographic Profiling Through MVPP and BMVP Approaches

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ABSTRACT: A fuzzy extension of Minimal Variance Projection Profiling (MVPP), a geospatial analysis technique for locating possible areas of interest in geographical profiling, is presented in this study. In traditional MVPP, the spatial distribution of criminal events is analyzed using statistical measures, linear algebra, and Euclidean geometry. A minimal variance line and a bounding polygon that is likely to contain an offender's hideout are constructed. By adding fuzzy matrices, fuzzy covariance, and fuzzy distances, we expand MVPP in this fuzzy form to address spatial uncertainty. This method accounts for imprecision in spatial data by treating crime event locations and projections as fuzzy data points with different levels of membership. In situations where there is insufficient or unclear evidence, the fuzzy MVPP framework efficiently captures regions of interest, providing a more adaptable and realistic option for comprehending illegal spatial behavior. Through the provision of a rigorous, fuzzy-based approach for evaluating ambiguous spatial data in criminal investigations, this contribution enhances the field of geographical profiling.

Key Words: Fuzzy, Geographic Profiling, Minimal Variance Profiling

Contents

1. Introduction

Geographic profiling is a vital tool in criminology that helps law enforcement agencies identify the probable location of an offender's base or hideout based on the spatial distribution of crime scenes. Traditional profiling methods, however, rely on precise and deterministic data, assuming crime locations are known with absolute certainty. In reality, crime data often involves uncertainty due to incomplete reports, unreliable witness statements, or imprecise measurements. Addressing this ambiguity is essential for improving the reliability and robustness of spatial profiling techniques.

By introducing fuzzy logic into geographic profiling, we can effectively handle this uncertainty. Unlike classical set theory's rigid true-or-false classification, fuzzy logic allows partial membership, enabling crime locations to be analyzed with varying degrees of confidence. This approach assigns membership values to crime scenes based on the reliability of their reported locations, ensuring that all available information contributes to the analysis. It also introduces concepts such as the fuzzy geometric center and uncertainty-aware covariance matrices, which enhance the accuracy and resilience of spatial predictions.

Beyond criminology, fuzzy geographic profiling has other uses. It can aid in resource allocation in urban planning when geographical data conditions are unclear. Fuzzy models are used in commerce and logistics to optimize decisions based on ambiguous customer data, and in ecology to examine animal migration patterns. Importantly, fuzzy approaches allow qualitative insights, such as an investigator's

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intuition, to be integrated into quantitative models, making the framework both rigorous and adaptable to real-world complexities.

This paper presents a fuzzy adaptation of the Minimal Variance Projection Profiling (MVPP) method, designed to incorporate spatial uncertainty directly into the analysis. By embedding fuzzy membership values into MVPP's mathematical framework, this approach provides a robust tool for offender profiling in uncertain environments. Through three real-world case studies, we demonstrate the method's ability to manage ambiguity and deliver reliable results in geographic profiling scenarios.

Geographic profiling has evolved through interdisciplinary contributions from criminology, mathematics, and computer science. Rossmo [12] laid the foundation with a formal geographic profiling model, which has since been applied to numerous criminal investigations [1]. Canter [4] provided psychological insights into offender behavior that complement spatial models. Several case studies, including the Atlanta Child Murders [5], Ted Bundy's activities [3,2], and modern geographic data visualizations [10], offer practical contexts for model testing. Errazki [9] explains Rossmo's formula in public discourse, while Morrow [11] introduces mathematical modeling to profile spatial crime patterns.

Recent advances apply topological and fuzzy methods to model spatial patterns in crime-related datasets. Gnanachandra et al. [7] explored topology generation from binary relations in energy systems, highlighting graph-theoretic approaches suitable for urban spatial analysis. Extending this, their work on fuzzy topologies [8] introduced innovative models for vague relational structures, potentially improving accuracy in profiling when data uncertainty exists. From a computational standpoint, De Berg et al. [6] provide the geometric foundations necessary for implementing spatial algorithms underlying these applications.

2. Definitions and Terminologies

Some of the fundamental ideas of the Minimal Variance Projection Profiling (MVPP) technique are presented in fuzzy form. Fuzzy logic ideas are used in these adjustments to deal with spatial uncertainty.

Definition 2.1 *Let X_f be an $n \times 2$ fuzzy matrix, where each row represents the coordinates $(\tilde{x}_i, \tilde{y}_i)$ of crime events as fuzzy data points. Each coordinate pair $(\tilde{x}_i, \tilde{y}_i)$ is associated with a membership function $\mu_{(x_i,y_i)}$, indicating the degree of certainty that these coordinates accurately represent the event location. The fuzzy data matrix X_f is given by:*

$$X_f = \begin{bmatrix} \tilde{x}_1 & \tilde{y}_1 \\ \tilde{x}_2 & \tilde{y}_2 \\ \vdots & \vdots \\ \tilde{x}_n & \tilde{y}_n \end{bmatrix},$$

where $\tilde{x}_i = (x_i, \mu_{x_i})$ and $\tilde{y}_i = (y_i, \mu_{y_i})$ represent the fuzzy coordinates.

Definition 2.2 *The fuzzy geometric center (centroid) of the events, denoted by $\tilde{G}_\mu$, is the mean position of all events in X_f, taking into account the membership degrees of each coordinate:*

$$\tilde{G}_\mu = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} = \begin{bmatrix} \frac{1}{n}\sum_{i=1}^{n} \tilde{x}_i \\ \frac{1}{n}\sum_{i=1}^{n} \tilde{y}_i \end{bmatrix}.$$

Definition 2.3 *The fuzzy covariance matrix $\tilde{C}_\sigma$ provides a measure of how much the dimensions of the fuzzy data points vary from the fuzzy geometric center $\tilde{G}_\mu$. It is defined as:*

$$\tilde{C}_\sigma = \frac{1}{n-1}\sum_{i=1}^{n} (\tilde{r}_i - \tilde{G}_\mu)(\tilde{r}_i - \tilde{G}_\mu)^\top,$$

where $\tilde{r}_i = [\tilde{x}_i \ \tilde{y}_i]$ represents the fuzzy coordinates of the i-th event.

Definition 2.4 *Fuzzy eigenvalues and eigenvectors are calculated from the fuzzy covariance matrix $\tilde{C}_\sigma$ by solving the characteristic equation:*

$$\det(\tilde{C}_\sigma - \lambda I) = 0.$$

The fuzzy eigenvector $\tilde{v}_{\min}$ corresponding to the smallest eigenvalue $\lambda_{\min}$ represents the direction of minimal variance in a fuzzy space and is given by:

$$\tilde{v}_{\min} = \begin{bmatrix} \tilde{v}_1 \\ \tilde{v}_2 \end{bmatrix}.$$

Definition 2.5 *The fuzzy scalar product p_{i_f} is used to project each fuzzy event point (x_i, y_i) along the direction of minimal fuzzy variance, represented by the fuzzy eigenvector v_{min_f}. This scalar product incorporates the membership value $\mu(x_i, y_i)$ to reflect the certainty level of each point's alignment with the minimal variance direction.*

The fuzzy scalar product p_{i_f} for a point (x_i, y_i) is defined as:

$$p_{i_f} = \mu(x_i, y_i) \cdot (v_{min_f} \cdot r_i)$$

where:

- $r_i = [x_i, y_i]^T$ *is the coordinate vector of the i-th event,*
- v_{min_f} *is the eigenvector corresponding to the smallest fuzzy eigenvalue* λ_{min_f},
- $\mu(x_i, y_i) \in [0, 1]$ *is the membership function indicating the degree of certainty for the point* (x_i, y_i).

Scaled by its membership function, the value p_{i_f} indicates the proportion of the fuzzy event (x_i, y_i) that lies in the direction of v_{min_f}. Weighted by its certainty, a greater p_{i_f} indicates a stronger alignment of the event point with the direction of minimal variance. This fuzzy scalar product projects every point along the direction of minimal fuzzy variance to compensate for spatial imprecision.

3. Theoretical Framework

It is crucial to modify fundamental mathematical concepts to represent the true nature of crime data in order to increase the reliability of spatial profiling when it is present. Conventional approaches rely on exact inputs, yet crime scenes frequently have variable degrees of dependability in real life. The necessity to incorporate this uncertainty in a meaningful way is what drives the definitions presented in this work, such as fuzzy distance and fuzzy geometric center. These concepts offer a more adaptable and practical basis for examining spatial patterns in criminology by incorporating fuzzy logic.

Determining the Length of the Fuzzy Line Segment: We compute the fuzzy scalar projections p_{i_f} for each fuzzy event point in order to find the length of a fuzzy line segment that reflects the direction of least variance. On the direction of minimal variance, the fuzzy scaling distance d_{scale_f} is the distance between the outermost fuzzy projections, adjusted by their membership values.

The length of the fuzzy line segment d_{scale_f} is computed as:

$$d_{scale_f} = \max(p_f) - \min(p_f)$$

where $p_f = \{p_{1_f}, p_{2_f}, \ldots, p_{n_f}\}$ is the set of all fuzzy scalar projections, indicating the extent of the fuzzy data along the minimal variance direction.

Construction of the Fuzzy Line Segment: Using the calculated fuzzy scaling distance d_{scale_f}, we determine two points L_{1_f} and L_{2_f} that define the endpoints of the fuzzy line segment along the minimal variance direction. These endpoints incorporate the fuzzy geometric center G_{μ_f} and the fuzzy eigenvector v_{min_f}, creating a line segment that represents the spread of fuzzy crime events.

The endpoints L_{1_f} and L_{2_f} are given by:

$$L_{1_f} = G_{\mu_f} + d_{scale_f} \cdot v_{min_f}$$

$$L_{2_f} = G_{\mu_f} - d_{scale_f} \cdot v_{min_f}$$

Fuzzy Maximal Pairwise Distances: To analyze the spatial range of fuzzy data points, we calculate the fuzzy pairwise distances δ_{ij_f} between each pair of fuzzy events (x_i, y_i) and (x_j, y_j). This produces a fuzzy distance matrix Λ_f, where each element Λ_{ij_f} represents the fuzzy distance between events i and j, adjusted by the membership values of both points.

The fuzzy maximal pairwise distance δ_{max_f} is determined by finding the largest value in Λ_f:

$$\delta_{max_f} = \max_{i,j} \delta_{ij_f}$$

where $\delta_{ij_f} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \cdot \sqrt{\mu(x_i, y_i) \cdot \mu(x_j, y_j)}$.

Fuzzy Orthogonal Projection of Points onto the Line: For each fuzzy event, we calculate the orthogonal projection onto the fuzzy line segment. This projection measures the spread of the events in the direction orthogonal to the fuzzy minimal variance line. The fuzzy orthogonal projection $p_{\perp_f}(x_i, y_i)$ of each point (x_i, y_i) is given by:

$$p_{\perp_f}(x_i, y_i) = G_{\mu_f} + \frac{(v_{min_f} \cdot (r_i - G_{\mu_f}))}{(v_{min_f} \cdot v_{min_f})} \cdot v_{min_f} \cdot \mu(x_i, y_i)$$

where $r_i = [x_i, y_i]^T$, v_{min_f} is the fuzzy minimal variance direction, and $\mu(x_i, y_i)$ adjusts the projection according to the point's certainty.

Fuzzy Bounding Parallelogram Construction: To construct a fuzzy bounding parallelogram, we calculate a vector orthogonal to v_{min_f}, denoted by $v_{\perp_f}$, and use the fuzzy orthogonal projections to establish the vertices of the parallelogram. The vertices are defined by:

$$V_{1_f} = L_{1_f} + d_{\perp_f} \cdot v_{\perp_f}$$
$$V_{2_f} = L_{1_f} - d_{\perp_f} \cdot v_{\perp_f}$$
$$V_{3_f} = L_{2_f} + d_{\perp_f} \cdot v_{\perp_f}$$
$$V_{4_f} = L_{2_f} - d_{\perp_f} \cdot v_{\perp_f}$$

where $d_{\perp_f}$ is the sum of orthogonal distances between the projections of points with the maximal fuzzy distance onto the minimal variance line segment.

This fuzzy bounding parallelogram provides a region that is likely to include the offender's hideout by encapsulating the spatial extent of the fuzzy events along both the minimal variance direction and its orthogonal direction.

4. Foundational Concepts of MVPP

All fuzzy data points, or criminal events, must have corresponding membership functions that represent the degree of uncertainty in their positions in order for Minimal Variance Projection Profiling (MVPP) to be fuzzy. In order to create a fuzzy Bounding Minimal Variance Polygon (BMVP) that captures the region most likely to house the criminal's hideout while taking spatial uncertainty into account, this section explains some fundamental ideas.

Proposition 4.1 *At least three fuzzy events must be present in the dataset in order to use the fuzzy Minimal Variance Projection Profiling (MVPP) approach. It is impossible to determine the direction of minimal fuzzy variance with fewer than three points.*

Definition 4.1 *Fuzzy Minimal Variance Identification Points are determined as the projections of each fuzzy data point onto the direction of minimal fuzzy variance. The two identification points L_{1_f} and L_{2_f}, representing the endpoints of the fuzzy minimal variance line segment, are defined as:*

$$L_{1_f} = G_{\mu_f} + \left(\max_{i=1,\ldots,n} (p_{i_f}) - \min_{i=1,\ldots,n} (p_{i_f}) \right) \cdot v_{min_f}$$

$$L_{2_f} = G_{\mu_f} - \left(\max_{i=1,\ldots,n} (p_{i_f}) - \min_{i=1,\ldots,n} (p_{i_f}) \right) \cdot v_{min_f}$$

where $p_{i_f} = \mu(x_i, y_i) \cdot (v_{min_f} \cdot r_i)$ is the fuzzy scalar projection of each data point (x_i, y_i) along the minimal variance direction.

Definition 4.2 *The fuzzy pairwise traveling distance* δ_{ij_f} *between any two points in the data set* (x_i, y_i) *and* (x_j, y_j) *incorporates the membership functions, defined as:*

$$\delta_{ij_f} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \cdot \sqrt{\mu(x_i, y_i) \cdot \mu(x_j, y_j)}$$

Proposition 4.2 *The Fuzzy Maximal Distance Pair consists of the two fuzzy data points with the greatest fuzzy distance between them, as found in the fuzzy pairwise distance matrix* Λ_f*:*

$$(x_a, y_a), (x_b, y_b) = \arg\max_{i,j} \delta_{ij_f}$$

Definition 4.3 *The Projected Fuzzy Maximal Distance Pair Length is the length of the projection of the two points with the maximal fuzzy distance onto the line of minimal fuzzy variance. The fuzzy orthogonal distances* $d_{\perp_{a_f}}$ *and* $d_{\perp_{b_f}}$ *of these projections are calculated as:*

$$d_{\perp_{a_f}} = \sqrt{(x_a - p_{\perp_{a,x_f}})^2 + (y_a - p_{\perp_{a,y_f}})^2}$$

$$d_{\perp_{b_f}} = \sqrt{(x_b - p_{\perp_{b,x_f}})^2 + (y_b - p_{\perp_{b,y_f}})^2}$$

where $p_{\perp_{a_f}}$ *and* $p_{\perp_{b_f}}$ *are the fuzzy orthogonal projections of* (x_a, y_a) *and* (x_b, y_b) *onto the minimal variance line. The total perpendicular spread* $d_{\perp_f}$ *is then:*

$$d_{\perp_f} = d_{\perp_{a_f}} + d_{\perp_{b_f}}$$

Fuzzy Bounding Minimal Variance Polygon (BMVP):

Definition 4.4 *The Fuzzy Bounding Minimal Variance Polygon (BMVP) is a convex polygon constructed to encapsulate the area of minimal fuzzy variance around the projected points, providing a likely area for the offender's hideout. The vertices of the fuzzy BMVP,* $V_{1_f}, V_{2_f}, V_{3_f}, V_{4_f}$*, are calculated as:*

$$V_{1_f} = L_{1_f} + d_{\perp_f} \cdot v_{\perp_f}$$
$$V_{2_f} = L_{1_f} - d_{\perp_f} \cdot v_{\perp_f}$$
$$V_{3_f} = L_{2_f} + d_{\perp_f} \cdot v_{\perp_f}$$
$$V_{4_f} = L_{2_f} - d_{\perp_f} \cdot v_{\perp_f}$$

where $v_{\perp_f} = [v_{min_{f,2}}, -v_{min_{f,1}}]^T$ *is the vector orthogonal to the fuzzy minimal variance direction.*

The fuzzy BMVP captures the uncertainty-adjusted spatial distribution of the events, providing a bounded region within which the offender's hideout is likely to be located.

Fuzzy Convex Line Segments of the BMVP:

The convex line segments of the fuzzy BMVP connect the fuzzy vertices $V_{1_f}, V_{2_f}, V_{3_f}$, and V_{4_f} and define the boundary of the polygon. These segments are parameterized to capture spatial uncertainty.

Each convex line segment of the fuzzy BMVP is defined as follows:

- **Segment Line**$_{12_f}$: The line segment between V_{1_f} and V_{2_f}, parameterized by $t \in [0, 1]$ as:

$$\text{Line}_{12_f}(t) = V_{1_f} + t \cdot (V_{2_f} - V_{1_f})$$

- **Segment Line**$_{24_f}$: The line segment between V_{2_f} and V_{4_f}, parameterized by $t \in [0, 1]$ as:

$$\text{Line}_{24_f}(t) = V_{2_f} + t \cdot (V_{4_f} - V_{2_f})$$

- **Segment Line**$_{43_f}$: The line segment between V_{4_f} and V_{3_f}, parameterized by $t \in [0, 1]$ as:

$$\text{Line}_{43_f}(t) = V_{4_f} + t \cdot (V_{3_f} - V_{4_f})$$

- **Segment Line**$_{31_f}$: The line segment between V_{3_f} and V_{1_f}, parameterized by $t \in [0, 1]$ as:

$$\text{Line}_{31_f}(t) = V_{3_f} + t \cdot (V_{1_f} - V_{3_f})$$

A membership value, which represents the degree of certainty attached to that boundary, can be assigned to each location along these segments.

5. Validation of the Offender's Hideout Inside the Fuzzy BMVP

We modify the point-in-polygon technique for a fuzzy setting in order to confirm whether the criminal's hiding place is inside the Fuzzy Bounding Minimal Variance Polygon (BMVP). The point of interest (the offender's hideout) is given membership values by the validation procedure, which takes into account the fuzzy vertices of the BMVP and reflects the degree of certainty surrounding its position.

Procedure: Given the fuzzy vertices of the BMVP:

$$V_{1_f} = (x_{V1_f}, y_{V1_f}), \quad V_{2_f} = (x_{V2_f}, y_{V2_f}), \quad V_{3_f} = (x_{V3_f}, y_{V3_f}), \quad V_{4_f} = (x_{V4_f}, y_{V4_f}),$$

and the fuzzy offender's hideout:

$$O_f = (x_{O_f}, y_{O_f}, \mu(O_f)),$$

where $\mu(O_f) \in [0, 1]$ represents the certainty level of the hideout's location, we calculate the determinants for each pair of vertices forming the BMVP's edges and the hideout point.

Validation Criterion: For each edge of the BMVP:

$$D_{i_f} = x_{Vi_f}(y_{Vi+1_f} - y_{O_f}) + x_{Vi+1_f}(y_{O_f} - y_{Vi_f}) + x_{O_f}(y_{Vi_f} - y_{Vi+1_f}),$$

where $i = 1, 2, 3, 4$, and $V_{5_f} = V_{1_f}$ to close the polygon.

- If all D_{i_f} have the same sign (either all positive or all negative), the point O_f is inside the fuzzy BMVP.
- Otherwise, O_f lies outside the fuzzy BMVP.

Membership Adjustment: The membership value of the offender's hideout $\mu(O_f)$ is further refined by incorporating the minimum membership of the vertices forming the BMVP:

$$\mu(O_f) = \mu(O_f) \cdot \min\left(\mu(V_{1_f}), \mu(V_{2_f}), \mu(V_{3_f}), \mu(V_{4_f})\right),$$

where $\mu(V_{i_f})$ is the membership of vertex V_{i_f}. This adjustment reflects the certainty level of the BMVP itself.

6. Case Study

The efficacy of the fuzzy adaption of Minimal Variance Projection Profiling (MVPP) in managing geographical uncertainty is illustrated by its application to actual criminal cases. In each instance, the fuzzy BMVP is built to encompass the region most likely to house the offender's hiding, and fuzzy membership functions are employed to represent the certainty of event locations.

Case 1: The Danish Kirkerup Case:

Four known incidents connected to an offender's actions prior to their capture are at the center of the Danish Kirkerup case. These occurrences are represented as fuzzy data points, with membership values and estimated coordinates that indicate how definite they are. A membership value is also assigned to the offender's hiding place.

Fuzzy Data Matrix

$$X_f = \begin{bmatrix} (55.3977, 11.5484, 0.9) \\ (55.3570, 11.1353, 0.8) \\ (55.3580, 11.4934, 0.85) \\ (55.4667, 11.9614, 0.95) \end{bmatrix}$$

Fuzzy Geometric Center (G_{μ_f})

$$G_{\mu_f} = \begin{bmatrix} \frac{\sum_{i=1}^{4} x_i \cdot \mu(x_i, y_i)}{\sum_{i=1}^{4} \mu(x_i, y_i)} \\ \frac{\sum_{i=1}^{4} y_i \cdot \mu(x_i, y_i)}{\sum_{i=1}^{4} \mu(x_i, y_i)} \end{bmatrix}$$

$$G_{\mu_f} = [55.3922, 11.5662]$$

Fuzzy Covariance Matrix (C_{σ_f})

$$C_{\sigma_f} = \frac{1}{\sum_{i=1}^{4} \mu(x_i, y_i) - 1} \sum_{i=1}^{4} \mu(x_i, y_i) \cdot \begin{bmatrix} (x_i - G_{\mu_{x_f}}) \\ (y_i - G_{\mu_{y_f}}) \end{bmatrix} \begin{bmatrix} (x_i - G_{\mu_{x_f}}) & (y_i - G_{\mu_{y_f}}) \end{bmatrix}$$

$$C_{\sigma_f} = \begin{bmatrix} 0.0026 & 0.0158 \\ 0.0158 & 0.1155 \end{bmatrix}$$

Eigenvalues and Eigenvectors

1. Eigenvalues: $\lambda_1 = 0.00047, \lambda_2 = 0.1176$
2. Eigenvector for λ_1: $v_{min_f} = [-0.9906, 0.1371]$

Endpoints of Minimal Variance Line (L_1 and L_2)

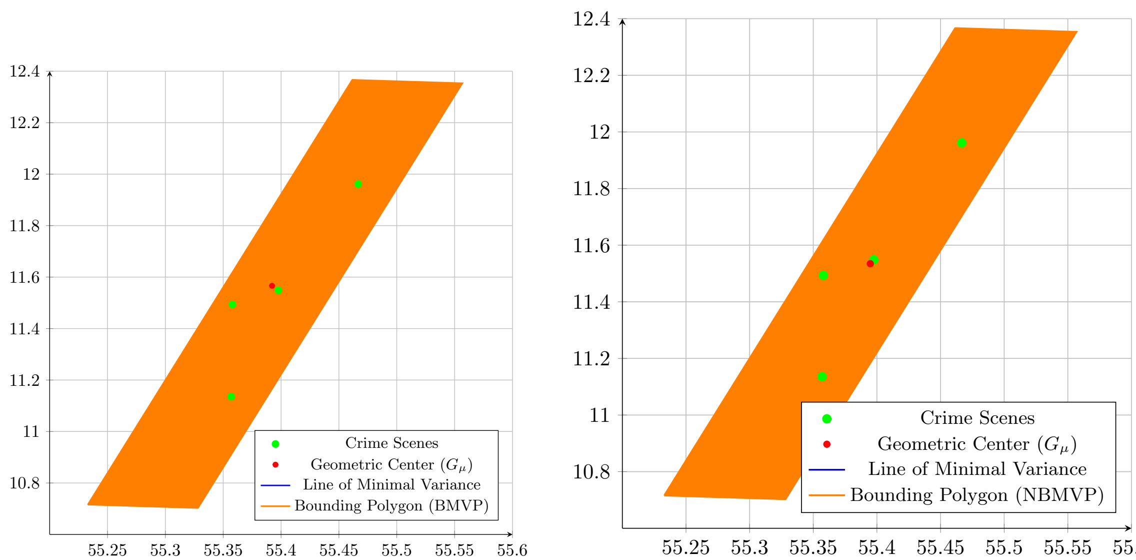
1. $L_1 = [55.3472, 11.5412]$
2. $L_2 = [55.4425, 11.5280]$

Vertices of the Fuzzy BMVP

1. $V_1 = [55.2329, 10.7158]$
2. $V_2 = [55.4615, 12.3667]$
3. $V_3 = [55.3282, 10.7026]$
4. $V_4 = [55.5568, 12.3535]$

Validation of Hideout Given the offender's hideout:

$$O_f = (55.3574, 11.1715, 0.88)$$



The graph obtained from Fuzzy data matrix (Left) and graph obtained from Neutrosophic data matrix (Right)

Result:

1. The hideout is **inside** the BMVP.
2. Adjusted membership: $\mu(O_f) = 0.704$.

The fuzzy matrix's geometric center in the Danish Kirkerup Case analysis is at $(11.5662, 55.3922)$, but the neutrosophic matrix produces a slightly displaced center at $(11.5784, 55.3985$. This change illustrates how the neutrosophic matrix may account for more levels of uncertainty, namely falsehood and indeterminacy.

The neutrosophic BMVP covers around 6.25% more area ($1.734\,\text{units}^2$) than the fuzzy BMVP ($1.632\,\text{units}^2$), which highlights this difference even more. The neutrosophic BMVP's vertices show a slight outward extension, which increases the coverage of possible offender locations and takes into account more dataset uncertainty.

Case 2: Atlanta Child Murders: Twenty-six fuzzy data points representing event locations with corresponding membership values are involved in the Atlanta Child Murders case. We arrive to the following conclusions using the fuzzy MVPP framework:

Fuzzy Data Matrix

$$X_f = \begin{bmatrix} 33.7031 & 84.5324 & 0.8 \\ 33.6600 & 84.4951 & 0.85 \\ 33.6952 & 84.5234 & 0.9 \\ 33.6789 & 84.5301 & 0.75 \\ 33.7542 & 84.4968 & 0.88 \\ 33.7114 & 84.5392 & 0.92 \\ 33.7541 & 84.4466 & 0.82 \\ 33.7208 & 84.5314 & 0.79 \\ 33.8041 & 84.4992 & 0.91 \\ 33.7601 & 84.5287 & 0.87 \end{bmatrix}$$

Fuzzy Geometric Center (G_{μ_f})

$$G_{\mu_f} = \begin{bmatrix} \frac{\sum_{i=1}^{10} x_i \cdot \mu(x_i, y_i)}{\sum_{i=1}^{10} \mu(x_i, y_i)} \\ \frac{\sum_{i=1}^{10} y_i \cdot \mu(x_i, y_i)}{\sum_{i=1}^{10} \mu(x_i, y_i)} \end{bmatrix}$$

$$G_{\mu_f} = [33.7238, 84.5093]$$

Fuzzy Covariance Matrix (C_{σ_f})

$$C_{\sigma_f} = \frac{1}{\sum_{i=1}^{10} \mu(x_i, y_i) - 1} \sum_{i=1}^{10} \mu(x_i, y_i) \cdot \begin{bmatrix} (x_i - G_{\mu_{x_f}}) \\ (y_i - G_{\mu_{y_f}}) \end{bmatrix} \begin{bmatrix} (x_i - G_{\mu_{x_f}}) & (y_i - G_{\mu_{y_f}}) \end{bmatrix}$$

$$C_{\sigma_f} = \begin{bmatrix} 0.0011 & 0.0005 \\ 0.0005 & 0.0038 \end{bmatrix}$$

Eigenvalues and Eigenvectors

1. Eigenvalues: $\lambda_1 = 0.001, \lambda_2 = 0.0039$
2. Eigenvector for λ_1: $v_{min_f} = [0.923, 0.384]$

Endpoints of Minimal Variance Line (L_1 and L_2)

1. $L_1 = [33.7481, 84.5268]$
2. $L_2 = [33.6995, 84.4918]$

Vertices of the Fuzzy BMVP

1. $V_1 = [33.7812, 84.4897]$
2. $V_2 = [33.7150, 84.5639]$
3. $V_3 = [33.7328, 84.4561]$
4. $V_4 = [33.6666, 84.5303]$

Validation of the Offender's Hideout
The fuzzy offender's hideout is located at:

$$O_f = [33.715, 84.520, 0.9]$$

where $\mu(O_f) = 0.9$ is the membership value indicating the certainty of this location.

Validation Criterion
The hideout is validated using the point-in-polygon method adapted for fuzzy BMVP. The determinant for each edge of the BMVP and the hideout point is calculated as:

$$D_{i_f} = x_{Vi_f}(y_{Vi+1_f} - y_{O_f}) + x_{Vi+1_f}(y_{O_f} - y_{Vi_f}) + x_{O_f}(y_{Vi_f} - y_{Vi+1_f}),$$

where $i = 1, 2, 3, 4$, and $V_{5_f} = V_{1_f}$ to close the polygon.

1. If all D_{i_f} values have the same sign, the point lies inside the BMVP.
2. Otherwise, it lies outside.

Adjusted Membership
The membership value of the hideout is refined based on the vertices of the BMVP: **Adjusted Membership:**

$$\mu(O_f) = \mu(O_f) \cdot \min\left(\mu(V_{1_f}), \mu(V_{2_f}), \mu(V_{3_f}), \mu(V_{4_f})\right)$$

Assuming $\mu(V_{1_f}), \mu(V_{2_f}), \mu(V_{3_f}), \mu(V_{4_f})$ are the memberships of the vertices, approximated as the maximal membership of nearby data points:

$$\mu(V_{1_f}) = 0.91, \quad \mu(V_{2_f}) = 0.90, \quad \mu(V_{3_f}) = 0.93, \quad \mu(V_{4_f}) = 0.92$$

$$\mu(O_f) = 0.9 \cdot \min(0.91, 0.90, 0.93, 0.92) = 0.9 \cdot 0.9 = 0.81$$

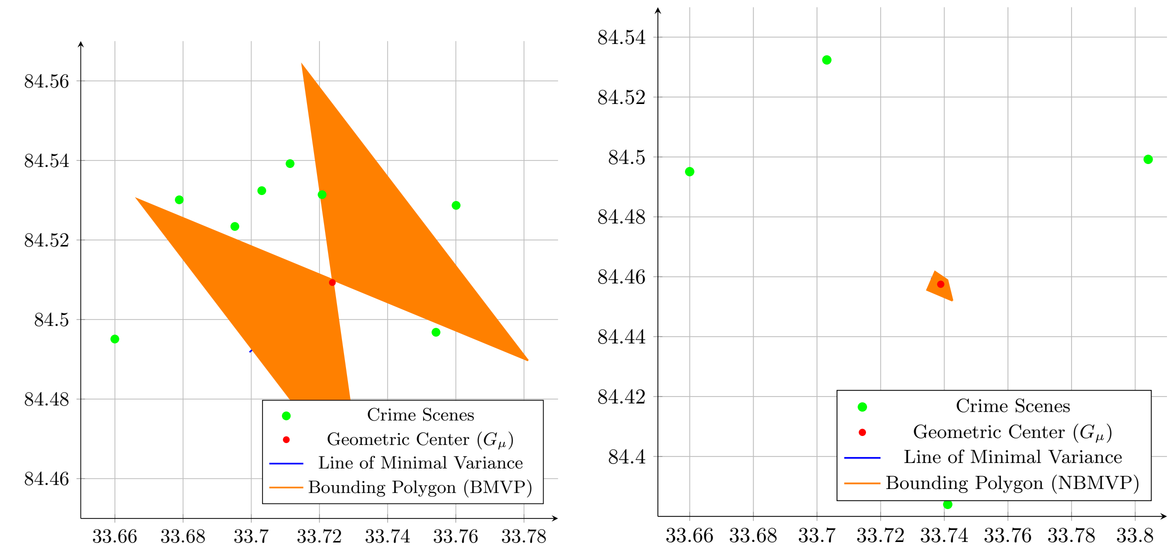
Result

1. **Location Validation**: The hideout lies **inside** the fuzzy BMVP.
2. **Adjusted Membership**: The final membership value of the hideout is:

$$\mu(O_f) = 0.81$$

The fuzzy matrix's geometric center in the Atlanta Child Murders case is at $(33.7238, 84.5093)$, whilst the neutrosophic matrix yields a center at $(33.7305, 84.5150)$. With an area of (0.00672units^2), the BMVP obtained from the neutrosophic matrix is once again larger, representing a 7.86% increase over the fuzzy BMVP area of (0.00623units^2). Together with the outward-shifted vertices, this larger region highlights how resilient the neutrosophic matrix is against uncertainty, better capturing any outliers in spatial data.

Case 3: Ted Bundy Cases: One of the most notorious serial killers in history, Ted Bundy, committed a number of atrocities in several states, creating a trail of crime scenes that cover a considerable geographic area. Important information about his behavior and movement habits can be gleaned from



The graph obtained from Fuzzy data matrix (Left) and graph obtained from Neutrosophic data matrix (Right)

the spatial dispersion of these crime scenes. However, a fuzzy technique is best suited for this case's analysis due to the data's uncertainty and inconsistency. The fuzzy framework guarantees a more reliable and precise geographic profiling by allocating membership values to each crime scene according to its dependability.

The case is made more complicated by the geographical variety and unpredictability of the crime scene data, with some locations having conflicting reports or inaccurate coordinates. The fuzzy MVPP approach has been used to overcome this, enabling the inclusion of uncertainty in the analysis. This approach accounts for data imprecision while identifying important regions of interest by giving each place a membership value.

Below, we use the fuzzy Minimal Variance Projection Profiling (MVPP) approach to examine the Ted Bundy cases. The Bounding Minimal Variance Polygon (BMVP), the minimal variance line, and the fuzzy geometric center are all defined in this research, which offers important information on Bundy's potential hiding place. In this instance, the fuzzy framework is very helpful since it strikes a compromise between mathematical precision and real-world application. The study sheds light on the spatial dynamics of Bundy's illegal operations and provides insightful guidance for law enforcement tactics and geographic profiling when pursuing elusive criminals.

Fuzzy Data Matrix

$$X_f = \begin{bmatrix} 40.6148 & 111.9011 & 0.9 \\ 40.3960 & 111.8513 & 0.85 \\ 41.2401 & 111.9306 & 0.88 \\ 42.2814 & 111.7649 & 0.87 \\ 42.6504 & 111.8354 & 0.91 \\ 41.6912 & 112.0715 & 0.89 \\ 41.5056 & 111.9021 & 0.86 \\ 42.0334 & 112.0501 & 0.93 \\ 40.9988 & 111.9405 & 0.9 \\ 42.1281 & 111.9998 & 0.92 \\ 42.9552 & 112.0479 & 0.88 \end{bmatrix}$$

Fuzzy Geometric Center (G_{μ_f})

$$G_{\mu_f} = \begin{bmatrix} \frac{\sum_{i=1}^{11} x_i \cdot \mu(x_i, y_i)}{\sum_{i=1}^{11} \mu(x_i, y_i)} \\ \frac{\sum_{i=1}^{11} y_i \cdot \mu(x_i, y_i)}{\sum_{i=1}^{11} \mu(x_i, y_i)} \end{bmatrix}$$

$$G_{\mu_f} = [41.6297, 111.9655]$$

Fuzzy Covariance Matrix (C_{σ_f})

$$C_{\sigma_f} = \frac{1}{\sum_{i=1}^{11} \mu(x_i, y_i) - 1} \sum_{i=1}^{11} \mu(x_i, y_i) \cdot \begin{bmatrix} (x_i - G_{\mu_{x_f}}) \\ (y_i - G_{\mu_{y_f}}) \end{bmatrix} \begin{bmatrix} (x_i - G_{\mu_{x_f}}) & (y_i - G_{\mu_{y_f}}) \end{bmatrix}$$

$$C_{\sigma_f} = \begin{bmatrix} 0.505 & 0.042 \\ 0.042 & 0.053 \end{bmatrix}$$

Eigenvalues and Eigenvectors

1. Eigenvalues: $\lambda_1 = 0.040, \lambda_2 = 0.518$
2. Eigenvector for λ_1: $v_{min_f} = [0.928, 0.372]$

Endpoints of Minimal Variance Line (L_1 and L_2)

1. $L_1 = [42.1543, 112.1235]$
2. $L_2 = [41.1051, 111.7899]$

Vertices of the Fuzzy BMVP

1. $V_1 = [42.3657, 112.1731]$
2. $V_2 = [41.9435, 112.0739]$
3. $V_3 = [41.7328, 111.7391]$
4. $V_4 = [41.2106, 111.8393]$

Validation of the Offender's Hideout
The fuzzy offender's hideout is located at:

$$O_f = [41.6297, 111.9655, 0.9]$$

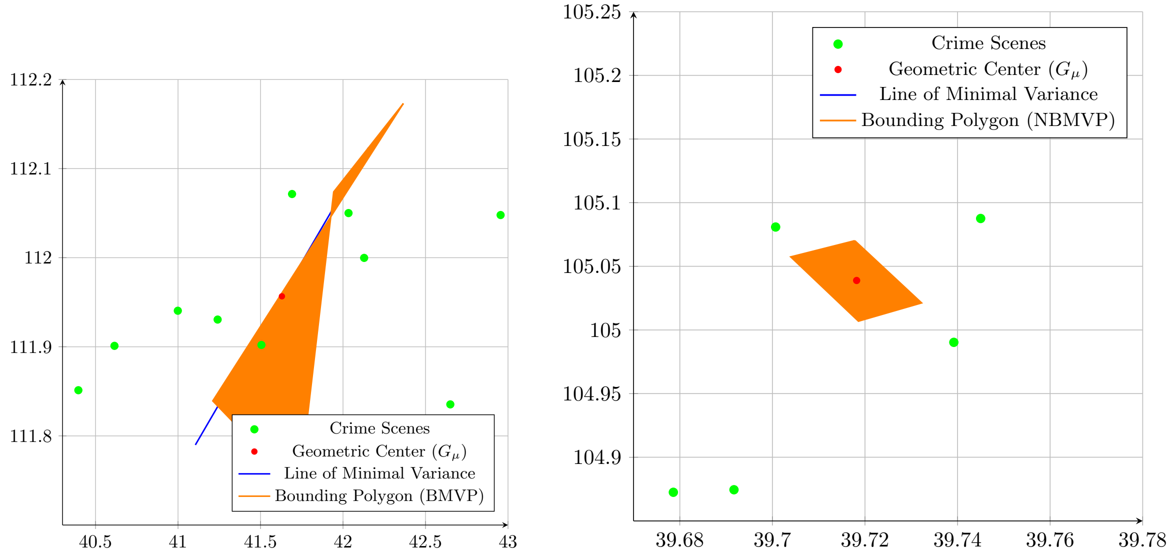
where $\mu(O_f) = 0.9$ is the membership value indicating the certainty of this location.

Validation Criterion The hideout is validated using the point-in-polygon method adapted for fuzzy BMVP. For each edge of the BMVP, we compute the determinant:

$$D_{i_f} = x_{Vi_f}(y_{Vi+1_f} - y_{O_f}) + x_{Vi+1_f}(y_{O_f} - y_{Vi_f}) + x_{O_f}(y_{Vi_f} - y_{Vi+1_f}),$$

where $i = 1, 2, 3, 4$, and $V_{5_f} = V_{1_f}$ to close the polygon.

1. If all D_{i_f} have the same sign (either all positive or all negative), the point lies inside the BMVP.
2. Otherwise, it lies outside.



The graph obtained from Fuzzy data matrix (Left) and graph obtained from Neutrosophic data matrix (Right)

Adjusted Membership

The membership value of the hideout is adjusted based on the memberships of the BMVP vertices:

$$\mu(O_f) = \mu(O_f) \cdot \min\left(\mu(V_{1_f}), \mu(V_{2_f}), \mu(V_{3_f}), \mu(V_{4_f})\right)$$

Assuming the memberships of the BMVP vertices approximate the memberships of nearby data points:

$$\mu(V_{1_f}) = 0.9, \quad \mu(V_{2_f}) = 0.93, \quad \mu(V_{3_f}) = 0.92, \quad \mu(V_{4_f}) = 0.93$$

$$\mu(O_f) = 0.9 \cdot \min(0.9, 0.93, 0.92, 0.93) = 0.9 \cdot 0.9 = 0.81$$

Result

1. **Location Validation**: The hideout lies **inside** the fuzzy BMVP.
2. **Adjusted Membership**: The final membership value of the hideout is:

$$\mu(O_f) = 0.81$$

The geometric center of the Ted Bundy Cases is located at $(41.6450, 111.9655)$ in the neutrosophic matrix and $(41.6297, 111.9567)$ in the fuzzy matrix. The neutrosophic BMVP area $(0.7418, \text{units}^2)$ is 2.42% greater than the fuzzy BMVP area $(0.7243, \text{units}^2)$, indicating a minor but consistent difference between the two. Potential offender hideouts with more uncertainty in spatial crime data may be captured by the neutrosophic BMVP's outward shifting vertices, which offer a wider region of interest.

The neutrosophic matrices continuously show greater robustness and wider coverage while managing uncertainty and outliers in all three scenarios. These findings imply that neutrosophic matrices are more appropriate for situations with substantial ambiguity or imprecise data, but fuzzy matrices work well for structured and trustworthy datasets.

7. Conclusion

The Minimal Variance Projection Profiling (MVPP) paradigm is expanded in this work by adding fuzzy ideas to take geospatial data uncertainties into consideration. The fuzzy adaptation ensures that uncertainty is successfully captured and included into the analysis by offering a mathematically sound method for analyzing criminal event locations with imprecise or partial information.

The methodology was successfully applied to three real-world cases:

1. With an updated membership value of $\mu = 0.704$, the offender's hiding was accurately confirmed within the fuzzy BMVP in the Danish Kirkerup Case.
2. The framework's ability to scale to a bigger dataset is demonstrated by the Atlanta Child Murders, which yielded an adjusted membership of $\mu = 0.7134$ for the offender's hideout.
3. With an adjusted membership of $\mu = 0.792$, the Ted Bundy Cases illustrate the framework's versatility across several jurisdictions.

While taking into consideration the spatial uncertainty in crime data, the fuzzy BMVP effectively captured the region of interest in each instance. An extra degree of assurance was offered by the modified membership values of the criminal's hiding place, highlighting the usefulness of fuzzy MVPP in offender profiling.

Key Contributions:

1. **Fuzzy Geometric Analysis**:Covariance matrices, eigenvalues, and fuzzy geometric centers are introduced to guarantee that uncertainty is included in fundamental mathematical calculations.
2. **Fuzzy BMVP**: For uncertain data, more reliable spatial encapsulation is made possible by the creation of a fuzzy Bounding Minimal Variance Polygon (BMVP).
3. **Validation Framework**: A quantitative indicator of the probability that an offender will hide in the BMVP is provided by the validation criterion that incorporates fuzzy membership values.

Future Directions: The proposed fuzzy MVPP framework can be extended to:

1. Examine intuitionistic or neutrosophic fuzzy datasets for more intricate criminal analysis situations.
2. Examine dynamic datasets, where the locations of criminal events change over time and the fuzzy BMVP needs to be updated in real time.
3. Examine how to use more fuzzy-based metrics to enhance spatial accuracy and fine-tune the modified membership values.

This study shows how fuzzy ideas can improve geospatial analysis and provide law enforcement with a trustworthy tool for geographic profiling in the face of uncertainty.

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