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Optimal pricing strategies of green innovative products in a dual-channel supply chain

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ABSTRACT: Due to the significant increase in internet usage, customers are increasingly favoring the online market. As a result, manufacturers are drawn to establishing an online channel that includes retail channel options. In recent years, the emphasis on environmental awareness and government regulations regarding the environment have motivated manufacturers to introduce and produce eco-friendly products while maintaining their economic viability. This article presents a dual-channel supply chain management model involving a manufacturer and a retailer. Under a Centralized policy, the article determines the optimal online prices, retail prices, wholesale prices, and level of green improvements. The optimal results are obtained and compared using a numerical example. Additionally, a sensitivity analysis is conducted to examine the impact of various significant parameters. It is found that the own-channel price sensitivity parameter negatively impacts supply chain profit and members' earnings, while increasing cost coefficients of green innovation rapidly decrease optimal levels and maximum profit, as demonstrated by a hypothetical data set simulation.

Key Words: Dual-channel supply chain; Green innovation; Game theory;

Contents

1. Introduction

Sustainability is an integral aspect of today's world and there is a close relationship between sustainability and its importance. Environmental problems are serious and most countries are moving in different directions towards sustainable development ([1],[9]). Innovative green product manufacturing is one of the key features of these themes. Products that have a relatively low environmental impact compared to conventional products are called environmentally friendly products or Green innovative products. The concept of eco-friendly products was first introduced by [18] to improve the balance with environmental aspects without harming the quality and efficacy of the product. The industry also produces greener and

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more innovative products due to people's environmental awareness. Additionally, Government regulations put pressure on the industry to produce these kinds of products [17]. In recent years, green supply chain (SC) management has increasingly had a positive impact on environments as it reduces the carbon emission ([7],[26]). In this study, we consider a common manufacturer who can produce green products and sell them through online channels or retail channels. Improvement of the convenience of Internet users, changes in customer shopping behavior, People's busy lifestyles are the main factors driving the growth of the online market. Potential advantages of online channels are: First, online channels directly provide convenient products for customers. Online prices are lower than retail prices as there are no third parties involved. Second, customers have more choice than any retail store and third, we can get the accessories you want right away. The market is also open 24 hours a day, 7 days a week [21]. Retail channel also has few points to choose by customers. First, retailers can offer retail services to the customers. For example, a customer may physically experience retail stores, free demonstration experiences, etc. Retail channel provides immediate delivery of their products to the customers ([11],[14]). In addition, customers can purchase products with confidence from professional vendors. These factors prompted the manufacturer to open his online channel and retail channel together. As a result, most well-known manufacturers have opened online channels and traditional retail channels offer products [12]. So far, the literature on SCs has mainly focused on analyzing the Dual-channel (DC) of retailers and manufacturers and their optimal choices and economic goals. [32] recently developed a model consisting of two suppliers and a traditional retailer to study the optimal pricing strategies and GI levels under different decision-making power strategies. This present study analyzes how environmental considerations and alternative product choices affect market demand. The research investigated how a product's environmental friendliness affects player demand and pricing decisions to maximize player profitability strategies for SC members (see Figure 1). Many circumstances in the real world that are pertinent to the suggested paradigm can be solved with DC SCN. For instance, the manufacturing company Maytag sells its goods through the merchant Sears. A common retailer can helps customers find the items on their wish lists, which will boost sales for both the producer and the store [16]. Additionally, a footwear manufacturing company creates footwear and sells it through a retail store or his own online store. Manufacturers aim to utilize eco-friendly packaging materials and organic raw materials whenever possible these days because consumers are more conscious of the environment. SCN of a similar nature is seen in the apparel and textile sectors. Assume P and Q are two textile industries that manufacture yarns and market them online or in brick-and-mortar stores. Both the yarn from industries P and Q may be found in a retail establishment. Retailers are common to industries P and Q. Then, in order to maximize earnings, they can choose which GI level and what price to establish for the goods. The electronics sector also uses SC models of this type. The following research questions are put forward in order to fill up the gaps in this direction:

1. What are the optimal prices, Green improvement values, and utilities for different game theoretic approaches in that DC SC ?
2. What impact do Green improvement's cost coefficient parameter and price sensitivity parameters have on decision variables and profits of the players and the SC?
3. How will the two different channels affect the interests and decision-making of chain members?

The order of this article is as follows:
Literature review is presented in Section 2. The topic is then discussed in Section 3 along with the presumptions underlying this study. In Section 4, the model is formulated, and the methodology is also discussed. The sensitivity analysis of the parameters and numerical examples are covered in Section 5. Finally, Section 6 presents the conclusions of the research findings and proposes potential future directions.

2. Literature review

This section illustrated the research gaps and mostly focused on the vital research that has been done in this direction. This section begins with an overview of recent research on the DC SC, follows with observations of various Green Improvement research, and then suggests an overview and research gaps.

2.1. Dual-channel supply chain

In different power-making scenarios, [3] evaluates the impact of pricing choices and advertising techniques on the DC SC. The most beneficial amount of advertising, investment, and selling pricing are also examined. A maker, a merchant, and a collector were included in a DC three-level closed-loop SC developed by [21]. Additionally, they evaluate the players' ideal choices in various leadership game models. The topics of price, green level, and channel coordination in a DC SC were covered by [1]. Additionally, a transshipment contrast is suggested to examine its uses and fulfilments. In a DC SC, where the government mandated a minimum Green improvement level to be maintained, [8] examined the situation to study about the Greenness affects on the members decisions. The best choices made by members of a DC SC were examined by [5] in various game-theoretic frameworks. Also, they concentrate on the quality effort, manufacturer service, and retail service and gain certain essential details. Subsequently, [22] investigated a multi-channel SC in which a single producer operates two stores and also establishes an online channel. Also, the company has offered direct service to consumers. The best profitable approach is then identified when the equilibrium decisions have been acquired. In a closed-loop DC SC, [2] analyzed the pricing and production decisions under two different subsidy strategies. They also look at the most profitable strategy . In order to deal with the demand functions of the consumers, [6] offered a mixed-integer nonlinear model to investigate the production systems in a three-stage SC and finally the important insights are provided by analyzing sensitivity and a real-world example. Also, [28] uses a DC SC made up of a retailer and a supplier to explore the best strategies that SC members should take in decentralized models with demand disruption. The outcomes demonstrate that by enhancing the revenue-sharing arrangement, SC's performance can be enhanced. [25] investigate the impact of product cost and quality on the return policy. The numerical example is solved using a solution algorithm, and the sensitivity of the parameters is examined. However,[4], [20], [24] and [30] have also analyzed direct channel SC models and looked into the members' best course of action.

2.2. Green innovations

The articles on green innovation (GI) encompass the methods to increase competition and increase market share in the production of innovative products. According to [19], a producer can be able to raise a product's GI level and it can boost environmental concerns while increasing SC profits. [9] looked into a SC involving a manufacturer and a retailer to study the pricing choices, GI level, and profits for various channel tactics. A carbon-neutral DC SC was demonstrated by [33] involving a manufacturer and a retailer. The single-channel and DC SC's best practices are being assessed and contrasted. A DC SC with uncertain demand was considered by [27], and the retailer provides the promotional efforts. The ideal degree of GI is found for the manufacturer, and the ideal pricing selections are computed in the article. In two separate choice scenarios, [31] builded a two-stage DC SC and look into the optimum pricing and greening procedures. In a DC SC, [8] investigated two different types of green products and determine the players' optimal decisions. These significant findings show how many products are affected by the government's eco-level policies. A closed-loop SC model with a socially conscious manufacturer, a retailer, and a third-party recycler was investigated by [29] under four distinct decision-making frameworks. As a result of the findings, the rate of waste recycling has increased, and the manufacturer's corporate social responsibility has improved. [25] investigated how remanufacturing and carbon emissions affect a DC SC in terms of both logistics and manufacturing. Investigated are the best pricing and collecting methods. [15] investigated the ideal decision variables and profit function of green SC's members playing the Stackelberg game with different information patterns. Here, a manufacturer creates eco-friendly products and sells them via two rival stores. [13] considered a two-echelon green SC with a supplier and a retailer under hazy uncertainty. After that, a numerical example is provided, and sensitivity analysis is carried out to gather crucial managerial insights. Some research gaps are noted beyond these literature studies. A few papers that are relevant when examining the vertical and horizontal competition in a DC SC constitute the primary research gaps and contributions of the current study. There is, however, no literature that takes into account both competitions in a DC green SC. This study makes the assumption that both channels compete in a green SC in an effort to close the gap.

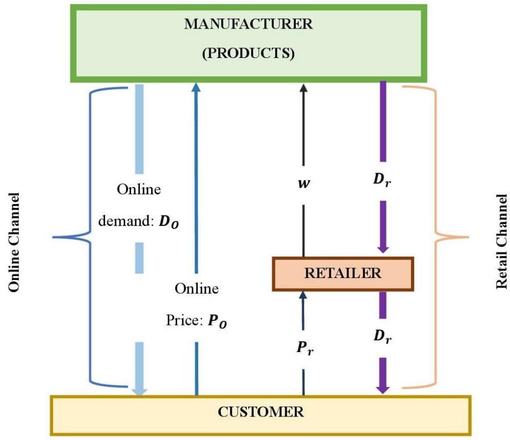

Figure 1: Pictorial representation of the proposed model

3. Problem Description

The present research investigates a DC green SC, including one manufacturer and one retailer (See Fig. 1). The manufacturer produces their desirable green improved products and sells them through either online or retail channels. The Manufacturer has the power to control the level of GI of the products. Also, the manufacturer decides the online and the wholesale price of the products and the retailer determines the retail price of the same products. In centralized policy (CP), the members will make their decisions collaboratively. Furthermore, we will calculate the decision variables and the profits of the members in CP. Also, the sensitivity of the profit functions with respect to the key parameters will be studied. Here, Table 1 describes the used notation of the parameters, decision variables and the profit functions throughout the paper.

Table 1: Notation

Parameters	
a_1	Market potential of the online channel
a_2	Market potential of the retail channel
b_1	Online channel sensitivities on demand rates
b_2	Retail channel sensitivities on demand rates
c_1	Online demand sensitivities of green improvement
c_2	Retail demand sensitivities of green improvement
Decision variables	
θ	Green improvement level of Product P ($0 < \theta < \theta_{\max}$)
P_r	Retail price of P (\$ per unit)
W	Wholesale price of P (\$ per unit)
Dependent variables	
D_1	Customer demand for P in online channels
D_2	Customer demand of P in the retail channel
π_m, π_r	Manufacturers and retailers profit functions (PFs)

The following assumptions have been made to validate the proposed model.

3.1. Assumptions

1. A manufacturer produces a product with the green improvement level θ and sells it through the wholesale price W, and sells directly to the customer through the online channel at price P_o. The

retail price of the product is P_r.

2. All the demands are taken to be linear in pricing decisions, namely online price P_o, retail price P_r, and level of green improvement θ. It is assumed that demands are downward sloping in their own price, upward sloping in the cross-channel price, and upward sloping in the products green improvement. For simplicity, the production cost M is considered as 0. Let D_1 be the customer demand through the online channel of the product. The customer demand of the product through the retail channel is D_2, and the demand functions can be expressed as follows:

$$D_1 = a_1 - b_1 P_o + c_1\theta, \tag{3.1}$$
$$D_2 = a_2 - b_2 P_r + c_2\theta. \tag{3.2}$$

3. The impact of self-price sensitivity parameters is greater than the sensitivity parameter of green improvement of demand rate $c_i,\ i = 1, 2$, i.e., $b_i > c_i$.

4. The manufacturer is not subjected to the effects of marginal cost. Moreover, it has some fixed cost for imposing the green improvement on the manufacturer. The cost function of the green improvement is considered as a convex function:

   $c(\theta) = \frac{1}{2}\eta\theta^2$

   where η is the green cost coefficient.

5. The manufacturer will sell their products to the retailer with a fixed discount k on the online price, i.e.,

   $w = kP_o$, where $k < 1$.

4. Model Formulation

With the help of assumptions, we obtain the profit functions of the players of the model as follows:

$$\pi_m = P_o D_1 + W D_2 - \tfrac{1}{2}\eta\theta^2 \tag{4.1}$$

$$\pi_r = (P_r - W) D_2 \tag{4.2}$$

where the notations are described previously and D_1 and D_2 are taken from (3.1) and (3.2). The subscripts r and m represent the retailer and manufacturer respectively. The total profit of the supply chain is the addition of the individual players profits and is obtained as

$$\begin{aligned}\pi_t &= \pi_m + \pi_r\\ &= P_o D_1 + W D_2 - \tfrac{1}{2}\eta\theta^2 + (P_r - W) D_2\\ &= P_o D_1 + W D_2 - \tfrac{1}{2}\eta\theta^2 + P_r D_2 - W D_2\\ &= P_o D_1 + P_r D_2 - \tfrac{1}{2}\eta\theta^2. \end{aligned}\tag{4.3}$$

Now, we have discussed the proposed model under CP and Manufacturer Stackelberg.

4.1. CENTRALIZED POLICY

All SC players act as a single-player under this strategy, i.e., there is a central decision maker to make the decision for the system. There is only one overall profit for the entire SC. The total profit of the SC is obtained from equation (4.3), and it depends on the retail price P_r, online price P_o, and the green improvement level θ of the product.

Simplifying equation (4.3), we have

$$\pi_t = a_1 P_o - b_1 P_o^2 + a_2 P_r - b_2 P_r^2 + c_1 P_o\theta + c_2 P_r\theta - \tfrac{1}{2}\eta\theta^2 \tag{4.4}$$

Proposition 4.1 *The profit function is maximum at*

$$P_o^* = \frac{-a_2c_1c_2 - a_1c_2^2 + 2a_2b_1\eta}{2(b_1c_2^2 + b_2c_1^2 - 2b_1b_2\eta)}$$
$$P_r^* = \frac{-a_1c_1c_2 - a_2c_1^2 + 2a_1b_2\eta}{2(b_1c_2^2 + b_2c_1^2 - 2b_1b_2\eta)}$$
$$\theta^* = \frac{a_1c_2 + a_2c_1}{(b_1c_2^2 + b_2c_1^2 - 2b_1b_2\eta)}$$

if the inequality

$$2b_2c_1^2 + 2b_1c_2^2 - 4b_1b_2\eta < 0$$

holds.

Proof: Firstly, equating the first-order partial derivatives of the profit function to zero, we have

$$\frac{\partial \pi_t}{\partial P_o} = a_1 - 2b_1P_o + c_1\theta = 0$$

$$\frac{\partial \pi_t}{\partial P_r} = a_2 - 2b_2P_r + c_2\theta = 0$$

$$\frac{\partial \pi_t}{\partial \theta} = c_1P_o + c_2P_r + \eta\theta = 0$$

Solving the above equations with respect to the decision variables P_o, P_r, and θ, we have

$$P_o^* = \frac{-a_2c_1c_2 - a_1c_2^2 + 2a_2b_1\eta}{2(b_1c_2^2 + b_2c_1^2 - 2b_1b_2\eta)}$$

$$P_r^* = \frac{-a_1c_1c_2 - a_2c_1^2 + 2a_1b_2\eta}{2(b_1c_2^2 + b_2c_1^2 - 2b_1b_2\eta)}$$

$$\theta^* = \frac{a_1c_2 + a_2c_1}{(b_1c_2^2 + b_2c_1^2 - 2b_1b_2\eta)}$$

Now, we calculate the Hessian Matrix HM of $\pi_t(P_o, P_r, \theta)$ in (6) as follows:

$$HM = \begin{bmatrix} \frac{\partial^2 \pi_t}{\partial P_o^2} & \frac{\partial^2 \pi_t}{\partial P_o \partial P_r} & \frac{\partial^2 \pi_t}{\partial P_o \partial \theta} \\ \frac{\partial^2 \pi_t}{\partial P_r \partial P_o} & \frac{\partial^2 \pi_t}{\partial P_r^2} & \frac{\partial^2 \pi_t}{\partial P_r \partial \theta} \\ \frac{\partial^2 \pi_t}{\partial \theta \partial P_o} & \frac{\partial^2 \pi_t}{\partial \theta \partial P_r} & \frac{\partial^2 \pi_t}{\partial \theta^2} \end{bmatrix}$$

$$HM = \begin{bmatrix} -2b_1 & 0 & c_1 \\ 0 & -2b_2 & c_2 \\ c_1 & c_2 & -\eta \end{bmatrix}$$

Hence the function π_t is maximum at the point (P_o^*, P_r^*, θ^*) if the Hessian Matrix is negative definite at the same point. Since the Hessian Matrix is independent of the decision variables, we have to show that the principal minors of the Hessian Matrix are alternatively negative and positive. Hence, if the following conditions hold, then π_t is maximum at that point:

1. $-2b_1 < 0$ (which is always true as b_1 is assumed to be positive),
2. $4b_1b_2 > 0$ (which is always true as b_1 and b_2 are assumed to be positive),

3. $2b_2c_1^2 + 2b_1c_2^2 - 4b_1b_2\eta < 0$.

Hence the proof.

The optimal profit of the SC can be obtained by substituting the values of the decision variables from the proposition in equation (4.4).

□

4.2. Manufacturers' Stackelberg

In the Stackelberg model, all the members of the SC optimize their corresponding decisions one by one according to the decision-making power. The manufacturer acts as a single-player in this game and leads the SC. In the end, the retailer makes his decision following the manufacturer. Member leadership mainly depends on the members' ability to make decisions and has a truthful effect on the SC. In this study, the level of Green improvement is controlled by the manufacturer to offer a new eco-friendly product to the customers, so the manufacturer has more ability to make decisions and is considered as a leader. According to the Stackelberg model principle, the optimal response to the follower (i.e., the retailer) is derived. Then, using these optimal responses in the leader's PFs (i.e., manufacturer), the optimal decisions of the leader are determined. Therefore the formulation of the model according to the decision making power is as follows:
L1 : Profit of Manufacturer
L2 : Profit of retailer

$$
\begin{aligned}
L1: \quad & \pi_{ms} = P_o D_1 + W D_2 - \tfrac{1}{2}\eta\theta^2 \\
L2: \quad & \pi_r = (P_r - W) D_2
\end{aligned}
\tag{4.5}
$$

Proposition 4.2 *The profit function of the retailer is maximum at the point:*

$$P_r^* = \frac{a_2(-2c_1^2 - c_1c_2k + 4b_1\eta + 3b_1k^2\eta) + a_1(2c_1c_2 + k(c_2^2 + 2b_2\eta))}{b_2(-4c_1^2 - 4c_1c_2k - c_2^2k^2 + 8b_1\eta + 4b_1k^2\eta)}$$

$$P_o^* = \frac{2(a_1c_2 + a_2k\eta)}{-4c_1^2 - 4c_1c_2k - c_2^2k^2 + 8b_1\eta + 4b_1k^2\eta}$$

$$\theta^* = \frac{(2a_1 + a_2k)(c_1 + c_2k)}{4c_1^2 + 4c_1c_2k + c_2^2k^2 - 8b_1\eta - 4b_1k^2\eta}$$

if the inequality $2b_1\eta - c_1^2 > 0$ *holds.*

Proof: Firstly, equating the first-order partial derivatives of the profit function π_r to zero, we have

$$\frac{\partial \pi_r}{\partial P_r} = 0 \;\Rightarrow\; P_r^* = \frac{a_2(-2c_1^2 - c_1c_2k + 4b_1\eta + 3b_1k^2\eta) + a_1(2c_1c_2 + k(c_2^2 + 2b_2\eta))}{b_2(-4c_1^2 - 4c_1c_2k - c_2^2k^2 + 8b_1\eta + 4b_1k^2\eta)}.$$

Now, if the Hessian Matrix of the function π_r is negative definite at that point (P_r^*), then π_r is maximum at the same point. The Hessian Matrix of π_r is simply $(-2b_2)$, which is negative since $b_2 > 0$. Hence, the first condition holds.

Now, the value of $P_r = P_r^*$ is taken from (3) and substituted in π_{ms}, transforming it into π_{ms}^*. Then, equating the first-order partial derivatives of π_{ms}^* to zero, we get

$$\frac{\partial \pi_{ms}^*}{\partial P_o} = 0 \Rightarrow P_o^* = \frac{2(c_a + a_c)\eta}{-4c_1^2 - 4c_1c_2k^2 + c_2^2k^2 + 8b_1\eta + 4b_2k^2\eta}$$

$$\frac{\partial \pi_{ms}^*}{\partial \theta} = 0 \Rightarrow \theta^* = \frac{(2a_1 + 2c_1)(c_c + k)}{4c_1^2 + 4c_1c_2k^2 + c_2^2k^2 - 8b_1\eta - 4b_2k^2\eta}$$

The Hessian Matrix with respect to π^*_{ms} is given by

$$\begin{bmatrix} \frac{\partial^2 \pi^*_{ms}}{\partial P_o^2} & \frac{\partial^2 \pi^*_{ms}}{\partial P_o \partial \theta} \\ \frac{\partial^2 \pi^*_{ms}}{\partial \theta \partial P_o} & \frac{\partial^2 \pi^*_{ms}}{\partial \theta^2} \end{bmatrix} \Rightarrow \begin{bmatrix} -2b_1 & c_1 \\ c_1 & \eta \end{bmatrix}$$

If the Hessian matrix of the function π^*_{ms} is negative definite at $P_o = P_o^*$ and $\theta = \theta^*$, then the function is maximum at the same value P_o, θ. Since the Hessian Matrix is independent of the decision variables, we have to show that the principal minors of the Hessian Matrix are alternatively negative and positive. Hence, if the following conditions hold, then π_{ms} is maximum at that point:

1. $-2b_1 < 0$ (which is always true as b_1 is assumed to be positive),
2. $2b_1\eta - c_1^2 > 0$

Hence the proof. □

Using the optimal decisions of the members, the profits of the members can be maximized.

5. Numerical example

Here, a numerical illustration and validation of the suggested model are provided. Due to the difficulties of gathering data from actual industrial settings, some data is used from prior research, and the remaining data is assumed hypothetically in order to validate the suggested model. To the greatest extent possible, the hypothetical data in this study are consistent with the published literature by [23]; however, it is impossible to assume an identical data set with any previous literature because our model is specially developed and has never been investigated previously. Consequently, the following is how the hypothetical example has been created:

Let us consider a commercial situation with a manufacturer and a retailer. The manufacturer produces products and sells them through either personal own channel or retail channel. Let the market potential of own channel of manufacturer be 410 units/unit time (i.e., $a_1 = 410$) and market potential of the retail channel of manufacturer be 425 units/unit time (i.e., $a_2 = 425$). The own channel price sensitivity of the own channel of M is 5.6 unit/unit \$ (i.e., $b_1 = 5.6$) and the retail channel of M is 5.5 unit/unit \$ (i.e., $b_2 = 5.5$). The demand sensitivity of the coefficient of θ of both own channel and retail channel be 0.75 and 0.73 (i.e., $c_1 = 0.75$ and $c_2 = 0.73$) respectively and the cost coefficient of the level Green improvement θ be \$1.85 ($\eta = 1.85$). Let the manufacturer provide a discount of 10% on the online price to sell the similar product to the retailer (i.e., $k = 0.9$).

Hence the data set is as follows:

$$a_1 = 410, \quad a_2 = 425, \quad b_1 = 5.6, \quad b_2 = 5.5, \quad c_1 = 0.75, \quad c_2 = 0.73, \quad \eta = 1.85, \quad k = 0.9$$

Now we will check the conditions of optimality and also find the optimal results under CP:

$$P_o = \$38.7354, \quad P_r = \$40.7455, \quad \theta = 31.7815, \quad \pi_t = \$16599.2$$

All the eigenvalues of the Hessian matrix of the profit functions are negative (i.e. -11.2751,-11.04187,-1.73308). Hence the optimality condition holds. Therefore, the values of the decision variables are optimal, and the profit function is maximum at the point. Now we will check the conditions of optimality and also find the optimal results under the Manufacturer Stackelberg policy. The optimal decisions are as follows: $P_o = 41.8586, P_r = 54.7967, \theta = 22.7507$
$\pi_m s = \$11519$ (Manufacturer Stackelberg Profit)
$\pi_r = \$3575.16$ (Retailer Profit)
$\pi_t = \$15094.2$ (Total Profit),
and all the eigenvalues of the Hessian matrix of the profit functions π_r and $\pi_m s$ are negative (-11) and (-11.0611,-1.78893) respectively. Hence the optimality condition holds. Therefore, the values of the decision variables are optimal, and the profit function is maximum at the point.

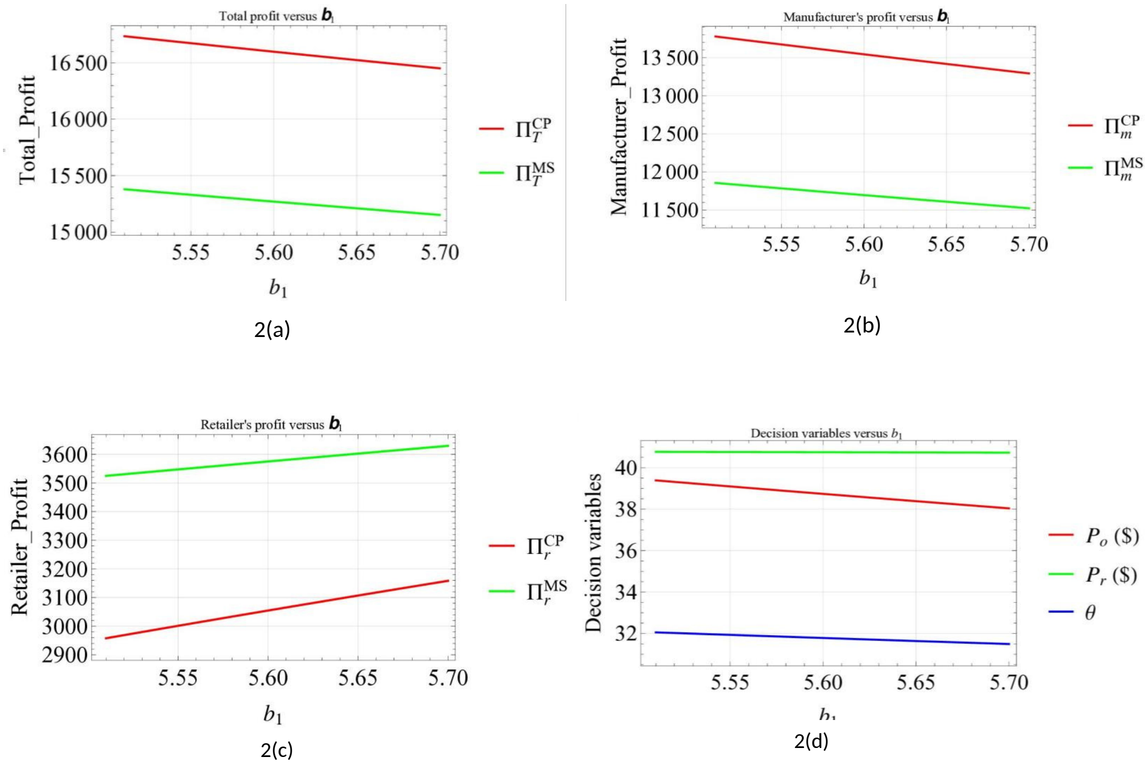

Figure 2: Impact of the parameter b_1

6. Sensitivity Analysis

This section focuses on analyzing the impact of parameter sensitivity on the dependent variables and profits. The changes in variable values under various parameter values for each gaming strategy are shown and discussed here. The sensitivity of the parameters on different performance factors is depicted in figures 2, 3, and 4. Based on the information presented in the tables and figures, the following subsections highlight the observations made.

6.1. Impact of the parameter b_1

Only the value of parameter b_1 will change at a time in the previous numerical example in order to analyze the impact of the parameter online channel sensitivity (b_1). Figure 2 shows how the parameter affects the players' profit. Figures 2(a), 2(b), 2(c), and 2(d) illustrate the effect of b_1. According to the figures 2(a) and 2(b), it can be observed that the total profit and manufacturer profit of the chain decreases with the increment of b_1 and from figures 2(c) we can observe that the retailer profit of the chain increases with the increment of b_1. Additionally, CP yields the highest profit. Figure 2(d) shows that when online channel sensitivity increases (b_1), the value of the online price P_o falls. The retail price P_r stays neatly constant while b_1 has increased. Again θ decreases slightly with the increasing value of b_1.

6.2. Impact of the parameter η

The parameters in the prior numerical example are held constant, and only the value of the parameter green cost coefficient(η) will change over time in order to analyze the effect of the parameter. Figure 3 shows how the parameter affects the players' earnings. Figures 3(a), 3(b), 3(c), and 3(d) illustrate the effect of the green cost coefficient(η). Figures 3(a) and 3(b) show that the chain's overall profit and manufacturer profit both decline as the green cost coefficient increases, while Figure 3(c) shows that the

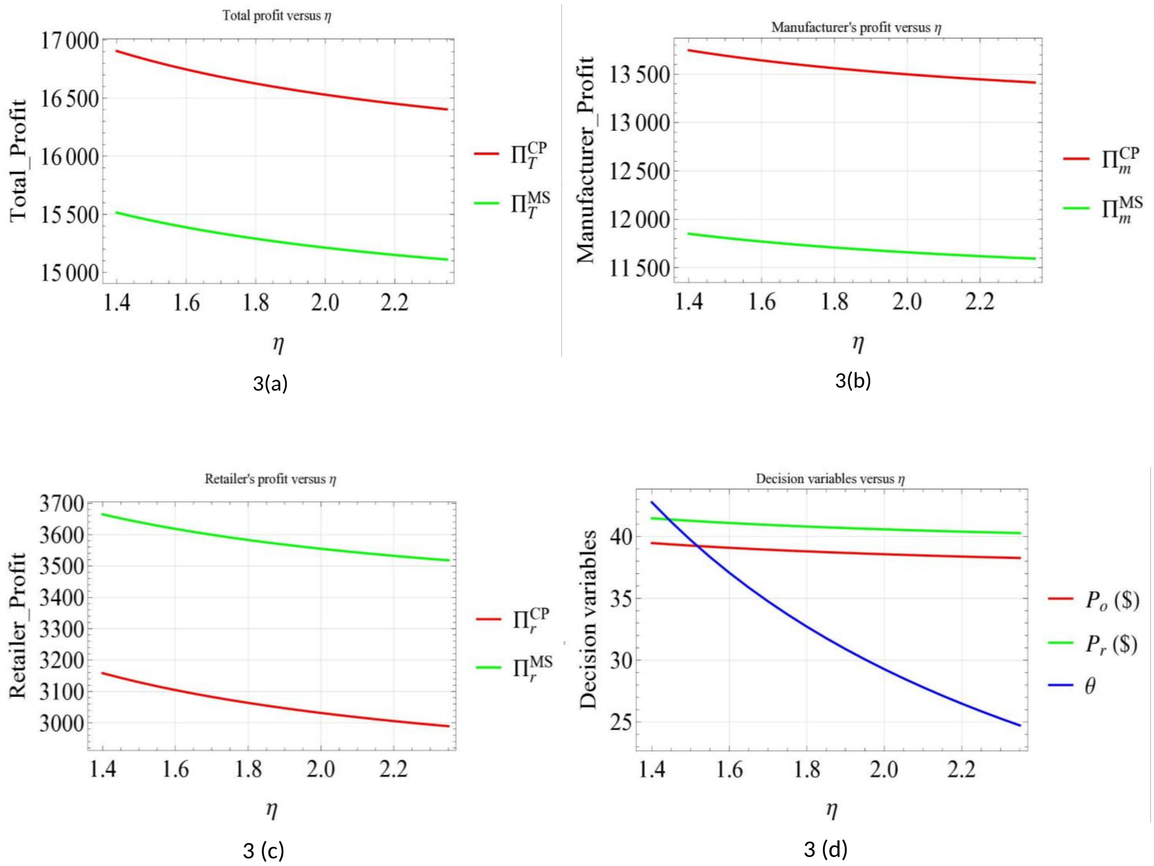

Figure 3: Impact of the parameter η

chain's retailer profit rises as the green cost coefficient increases. Additionally, CP yields the highest profit. Figure 3(d) shows that the value of the online price P_o and θ falls as the green cost coefficient (η) increases. The retail price P_r stays constant while the green cost coefficient(η) has increased.

7. Conclusion

This article formulates a DC green improved supply chain with a Manufacturer and a Retailer. The Manufacturer produces green innovative products and sells through either own-online channel or retail channel. The demand rates are assumed to be linear depending on the pricing decisions and the GI level. The Manufacturer determines the GI level and the online price whereas the Retailer decides the retail price of the same. The manufacturer provides a fixed discount to the retailer on the online price as the form of wholesale price. Two game theoretic approaches are considered and formulated. The Stackelberg theory and classical optimization technique are utilized to determine the optimal decisions of the members within each decision model. The analysis of the findings indicates that adopting the CP proves to be the most lucrative approach for the SC, thereby assisting in achieving the desired economic objectives.

The SC's overall profit as well as the earnings of the corresponding members are adversely affected by the own-channel price sensitivity parameter. Also the increment of cost coefficients of the GI (η) decreases the optimal level of GI very rapidly and maximum profit of the SC also decreases. With the help of the hypothetical data set, a simulation of the present market scenario is presented and with respect to the example the optimal results are also derived.

Naturally, our model can be extended to model stochastic demand and expand to multi-retailer or multi-manufacturer competition, despite its limitations, which include deterministic, linear demand and a single manufacturer-retailer setup. Future studies could also look at how promotional activities and return policies affect the profitability and best choices made by channel members.

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