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Orthogonal Jordan Derivations on Γ-semihyperrings*

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ABSTRACT: The present paper introduces the concept of orthogonal derivation on Γ-semihyperrings and explores some fundamental properties of orthogonal Jordan derivation. It is shown that a non-zero derivation is not necessarily orthogonal to itself and the specific condition under which such a derivation becomes orthogonal to itself is established. Furthermore, the sum of two orthogonal derivations remains orthogonal to each of its summands is proved. A necessary and sufficient condition for two derivations to be orthogonal is also derived. The study concludes with an in-depth examination of orthogonal Jordan derivations, revealing important results regarding their behaviour in the context of Γ-semihyperrings.

Key Words: Γ-semihyperring, idempotent Γ-semihyperring, orthogonal derivation, orthogonal Jordan derivation, etc.

Contents

1. Introduction

Algebraic hyperstructures stem from classical algebraic structures, distinguished by the central role of hyperoperations. The foundation of this field was laid by Marty in 1934, when he introduced the notion of hypergroups during the Eighth Congress of Scandinavian Mathematicians in Stockholm [11]. Since this pioneering work, a wide variety of algebraic hyperstructures have emerged, such as semihypergroups, hypergroups, Γ-semihypergroups, hyperrings and Γ-hyperrings. A key difference from traditional algebraic structures is that, in hyperstructures, an operation between two elements results in non-empty sets rather than a single outcome. This defining feature captures the core idea of hyperstructures. The extensive studies by Corsini and Leoreanu [6,7] have revealed numerous applications of hyperstructures in multiple scientific fields, highlighting their exceptional versatility and broad relevance.

Davvaz and Leoreanu-Fotea [7] explored the framework of hyperrings in detail. Building on this foundation, Davvaz and Dehkordi later introduced the concept of Γ-semihyperrings [8], a significant generalization that encompasses semirings, semihyperrings and Γ-semirings. In recent years, research in the domain of Γ-semihyperrings has seen substantial progress. Pawar et al. [16] contributed by defining the notions of regular and strongly regular Γ-semihyperrings, incorporating regularity conditions with respect to the ideals within these structures. Furthermore, in 2021, the study of prime and semiprime ideals in the context of Γ-semihyperrings was carried out in [15]. Collectively, these studies have significantly enriched the theoretical understanding and development of Γ-semihyperrings.

In [16], the notion of an idempotent element of a Γ-semihyperring is introduced where the characterization of quasi-ideals of an idempotent Γ-semihyperring is given with the help of ideals of a Γ-semihyperring.

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The concept of derivation in ring theory was first introduced by Posner in 1957 [18], who laid the groundwork by establishing fundamental results related to commutativity in rings possessing derivations. This pioneering work sparked widespread interest, leading to the extension of derivation theory to various algebraic structures. In 1987, Jing [10] extended the study to Γ-rings by defining derivations on it. The study of derivations within hyperstructures progressed notably in 2013, when Asokkumar [4] investigated derivations on Krasner hyperrings, focusing on their behaviour in prime hyperrings. Building on this, the exploration of derivations was broadened to include other hyperstructures, particularly Γ-semihyperrings. Significant contributions in this direction were made by Davvaz and Ardekani [1], who presented key results concerning to derivations on both Γ-semihyperrings and Γ-hyperrings. Further development appeared in the submitted article [14], where various essential results related to derivations on Γ-semihyperrings were analyzed. More recently, the novel concept of (σ, τ)-derivations was introduced in [12] for ordered Γ-semihyperrings, where their properties were thoroughly examined with particular emphasis given on their behaviour in the context of ordered settings. $\star$-derivation and $\star$-f-derivation were introduced and examined under specific constraints to determine whether the MA-Γ-semihyperring exhibits skew commutativity or weak commutativity in the under reviewed paper [13].

The concept of Jordan derivation in ring theory was initially introduced by Herstein in 1957 [9]. Later, in 1997, Sapanci and Nakajima [19] extended this idea to completely prime Γ-rings. Since then, various researchers have explored and examined Jordan derivations in greater depth, particularly in semiprime rings and Γ-rings. The study of Jordan derivations on Γ-semihyperrings was expanded in the submitted article [17], which offered a detailed characterization of such derivations on Γ-semihyperrings.

Brešar and Vukman [5] introduced the concept of orthogonal derivations on rings in 1989. They established orthogonality conditions for a pair of derivations (d, g) in semiprime rings. They provided several necessary and sufficient criteria under which the derivations d and g are orthogonal, along with results that connect their findings to the classical theorem of Posner [18]. Later, Ashraf and Jamal [3] extended the concept of orthogonality to derivations on Γ-rings. They also derived various conditions characterizing when a pair (d, g) of derivations can be considered orthogonal. Additionally, Argaç et al. [2] contributed to this area by presenting results on orthogonal generalized derivations.

The present paper comprises four main components. First is the introduction and review of the paper. Second, it presents a collection of fundamental definitions from [8], which are essential for understanding the structure of a Γ-semihyperring. Next, it introduces the concept of orthogonal derivations on Γ-semihyperrings, accompanied by an illustrative example and a discussion of several basic properties. Finally, it investigates key results related to orthogonal Jordan derivations on Γ-semihyperrings.

2. Preliminaries

This section presents a succinct exposition of the fundamental terminologies required for a coherent understanding of the subsequent discussions. For an in-depth understanding, readers are referred to [8].

Definition 2.1 *[8] Let H be a nonempty set and let $\bullet : H \times H \to \mathcal{P}^*(H)$ be a hyperoperation, where $\mathcal{P}^*(H)$ is a family of all non-empty subsets of H. Then a pair $(H, \bullet)$ is known as hypergroupoid. For any two non-empty subsets A and B of H and $x \in H$, we have, $A \bullet B = \bigcup_{(a \in A, b \in B)} a \bullet b$, $A \bullet \{x\} = A \bullet x$ and $\{x\} \bullet A = x \bullet A$.*

Definition 2.2 *[8] A hypergroupoid for which $(x \bullet y) \bullet z = x \bullet (y \bullet z)$, that is, $\bigcup_{(u \in x \bullet y)} u \bullet z = \bigcup_{(v \in y \bullet z)} x \bullet v$, for all x, y, $z \in H$ is called a semihypergroup, also, if for every $x \in H$, $x \bullet H = H = H \bullet x$, then $(H, \bullet)$ is called a hypergroup.*

For a comprehensive study of hypergroups and semihypergroups, the reader is directed to [7].

Definition 2.3 *[8] Let S and Γ be two nonempty sets. Then S is called a Γ-semihypergroup if $x\alpha y \subseteq S$ and $x\alpha(y\beta z) = (x\alpha y)\beta z$, for all $\alpha, \beta \in \Gamma$ and $x, y, z \in S$.*

Definition 2.4 *[8] The algebraic structure satisfying the ring like postulates is called a hyperring, that is, $(R, +, \cdot)$ is a hyperring if '$+$' and '$\cdot$' are two hyperoperations such that $(R, +)$ is a hypergroup and '$\cdot$' is an associative hyperoperation, which is distributive with respect to '$+$'.*

Hyperrings are classified into various types, including Krasner hyperrings, multiplicative hyperrings and Γ-hyperrings. Foundational studies on these structures have been conducted by Davvaz and Leoreanu-Fotea in [7].

Definition 2.5 *[8] $(R, +, \Gamma)$ is known as a Γ-semihyperring wherein R is a commutative semihypergroup and Γ is a commutative group if there is a map $R \times \Gamma \times R \to \mathcal{P}^*(R)$ such that for each $x, y, z \in R$, $\alpha \in \Gamma$, $x\alpha y$ denotes the images and $\mathcal{P}^*(R)$ stands for a cluster of all nonempty subsets of R fulfilling the constraints:*

(1) $x\alpha(y + z) = x\alpha y + x\alpha z$;

(2) $(x + y)\alpha z = x\alpha z + y\alpha z$;

(3) $x\alpha(y\beta z) = (x\alpha y)\beta z$;

(4) $x(\alpha + \beta)y = x\alpha y + x\beta y$.

Definition 2.6 *[8] If R is a semigroup in the above definition, then R is called a multiplicative Γ-semihyperring.*

Definition 2.7 *[8] A Γ-semihyperring R is called commutative (weak commutative, respectively) if $x\alpha y = y\alpha x$ ($x\alpha y \bigcap y\alpha x \neq \phi$, respectively), for all $x, y \in R$ and $\alpha \in \Gamma$.*

Definition 2.8 *[8] R is called a Γ-semihyperring with zero if there exists $0 \in R$ such that $x \in x + 0$, $0 \in 0\alpha x$ and $0 \in x\alpha 0$, for all $x \in R$ and $\alpha \in \Gamma$.*

Definition 2.9 *[1] A Γ-semihyperring R with zero is known as prime Γ-semihyperring if $0 \in x\alpha r\beta y$, for all $r \in R$ and $\alpha, \beta \in \Gamma$ implies that either $x = 0$ or $y = 0$.*

Definition 2.10 *[1] Let R be a Γ-semihyperring with zero. Then R is called n-torsion free if $0 \in nx = \underbrace{x + \ldots + x}_{n}$, $x \in R$, implies that $x = 0$, where n is a positive integer.*

Definition 2.11 *[1] Let $(R, +, \Gamma)$ be a Γ-semihyperring with zero. Then R is said to be of characteristic n if n is the smallest positive integer such that $0 \in nx = \underbrace{x + \ldots + x}_{n}$, for all $x \in R$. If there are no n of this kind, then R is said to be of characteristic 0.*

Definition 2.12 *[16] An element $e \in R$ is said to be an idempotent element of a Γ-semihyperring if $e \in e\alpha e$, for any $\alpha \in \Gamma$.*

Definition 2.13 *[16] A Γ-semihyperring R is said to be an idempotent Γ-semihyperring if every element of R is an idempotent element.*

Definition 2.14 *[14] Let $(R, +, \Gamma)$ be a multiplicative Γ-semihyperring. A function $d : R \to R$ satisfying the following conditions is called derivation if for all $x, y \in R$, $\alpha \in \Gamma$,*

(1) $d(x + y) = d(x) + d(y)$;

(2) $d(x\alpha y) \subseteq d(x)\alpha y + x\alpha d(y)$.

Definition 2.15 *[14] A derivation d on R is known as a homoderivation (strong homoderivation) on R if $d(x\alpha y) \subseteq d(x)\alpha d(y)(d(x\alpha y) = d(x)\alpha d(y))$.*

Definition 2.16 *[17] Let $(R, +, \Gamma)$ be a multiplicative Γ-semihyperring. A function $d : R \to R$ satisfying the following conditions is called Jordan derivation if for all $x, y \in R$, $\alpha \in \Gamma$,*

(1) $d(x + y) = d(x) + d(y)$;

(2) $d(x\alpha x) \subseteq d(x)\alpha x + x\alpha d(x)$.

Henceforth, R is assumed to be an idempotent multiplicative Γ-semihyperring.

3. Orthogonal Derivations

In this section, we introduce the notion of an orthogonal derivation on a Γ-semihyperring with examples and examine some basic properties.

Definition 3.1 *Let R be a Γ-semihyperring. The derivations $d, g : R \to R$ is said to be orthogonal if $0 \in d(x)\Gamma R\Gamma g(y)$ and $0 \in g(x)\Gamma R\Gamma d(y)$, for all $x, y \in R$.*

Example 3.1 Let $R = \{a, b, c, d\}$. Then R is a commutative semigroup with '+'.

+	a	b	c	d
a	{a}	{b}	{c}	{d}
b	{b}	{b}	{c}	{d}
c	{c}	{c}	{c}	{d}
d	{d}	{d}	{d}	{d}

Let '·' be the hyperoperation such that $x\alpha y \mapsto x \cdot y$, for all $x, y \in R$ and $\alpha \in \Gamma$.

·	a	b	c	d
a	{a}	{a}	{a}	{a}
b	{a}	{b}	{b, c}	{b, c, d}
c	{a}	{b, c}	{c}	{c, d}
d	{a}	{b, c, d}	{c, d}	{d}

Then $(R, +, \Gamma)$ is an idempotent Γ-semihyperring.

Theorem 3.1 *Let x and y be two elements of a prime Γ-semihyperring R with zero. Then followings are equivalent for all $r \in R$.*

(1) $0 \in x\Gamma r\Gamma y$;

(2) $0 \in y\Gamma r\Gamma x$.

Moreover, $0 \in x\Gamma r\Gamma y + y\Gamma r\Gamma x$ *and* $x = 0$ *or* $y = 0$.

Proof: Applying $y\Gamma r\Gamma$ on the left and $\Gamma r\Gamma x$ on the right on $0 \in x\Gamma r\Gamma y$, for all $x, y, r \in R$, we get $0 \in y\Gamma r\Gamma x\Gamma r\Gamma y\Gamma r\Gamma x$ i.e., $0 \in (y\Gamma r\Gamma x)\Gamma r\Gamma(y\Gamma r\Gamma x)$. As R is a prime Γ-semihyperring, we get, $0 \in y\Gamma r\Gamma x$. Therefore, $0 \in x\Gamma r\Gamma y \implies 0 \in y\Gamma r\Gamma x$. Thus, (1) $\implies$ (2). Similarly, we prove (2) $\implies$ (1).

Now, from (1) and (2), $0 \in x\Gamma r\Gamma y$ and $0 \in y\Gamma r\Gamma x$. Hence, $0 \in x\Gamma r\Gamma y + y\Gamma r\Gamma x$. Moreover, $0 \in x\Gamma r\Gamma y$, as R is prime, $x = 0$ or $y = 0$. □

Proposition 3.1 *Let d be any non-zero Jordan derivation on a Γ-semihyperring R and $g(x) = 0$, for all $x \in R$. Then d and g are orthogonal.*

Proof: Consider for all $x, y \in R$, $d(x)\Gamma R\Gamma g(y) = d(x)\Gamma R\Gamma 0$, which includes zero. Similarly, it can be shown that $0 \in g(x)\Gamma R\Gamma d(y)$, for all $x, y \in R$. Hence, zero derivation is orthogonal to any derivation. □

Proposition 3.2 *Every non-zero Jordan derivation on a Γ-semihyperring R may not be orthogonal to itself.*

Proof: Let d be any non-zero Jordan derivation on a Γ-semihyperring R. Then $0 \notin d(x)\Gamma R\Gamma d(y)$, for all $x, y \in R$. Thus, d is not orthogonal to itself. □

The following theorem gives the condition under which non-zero derivation becomes orthogonal to itself.

Theorem 3.2 *Let d be any non-zero Jordan derivation on a Γ-semihyperring R with zero. Then d is orthogonal to itself.*

Proof: Consider for all $x, y \in R$, $d(x)\Gamma R\Gamma d(y)$, which includes zero. Thus, d is orthogonal to itself. □

Example 3.2 Let $R = \{a, b\}$. Then R is a commutative semigroup with '+'.

+	a	b
a	{a}	{a}
b	{a}	{b}

Let '·' be such that $x\alpha y \mapsto x \cdot y$, for all $x, y \in R$ and $\alpha \in \Gamma$.

·	a	b
a	{a}	{a, b}
b	{a, b}	{b}

Here, R is an idempotent Γ-semihyperring. Let $d(x) = x$, for all $x \in R$. Then d is a derivation on R which is a Jordan derivation on R by Remark 3.1 in [17]. Therefore, d is orthogonal to itself.

Example 3.3 Consider Example 3.2, $d_1(x) = a$, for all $x \in R$ and $d_2(x) = x$, for all $x \in R$. Then d_1 and d_2 are Jordan derivations on R. Moreover, d_1 and d_2 are orthogonal Jordan derivations on R. Consider $d_1(x)\Gamma R\Gamma d_1(y) = a\Gamma R\Gamma a$, which includes zero. Thus, d_1 is orthogonal to itself.

The following theorem describes that orthogonality is preserved under addition.

Theorem 3.3 *Let d and g be any two non-zero orthogonal Jordan derivations on a Γ-semihyperring R with zero. Then $h = d + g$ is orthogonal to g and d.*

Proof: Let $h = d + g$. Consider for all $x, y \in R$, $h(x)\Gamma R\Gamma g(y) = (d + g)(x)\Gamma R\Gamma g(y) = (d(x) + g(x))\Gamma R\Gamma g(y) = d(x)\Gamma R\Gamma g(y) + g(x)\Gamma R\Gamma g(y)$, which contains zero, as every derivation is orthogonal to itself. Similarly, $0 \in g(x)\Gamma R\Gamma h(y)$. Hence, h is orthogonal to g. On the similar lines, it can be proved that h is orthogonal to d. □

Theorem 3.4 *Let R be a Γ-semihyperring with zero and d_1, d_2, g_1 and g_2 be any non-zero Jordan derivations on R such that d_1 is orthogonal to d_2 and g_1 is orthogonal to g_2. Then $d_1 + g_1$ is orthogonal to $d_2 + g_2$.*

Proof: Consider for all $x, y \in R$, $(d_1 + g_1)(x)\Gamma R\Gamma(d_2 + g_2)(y) = (d_1(x) + g_1(x))\Gamma R\Gamma(d_2(y) + g_2(y)) = d_1(x)\Gamma R\Gamma d_2(y) + d_1(x)\Gamma R\Gamma g_2(y) + g_1(x)\Gamma R\Gamma d_2(y) + g_1(x)\Gamma R\Gamma g_2(y)$, which includes zero. Similarly, we can show that $0 \in (d_2 + g_2)(x)\Gamma R\Gamma(d_1 + g_1)(y)$. Thus, $d_1 + g_1$ is orthogonal to $d_2 + g_2$. □

Theorem 3.5 *Let R be a two torsion free commutative Γ-semihyperring with zero and characteristic two and $x + x = x$, for all $x \in R$ and d_1, d_2, g_1 and g_2 be any non-zero Jordan derivations on R such that d_1 is orthogonal to d_2 and g_1 is orthogonal to g_2. Then $d_1(g_1)$ is orthogonal to $d_2(g_2)$.*

Proof: Consider for all $x, y \in R$, $d_1(g_1)(x)\Gamma R\Gamma d_2(g_2)(y)$. This includes zero. From Theorem 3.3 of [17], $d_1(g_1)$ and $d_2(g_2)$ are also Jordan derivations. Thus, $d_1(g_1)$ is orthogonal to $d_2(g_2)$. □

4. Main Results

This section explores key results concerning orthogonal Jordan derivations on a Γ-semihyperring. For this section, we consider a Γ-semihyperring R with zero and $x\alpha 0 = 0\alpha x = \{0\} = 0\alpha 0$, for all $x \in R$ and $\alpha \in \Gamma$.

Theorem 4.1 *Let R be a prime Γ-semihyperring and d and g are Jordan derivations on R.*

(1) If d and g are orthogonal derivations, then $0 \in d(y)\Gamma g(x)$ and $0 \in g(y)\Gamma d(x)$, for all $x, y \in R$.

(2) If d and g are homo-Jordan derivations on R and $0 \in d(y)\Gamma g(x)$ and $0 \in g(y)\Gamma d(x)$, for all $x, y \in R$, then d and g are orthogonal.

Proof:

(1) Let d and g be orthogonal Jordan derivations on R. Then $0 \in d(x)\Gamma R\Gamma g(y)$, for all $x, y \in R$, pre applying $g(y)\Gamma$ on both sides, we get, $0 \in g(y)\Gamma d(x)\Gamma R\Gamma g(y)$. Similarly, post applying $\Gamma d(x)$ on both sides, we get, $0 \in (g(y)\Gamma d(x))\Gamma R\Gamma(g(y)\Gamma d(x))$. As R is prime, $0 \in g(y)\Gamma d(x)$. Similarly, it can be shown that, $0 \in d(y)\Gamma g(x)$.

(2) Let $0 \in d(y)\Gamma g(x)$, for all $x, y \in R$. As R is an idempotent Γ-semihyperring, for $\alpha \in \Gamma$, $y \in y\alpha y$, gives $0 \in d(y\alpha y)\Gamma g(x)$. Here, d is a homo-Jordan derivation, $0 \in d(y)\alpha d(y)\Gamma g(x)$, therefore, $0 \in d(y)\alpha R\Gamma g(x)$, i.e., $0 \in d(y)\Gamma R\Gamma g(x)$. Similarly, it can be proved that, $0 \in g(y)\Gamma R\Gamma d(x)$. Thus, d and g are orthogonal.

□

Theorem 4.2 *Let R be a prime Γ-semihyperring and d and g be non-zero homo-Jordan derivations on R. If d and g are orthogonal Jordan derivations, then*

(1) $0 \in d(d)$ or $0 \in d(g)$.

(2) $0 \in g(g)$ or $0 \in g(d)$.

Proof:

1. Let d and g be orthogonal derivations. Then for all $x, y \in R$, $0 \in d(x)\Gamma g(y)$. Hence, for $\alpha \in \Gamma$, $0 \in d(x)\alpha g(y)$. Therefore, $d(0) \in d(d(x)\alpha g(y))$. From Proposition (4.2) of [14], $d(0) = 0$. Hence, $0 \in d(d(x)\alpha g(y)) \subseteq d(d(x))\alpha d(g(y)) \subseteq d(d(x\alpha x))\alpha d(g(y)) \subseteq d(d(x))\alpha d(d(x))\alpha d(g(y))$. Hence, $0 \in d(d(x))\alpha R\alpha d(g(y))$. R is prime, thus, $0 \in d(d)$ or $0 \in d(g)$.

2. On the similar lines, we can prove that $0 \in g(g)$ or $0 \in g(d)$.

□

Theorem 4.3 *Let R be a two torsion free prime Γ-semihyperring with zero and d be Jordan homo-derivation such that $d^2 = 0$. Then $d = 0$.*

Proof: Consider $d^2 = 0$, therefore, for $\alpha \in \Gamma$ and $x \in R$, $0 = d^2(x\alpha x) \subseteq d^2(x)\alpha x + 2d(x)\alpha d(x) + x\alpha d^2(x)$. As $d^2 = 0$, $0 \in 0\alpha x + 2d(x)\alpha d(x) + x\alpha 0 \subseteq 2d(x)\alpha d(x)$. Since R is two torsion free, $0 = d(x)\alpha d(x)$. As R is an idempotent Γ-semihyperring, $0 \in d(x)\alpha d(x\alpha x) \subseteq d(x)\alpha d(x)\alpha d(x)$. Hence, $0 \in d(x)\alpha R\alpha d(x)$. R is prime, gives $d(x) = 0$, for all $x \in R$. Thus, $d = 0$. □

Theorem 4.4 *Let R be a two torsion free prime Γ-semihyperring with zero and $x + x = x$, for all $x \in R$ and d be Jordan homo-derivation such that $d(d) = 0$. Then $d = 0$.*

Proof: Consider $d(d) = 0$, therefore, for $\alpha \in \Gamma$ and $x \in R$, $0 = d(d(x\alpha x)) \subseteq d(d(x)\alpha x + x\alpha d(x)) \subseteq d(d(x))\alpha x + d(x)\alpha d(x) + d(x)\alpha d(x) + x\alpha d(d(x))$. As $d(d) = 0$, $0 \in 0\alpha x + 2d(x)\alpha d(x) + x\alpha 0 \in 2d(x)\alpha d(x)$. Since R is two torsion free, $0 = d(x)\alpha d(x)$. As R is an idempotent Γ-semihyperring, $0 \in d(x)\alpha d(x\alpha x) \in d(x)\alpha d(x)\alpha d(x)$. Hence, $0 \in d(x)\alpha R\alpha d(x)$. R is prime, gives $d(x) = 0$, for all $x \in R$. Thus, $d = 0$. □

Theorem 4.5 *Let R be a two torsion free commutative Γ-semihyperring with zero and characteristic two with $x + x = x$, for all $x \in R$ and d and g are orthogonal Jordan derivations such that $d(d) = 0$ and $g(g) = 0$. Then d and $d(g)$ are orthogonal. Moreover, g and $g(d)$ are also orthogonal.*

Proof: Consider for all $x, y \in R$, $0 \in d(x)\Gamma g(y)$. Hence, for $\alpha \in \Gamma$, $0 \in d(x)\alpha g(y)$, therefore, $d(0) \in d(d(x)\alpha g(y))$. From Proposition (4.2) of [14], $d(0) = 0$. Hence, $0 \subseteq d(d(x))\alpha g(y) + d(x)\alpha d(g(y)) \subseteq 0\alpha g(y) + d(x)\alpha d(g(y)) \subseteq d(x)\alpha d(g(y))$, i.e., $0 \in d(x)\alpha d(g(y))$. Similarly, it can be shown that $0 \in d(g(x))\alpha d(y)$. From Theorem 3.3 of [17], $d(g)$ is also a Jordan derivation. Thus, d and $d(g)$ are orthogonal. On the similar lines, by applying g on $0 \in d(x)\alpha g(y)$, we can prove that, g and $g(d)$ are also orthogonal. □

Theorem 4.6 *Let R be a two torsion free commutative Γ-semihyperring with zero and characteristic two with $x + x = x$, for all $x \in R$ and d and g are non-zero orthogonal Jordan homo-derivations. Then $d(d)$ and $d(g)$ are orthogonal. Moreover, $g(g)$ and $g(d)$ are also orthogonal.*

Proof: Consider for all $x, y \in R$, $0 \in d(x)\Gamma R\Gamma g(y)$. Hence, for $\alpha \in \Gamma$, $0 \in d(x)\alpha R\alpha g(y)$, therefore, $d(0) \in d(d(x)\alpha R\alpha g(y))$. Hence, $0 \subseteq d(d(x))\alpha d(R)\alpha d(g(y))$, i.e., $0 \in d(d(x))\alpha R\alpha d(g(y))$. Similarly, it can be shown that $0 \in d(g(x))\alpha R\alpha d(d(y))$. From Theorem 3.3 of [17], $d(d)$ and $d(g)$ are Jordan derivations. Thus, $d(d)$ and $d(g)$ are orthogonal. On the similar lines, by applying g on $0 \in d(x)\alpha R\alpha g(y)$, we can prove that, $g(g)$ and $g(d)$ are also orthogonal. □

5. Conclusion

The notion of orthogonal Jordan derivation is introduced and key results characterizing its structure are established within Γ-semihyperrings. An equivalent condition ensuring the orthogonality of two derivations has been formulated.

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