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Novel Coding Inequalities for Mean Codeword Length and Generalized Entropy using Noiseless Communication

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ABSTRACT: Shannon's entropy forms the basis for almost every aspect of information theory. It formulates the foundational stone for various source coding theorems assuming statistical independence and extensive systems. This research aims to investigate the possibility of deriving novel entropy measures using noiseless coding theorem. The obtained results find a widespread application in information theory and applied mathematics. To accomplish this, a novel expression for mean codeword length has been illustrated. Besides, established relation between entropy measure and its corresponding codeword length. The results obtained pave the way for a new avenue for entropy-based coding in non-extensive and information-rich environments.

Key Words: Noiseless coding theorem, Mean codeword length, Entropy measure, Holder's inequalities, Kraft inequalities, Optimal code length, Power Probabilities

Contents

1. Introduction

Inequalities are used in mathematics to examine the overall magnitude of quantities. They may be used to compare various mathematical expressions, factors, and numbers. When it comes to solving problems involving the least or greatest potential attributes, inequality is quite useful. While indicates that is strictly more than , indicates that is less than or equal to , and indicates that is higher than or equal to , the idea means that is strictly smaller modest in size than . Not everything in mathematics is about "equals"; sometimes we merely realise that something is more notable or not. Inequalities are often used by mathematicians to constrain values that have certain recipes that are not easily handled. There are many such inequalities including Chebyshev's inequality, Bernoulli, Holder's and Cauchy-Schwarz. One of the eminent inequalities used in the literature of information theory is Jensen inequality, given by Johan Jensen, a Danish mathematician. Initially demonstrated in 1906, Jensen inequality establishes the link between the estimation of a convex function of an integral and its integral. Although its apparent simplicity, the inequality can take several forms based on the mathematical context in which it is utilized. The idea of entropy was first presented by Shannon [1] and has since become a foundation for assessing uncertainty in random variables. According to several scholarly references, including online encyclopedias, entropy fundamentally describes the degree of unpredictability or randomness related to a given variable. Shannon's theory defines entropy as the predicted amount of information contained in a message, which is commonly measured in bits, where a message is considered as the precise outcome of a random variable. Shannon's formulation provides a method for calculating the average information loss caused by the variable's implicit uncertainty in behavior. Entropy has played a significant impact on current communication theory during the last few decades. Notably, [2] made substantial contributions to our knowledge of probabilistic instability, which led to a wide range of applications. Subsequently, the entropy theory finds its utility in various domains of information and coding theory including operational

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research, decision making, image construction, Error detection and correction, regional planning, bioinformatics, queuing theory, industrial productions and many more. Efficient data compression is essential to information theory because it enables the removal of redundancy in data transfer while retaining information. The Noiseless Coding Theorem (Shannon's Source Coding Theorem) is a basic result in this discipline which establishes basic lower bound on the average length of codewords in terms of source entropy. This theorem assures that no unique decodable code may compress on average, below the source's entropy. However, with growing demands for more efficient communication particular in situations involving skewed or complex distributions, classical entropy measure and corresponding bounds may fall short of capturing real word subtleties. Keeping in mind that the RV collection $F = (\vartheta_1, \ldots, \vartheta_M; \rho_1, \ldots, \rho_M)$ and its corresponding probabilities $(\rho_1, \ldots, \rho_M)$, $\rho_t \geq 0$, $\sum_t \rho_t = 1$, are a subset of the discrete RV $\vartheta_1, \ldots, \vartheta_M$, and so forth. For the finite information scheme, the uncertainty measure or the entropy measure is given as:

$$h(\rho_\vartheta) = -\sum_{t=1}^{M} \vartheta_t \, \rho_t \, \log \rho_t \tag{1.1}$$

For the code word length (to be conveyed) $(m_1, m_2, \ldots, m_M)$, the set of probabilities are given by $(\rho_1, \rho_2, \ldots, \rho_M)$. Then let us assume for D being the coding alphabet size, the Kraft inequality satisfies [3]:

$$\sum_{t=1}^{M} D^{-m_t} \leq 1 \tag{1.2}$$

Using the characteristics of [1], the constraints for mean code word can be obtained by solving:

$$I_\vartheta = \sum_{t=1}^{M} \vartheta_t \, \rho_t \, m_t \tag{1.3}$$

It is located in the range of $h(\rho)$ and $h(\rho) + 1$. According to Feinstein [4], this is the codeword length for a code that satisfies (1.2):

$$I_\vartheta \geq h(\rho_\vartheta) \tag{1.4}$$

where equality holds if and only if

$$m_t = -\log_D(\vartheta_t \rho_t), \quad t = 1, 2, \ldots, M \tag{1.5}$$

The average length can be obtained close to $h(\rho_\vartheta)$. This result can be understood by noiseless coding theorem given by Shannon. In correspondence to this, Campbell [5] established an equivalent coding theorem using Renyi's [6] entropy and noiseless coding theorem. The authors also obtained bounds for the results in terms of the expression:

$$h_r(\rho_\vartheta) = \frac{1}{1-r} \log_D \Big(\sum_t (\vartheta_t \rho_t^r) \Big), \quad r > 0 \; (r \neq 1)$$

Further, Kieffer [7] deals with source coding with side information or multiple source coding, and more specifically with choosing between two sources for coding data in such a way that the expected cost per symbol is minimized in the asymptotic limit. Jelinek [8] worked on the problem of buffer overflow with real time data encoding. He used Campbell [5] mean length and created source symbols in case when symbols are stored in a finite buffer. The extension of mean length given by Campbell were also illustrated by Hooda and Bhaker [9]

$$I_{s\vartheta} = \frac{1}{p} \log_D \left\{ \frac{\sum_{t=1}^{M} (\vartheta_t \rho_t)^s D^{-m_t p}}{\sum_{t=1}^{M} (\vartheta_t \rho_t)^s} \right\}, \qquad s \geq 1 \tag{1.6}$$

And proved as a result, (1.6)'s smallest value is between $h_r(\rho_\vartheta)$ and $h_r(\rho_\vartheta)+1$.

$$h_{rs}(\rho_\vartheta) \;\le\; I_{s\vartheta} \;<\; h_{rs}(\rho_\vartheta)+1, \quad r>0,\ r\neq 1,\ s\geq 1$$

Under the condition

$$\sum_{t=1}^{M}(\vartheta_t\rho_t)^{s-1}D^{-m_t} \;\le\; \sum_{t=1}^{M}(\vartheta_t\rho_t)^{s}$$

This is in line with the Tsallis entropy [10]. Numerous scholars have examined a range of generalized entropy measures. Considering uniquely decipherability, generalized CWL and theorems were developed based on these measures. A case in point is [11]. By applying the weighted entropy that was explained in [12], [13] was able to determine the least value of a meaningful MCWL. In [14], the Noiseless Coding Theorem (NCT) was developed. The average and lowest values of CWL were also covered. [15] examined the limit and average codeword length (CWL), which is significant data. Many applications of significant generalized theorems in coding information theory are discussed by well-known authors, such as [16,17,18],and [19,20,21,22]. Some of the important results were also examined in [23].

In this research, we examine numerous coding theorems by proposing a novel function that depends on parameters. We are looking at this new function since it generalizes a number of entropy functions that are already known from the literature, including the entropy used by [24,25,26,27] in physics.

2. Noiseless Coding Theorem

We define a completely new measure that is described as follows:

$$h_{rs}(\rho_\vartheta)=\frac{1}{r-1}\left\{1-\frac{\sum_{t=1}^{M}(\vartheta_t\rho_t^r)^s}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s}\right\}, \quad r>0,\ r\neq 1,\ s>0,\ \vartheta_t>0,\ \rho_t\geq 0,\ t=1,2,\ldots,M \tag{2.1}$$

Since ρ_t gives the probability distribution, we have $\sum_{t=1}^{M}\rho_t=1$.

Assume $s=1$, the equation (2.1) takes the form discussed in [24]:

$$h_r(\rho_\vartheta)=\frac{1}{r-1}\left\{1-\sum_{t=1}^{M}\vartheta_t\rho_t^r\right\} \tag{2.2}$$

Equation (2.1) relates to [1] with $s=1$ and $r\to 1$:

$$h(\rho_\vartheta)=-\sum_{t=1}^{M}\vartheta_t\rho_t\,\log\rho_t \tag{2.3}$$

If $\vartheta_t=1$ and $r\to 1$, (2.1) reduces to a measure of "useful" information as in [19]:

$$h_s(\rho)=-\frac{\sum_{t=1}^{M}(\rho_t)^s\log(\rho_t^s)}{\sum_{t=1}^{M}(\rho_t)^s} \tag{2.4}$$

Definition 2.1 *In accordance with the entropy measure, the useful mean length is denoted and defined as:*

$$I_{rs\vartheta}=\frac{1}{r-1}\left[1-\left\{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s\left(\frac{1}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s}\right)^{\frac{1}{r}}D^{-\frac{m_t(r-1)}{s}}\right\}^{r}\right], \tag{2.5}$$

where $r>0$ with $r\neq 1$, $s>0$, $\vartheta_t>0$, and $\rho_t\geq 0$ for $t=1,2,\ldots,M$. Since ρ_t gives the probability distribution, we also have $\sum_{t=1}^{M}\rho_t=1$.

For $s=1$ and $r\to 1$, $I_{rs\vartheta}$ becomes the optimal code length as defined by [1]:

$$I_\vartheta = \sum_{t=1}^{M} m_t\, \vartheta_t \rho_t.$$

If so, the MCWL (2.5) becomes the new MCL:

$$I_{r\vartheta} = \frac{1}{r-1}\left[1 - \left\{\sum_{t=1}^{M} \vartheta_t \rho_t\, D^{-\frac{m_t(r-1)}{r}}\right\}^r\right]. \tag{2.6}$$

Theorem 2.1 *The code word length $D > 1$ satisfies the following for all integers:*

$$I_{rs\vartheta} \;\geq\; h_{rs}(\rho_\vartheta), \tag{2.7}$$

under the condition

$$\sum_{t=1}^{M} D^{-m_t} \leq 1,$$

where equality holds if and only if

$$m_t = -\log_D\left(\frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M}(\vartheta_t \rho_t^r)^s}\right). \tag{2.8}$$

Here,

$$I_{rs\vartheta} = \frac{1}{r-1}\left[1 - \left\{\sum_{t=1}^{M} (\vartheta_t \rho_t)^s \left(\frac{1}{\sum_{t=1}^{M}(\vartheta_t \rho_t)^s}\right)^{\frac{1}{r}} D^{-\frac{m_t(r-1)}{r}}\right\}^r\right],$$

and

$$h_{rs}(\rho_\vartheta) = \frac{1}{r-1}\left\{1 - \frac{\sum_{t=1}^{M}(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M}(\vartheta_t \rho_t)^s}\right\}.$$

Proof: Hölder's disparity is known to be caused by

$$\left(\sum_{t=1}^{M}(a_t)^e\right)^{1/e}\left(\sum_{t=1}^{M}(b_t)^f\right)^{1/f} \leq \sum_{t=1}^{M} a_t b_t, \tag{2.9}$$

for all $a_t \geq 0, b_t \geq 0,\ t = 1, 2, \ldots, M$, with equality if and only if there exists a positive number e such that

$$a_t^\rho = e b_t^\rho, \tag{2.10}$$

where

$$e^{-1} + f^{-1} = 1, \quad e = \frac{r-1}{r}, \quad f = 1 - r.$$

For the present case, we choose

$$a_t = (\vartheta_t \rho_t)^{\frac{rs}{r-1}}\left(\frac{1}{\sum_{t=1}^{M}(\vartheta_t \rho_t)^s}\right)^{\frac{1}{r-1}} D^{-m_t},$$

and

$$b_t = (\vartheta_t \rho_t)^{\frac{rs}{1-s}}\left(\frac{1}{\sum_{t=1}^{M}(\vartheta_t \rho_t)^s}\right)^{\frac{1}{1-r}}.$$

The equality holds if and only if

$$D^{-m_t} = \frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s},$$

which gives

$$m_t = -\log_D \left(\frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s} \right). \tag{2.1}$$

□

Theorem 2.2 *I_{rs} can satisfy the subsequent condition if the lengths $m_1, \ldots, m_M$ in Theorem 2.1 code are chosen correctly:*

$$I_{rs,\vartheta} < D^{(1-r)} h_{rs}(\rho_\vartheta) + \frac{1}{r-1} \left(1 - D^{(1-r)} \right), \tag{2.11}$$

where

$$I_{rs,\vartheta} = \frac{1}{r-1} \left[1 - \left\{ \sum_{t=1}^{M} (\vartheta_t \rho_t)^s \left(\frac{1}{\sum_{t=1}^{M} (\vartheta_t \rho_t)^s} \right)^{1/r} D^{-m_t \frac{(r-1)}{r}} \right\}^r \right],$$

and

$$h_{rs}(\rho_\vartheta) = \frac{1}{r-1} \left\{ 1 - \frac{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t)^s} \right\}, \qquad r > 0,\ s > 0.$$

Proof: Let m_t be a positive integer that fulfils

$$-\log_D \left(\frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s} \right) \le m_t \le -\log_D \left(\frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s} \right) + 1. \tag{2.12}$$

Consider the interval of length 1

$$\partial_t = \left[-\log_D \left(\frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s} \right),\ -\log_D \left(\frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s} \right) + 1 \right]. \tag{2.13}$$

Then, for each ∂_t there exists exactly one positive integer m_t such that

$$0 < -\log_D \left(\frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s} \right) \le m_t \le -\log_D \left(\frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s} \right) + 1. \tag{2.14}$$

From the upper bound in (2.14) we obtain

$$m_t < -\log_D \left(\frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s} \right) + 1,$$

hence

$$D^{-m_t} > \frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s} D^{-1},$$

and therefore

$$D^{-\frac{m_t(r-1)}{r}} > \left(\frac{(\vartheta_t \rho_t^r)^s}{\sum_{t=1}^{M} (\vartheta_t \rho_t^r)^s} \right)^{\frac{r-1}{r}} D^{\frac{1-r}{r}}. \tag{2.15}$$

Multiplying both sides of (2.15) by

$$(\vartheta_t \rho_t)^s \left(\frac{1}{\sum_{t=1}^{M} (\vartheta_t \rho_t)^s} \right)^{1/r},$$

summing over $t = 1, \dots, M$, and simplifying (and then taking the factor $1/(r-1)$ for $r > 1$) yields the inequality (2.11).

□

Theorem 2.3 *For every codeword length m_t, $I_{rs,\vartheta}$ must satisfy the following inequality, according to Theorem 2.1:*

$$I_{rs,\vartheta} \;\geq\; h_{rs}(\rho_\vartheta) \;>\; h_{rs}(\rho_\vartheta)D \;+\; \frac{1}{r-1}(1-D). \tag{2.16}$$

Here

$$I_{rs,\vartheta} = \frac{1}{r-1}\left[1 - \left\{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s\left(\frac{1}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s}\right)^{\frac{1}{r}} D^{-\frac{m_t(r-1)}{s}}\right\}^r\right],$$

and

$$h_{rs}(\rho_\vartheta) = \frac{1}{r-1}\left\{1 - \frac{\sum_{t=1}^{M}(\vartheta_t\rho_t^r)^s}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s}\right\}, \qquad r > 0,\; s > 0.$$

Proof: Suppose

$$\overline{m_t} = -\log_D\left\{\frac{(\vartheta_t\rho_t^r)^s}{\sum_{t=1}^{M}(\vartheta_t\rho_t^r)^s}\right\}. \tag{2.17}$$

It can be clearly seen that equation (2.9) can be satisfied by $\overline{m_t}$ and $\overline{m_t}+1$ with the help of Hölder's inequality. Let us assume m_t is the only integer between $\overline{m_t}$ and $\overline{m_t}+1$, then we have the following result:

$$\begin{aligned}
&\left[\sum_{t=1}^{M}(\vartheta_t\rho_t)^s\left(\frac{1}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s}\right)^{\frac{1}{r}} D^{-m_t(r-1)/r}\right]^r \\
&\leq \left[\sum_{t=1}^{M}(\vartheta_t\rho_t)^s\left(\frac{1}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s}\right)^{\frac{1}{r}} D^{-\overline{m_t}(r-1)/r}\right]^r \\
&< D\left[\sum_{t=1}^{M}(\vartheta_t\rho_t)^s\left(\frac{1}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s}\right)^{\frac{1}{r}} D^{-\overline{m_t}(r-1)/r}\right]^r .
\end{aligned} \tag{2.18}$$

Hence, since

$$\left[\sum_{t=1}^{M}(\vartheta_t\rho_t)^s\left(\frac{1}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s}\right)^{\frac{1}{r}} D^{-\overline{m_t}(r-1)/r}\right]^r = \frac{\sum_{t=1}^{M}(\vartheta_t\rho_t^r)^s}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s},$$

equation (2.18) turns into

$$\left[\sum_{t=1}^{M}(\vartheta_t\rho_t)^s\left(\frac{1}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s}\right)^{\frac{1}{r}} D^{-m_t(r-1)/r}\right]^r \leq \frac{\sum_{t=1}^{M}(\vartheta_t\rho_t^r)^s}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s} < D\,\frac{\sum_{t=1}^{M}(\vartheta_t\rho_t^r)^s}{\sum_{t=1}^{M}(\vartheta_t\rho_t)^s}.$$

The outcome is equation (2.16).

□

3. Conclusion

The key objective of encoding is to maximize the number of messages transferred in a certain stretch of time and minimize the data loss during the transmissions. To deal with this, we present generalized noiseless coding theorems based on the generalized mean codeword length and an inaccuracy measure that we established. These expansions provide modelling flexibility for source distributions, particularly when standard entropy measurements are insufficient. Our approach includes adjustable parameters, allowing for more precise control over codeword allocation based on probability irregularities. The theorems provided keep crucial characteristics that include different decodability and prefix-freeness while improving mean codeword constraints. The generalized entropy-based approach promotes the accuracy of coding schemes, which makes them suitable for applications in real-life data compression for safe digital communication.

4. Future Research Endeavours

Experts provide information based on their understanding of the system to assist in resolving the decision-making dilemma. In addition to the study mentioned above, algorithms for use in decision-making can also be investigated in parallel with mathematical studies. It is also possible to illustrate and compare alternative information estimations in the context of the noiseless theorem and the best 1:1 code. Using the noiseless coding theorem, R-Norm entropy metrics are described and used.

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