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Impact of Socio-Economic Drivers on Environmental Complexity Through Chaotic Synchronization

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ABSTRACT: In the field of environmental research, investigating the complexity of interactions between environmental systems and socio-economic drivers, such as financial market fluctuations, requires a variety of sophisticated scientific methodologies. This study aimed to examine the synchronization and anti-synchronization phenomena between two distinct dissipative systems using an active control method, which will offer a prospect in modelling complex environmental interactions. The first chaotic system, introduced by Huang and Li (1993), and the second system, proposed by P. Kumar and S. Jha (2022), are analyzed in depth. By employing phase portraits and Poincaré sections across a range of parameters and initial conditions, we confirm the chaotic nature of both systems. Subsequently, a set of active control laws is designed and implemented to control the intended dynamical phenomena. To validate the theoretical results, comprehensive numerical simulations are conducted, which highlight the effectiveness of the proposed control scheme. This research highlights the relevance of coordination strategies of dissipative systems and managing the complexity inherent in environmental systems influenced by socio-economic dynamics.

Key Words: Attractor, demand, Lyapunov exponent, proliferation, supply, virus.

Contents

1. Introduction

With rapid industrial growth,human civilization faces challenging concerns about the irregularity of the financial system and environmental issues. The irregularities are induced by the dissipative behaviour of the financial systems and the herculean task of managing the financial data. Normally, research in the financial area uses a variety of sophisticated scientific approaches to investigate the complexity of the financial system and study the dynamics of the financial market [1,2,3,4]. Over the years, researchers have focused on the relation between dynamics and time data [5,6,7]. Chen et al. [8] used the Hankel matrix approach and discussed dynamical systems which identified the time series data [9]. Based on noise observation data, Lu et al. reconstructed the dynamic system in 2003 [10], using the least squares method. Time parameter identification and dynamic system synchronization were topics covered by Yu et al. in 2007 [11]. In addition, Liu et al. (2013) rebuilt time series data [12] using a recursive graph of the power system.

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The existing literature has a significant gap on the internal framework of financial data as a dissipative dynamical system [13] and the market index has not been examined for chaotic dynamics at various time scales. In this research work we examine the nonlinear qualities of the model by Huang-Li [13] and follow it by using an active control synchronization scheme to synchronize this finance model with an environmental model. Chaos is a very intriguing phenomenon in nature. Chaos was initially discovered by Lorenz [14] in the context of atmospheric convection, and since then it has found use in a number of scientific domains, both theoretically and practically. As a result, over the past five decades, intensive research by researchers has led to the creation of new chaotic models for applications. Some of the well-known models are the logistic and Henon maps [15], Chua's circuits [16], the Lorenz-like systems developed by Chen and Lu et. al. in [17,18] , and more contemporary ones include the $4D$ chaotic Duffing system [19].

The long-term repercussions of economic actions cannot be predicted due to the great sensitivity of chaotic systems to changes in the initial conditions. Therefore, the crucial question is whether there is a set of parameter values for which the dynamics is regular and the system under consideration is integrable to enhance accurate economic predictions.

Recently, synchronization of chaotic systems has become a blooming area of research. In most of the chaos synchronization approaches, the systems considered as master and slave systems, and synchronization aims to use the dynamics of the master system for controlling the slave system as the difference of their outputs decays converges to zero asymptotically. Chaotic systems naturally resist synchronization, even two identical systems that begin with slightly dissimilar initial conditions evolve over time in an unsynchronized fashion, which means that the disparities between the systems' state's increase exponentially. As experimental initial circumstances can be practically challenging, therefore, this is an application orientation issue. To achieve any sort of collective (synchronized) behaviour is a highly consequential and intriguing problem. In the $17th$ century, when Huygen observed that two pendulum clocks which were very weakly connected (suspended from the same beam) happened to be synchronized in a phase [20]. Early discoveries also include the synchronized lighting of fireflies and the peculiarity of those neighbouring organ pipes, that can virtually speak in absolute synchrony or quiet one another, as in [21].

In the last three decades, we have seen a great deal of research in this direction. Numerous researchers have worked on this problem with focus on either complete or identical synchronization. Complete synchronization described by [22], appears to be the elementary type of synchronization method. These revolutionary studies encourage the search for synchronization phenomena in different artificial or natural systems. Despite decades of research, forecasting climate change continues to pose a significant challenge, largely owing to the human-induced variability in CO_2 emissions. Key indicators of atmospheric climate change include variations in temperature, barometric pressure, humidity levels, wind speed, and the concentration of trace gases. Current literature has largely neglected the role of environmental microbes—key players in carbon and nutrient dynamics—in relation to changing climate conditions. The cause of variability in climate conditions by some financial aspects was recently explored by [23]. In their investigation, the financial condition of society is also a factor in the environment's temperature shift.

In this paper, we focus on the variation in climate conditions due to these financial aspects. We have used a model illustrating the dissipating behavior in the environment [24], and have synchronizing it with a financial model.Thus we have tried to investigate the intertwining relation between the financial conditions and the environmental conditions. We employ the active control strategy developed by [25] to synchronize and anti-synchronize our models, which consist of two dissimilar dissipative systems that are not identical. Using the synchronization approach and numerical simulations, we have demonstrated synchronization of environmental aspects with the financial aspects.

2. Mathematical Model

Recent advances in research have focused on the Huang–Li system, which serves as a representative model for chaotic dynamics in financial systems. It is given by the following system of equations:

$$\frac{dg_1}{dt} = i_1 + (h_1 - a)\, g_1,$$
$$\frac{dh_1}{dt} = 1 - b\, h_1 - g_1^2, \tag{2.1}$$
$$\frac{di_1}{dt} = -g_1 - c\, i_1.$$

where (g_1, h_1, i_1) are time-dependent variables which were initialize as $g_1(0) = g_{1_0}, h_1(0) = h_{1_0}, i_1(0) = i_{1_0}$, and (a, b, c) are real nonnegative parameters. Here, g_1 represents the interest rate, h_1 is the investment demand,and z_1 is the price index. Parameters (a, b, c) denote the saving amount, the cost per investment and the elasticity demand of commercial markets, respectively. The complex behavior of system [2.1] was first noted by Ma and Chen in 2001 [26].

We now consider a dynamical system proposed by Pardeep et al. [24], which captures the complex interactions between microorganisms and the atmosphere, specifically the interplay between viral and bacterial populations within a suitable environmental context. This system is mathematically formulated as a set of three coupled ordinary differential equations:

$$\frac{dg_2}{dt} = -\alpha\, g_2 + h_2,$$
$$\frac{dh_2}{dt} = \beta\, h_2 - g_2 i_2, \tag{2.2}$$
$$\frac{di_2}{dt} = \alpha\, g_2 h_2 - \gamma\, i_2,$$

where (g_2, h_2, i_2) are time-dependent variables and (α, β, γ) are real nonnegative parameters. Here, α represents the rate of variation in temperature, β is the rate of variation in population of bacteria,and γ is the rate of variation in virus population, respectively. Also, the temperature and the population of bacteria and viruses are initialized as $g_2(0) = g_{2_0}, h_2(0) = h_{2_0}, i_2(0) = i_{2_0}$.

3. Chaotic Dynamics

Lyapunov exponents quantify the chaotic behavior of dynamical systems, as it is a measure of sensitivity to tiny changes in initial circumstances can be used to define chaotic behaviour and estimated by using separation (d_0) initially, as follows:

$$\lambda = \lim_{n\to\infty} \left(\frac{1}{n\Delta t} \sum_{i=1}^{n} \ln \left| \frac{d_i}{d_0)} \right| \right). \tag{3.1}$$

Where di is the distance between neighbouring points at step i [27]. Only the maximal Lyapunov exponent (MLE) will be explored in this work, as "$MLE > 0$ implies chaotic behaviour".

The Lyapunov spectrum $(0.0918, 0.0133, -1.1107)$ corresponding to the parameter values $a = 0.1$, $b = 0.3$, and $c = 1.5$ unequivocally demonstrates that System (2.1) exhibits chaotic dynamics, as depicted in Figure 2. In addition Figures (3-5) depicts the projections of financial model in $g_1 i_1$-plane, $h_1 i_1$-plane and $g_1 h_1$-plane.

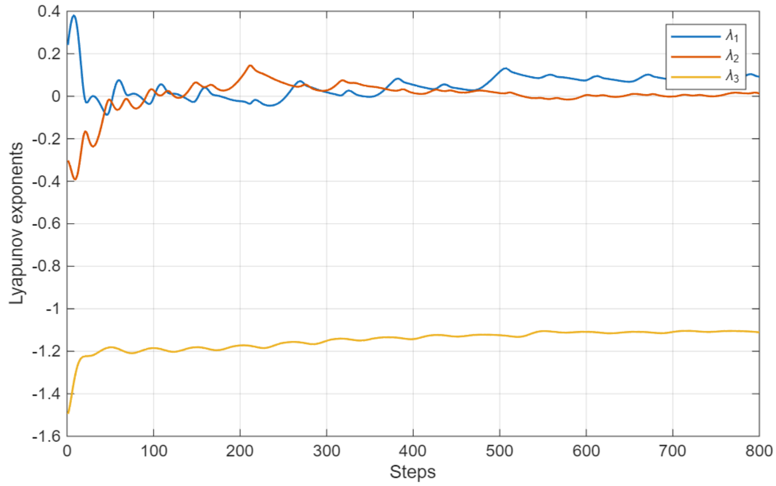

Figure 1: Lyapunov spectrum for system (2.1).

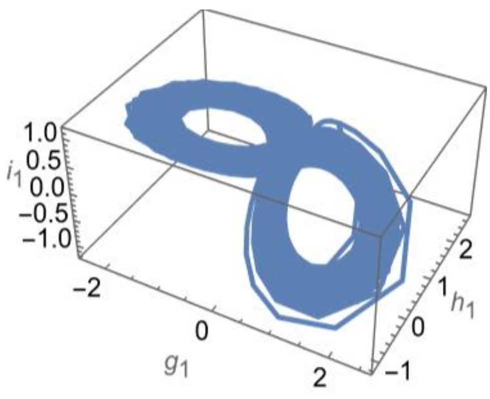

Figure 2: Chaotic Attractor of Financial Model when $g_{1_0} = 1, h_{2_0} = 11, i_{1_0} = 11$

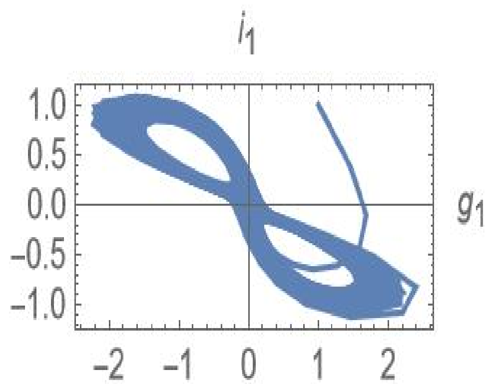

Figure 3: Projection of Figure 2 in $g_1 i_1$-plane

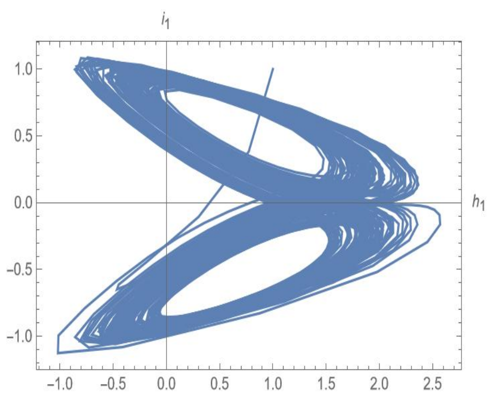

Figure 4: Projection of Figure 2 in $h_1 i_1$-plane

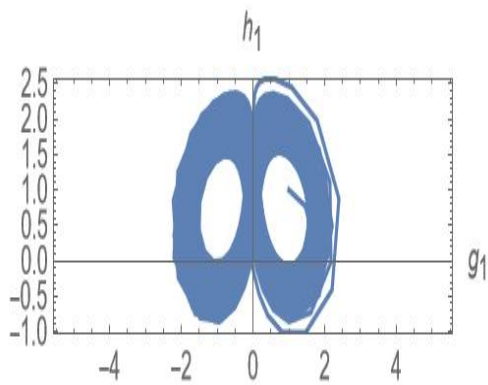

Figure 5: Projection of Figure 2 in $g_1 h_1$-plane

The positive Lyapunov exponent (0.1453) obtained for System (2.2) with $\alpha = 36$, $\beta = 13$, and $\gamma = 4$ indicates chaotic behavior, which is corroborated by the corresponding phase portrait shown in Figure (7). In addition Figures (8-10) depicts the projections of tumor in $g_2 i_2$-plane, $h_2 i_2$-plane and $g_2 h_2$-plane.

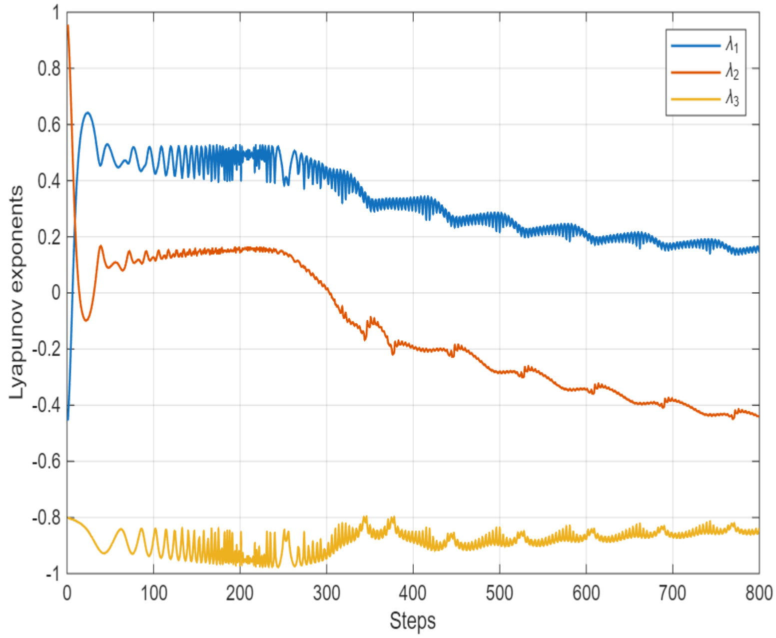

Figure 6: Lyapunov spectrum for system (2.2).

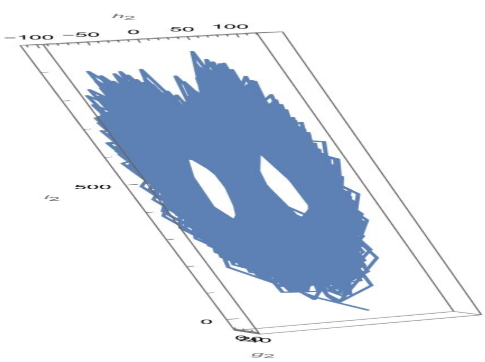

Figure 7: Chaotic Attractor of Environmental Model when $g_{2_0} = 20, h_{2_0} = 10, i_{1_0} = 10$

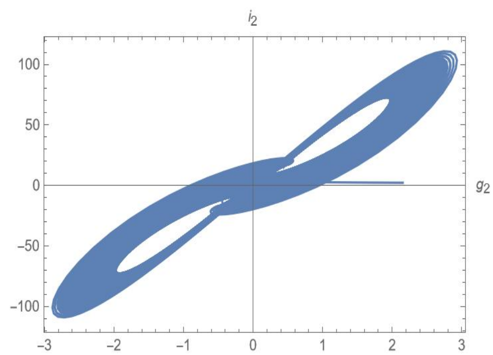

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Figure 8: Projection of Figure 7 in g_2i_2-plane

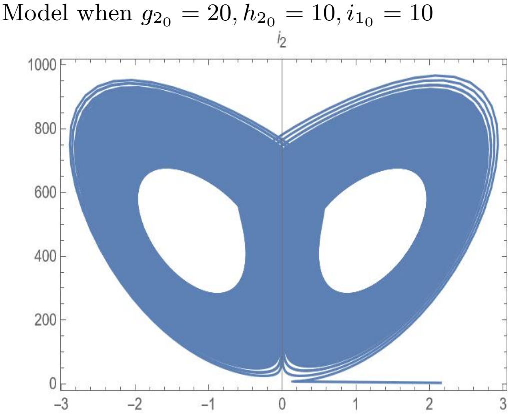

Figure 9: Projection of Figure 7 in h_2–i_2 plane.

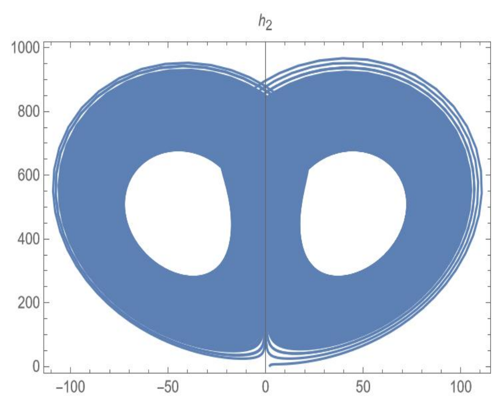

Figure 10: Projection of Figure 7 in g_2–h_2 plane.

4. Designing of Controller to achieve Synchronization

Describing drive and the corresponding response systems as follows:

$$\begin{cases} \dot{g_1} = i_1 + (h_1 - a)g_1, \\ \dot{h_1} = 1 - b\, h_1 - g_1^2, \\ \dot{i_1} = -\, g_1 - c\, i_1, \end{cases} \tag{4.1}$$

$$\begin{cases} \dot{g_2} = -\alpha\, g_2 + h_2 + v_1(\mathrm{t}), \\ \dot{h_2} = \beta\, h_2 - g_2 i_2 + v_2(\mathrm{t}), \\ \dot{i_2} = \alpha\, g_2 h_2 - \gamma\, i_2 + v_3(\mathrm{t}), \end{cases} \tag{4.2}$$

where, $v_1(\mathrm{t}), v_2(\mathrm{t})$ and $v_3(\mathrm{t})$ are control functions to be determined. The error dynamical system corresponding to (4.1) and (4.2) is described as

$$\begin{cases} e_1 = g_2 - g_1, \\ e_2 = h_2 - h_1, \\ e_3 = i_2 - i_1. \end{cases} \tag{4.3}$$

This research work's decisive purpose is to develop active nonlinear control functions $v_i(t)$, $(i = 1, 2, 3)$ such that the state error variable set out in (4.3) assure that

$$\lim_{t\to\infty} e_i(t) = 0 \quad \text{for} \quad (i = 1, 2, 3)$$

Using (4.2) and (4.1),the emerging error dynamical System is

$$\begin{cases} \dot{e}_1 = -\alpha\, g_2 + h_2 - i_1 - (h_1 - a)g_1 + v_1(t), \\ \dot{e}_2 = \beta\, h_2 - g_2 i_2 - 1 + b\, h_1 + g_1^2 + v_2(t), \\ \dot{e}_3 = \alpha\, g_2 h_2 - \gamma\, i_2 + g_1 + c\, i_1 + v_3(t). \end{cases} \tag{4.4}$$

Active control functions $v_1(\mathrm{t}), v_2(\mathrm{t})$ and $v_3(\mathrm{t})$ are defined as follows:

$$\begin{cases} v_1(\mathrm{t}) = w_1(\mathrm{t}) - h_2 + i_1 + g_1 h_1 - a\, g_1 + \alpha\, g_1 \\ v_2(\mathrm{t}) = w_2(\mathrm{t}) + g_2 i_2 + 1 - \beta\, h_1 - g_1^2 - b h_1 \\ v_3(\mathrm{t}) = w_3(\mathrm{t}) - \alpha\, g_2 h_2 + \gamma\, i_1 - g_1 - c\, i_1. \end{cases} \tag{4.5}$$

where $w_1(\mathrm{t}), w_2(\mathrm{t})$ and $w_3(\mathrm{t})$ are control inputs ,which are a function of error state variables e_1, e_2 and e_3, to be determined.

Using System (4.5), and (4.4), we have

$$\begin{cases} \dot{e}_1 = -\alpha\, e_1 + w_1(\mathrm{t}), \\ \dot{e}_2 = \beta\, e_2 + w_2(\mathrm{t}), \\ \dot{e}_3 = -\gamma\, e_3 + w_3(\mathrm{t}). \end{cases} \tag{4.6}$$

Stabilization of the system (4.6) through the appropriately designed control inputs $w_1(t)$, $w_2(t)$, and $w_3(t)$ ensures that the error states e_1, e_2, and e_3 asymptotically converge to zero as $t \to \infty$. This asymptotic convergence directly implies that Systems (2.1) and (2.2) achieve complete synchronization. To realize this objective, we define the control inputs as follows:

$$\begin{bmatrix} w_1(\mathrm{t}) \\ w_2(\mathrm{t}) \\ w_3(\mathrm{t}) \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}. \tag{4.7}$$

where A is 3×3 constant matrix. For System (10) to exhibit asymptotic stability, it is required that all eigenvalues of its system matrix possess strictly negative real parts. To achieve this, let us consider the Lyapunov function V as:

$$V(t) = \frac{1}{2}\left[e_1^2 + e_2^2 + e_3^2\right],$$

which implies that V is positive definite. On differentiating the Lyapunov function V, we can get

$$\dot{V}(t) = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3. \tag{4.8}$$

Using (4.6) and (4.7) in (4.8), we can get

$$\dot{V}(t) = -2\alpha\, e_1^2 - \beta\;\, e_2^2 - 2\,\gamma\, e_3^2 < 0 \tag{4.9}$$

Using (4.9), matrix A can be determined as:

$$A = \begin{bmatrix} -2\,\alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & -2\,\gamma \end{bmatrix}. \tag{4.10}$$

Then, by using (4.10) the error dynamical System (4.4) can converge to origin asymptotically that is $\lim_{t\to\infty} ||e(t)|| = 0$, which implies that the synchronization between Systems (2.1) and (2.2) is achieved.

5. Numerical Simulations for Synchronization

The numerical simulations were performed using the software Mathematica. To perform the investigation, the parameters of the drive System (4.1) and response System (4.2) are taken as $a = 0.1, b = 0.3, c = 1.5$, and $\alpha = 36, \beta = 14, \gamma = 5$. Initially, $g_1(0) = 1, h_1(0) = 1, i_1(0) = 1$ and $g_2(0) = 50, h_2(0) = 10, i_2(0) = 10$. The initial states of the error system (4.3) are $e_1(0) = 49, e_2(0) = 9$, and $e_3(0) = 9$. Figures (11), (12), and (13) depicts the synchronization of the systems (4.1) and (4.2). Figure (14), exhibits the synchronization of error trajectories e_1, e_2 and e_3 converge to zero.

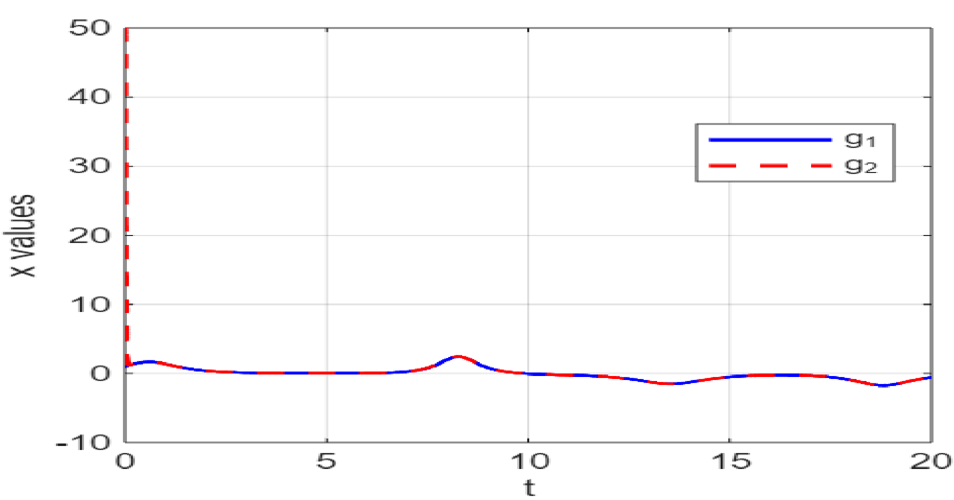

Figure 11: Synchronization of g_1 and g_2

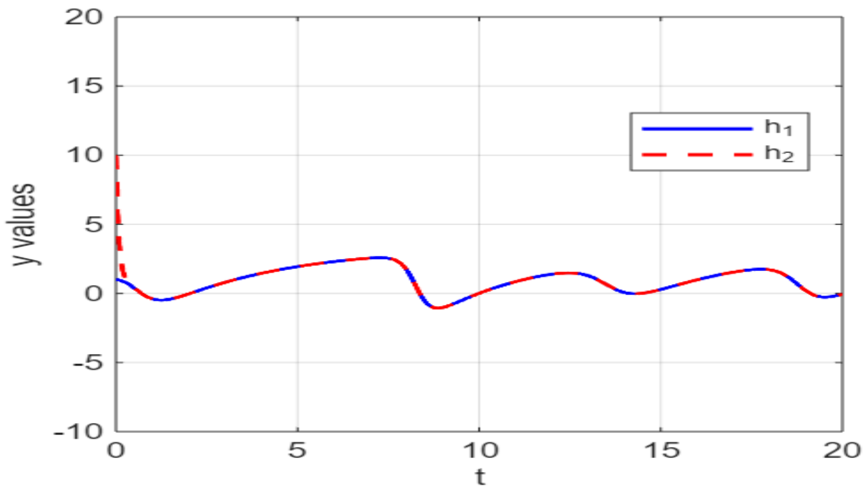

Figure 12: Synchronization of h_1 and h_2

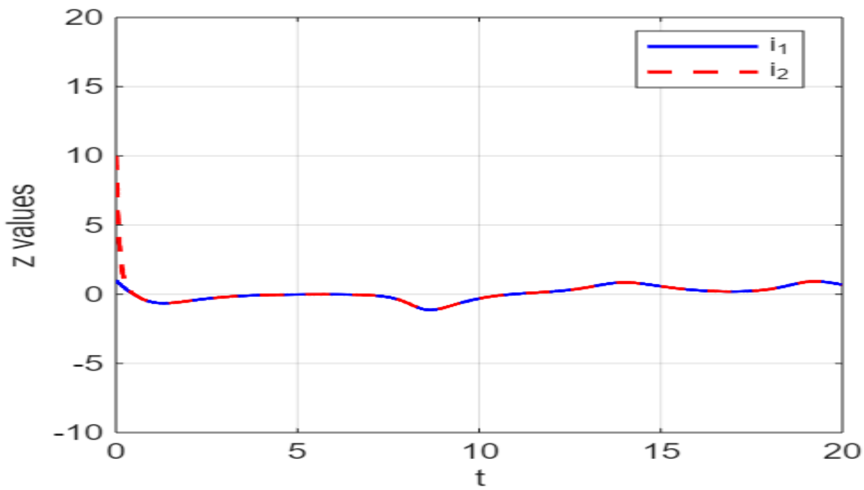

Figure 13: Synchronization of i_1 and i_2

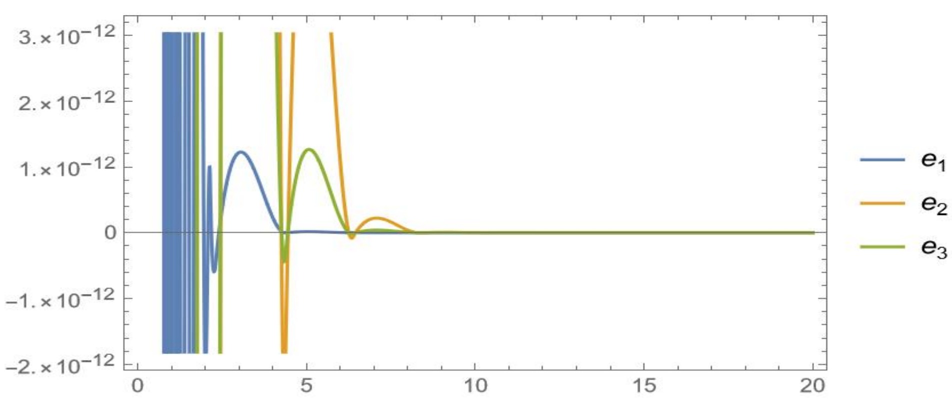

Figure 14: Dynamics of Synchronization of error states (e_1, e_2, e_3) for Systems (2.1) and (2.2)

6. Anti-synchronization between the Systems (2.1) and (2.2)

Let the drive and the corresponding response system be as follows:

$$\begin{cases} \dot{g_1} = i_1 + (h_1 - a)g_1, \\ \dot{h_1} = 1 - b\, h_1 - g_1^2, \\ \dot{i_1} = -\, g_1 - c\, i_1, \end{cases} \tag{6.1}$$

$$\begin{cases} \dot{g_2} = -\alpha\, g_2 + h_2 + p_1(\mathrm{t}), \\ \dot{h_2} = \beta\, h_2 - g_2 i_2 + p_2(\mathrm{t}), \\ \dot{i_2} = \alpha\, g_2 h_2 - \gamma\, i_2 + p_3(\mathrm{t}), \end{cases} \tag{6.2}$$

where, $p_1(\mathrm{t}), p_2(\mathrm{t})$ and $p_3(\mathrm{t})$ are control functions to be determined. The state error variables between the Systems (6.1) and (6.2) are described as

$$\begin{cases} E_1 = g_2 + g_1, \\ E_2 = h_2 + h_1, \\ E_3 = i_2 + i_1. \end{cases} \tag{6.3}$$

Now we will develop active nonlinear control functions $p_i(t)$, $(i = 1, 2, 3)$ such that the state error variable set out in (6.3) assure that

$$\lim_{t\to\infty} E_i(t) = 0 \quad \text{for} \quad (i = 1, 2, 3)$$

Adding System (6.1) and System (6.2),the emerging error dynamical System corresponding to systems(6.1) and (6.2) is

$$\begin{cases} \dot{E}_1 = -\alpha\, g_2 + h_2 + i_1 + (h_1 - a)g_1 + p_1(t), \\ \dot{E}_2 = \beta\, h_2 - g_2 i_2 + 1 - b\, h_1 - g_1^2 + p_2(t), \\ \dot{E}_3 = \alpha\, g_2 h_2 - \gamma\, i_2 - g_1 - c\, i_1 + p_3(t). \end{cases} \tag{6.4}$$

Active control functions $p_1(\mathrm{t}), p_2(\mathrm{t})$ and $p_3(\mathrm{t})$ are defined as follows:

$$\begin{cases} p_1(\mathrm{t}) = q_1(\mathrm{t}) - h_2 - i_1 - g_1 h_1 + a\, g_1 - \alpha\, g_1 \\ p_2(\mathrm{t}) = q_2(\mathrm{t}) + g_2 i_2 - 1 + \beta\, h_1 + g_1^2 + b h_1 \\ p_3(\mathrm{t}) = q_3(\mathrm{t}) - \alpha\, g_2 h_2 - \gamma\, i_1 + g_1 + c\, i_1. \end{cases} \tag{6.5}$$

where $q_1(\mathrm{t}), q_2(\mathrm{t})$ and $q_3(\mathrm{t})$ are control inputs that are the function of error state variables E_1, E_2 and E_3 to be determined.

Using System (6.4), and (6.5), we have

$$\begin{cases} \dot{E}_1 = -\alpha\, E_1 + q_1(\mathrm{t}), \\ \dot{E}_2 = \beta\, E_2 + q_2(\mathrm{t}), \\ \dot{E}_3 = -\gamma\, E_3 + q_3(\mathrm{t}). \end{cases} \tag{6.6}$$

When the system (6.6) is stabilised by control inputs $q_1(\mathrm{t}), q_2(\mathrm{t})$ and $q_3(\mathrm{t})$ then, E_1, E_2 and E_3 will converge to zero as time $t \to \infty$, which implies that the Systems (6.1) and (6.2) are Anti-synchronized. To achieve this goal, we choose the control inputs as

$$\begin{bmatrix} q_1(\mathrm{t}) \\ q_2(\mathrm{t}) \\ q_3(\mathrm{t}) \end{bmatrix} = B \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}. \tag{6.7}$$

where B is 3×3 constant matrix. For System (6.7) to be asymptotically stable, its system characteristic matrix must have all its eigenvalues with negative real parts. To achieve this, we need to determine the matrix B. For this, let us consider the Lyapunov function W as:

$$W(t) = \frac{1}{2}\left[E_1^2 + E_2^2 + E_3^2\right],$$

which implies that W is positive definite. On differentiating the Lyapunov function W, we can get

$$\dot{W}(t) = E_1\dot{E}_1 + E_2\dot{E}_2 + E_3\dot{E}_3. \tag{6.8}$$

Using (6.6) and (6.7) in (6.8), we can get

$$\dot{W}(t) = -2\alpha\, E_1^2 - \beta\ \ E_2^2 - 2\,\gamma\, E_3^2 < 0 \tag{6.9}$$

Using (6.9), we can have B as:

$$B = \begin{bmatrix} -\alpha & 0 & 0 \\ 0 & -2\,\beta & 0 \\ 0 & 0 & -\gamma \end{bmatrix}. \tag{6.10}$$

Using (6.10), we observe that all the three eigenvalues of closed loop system (6.7) have a negative real part. Therefore, the error dynamical system (6.4) converges to origin asymptotically that is $\lim_{t\to\infty} ||E(t)|| = 0$, which implies that the anti-synchronization between systems (6.1) and (6.2) is achieved.

7. Numerical Simulations for Anti-synchronization

To perform the numerical investigation, the parameters of the drive System (6.1) and response System (6.2) are taken as $a = 0.1, b = 0.3, c = 1.5$, and $\alpha = 36, \beta = 14, \gamma = 5$ with initial conditions as $g_1(0) = 1, h_1(0) = 1, i_1(0) = 1$ and $g_2(0) = 50, h_2(0) = 10, i_2(0) = 10$. The initial states of the error system (6.3) are $E_1(0) = 49, E_2(0) = 9$, and $E_3(0) = 9$. Figures (15), (16), and (17) illustrate anti-synchronization between the systems (6.1) and (6.2). Figure 18, exhibits the anti-synchronization of error trajectories E_1, E_2 and E_3 converge to zero .

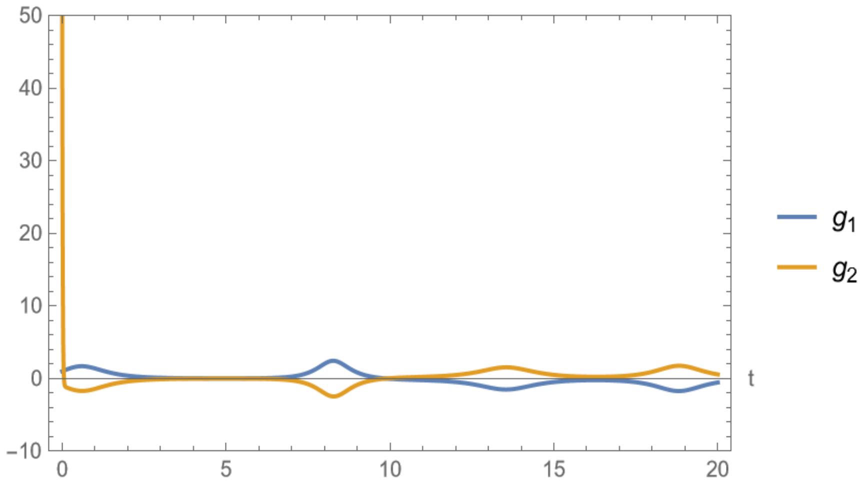

Figure 15: Anti-synchronization of g_1 and g_2

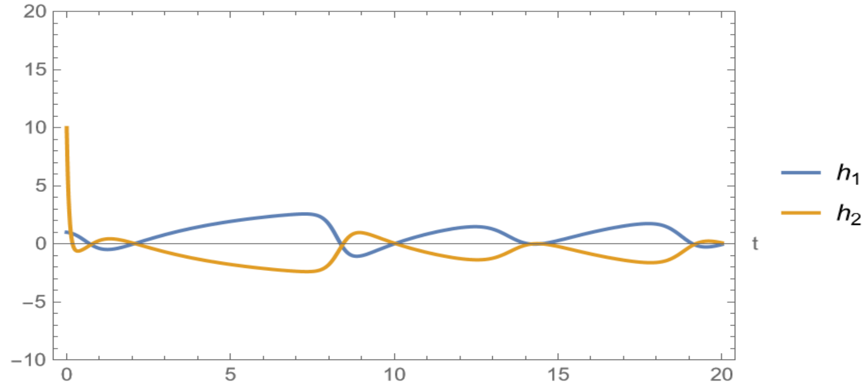

Figure 16: Anti-synchronization of h_1 and h_2

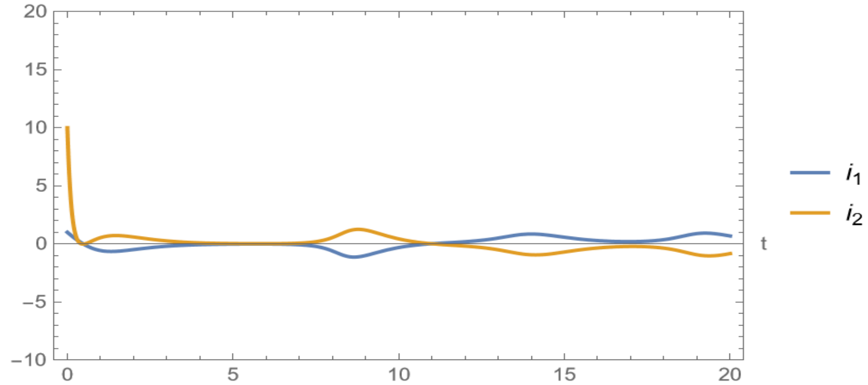

Figure 17: Anti-synchronization of i_1 and i_2

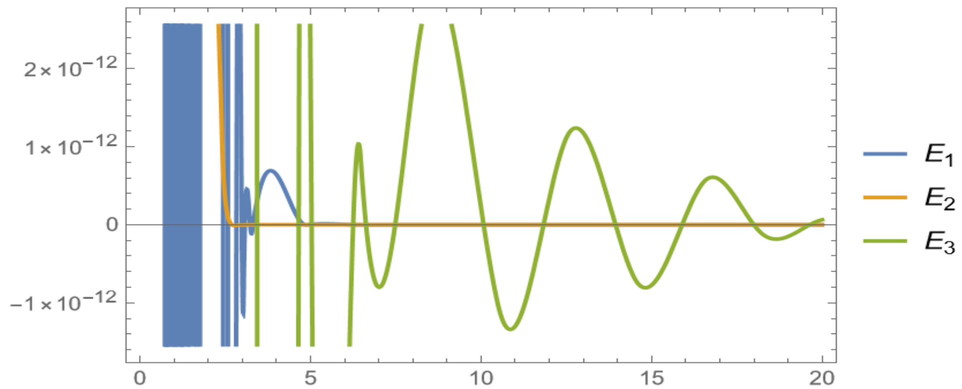

Figure 18: Dynamics of Anti-synchronization of error states (E_1, E_2, E_3) for Systems (2.1) and (2.2)

8. Conclusion

Synchronizing chaotic financial systems with chaotic environmental dynamics offers insights for holistic risk management and sustainable development. By studying how environmental factors impact financial markets and vice versa, we can develop more resilient investment strategies, mitigate systemic risks, and ensure long-term economic stability. Additionally, it promotes cross-sector collaboration, enhancing adaptive capacity to address emerging challenges such as carbon emissions, resource depletion, and ecosystem degradation. Our investigation demonstrates that some variation in financial conditions may control the temperature in the environment by synchronizing them. Also, it has been observed that financial constraints may regulate the population of microorganisms. The solution of anti-synchronization problem encourages the adoption of environmentally sustainable practices, incentivizes green innovation, and facilitates the transition to a low-carbon economy, and our analytical results are in excellent agreement with the numerical results.

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