Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2025 (43) 3** : 1–9.
ISSN-0037-8712
doi:10.5269/bspm.78437

Antisymmetric Electromagnetic Lorentz Force Tensor Formulation of Electrodynamics

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ABSTRACT: The usual Lorentz force $\mathbf{F} = q\left[\mathbf{E} + (\mathbf{v} \times \mathbf{B})\right]$ predicts an independent structure of electrodynamics in terms of electric force and magnetic force. As the electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ are the building blocks of the antisymmetric second-rank electromagnetic field tensor $F^{\mu\nu}$, correspondingly, the electric force $q\mathbf{E}$ and magnetic force $q(\mathbf{v} \times \mathbf{B})$ are the building blocks of the antisymmetric second-rank electromagnetic force tensor $f^{\mu\nu}$. In other words, the Lorentz force is a tensorial force by birth. A new type of electrodynamics emerges that possesses the same structure as standard Maxwell's theory, such that electrodynamical laws are now force laws. It obeys the principle of relativity, conservation law, and symmetry. We designate this model as *Lorentz Force Electrodynamics (LFE)*. The matrix method is employed in tensor notation. The model consists of force field equations, Lorentz force Maxwell's equations, conservation law, and 4D waves of electric force and magnetic force. Similarly, the dual of LFE is worked out, and the conservation law is valid in both cases.a consequence of this model, we have developed antisymmetric mechanical Lorentz force tensor and gravitational Lorentz force tensor. This model is expected to facilitate electrical, electronic, mechanical, and space engineers, as well as theoretical physicists, in the study of astrophysics, cosmology, and magnetohydrodynamics.

Key Words:: Lorentz Force Tensor, Force Maxwell's Equations, Conservation law, Electric Force Wave Equation, Magnetic force Wave Equation.

Contents

1. Introduction

Electric force and magnetic force are empirically established concepts [1-3]. Any physical theory based on established facts will naturally appeal to everyone working in the field. The non-relativistic relation for the Lorentz force

$$\mathbf{F} = q\left[\mathbf{E} + (\mathbf{v} \times \mathbf{B})\right]$$

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Submitted August 15, 2025. Published October 02, 2025
2010 *Mathematics Subject Classification:* 05C90, 05C07.

in the contemporary literature has not been recognized as it possesses an independent relativistic framework of force electrodynamics. It represents an antisymmetric second-rank tensor $f^{\mu\nu}$ whose components are the electric force $q\mathbf{E}$ and magnetic force $q(\mathbf{v}\times\mathbf{B})$. We designate this model as *Lorentz Force Electrodynamics (LFE).*

The Maxwellian electromagnetic field tensor $F^{\mu\nu}$ represents partially the force field $q\mathbf{E}$ and partially field theory, as $q\mathbf{B}$ is not a magnetic force, though the theory has its own importance. The Lorentz force governs the motion of an electric charge under the influence of electric force and magnetic force. It is one of the most applied concepts in electrodynamics of rotating frames, particle accelerators, isotope separation, the Large Hadron Collider (LHC), magnetohydrodynamics, and spacecraft propulsion [4-10].

After the work of J. C. Maxwell, Hendrik Lorentz introduced the Lorentz force or electromagnetic force in 1895. Their contribution to the development of electromagnetic theory is a great gift for humanity. All the communication systems and power generation in our modern world are due to these legends, including Gauss, Ampère, and Faraday. In this paper, we shall develop the structure of LFE exactly like Maxwellian electrodynamics. This framework differs from Maxwell in the case of the magnetic field, where B is replaced by $q(\mathbf{v}\times\mathbf{B})$. At the moment, we present the LFE field components, Lorentz force Maxwell's equations, conservation law, and wave equations for electric force and magnetic force. The dual of LFE, denoted by ${}^*f^{\mu\nu}$, differs from the usual technique (where $E \to B$ and $B \to -E$). In the case of ${}^*f^{\mu\nu}$, the electric force qE goes to the magnetic force $q(\mathbf{v}\times\mathbf{B})$ and $q(\mathbf{v}\times\mathbf{B})$ goes to $-qE$, otherwise the wave equations cannot be derived in terms of electric force and magnetic force. The dual of electrodynamics obeys the conservation law. The electric force wave equation is obtained by taking the time derivative of Ampere's law with a minus sign on both sides and utilizing the curl of Faraday's law. The magnetic force wave equation is derived by taking the curl of Ampere's law on both sides and by using the curl of Faraday's law. The time-varying spatial force density $-f^{i\nu}_{,\nu 0}$ and the curl of spatial force density $\nabla\times f^{i\nu}_{,\nu}$ play the role of sources for the 4D wave of electric force and magnetic force, respectively. In usual electrodynamics, the time-varying current density $-J^{i}_{,0}$ and the curl of current density $\nabla\times J$ are the sources of the 4D wave of the electric field and magnetic field. This model is also valid for Lorentz force density $\rho E + (J\times B) = \rho\big[E + (\mathbf{v}\times\mathbf{B})\big]$ such that ρE and $\rho(\mathbf{v}\times\mathbf{B})$ become the components of the Lorentz force density tensor.Furthermore, we have obtained antisymmetric mechanical Lorentz force tensor and gravitational Lorentz force tensor. They possess the same mathematical framework but physically different. We shall present their formulation in a separate paper. Here they will be presented as consequences with basic formulae only. Since the model is entirely new, we have compared the results with standard Maxwell's theory of electrodynamics in Table 1. Notations in this model are adopted according to the modern approach of relativity. Greek alphabets $\mu,\nu,\alpha,\beta,\ldots$ run from 0 to 3 and Latin letters $i,j,k,\ldots$ run from 1 to 3. A comma (,) denotes partial differentiation, e.g. $E_{,0}=\frac{\partial E}{\partial t}$ represents the partial derivative of the electric field with respect to time, and $E_{,1}=\frac{\partial E}{\partial x}$ represents the partial derivative of the electric field with respect to the x-axis. The notation $F^{\mu\nu}{}_{,\nu}$ means the spacetime divergence of the electromagnetic force tensor. The 4-dimensional coordinates are defined as

$$x^{\mu} = (x^0, x^1, x^2, x^3) = (ct, x, y, z) = (ct, x^i),$$

with $x^0 = ct$ and $x^i = (x, y, z)$. The time component ct is a scalar, while the space components x^i form a vector, such that x^{μ} is the unification of time and space. The dimensions of all components are those of length.

2. Development of Lorentz Force Field Electrodynamics

The Lorentz force tensor $f^{\mu\nu}$ is related to its components, electric force $q\mathbf{E}$ and magnetic force $q(\mathbf{v}\times\mathbf{B})$, as follows:

$$f^{0i} = qE^i, \qquad f^{ij} = q\,\epsilon^{ijk}(\mathbf{v}\times\mathbf{B})_k. \tag{2.1}$$

2.1. Lorentz Force Field Tensor in Matrix Form

$$f^{\mu\nu} = q \begin{bmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & (\mathbf{v}\times\mathbf{B})^3 & -(\mathbf{v}\times\mathbf{B})^2 \\ -E^2 & -(\mathbf{v}\times\mathbf{B})^3 & 0 & (\mathbf{v}\times\mathbf{B})^1 \\ -E^3 & (\mathbf{v}\times\mathbf{B})^2 & -(\mathbf{v}\times\mathbf{B})^1 & 0 \end{bmatrix} \tag{2.2}$$

which may be expressed as

$$\left(f^{0\nu}, f^{i\nu}\right) = q\Big(E^i,\ \ [(\mathbf{v}\times\mathbf{B})^3 - (\mathbf{v}\times\mathbf{B})^2 - E^1] + [(\mathbf{v}\times\mathbf{B})^1 - (\mathbf{v}\times\mathbf{B})^3 - E^2] + [(\mathbf{v}\times\mathbf{B})^2 - (\mathbf{v}\times\mathbf{B})^1 - E^3]\Big). \tag{2.3}$$

Electric Force Field:

$$f^{0i} = qE^i \tag{2.4}$$

Tensorial Magnetic Force Field:

$$f^{i\nu} = q(\mathbf{v}\times\mathbf{B}) - q\big[\mathbf{E} + (\mathbf{v}\times\mathbf{B})\big] \tag{2.5}$$

Lorentz Force Field Tensor:

$$f^{\mu\nu} = q\big[\mathbf{E} + (\mathbf{v}\times\mathbf{B})\big] - q\big[\mathbf{E} + (\mathbf{v}\times\mathbf{B})\big] = 0 \tag{2.6}$$

Thus, the Lorentz force field tensor remains antisymmetric due to the opposite sign of the Lorentz force.

2.2. Maxwell's Force Equations in Tensor Components Form

The tensor form of Maxwell's force equations can be written as

$$f^{\mu\nu}{}_{,\nu} = \begin{bmatrix} f^{0\nu}{}_{,\nu} \\ f^{1\nu}{}_{,\nu} \\ f^{2\nu}{}_{,\nu} \\ f^{3\nu}{}_{,\nu} \end{bmatrix} = q \begin{bmatrix} 0 & E^1{}_{,1} & E^2{}_{,2} & E^3{}_{,3} \\ -E^1{}_{,0} & 0 & (\mathbf{v}\times\mathbf{B})^3{}_{,2} & -(\mathbf{v}\times\mathbf{B})^2{}_{,3} \\ -E^2{}_{,0} & -(\mathbf{v}\times\mathbf{B})^3{}_{,1} & 0 & (\mathbf{v}\times\mathbf{B})^1{}_{,3} \\ -E^3{}_{,0} & (\mathbf{v}\times\mathbf{B})^2{}_{,1} & -(\mathbf{v}\times\mathbf{B})^1{}_{,2} & 0 \end{bmatrix}. \tag{2.7}$$

Equivalently,

$$\left(f^{0\nu}{}_{,\nu}, f^{i\nu}{}_{,\nu}\right) = q\Big(\nabla\cdot\mathbf{E},\ [\nabla\times(\mathbf{v}\times\mathbf{B})] - \mathbf{E}_{,0}\Big).$$

Gaussian Force Law (divergence of electric force):

$$f^{0i}{}_{,i} = \nabla\cdot(q\mathbf{E}) \tag{2.8}$$

where the term $f^{0i}{}_{,i}$ plays the role of charge density in Maxwellian electrodynamics.
Amperian Force Law:

$$f^{i\nu}{}_{,\nu} = q\big[\nabla\times(\mathbf{v}\times\mathbf{B})\big] - \frac{\partial}{\partial t}(q\mathbf{E}) \tag{2.9}$$

In usual notation, Ampere's law becomes

$$q\big[\nabla\times(\mathbf{v}\times\mathbf{B})\big] = f^{i\nu}{}_{,\nu} + \frac{\partial}{\partial t}(q\mathbf{E}), \tag{2.10}$$

where $f^{i\nu}{}_{,\nu}$ represents force current density.

Thus, Maxwell's force equations in unified tensor form are

$$f^{\mu\nu}{}_{,\nu} = \nabla\cdot(q\mathbf{E}) + q\Big[\nabla\times(\mathbf{v}\times\mathbf{B}) - \frac{\partial}{\partial t}(\mathbf{E})\Big]. \tag{2.11}$$

2.3. Electromagnetic Force Conservation Law in Tensor Components

The conservation form is written as

$$\begin{bmatrix} f^{0\nu}{}_{,\nu 0} \\ f^{1\nu}{}_{,\nu 1} \\ f^{2\nu}{}_{,\nu 2} \\ f^{3\nu}{}_{,\nu 3} \end{bmatrix} = q \begin{bmatrix} 0 & E^1{}_{,10} & E^2{}_{,20} & E^3{}_{,30} \\ -E^1{}_{,01} & 0 & (\mathbf{v}\times\mathbf{B})^3{}_{,21} & -(\mathbf{v}\times\mathbf{B})^2{}_{,31} \\ -E^2{}_{,02} & -(\mathbf{v}\times\mathbf{B})^1{}_{,32} & 0 & -(\mathbf{v}\times\mathbf{B})^3{}_{,12} \\ -E^3{}_{,03} & (\mathbf{v}\times\mathbf{B})^2{}_{,13} & -(\mathbf{v}\times\mathbf{B})^1{}_{,23} & 0 \end{bmatrix}. \tag{2.12}$$

Or equivalently,

$$\left(f^{0\nu}{}_{,\nu 0}, f^{i\nu}{}_{,\nu i}\right) = q\Big(\nabla\cdot\mathbf{E}_{,0},\ \nabla\cdot\big[\nabla\times(\mathbf{v}\times\mathbf{B}) - \mathbf{E}_{,0}\big]\Big).$$

Conservation of Gauss's Force Law:

$$f^{0\nu}{}_{,\nu 0} = \nabla\cdot(q\mathbf{E})_{,0} \tag{2.13}$$

which represents the divergence of electric Yank (rate of change of electric force).
Conservation of Ampere's Force Law:

$$f^{i\nu}{}_{,\nu i} = q\Big[\nabla\cdot(\nabla\times\mathbf{v}\times\mathbf{B}) - \mathbf{E}_{,0}\Big] \tag{2.14}$$

Total Conservation of Lorentz Force Field Tensor:

$$f^{\mu\nu}{}_{,\nu\mu} = \nabla\cdot(q\mathbf{E})_{,0} + \Big[\nabla\cdot q(\nabla\times\mathbf{v}\times\mathbf{B}) - \nabla\cdot(q\mathbf{E})_{,0}\Big] = 0 \tag{2.15}$$

In tensor component form, the continuity or conservation law can be expressed as

$$\nabla\cdot f^{i\nu}{}_{,\nu} + \frac{\partial}{\partial t} f^{0\nu}{}_{,\nu} = 0, \tag{13a}$$

which states that the divergence of Amperian force current density plus the rate of change of Gaussian force density is equal to zero.

2.4. Dual of Lorentz Force Field

The dual transformation of the Lorentz force components is given by

$$qE^i \longrightarrow q(\mathbf{v}\times\mathbf{B})^i, \qquad q(\mathbf{v}\times\mathbf{B})^i \longrightarrow -qE^i. \tag{2.16}$$

The dual Lorentz force field tensor can be expressed as

$$[*f^{\mu\nu}] = q\begin{bmatrix} 0 & (v\times B)^1 & (v\times B)^2 & (v\times B)^3 \\ -(v\times B)^1 & 0 & -E^3 & E^2 \\ -(v\times B)^2 & E^3 & 0 & -E^1 \\ -(v\times B)^3 & -E^2 & E^1 & 0 \end{bmatrix}.$$

The dual of the electric force field is equal to the magnetic force,

$$*f^{0i} = q(\mathbf{v}\times\mathbf{B}), \tag{2.17}$$

while the dual of the magnetic force field is obtained as the difference of electric force and Lorentz force,

$$*f^{i\nu} = qE - q\big[E + (\mathbf{v}\times\mathbf{B})\big]. \tag{2.18}$$

Thus, the dual of the Lorentz force field tensor becomes

$$*f^{\mu\nu} = q\big[E + (\mathbf{v}\times\mathbf{B})\big] - q\big[E + (\mathbf{v}\times\mathbf{B})\big] = 0. \tag{2.19}$$

2.5. Dual of Lorentz Force Maxwell's Equations

The divergence form of the dual tensor reads

$$[*f^{\mu\nu}{}_{,\nu}] = q \begin{bmatrix} 0 & (v\times B)^1{}_{,1} & (v\times B)^2{}_{,2} & (v\times B)^3{}_{,3} \\ -(v\times B)^1{}_{,0} & 0 & -E^3{}_{,2} & E^2{}_{,3} \\ -(v\times B)^2{}_{,0} & E^3{}_{,1} & 0 & -E^1{}_{,3} \\ -(v\times B)^3{}_{,0} & -E^2{}_{,1} & E^1{}_{,2} & 0 \end{bmatrix}.$$

From this representation, the Gaussian force law for magnetism is written as

$$*f^{0i}{}_{,i} = \nabla\cdot q(\mathbf{v}\times\mathbf{B}), \tag{2.20}$$

and Faraday's force law as

$$*f^{i\nu}{}_{,\nu} = -\Big[(\nabla\times qE) + \frac{\partial}{\partial t}q(\mathbf{v}\times\mathbf{B})\Big]. \tag{2.21}$$

Faraday's law implies that the curl of the electric force is equal to the negative of the time-varying magnetic force, where $*f^{i\nu}{}_{,\nu}$ represents the source of magnetic current in terms of the dual spatial force density. Therefore, Faraday's law can be expressed as

$$\nabla\times qE = -*f^{i\nu}{}_{,\nu} - \frac{\partial}{\partial t}q(\mathbf{v}\times\mathbf{B}). \tag{2.22}$$

Equations (18) and (19a) thus represent the existence of magnetic sources in terms of dual force density and dual force current density.

2.6. Dual of Lorentz Force Conservation Law

The dual conservation law matrix takes the form

$$[*f^{\mu\nu}{}_{,\mu}] = q \begin{bmatrix} 0 & (v\times B)^1{}_{,10} & (v\times B)^2{}_{,20} & (v\times B)^3{}_{,30} \\ -(v\times B)^1{}_{,01} & 0 & -E^3{}_{,21} & E^2{}_{,31} \\ -(v\times B)^2{}_{,02} & E^3{}_{,12} & 0 & -E^1{}_{,32} \\ -(v\times B)^3{}_{,03} & -E^2{}_{,13} & E^1{}_{,23} & 0 \end{bmatrix}.$$

The corresponding dual conservation law is expressed as

$$*f^{\mu\nu}{}_{,\nu\mu} = \frac{\partial}{\partial t}\big(\nabla\cdot q(\mathbf{v}\times\mathbf{B})\big) - \nabla\cdot(\nabla\times qE) - \frac{\partial}{\partial t}\big[\nabla\cdot q(\mathbf{v}\times\mathbf{B})\big] = 0. \tag{2.23}$$

In tensor component form, the dual continuity equation becomes

$$\nabla\cdot(*f^{i\nu}{}_{,\nu}) + \frac{\partial}{\partial t}(*f^{0\nu}{}_{,\nu}) = 0.$$

2.7. Wave Equations for Electric Force and Magnetic Force

We derived the electric force wave equation by utilizing the negative time derivative of Ampere's law (8) and the curl of Faraday's law (19), and by substituting the value of $\frac{\partial}{\partial t}(\nabla\times q\mathbf{E})$ into the curl of (8). For simplicity, we present them in source-free form.

Electric Force Wave Equation

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) q\mathbf{E} = 0 \quad \text{or} \quad \Box^2 q\mathbf{E} = 0 \tag{2.24}$$

We derived the magnetic force wave equation by taking the curl of Ampere's law (8) and the time derivative of Faraday's law (19), and by substituting the value of $\frac{\partial}{\partial t}(\nabla\times(q\mathbf{v}\times\mathbf{B}))$ into the curl of (19).

Magnetic Force Wave Equation

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\big[q(\mathbf{v}\times\mathbf{B})\big] = 0 \quad \text{or} \quad \Box^2\big[q(\mathbf{v}\times\mathbf{B})\big] = 0 \tag{2.25}$$

d'Alembertian Operator

$$\Box^2 = \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) \tag{2.26}$$

Here, $\Box^2$ is called the four-dimensional wave operator or the *d'Alembertian operator*.

3. Consequences

3.1. Existence of Antisymmetric Mechanical Lorentz Force Tensor

$$f^{\mu\nu} = \begin{bmatrix} 0 & F^1 & F^2 & F^3 \\ -F^1 & 0 & (P\times\omega)^3 & -(P\times\omega)^2 \\ -F^2 & -(P\times\omega)^3 & 0 & (P\times\omega)^1 \\ -F^3 & (P\times\omega)^2 & -(P\times\omega)^1 & 0 \end{bmatrix} \tag{3.1}$$

$$f^{0i} = F^i, \qquad f^{ij} = \epsilon^{ijk}(P\times\omega)_k \tag{3.2}$$

The nonrelativistic mechanical Lorentz force f_{MLF} consists of Newton's second law of motion $F = ma$ and the Coriolis force $(P\times\omega)$, where P is linear momentum and ω is angular velocity.

$$f_{\text{MLF}} = F + (P\times\omega) \tag{3.3}$$

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3.2. Existence of Antisymmetric Gravitational Lorentz Force Tensor

$$G^{\mu\nu} = m\begin{bmatrix} 0 & g^1 & g^2 & g^3 \\ -g^1 & 0 & (g_t\times\omega)^3 & -(g_t\times\omega)^2 \\ -g^2 & -(g_t\times\omega)^3 & 0 & (g_t\times\omega)^1 \\ -g^3 & (g_t\times\omega)^2 & -(g_t\times\omega)^1 & 0 \end{bmatrix} \tag{3.4}$$

$$G^{0i} = mg^i, \qquad G^{ij} = m\epsilon^{ijk}(g_t\times\omega)_k \tag{3.5}$$

The nonrelativistic gravitational Lorentz force f_{GLF} consists of Newton's second law for gravitation $F = mg$ and the gravitational Coriolis force $m(g_t\times\omega)$, where mg_t represents gravitational linear momentum and ω is angular velocity.

$$f_{\text{GLF}} = m\big(g + (g_t\times\omega)\big) \tag{3.6}$$

4. Discussion and Comparison

One of the most exciting outcomes of our research is the realization that the Lorentz force $q\big[\mathbf{E}+(\mathbf{v}\times\mathbf{B})\big]$ possesses the complete structure of electrodynamics, where it naturally behaves as an antisymmetric force tensor.

In classical Maxwellian electrodynamics, $q\mathbf{E}$ represents the electric force; however, $q\mathbf{B}$ does not correspond to a true magnetic force. In contrast, within the Lorentz Force Electrodynamics (LFE) model, both $q\mathbf{E}$ and $q(\mathbf{v}\times\mathbf{B})$ provide a consistent and unified definition of the electric and magnetic forces.

Thus, Lorentz Force Electrodynamics establishes a fundamentally new framework, offering a reformulation of electrodynamics. In the following, we compare our model with the standard Maxwellian electrodynamics.

Table 1: Comparison of Lorentz Force Electrodynamics and Maxwellian Electrodynamics

Lorentz Force Electrodynamics	**Maxwellian Electrodynamics**
Structure of Lorentz Force Field Tensor $f^{\mu\nu}$ $$q\begin{bmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & (v\times B)^3 & -(v\times B)^2 \\ -E^2 & -(v\times B)^3 & 0 & (v\times B)^1 \\ -E^3 & (v\times B)^2 & -(v\times B)^1 & 0 \end{bmatrix}$$ $$f^{0i} = qE^i, \quad f^{ij} = q\epsilon^{ijk}(v\times B)_k$$	**Structure of Electromagnetic Field Tensor** $f^{\mu\nu}$ $$q\begin{bmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & B^3 & -B^2 \\ -E^2 & -B^3 & 0 & B^1 \\ -E^3 & B^2 & -B^1 & 0 \end{bmatrix}$$ $$F^{0i} = E^i, \quad F^{ij} = \epsilon^{ijk}B_k$$
Electric Force Field: $f^{0i} = qE^i$ **Magnetic Force Field:** $f^{i\nu} = -qE^i + q\big[(v\times B) - (v\times B)\big]$ **Lorentz Force Field Tensor:** $f^{\mu\nu} = q[E + (v\times B)] - q[E + (v\times B)] = 0$	**Electric Field:** $F^{0i} = E^i$ **Magnetic Field:** $F^{i\nu} = -E^i + \big[B - B\big]$ **Electromagnetic Field Tensor:** $F^{\mu\nu} = [E + B] - [E + B] = 0$
Lorentz Force Maxwell's Equations: Gauss's Law: $f^{0i}{}_{,i} = \nabla\cdot(qE)$ Ampère's Law: $f^{i\nu}{}_{,\nu} = q\big[\nabla\times(v\times B) - E_{,0}\big]$ Gauss's Law for Magnetism: ${}^*f^{0i}{}_{,i} = \nabla\cdot q(v\times B)$ Faraday's Law: ${}^*f^{i\nu}{}_{,\nu} = -(\nabla\times qE) - \frac{\partial}{\partial t}\big[q(v\times B)\big]$ Tensor Form: $f^{\mu\nu}{}_{,\nu} = \nabla\cdot qE + q[(\nabla\times v\times B) - E_{,0}]$	**Maxwell's Equations:** Gauss's Law: $F^{0\nu}{}_{,\nu} = \nabla\cdot E$ Ampère's Law: $F^{i\nu}{}_{,\nu} = (\nabla\times B) - E_{,0}$ Gauss's Law for Magnetism: ${}^*F^{0\nu}{}_{,\nu} = \nabla\cdot B$ Faraday's Law: ${}^*F^{i\nu}{}_{,\nu} = -[(\nabla\times E) + B_{,0}]$ Tensor Form: $F^{\mu\nu}{}_{,\nu} = \nabla\cdot E + [(\nabla\times B) - E_{,0}]$

Conservation Law: $$f^{\mu\nu}{}_{,\nu\mu} = (\nabla\cdot qE)_{,0} + \big[\nabla\cdot q(\nabla\times v\times B) - (qE)_{,0}\big] = 0$$	**Conservation Law:** $$F^{\mu\nu}{}_{,\nu\mu} = (\nabla\cdot E)_{,0} + \nabla\cdot[(\nabla\times B) - E_{,0}] = 0$$
Electric Force Wave Equation: $$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)(qE) = 0$$ **Magnetic Force Wave Equation:** $$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[q(v\times B)] = 0$$ **Poynting Force Vector:** $S = q^2[E\times(v\times B)]$	**Electric Field Wave Equation:** $$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)E = 0$$ **Magnetic Field Wave Equation:** $$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)B = 0$$ **Poynting Vector:** $S = E\times B$

5. Lorentz Force Maxwell's Equations

The Lorentz force Maxwell's equations can be reformulated in terms of electric and magnetic forces, providing a symmetric and consistent framework for electrodynamics. Gauss's law appears as the divergence of the electric force, expressed as

$$\nabla\cdot q\mathbf{E} = f^{0i}_{,i} \tag{5.1}$$

Ampere's law emerges as the curl of the magnetic force together with the curl of the time-varying electric force, written as

$$\nabla \times q(\mathbf{v} \times \mathbf{B}) = f^{i\nu}_{,\nu} + \frac{\partial q\mathbf{E}}{\partial t} \tag{5.2}$$

Similarly, Gauss's law for magnetism is expressed as the divergence of the magnetic force,

$$\nabla \cdot q(\mathbf{v} \times \mathbf{B}) = {}^*f^{0i}_{,i} \tag{5.3}$$

Faraday's law appears as the curl of the electric force together with the contribution of the time-varying magnetic force,

$$\nabla \times q\mathbf{E} = -{}^*f^{i\nu}_{,\nu} - \frac{\partial}{\partial t}[q(\mathbf{v} \times \mathbf{B})] \tag{5.4}$$

From these relations, the electric force wave equation naturally arises as

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) q\mathbf{E} = 0 \tag{5.5}$$

while the magnetic force wave equation follows as

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) [q(\mathbf{v} \times \mathbf{B})] = 0 \tag{5.6}$$

Adding the wave equations (28) and (29), we obtain the 4D wave equation for the Lorentz force:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) q[\mathbf{E} + \mathbf{v} \times \mathbf{B}] = 0. \tag{5.7}$$

Note that Eqs. (5.1)–(5.4) represent the electromagnetic force laws, from which a remarkable symmetry of Maxwell's equations emerges. In light of Faraday's law, one may recall his picture of magnetic lines of force. The present model also clarifies the distinction between the Lorentz force law and Ampère's force law.

The form invariance of this model can be exhibited using the relativistic framework discussed in [11–15], where the concepts underlying the electrodynamic laws are developed step by step.

6. Conclusion

Our model of Lorentz force field electrodynamics constitutes a significant breakthrough in the field of electrodynamics after the pioneering contributions of Maxwell and Lorentz. It adheres to the principles of relativity, conservation laws, and symmetry. In this framework, the Lorentz force is formulated as an antisymmetric tensor force field, where the electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ of Maxwellian electrodynamics are replaced by the electric force $q\mathbf{E}$ and the magnetic force $q(\mathbf{v} \times \mathbf{B})$. Consequently, electromagnetic laws are interpreted as force laws, and the conservation laws correspond to the conservation of forces. Furthermore, the classical wave equations are reformulated as electric force waves and magnetic force waves. These models provide powerful tools to study the motion of particles and celestial bodies in terms of electromagnetic forces. Applications of this approach extend naturally to electrical, electronic, mechanical, and space engineering, offering new perspectives in astrophysics, cosmology, and magnetohydrodynamics. Our subsequent research will focus on Lorentz force electrodynamics in noninertial coordinate metrics, based on a single transformation law for 4-vectors. This framework predicts a 7D wave of electrical energy within the expression of the conservation law.

Dedicated to H. A. Lorentz, J. C. Maxwell, C. F. Gauss, A. M. Ampere and Michael Faraday

Declaration: The authors declare no conflict of interest regarding the publication of this paper.

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