



Certain Properties of Bi-Starlikeness and Bi-Convexity Functions Associated with Horadam Polynomial Defined through (p, q) -Calculus

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ABSTRACT: In this research work, we explore the field of univalent analytic and bi-univalent functions, focusing on subclasses connected to Horadam polynomials. We introduce more generalized class that includes both bi-starlike and bi-convex functions. Our main objectives are to establish precise coefficient bounds and to derive Fekete-Szegő inequalities for functions in the newly defined class. Additionally, we present detailed observations and special cases to demonstrate the breadth and significance of our findings.

Keywords: Analytic function, Bi-univalent function, Bi-starlike function, coefficient estimates, Fekete-Szegő inequality, Horadam polynomial.

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1. Introduction and Motivations

Quantum calculus, also known as q -analysis, expands traditional calculus by removing the need for limits. Jackson's work [13] demonstrated the application and utility of q -calculus, highlighting its foundational concepts such as q -derivative and q -integral, which were presented in earlier studies. The versatility of the q -derivative has proven to be highly valuable in the field of geometric function theory.

In addition to q -calculus, researchers have explored (p, q) -calculus. The (p, q) -number, which was initially examined around 1991, has been the subject of detailed study. Applications include the investigation of Fibonacci oscillators, the development of a (p, q) -harmonic oscillator, the integration of different q -oscillator algebras, and the determination of (p, q) -Stirling numbers.

Since 1991, and building upon earlier research, the (p, q) -calculus has gained significant attention in various fields. The works presented in ([3], [6], [14], [27]) introduced a method for incorporating q -series into the framework of (p, q) -series. Further, exploration of (p, q) -hypergeometric series produced results that parallel the (p, q) -extensions of established q -identities. Notably, (p, q) -series can be derived through corresponding extensions of q -identities.

The concept of q -starlike functions was first explored by Ismail et al. [12], serving as a q -extension that sparked significant research into q -calculus within geometric function theory. For example, the q -analogue of the Ruscheweyh differential operator was defined, and its applications to multivalent functions were investigated. Further studies examined q -starlike functions in relation to generalized conic domains through convolution, as well as operators associated with the q -hypergeometric function. Recent publications continue to delve into various aspects of q -starlike functions.

Researchers have investigated the geometric properties of q -analytic functions, including the close-to-convexity of q -Mittag-Leffler functions ([24]) and coefficient inequalities for q -starlike and q -convex functions ([20], [21], [26]). Moreover, q -differential operators have been utilized to define new classes of analytic functions ([1], [16], [22]).

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2020 *Mathematics Subject Classification*: 30C45, 30C50, 30C55.

Submitted August 17, 2025. Published March 12, 2026

Inspired by the emerging field of (p,q) -analysis, which generalizes q -analysis, this paper aims to extend the concepts of q -starlikeness and q -convexity to (p,q) -bi-starlikeness and (p,q) -bi-convexity. Our primary objective is to establish Fekete-Szegő inequalities for these new classes.

2. Preliminaries

Let \mathcal{A} represent the class of functions \mathcal{F} defined by

$$\mathcal{F}(\chi) = \chi + \sum_{n=2}^{\infty} a_n \chi^n \quad (2.1)$$

and analytic within the open unit disk $\mathbb{U} = \{\chi \in \mathbb{C} : |\chi| < 1\}$. We denote the subclass of \mathcal{A} that consists of normalized analytic functions exhibiting univalence in \mathbb{U} by \mathcal{S} . It is a well-established fact that every function \mathcal{F} in \mathcal{S} has an inverse, denoted \mathcal{F}^{-1} , is known as Koebe one quarter theorem(see [7]) defined by the relations

$$\mathcal{F}^{-1}(\mathcal{F}(\chi)) = \chi \quad (\chi \in \mathbb{U})$$

and

$$\mathcal{F}(\mathcal{F}^{-1}(\xi)) = \xi \quad (|\xi| < r_0(\mathcal{F}); r_0(\mathcal{F}) \geq \frac{1}{4}).$$

Specifically, this inverse function $g = \mathcal{F}^{-1}$, can be expressed as follows:

$$g(\xi) = \mathcal{F}^{-1}(\xi) = \xi - a_2 \xi^2 + (2a_2^2 - a_3) \xi^3 - (5a_2^3 - 5a_2 a_3 + a_4) \xi^4 + \dots \quad (2.2)$$

This can also be represented as:

$$g(\xi) := \xi + \sum_{n=2}^{\infty} A_n \xi^n. \quad (2.3)$$

A function \mathcal{F} is regarded as bi-univalent in the open unit disk \mathbb{U} if it, along with its inverse \mathcal{F}^{-1} , demonstrates univalency within \mathbb{U} . The collection of these noteworthy functions is referred to as Σ .

The concept of analytic bi-univalent functions was initially introduced and explored by Lewin [15], who proved that if $\mathcal{F} \in \Sigma$ then $|a_2| < 1.51$. Brannan and Clunie [5] later refined this result, demonstrating that $|a_2| \leq \sqrt{2}$, and Netanyahu [19] established that $|a_2| \leq \frac{4}{3}$. Further investigations by Brannan and Taha [4], as well as Taha [25] alone, examined specific subclasses of bi-univalent functions, yielding non-sharp estimates for the first two Taylor-Maclaurin coefficients, $|a_2|$ and $|a_3|$.

For a concise history and notable examples of functions within the class Σ , refer to Srivastava et. al [23]. Their work significantly rekindled interest in the study of various subclasses of the bi-univalent function class Σ , prompting subsequent research by Frasin and Aouf [8], Hayami and Owa [9] and others.

It is worthy to mention that determining bounds for the general coefficient $|a_n|$ for $n > 3$ remains a challenge. This difficulty arises because the condition of bi-univalence adds unpredictability to the behavior of the coefficients of both the function \mathcal{F} and its inverse \mathcal{F}^{-1} . In this paper, we utilize Horadam polynomial expansions within the framework of a general subclass of analytic bi-univalent functions to derive estimates for the bounds.

Recent mathematical studies have focused on coefficient estimates for univalent and bi-univalent functions, particularly related to Horadam polynomials. This work examines estimates for the initial coefficients in the Taylor-Maclaurin series and explores Fekete-Szegő inequalities in bi-univalent functions defined by these polynomials.

Horadam polynomials are important because they include various polynomial sequences such as Lucas, Pell-Lucas, Fibonacci, Pell, and Chebyshev polynomials. This study will highlight the connections to previous research findings.

Bi-univalent functions are of significant interest in complex analysis and geometric function theory due to their applications in fields like mathematical physics and engineering. The Bieberbach conjecture, which proposed inequalities for the coefficients of bi-univalent functions, was proven by Louis de Branges in 1985, greatly impacting complex analysis.

Here are few examples that are member of Σ

$$\frac{\chi}{1-\chi}, \quad \frac{1}{2} \log \left(\frac{1+\chi}{1-\chi} \right), \quad \text{and} \quad -\log(1-\chi)$$

with the corresponding inverse functions

$$\frac{\xi}{1+\xi}, \quad \frac{e^{2\xi}-1}{e^{2\xi}+1}, \quad \text{and} \quad \frac{e^\xi-1}{e^\xi},$$

respectively. But the functions

$$\chi - \frac{\chi^2}{2}, \quad , \quad \frac{\chi}{1-\chi^2} \quad \text{and} \quad \frac{\chi}{(1-\chi)^2}$$

are not the member of the class Σ . For two analytic functions \mathcal{F} and g , if \mathcal{F} is subordinate to g and it can be written as $\mathcal{F} \prec g$ if there exists an analytic function β with $|\beta(\chi)| < 1$ and $\beta(0) = 0$ such that $\mathcal{F}(\chi) = g(\beta(\chi))$. It is well known fact that if g is univalent in \mathbb{U} then $\mathcal{F} \prec g \iff \mathcal{F}(0) = g(0)$ and $\mathcal{F}(\mathbb{U}) \subset g(\mathbb{U})$. To facilitate our discussion, we present some fundamental definitions related to (p, q) -calculus. In particular, the derivative of a function using (p, q) -calculus of a function $h(\chi) = \chi^m$ is given below:

$$D_{p,q}h(\chi) = [m]_{p,q}\chi^{m-1},$$

where $[m]_{p,q}$ denotes the twin number defined as:

$$[m]_{p,q} = \frac{p^m - q^m}{p - q}, \quad \text{where} \quad (0 < q < p \leq 1), \quad (2.4)$$

which serves as a generalization of the q -number (refer to 2.4), defined as:

$$[m]_q = \frac{1 - q^m}{1 - q}, \quad (q \neq 1).$$

Since, here we see that $[n]_{p,q}$ goes to n , when $p = 1$ and q goes to 1^- , therefore $D_{p,q}h(\chi)$ goes to $h'(\chi)$ as when $p = 1$ and q goes to 1^- , where $h'(\chi)$ known as the normal derivative of the function $h(\chi)$ w.r.t χ .

Remark 1. Using (p, q) -calculus the derivative of a function of the form (2.1) is defined as

$$D_{p,q}\mathcal{F}(\chi) = \frac{\mathcal{F}(p\chi) - \mathcal{F}(q\chi)}{(p-q)\chi} \quad \text{where} \quad \chi \neq 0; \quad 0 < q < p \leq 1;$$

Here $D_{p,q}\mathcal{F}(\chi)$ goes to $\mathcal{F}'(\chi)$ whenever $p = 1$ and $q \rightarrow 1^-$. Here \mathcal{F}' is called as normal derivative of the function \mathcal{F} .

For a function \mathcal{F} is in form of (2.1), we have

$$D_{p,q}\mathcal{F}(\chi) = 1 + \sum_{m=2}^{\infty} [m]_{p,q} a_m \chi^{m-1}, \quad (0 < q < p \leq 1), \quad (2.5)$$

where $[m]_{p,q}$ is given by (2.4).

Definition 2.1. A function \mathcal{F} of the form (2.1) is called (p, q) -starlike if the function hold the below subordination condition:

$$\frac{\chi D_{p,q}\mathcal{F}(\chi)}{\mathcal{F}(\chi)} \prec \phi(\chi), \quad (0 < q < p \leq 1), \quad (2.6)$$

where ϕ is analytic in \mathbb{U} with $\Re(\phi(\chi)) > 0$, $\phi(0) = 1$, and $\phi'(0) > 0$.

Definition 2.2. A function \mathcal{F} of the form (2.1) is called (p, q) -convex if the function hold the below subordination condition:

$$\frac{D_{p,q}(\chi D_{p,q}\mathcal{F}(\chi))}{D_{p,q}\mathcal{F}(\chi)} \prec \phi(\chi), \quad (0 < q < p \leq 1), \quad (2.7)$$

where ϕ is provided as above in Definition 2.1.

Horadam Polynomial: Horadam polynomial denoted by $h_n(v, a, b, y_1, y_2)$ or in short $h_n(v)$ are formulated through the following recurrence relation (see [10], [11]):

$$h_n(v) = y_1 v h_{n-1}(v) + y_2 h_{n-2}(v) \quad (v \in \mathbb{R}, n \geq 3) \quad (2.8)$$

with

$$h_1(v) = a \quad \text{and} \quad h_2(v) = bv \quad (2.9)$$

where a, b, y_1, y_2 are real constants. The characteristic equation of recurrence relation (2.8) is

$$t^2 - y_1 vt - y_2 = 0. \quad (2.10)$$

This equation has two real roots:

$$\alpha = \frac{y_1 v + \sqrt{y_1^2 v^2 + 4y_2}}{2}, \quad \beta = \frac{y_1 v - \sqrt{y_1^2 v^2 + 4y_2}}{2}. \quad (2.11)$$

From (2.8),(2.9), we give the following table.

n	$h_n(v, a, b; y_1, y_2)$
1	a
2	bv
3	$by_1 v^2 + ay_2$
4	$by_1^2 v^3 + (ay_1 y_2 + by_2)v$
5	$by_1^3 v^4 + (ay_1^2 y_2 + 2by_1 y_2)v^2 + ay_2^2$
6	$by_1^4 v^5 + (ay_1^3 y_2 + 3by_1^2 y_2)v^3 + (2ay_1 y_2^2 + by_2^2)v$
7	$by_1^5 v^6 + (ay_1^4 y_2 + 4by_1^3 y_2)v^4 + (3ay_1^2 y_2^2 + 3by_1 y_2^2)v^2 + ay_2^3$
8	$by_1^6 v^7 + (ay_1^5 y_2 + 5by_1^4 y_2)v^5 + (4ay_1^3 y_2^2 + 6by_1^2 y_2^2)v^3 + (3ay_1 y_2^3 + by_2^3)v$
\vdots	\vdots

The generating function of the Horadam polynomials $h_n(v)$ is given by (see [11]):

$$Q(v, \chi) = \sum_{n=1}^{\infty} h_n(v) \chi^{n-1} = \frac{a + (b - ay_1)v\chi}{1 - y_1 v\chi - y_2 \chi^2}. \quad (2.12)$$

By assigning specific fixed values to $a, b, y_1,$ and $y_2,$ the Horadam polynomial $h_n(v)$ generates various classes of polynomials. Here are some examples of these resulting polynomials :

1. If $a = 2$ & $b = y_1 = y_2 = 1,$ we get the **Lucas polynomial** $L_n(v),$ (see [2]).
2. If $a = b = y_1 = y_2 = 1,$ we get the **Fibonacci polynomial** $F_n(v),$ (see [17]).
3. If $a = b = y_1 = 2$ & $y_2 = 1,$ we have **pell-Lucas polynomials** $Q_n(v),$ (see [10]).
4. If $a = y_2 = 1$ & $b = y_1 = 2,$ we arrive at Pell polynomial $P_n(v),$ (see [10]).
5. If $a = 1, b = y_1 = 2$ and $y_2 = -1,$ we found **Chebyshev polynomials** $U_n(v)$ **of the second kind,**(see [18]).
6. If $a = b = 1$ and $y_1 = 2, y_2 = -1,$ we found **Chebyshev polynomials** $T_n(v)$ **of the first kind,** (see [18]).

Taking into account aforementioned works and making use of generating function of Horadam polynomial we introduce a novel subclass of (p, q) -analogue of bi-starlikeness and bi-convexity functions associated with the principle of subordination.

Definition 2.3. A function $\mathcal{F} \in \mathcal{A}$ given by (2.1) is said to be in the class $U_{\Sigma}(k, v)$ if it satisfies the following subordination conditions.

$$\left[\frac{\chi D_{p,q} \mathcal{F}(\chi)}{\mathcal{F}(\chi)} \right]^k \left[\frac{D_{p,q}(\chi D_{p,q} \mathcal{F}(\chi))}{D_{p,q} \mathcal{F}(\chi)} \right]^{1-k} \prec Q(v, \chi) - a + 1$$

and

$$\left[\frac{\xi D_{p,q} g(\xi)}{g(\xi)} \right]^k \left[\frac{D_{p,q}(\xi D_{p,q} g(\xi))}{D_{p,q} g(\xi)} \right]^{1-k} \prec Q(v, \xi) - a + 1$$

where $k \in [0, 1], v \in \mathbb{R}$ and a is a real constant, the function $g = \mathcal{F}^{-1}$ is given in (2.2).

Special Case-I: Taking $k = 1$ and imposing the condition $0 < q < p \leq 1$, we say a function \mathcal{F} consider to be a member of the bi-starlike class, denoted by $BS_{\Sigma}(k, v)$ if it satisfies following subordination conditions:

$$\frac{\chi D_{p,q} \mathcal{F}(\chi)}{\mathcal{F}(\chi)} \prec Q(v, \chi) - a + 1$$

and

$$\frac{\xi D_{p,q} g(\xi)}{g(\xi)} \prec Q(v, \xi) - a + 1$$

where the function $g = \mathcal{F}^{-1}$ is defined by the equation (2.2).

Special Case-II: Letting $k = 0$ and imposing $0 < q < p \leq 1$ in the Definition 2.3, we say a function \mathcal{F} belongs to the family $U_{\Sigma}(k, v)$ is consider to be a member of the bi-convex class, denoted by $BC_{\Sigma}(k, v)$ if it satisfies conditions. These conditions can be expressed as follows:

$$\frac{D_{p,q}(\chi D_{p,q} \mathcal{F}(\chi))}{D_{p,q} \mathcal{F}(\chi)} \prec Q(v, \chi) - a + 1$$

and

$$\frac{D_{p,q}(\xi D_{p,q} g(\xi))}{D_{p,q} g(\xi)} \prec Q(v, \xi) - a + 1$$

where the function $g = \mathcal{F}^{-1}$ is given by (2.2).

3. Main Results

In this section unless otherwise stated we assume through our sequel that $0 < q < p \leq 1, v \in \mathbb{R}, k \in [0, 1], a, b \in \mathbb{R}$.

Theorem 3.1. If a function $\mathcal{F} \in \mathcal{A}$ is in the class $U_{\Sigma}(k, v)$, then

$$|a_2| \leq \frac{\sqrt{2}|bv|\sqrt{|bv|}}{\sqrt{b^2 v^2 \left[([2]_{p,q} - 1)^4 k^2 + (-[2]_{p,q}^4 + 6[2]_{p,q}^3 - 8[2]_{p,q}^2 - 2[3]_{p,q}^2 + 2[2]_{p,q} + 4[3]_{p,q} - 1) k - 2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right] - 2(pbv^2 + qa) ((k-1)[2]_{p,q} - k)^2 ([2]_{p,q} - 1)^2}},$$

$$|a_3| \leq \frac{|bv|}{(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} + \frac{b^2 v^2}{([2]_{p,q} k - k - [2]_{p,q})^2 ([2]_{p,q} - 1)^2}.$$

Proof. Let the function \mathcal{F} given by the Taylor-Maclaurin expansion expansion of the form (2.1) be in the function class $U_{\Sigma}(k, v)$. Then there exists two analytic functions R, T given by

$$R(\chi) = r_1\chi + r_2\chi^2 + r_3\chi^3 + \dots (\chi \in \mathbb{U}), \quad (3.1)$$

$$T(\xi) = t_1\xi + t_2\xi^2 + t_3\xi^3 + \dots (\xi \in \mathbb{U}) \quad (3.2)$$

with

$$R(0) = T(0) = 0, \quad |R(\chi)| < 1, \quad |T(\xi)| < 1, \quad (\chi, \xi \in \mathbb{U}),$$

such that

$$\left[\frac{\chi D_{p,q}\mathcal{F}(\chi)}{\mathcal{F}(\chi)} \right]^k \left[\frac{D_{p,q}(\chi D_{p,q}\mathcal{F}(\chi))}{D_{p,q}\mathcal{F}(\chi)} \right]^{1-k} = \Pi(\nu, R(\chi)) + 1 - a, \quad (3.3)$$

and

$$\left[\frac{\xi D_{p,q}g(\xi)}{g(\xi)} \right]^k \left[\frac{D_{p,q}(\xi D_{p,q}g(\xi))}{D_{p,q}g(\xi)} \right]^{1-k} = \Pi(\nu, T(\xi)) + 1 - a \quad (3.4)$$

Using (3.1) in (3.3) and (3.2) in (3.4), and after simplification we get

$$\left[\frac{\chi D_{p,q}\mathcal{F}(\chi)}{\mathcal{F}(\chi)} \right]^k \left[\frac{D_{p,q}(\chi D_{p,q}\mathcal{F}(\chi))}{D_{p,q}\mathcal{F}(\chi)} \right]^{1-k} = 1 + h_2(v)r_1\chi + [h_2(v)r_2 + h_3(v)r_1^2]\chi^2 + \dots \quad (3.5)$$

and

$$\left[\frac{\xi D_{p,q}g(\xi)}{g(\xi)} \right]^k \left[\frac{D_{p,q}(\xi D_{p,q}g(\xi))}{D_{p,q}g(\xi)} \right]^{1-k} = 1 + h_2(v)t_1\xi + [h_2(v)t_2 + h_3(v)t_1^2]\xi^2 + \dots \quad (3.6)$$

Now expanding the series of the function in the left hand side of the equations (3.3), (3.4) we get

$$\begin{aligned} & \left[\frac{\chi D_{p,q}\mathcal{F}(\chi)}{\mathcal{F}(\chi)} \right]^k \left[\frac{D_{p,q}(\chi D_{p,q}\mathcal{F}(\chi))}{D_{p,q}\mathcal{F}(\chi)} \right]^{1-k} = 1 - a_2 ([2]_{p,q}k - k - [2]_{p,q}) ([2]_{p,q} - 1) \chi \\ & + \left[\left(\frac{1}{2}k^2[2]_{p,q}^4 - 2[2]_{p,q}^2[2]_{p,q}^3 - \frac{1}{2}k[2]_{p,q}^4 + 3k^2[2]_{p,q}^2 + 3k[2]_{p,q}^3 - 4k[2]_{p,q}^2 - [2]_{p,q}^3 + [2]_{p,q}^2 - 2k^2[2]_{p,q} \right. \right. \\ & \left. \left. + k[2]_{p,q} + \frac{1}{2}k^2 + \frac{1}{2}k \right) a_2^2 + (-[3]_{p,q}^2k + 2[3]_{p,q}k + [3]_{p,q}^2 - k - [3]_{p,q}) a_3 \right] \chi^2 + \dots, \end{aligned} \quad (3.7)$$

and

$$\begin{aligned} & \left[\frac{\xi D_{p,q}g(\xi)}{g(\xi)} \right]^k \left[\frac{D_{p,q}(\xi D_{p,q}g(\xi))}{D_{p,q}g(\xi)} \right]^{1-k} = 1 - a_2 ([2]_{p,q}k - k - [2]_{p,q}) ([2]_{p,q} - 1) \xi \\ & + \left[\left(\frac{1}{2}k^2[2]_{p,q}^4 - 2[2]_{p,q}^2[2]_{p,q}^3 - \frac{1}{2}k[2]_{p,q}^4 + 3k^2[2]_{p,q}^2 + 3k[2]_{p,q}^3 - 4k[2]_{p,q}^2 - [2]_{p,q}^3 + [2]_{p,q}^2 - 2k^2[2]_{p,q} \right. \right. \\ & \left. \left. + k[2]_{p,q} + \frac{1}{2}k^2 + \frac{1}{2}k \right) a_2^2 + (-[3]_{p,q}^2k + 2[3]_{p,q}k + [3]_{p,q}^2 - k - [3]_{p,q}) a_3 \right] \xi^2 + \dots. \end{aligned} \quad (3.8)$$

We know that if $|R(\chi)| < 1$ and $|T(\xi)| < 1, \chi, \xi \in \mathbb{U}$ then $|r_i| \leq 1$ and $|t_i| \leq 1 \forall i \in \mathbb{N}$. Now we equate like coefficients in (3.5), (3.7) and (3.6), (3.8) and simplifying, we get

$$-a_2 ([2]_{p,q}k - k - [2]_{p,q}) ([2]_{p,q} - 1) = h_2(v)r_1 \quad (3.9)$$

$$\begin{aligned} & \left(\frac{1}{2}k^2[2]_{p,q}^4 - 2[2]_{p,q}^2[2]_{p,q}^3 - \frac{1}{2}k[2]_{p,q}^4 + 3k^2[2]_{p,q}^2 + 3k[2]_{p,q}^3 - 4k[2]_{p,q}^2 - [2]_{p,q}^3 + [2]_{p,q}^2 - 2k^2[2]_{p,q} \right. \\ & \left. + k[2]_{p,q} + \frac{1}{2}k^2 + \frac{1}{2}k \right) a_2^2 + (-[3]_{p,q}^2k + 2[3]_{p,q}k + [3]_{p,q}^2 - k - [3]_{p,q}) a_3 = h_2(v)r_2 + h_3(v)r_1^2 \end{aligned} \quad (3.10)$$

and

$$a_2 ([2]_{p,q}k - k - [2]_{p,q}) ([2]_{p,q} - 1) = h_2(v)t_1 \quad (3.11)$$

$$\begin{aligned} & \left(\frac{1}{2}k^2[2]_{p,q}^4 - 2[2]_{p,q}^2[2]_{p,q}^3 - \frac{1}{2}k[2]_{p,q}^4 + 3k^2[2]_{p,q}^2 + 3k[2]_{p,q}^3 - 4k[2]_{p,q}^2 - [2]_{p,q}^3 + [2]_{p,q}^2 - 2k^2[2]_{p,q} \right. \\ & \left. + k[2]_{p,q} + \frac{1}{2}k^2 + \frac{1}{2}k \right) a_2^2 + (-[3]_{p,q}^2k + 2[3]_{p,q}k + [3]_{p,q}^2 - k - [3]_{p,q}) (2a_2^2 - a_3) = h_2(v)t_2 + h_3(v)t_1^2 \end{aligned} \quad (3.12)$$

By solving (3.9) and (3.11) we found that

$$r_1 = -t_1 \quad (3.13)$$

Squaring and adding (3.9) and (3.11), we have

$$\begin{aligned} & 2a_2^2 ([2]_{p,q}k - k - [2]_{p,q})^2 ([2]_{p,q} - 1)^2 = h_2^2(v)(r_1^2 + t_1^2) \\ \implies & a_2^2 = \frac{h_2^2(v)(r_1^2 + t_1^2)}{2([2]_{p,q}k - k - [2]_{p,q})^2 ([2]_{p,q} - 1)^2} \end{aligned} \quad (3.14)$$

$$\implies \frac{2a_2^2 ([2]_{p,q}k - k - [2]_{p,q})^2 ([2]_{p,q} - 1)^2}{h_2^2(v)} = (r_1^2 + t_1^2). \quad (3.15)$$

In addition, by adding (3.10) and (3.12) we have

$$\begin{aligned} & a_2^2 \left(([2]_{p,q} - 1)^4 k^2 + (-[2]_{p,q}^4 + 6[2]_{p,q}^3 - 8[2]_{p,q}^2 - 2[3]_{p,q}^2 + 2[2]_{p,q} + 4[3]_{p,q} - 1) k \right. \\ & \left. - 2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right) = h_2(v)(r_2 + t_2) + h_3(v)(r_1^2 + t_1^2). \end{aligned} \quad (3.16)$$

By making use of (3.15) in (3.16) it is reduced to

$$\begin{aligned} & a_2^2 \left(([2]_{p,q} - 1)^4 k^2 + (-[2]_{p,q}^4 + 6[2]_{p,q}^3 - 8[2]_{p,q}^2 - 2[3]_{p,q}^2 + 2[2]_{p,q} + 4[3]_{p,q} - 1) k \right. \\ & \left. - 2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right) = h_2(v)(r_2 + t_2) + h_3(v) \left(\frac{2a_2^2 ([2]_{p,q}k - k - [2]_{p,q})^2 ([2]_{p,q} - 1)^2}{h_2^2(v)} \right) \\ \implies & \left(h_2^2 \left[([2]_{p,q} - 1)^4 k^2 + (-[2]_{p,q}^4 + 6[2]_{p,q}^3 - 8[2]_{p,q}^2 - 2[3]_{p,q}^2 + 2[2]_{p,q} + 4[3]_{p,q} - 1) k \right. \right. \\ & \left. \left. - 2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right] - 2h_3((k-1)[2]_{p,q} - k)^2 ([2]_{p,q} - 1)^2 \right) a_2^2 = h_2^3(r_2 + t_2) \\ a_2^2 = & \frac{h_2^3(v)(r_2 + t_2)}{h_2^2(v) \left[([2]_{p,q} - 1)^4 k^2 + (-[2]_{p,q}^4 + 6[2]_{p,q}^3 - 8[2]_{p,q}^2 - 2[3]_{p,q}^2 \right.} \\ & \left. + 2[2]_{p,q} + 4[3]_{p,q} - 1) k - 2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right] - 2h_3(v)((k-1)[2]_{p,q} - k)^2 ([2]_{p,q} - 1)^2} \end{aligned} \quad (3.17)$$

Now taking modulus and using equations (2.9) in (3.17), we obtain

$$|a_2| \leq \frac{\sqrt{2}|bv|\sqrt{|bv|}}{\sqrt{b^2v^2 \left[([2]_{p,q} - 1)^4 k^2 + (-[2]_{p,q}^4 + 6[2]_{p,q}^3 - 8[2]_{p,q}^2 - 2[3]_{p,q}^2 + 2[2]_{p,q} + 4[3]_{p,q} - 1)k - 2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right] - 2(pbv^2 + qa)((k-1)[2]_{p,q} - k)^2 ([2]_{p,q} - 1)^2}}$$

Hence we get bound for $|a_2|$.

Subtracting (3.12) from (3.10) and making use of (3.13), gives

$$2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})(a_3 - a_2^2) = h_2(v)(r_2 - t_2). \quad (3.18)$$

Making use of (3.14) in (3.18) we get

$$\Rightarrow a_3 = \frac{h_2(v)(r_2 - t_2)}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} + \frac{h_2^2(v)(r_1^2 + t_1^2)}{2([2]_{p,q} k - k - [2]_{p,q})^2 ([2]_{p,q} - 1)^2}. \quad (3.19)$$

Now taking modulus and using (2.9) in (3.19), we get

$$\Rightarrow |a_3| \leq \frac{|bv|}{(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} + \frac{b^2v^2}{([2]_{p,q} k - k - [2]_{p,q})^2 ([2]_{p,q} - 1)^2}$$

This completes the proof of the Theorem 3.1. \square

Taking $k = 1$ in Theorem 3.1, we obtain the following result in form of corollary:

Corollary 3.2. *Let a function $\mathcal{F} \in \mathcal{A}$ given by (2.1) is in the class $BS_{\Sigma}(k, v)$. Then*

$$|a_2| \leq \frac{\sqrt{2}|bv|\sqrt{|bv|}}{\sqrt{b^2v^2 \left[([2]_{p,q} - 1)^4 + (-[2]_{p,q}^4 + 4[2]_{p,q}^3 - 6[2]_{p,q}^2 + 2[2]_{p,q} + 2[3]_{p,q} - 1) \right] - 2(pbv^2 + qa)([2]_{p,q} - 1)^2}},$$

$$|a_3| \leq \frac{|bv|}{(2[3]_{p,q} - 1 - [3]_{p,q})} + \frac{b^2v^2}{([2]_{p,q} - 1)^2}.$$

If we take $k = 0$ in Theorem 3.1, then we obtain the following result:

Corollary 3.3. *If a function $\mathcal{F} \in \mathcal{A}$ and of the form (2.1) is in the class $BC_{\Sigma}(k, v)$, then*

$$|a_2| \leq \frac{\sqrt{2}|bv|\sqrt{|bv|}}{\sqrt{b^2v^2 \left[-2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right] - 2(pbv^2 + qa)[2]_{p,q}^2 ([2]_{p,q} - 1)^2}},$$

$$|a_3| \leq \frac{|bv|}{([3]_{p,q}^2 - [3]_{p,q})} + \frac{b^2v^2}{[2]_{p,q}^2 ([2]_{p,q} - 1)^2}.$$

Letting $p = 1$ and $q \rightarrow 1^-$ in Corollary 3.2 gives the following:

Corollary 3.4. *Suppose the function $\mathcal{F} \in \mathcal{A}$ and of the form (2.1) is in the class $BS_{\Sigma}(k, v)$. Then*

$$|a_2| \leq \frac{|bv|\sqrt{|bv|}}{\sqrt{b^2v^2 - (pbv^2 + qa)}} \quad \text{and} \quad |a_3| \leq \frac{|bv|}{2} + b^2v^2.$$

If we take $p = 1$, then (p, q) -calculus reduced to q -calculus and $q \rightarrow 1^-$ in Corollary 3.3, we obtain the following result:

Corollary 3.5. *Let $\mathcal{F} \in \mathcal{A}$ and of the form (2.1) be in the class $BC_{\Sigma}(k, v)$. Then*

$$|a_2| \leq \frac{\sqrt{2}|bv|\sqrt{|bv|}}{\sqrt{4b^2v^2 - 8(pbv^2 + qa)}} \quad \text{and} \quad |a_3| \leq \frac{|bv|}{6} + \frac{b^2v^2}{4}.$$

4. Fekete Szegő functional

Theorem 4.1. *Let the function \mathcal{F} of the form (2.1) be a member of the class $U_{\Sigma}(k, v)$. Then for any complex parameter ρ , we have*

$$|a_3 - \rho a_2^2| \leq \begin{cases} \frac{|h_2(v)|}{(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})}, & |\rho - 1| \leq |T| \\ 2|h_2(v)||M(\rho, v)|, & |\rho - 1| \geq |T| \end{cases}$$

where

$$M(\rho, v) = \frac{(1 - \rho)h_2^2(v)}{h_2^2(v) \left[([2]_{p,q} - 1)^4 k^2 + (-[2]_{p,q}^4 + 6[2]_{p,q}^3 - 8[2]_{p,q}^2 - 2[3]_{p,q}^2 + 2[2]_{p,q} + 4[3]_{p,q} - 1) k - 2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right] - 2h_3(v) ((k - 1) [2]_{p,q} - k)^2 ([2]_{p,q} - 1)^2}$$

and

$$T = \frac{b^2v^2 \left[([2]_{p,q} - 1)^4 k^2 + (-[2]_{p,q}^4 + 6[2]_{p,q}^3 - 8[2]_{p,q}^2 - 2[3]_{p,q}^2 + 2[2]_{p,q} + 4[3]_{p,q} - 1) k - 2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right] - 2(pbv^2 + qa) ((k - 1) [2]_{p,q} - k)^2 ([2]_{p,q} - 1)^2}{(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q}) b^2v^2}.$$

Proof. It follows from the relation (3.18) that

$$a_3 - a_2^2 = \frac{h_2(v)(r_2 - t_2)}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})}.$$

Therefore,

$$\begin{aligned} a_3 - \rho a_2^2 &= \frac{h_2(v)(r_2 - t_2)}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} + a_2^2 - \rho a_2^2 \\ &= \frac{h_2(v)(r_2 - t_2)}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} + (1 - \rho)a_2^2 \end{aligned} \quad (4.1)$$

Now using (3.17) in (4.1) we get

$$\begin{aligned} a_3 - \rho a_2^2 &= \frac{h_2(v)(r_2 - t_2)}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} \\ &\quad + (1 - \rho) \frac{h_2^3(v)(r_2 + t_2)}{h_2^2(v) \left[([2]_{p,q} - 1)^4 k^2 + (-[2]_{p,q}^4 + 6[2]_{p,q}^3 - 8[2]_{p,q}^2 - 2[3]_{p,q}^2 + 2[2]_{p,q} + 4[3]_{p,q} - 1) k - 2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right] - 2h_3(v) ((k - 1) [2]_{p,q} - k)^2 ([2]_{p,q} - 1)^2} \end{aligned}$$

$$\begin{aligned}
&= h_2(v) \left[\frac{(r_2 - t_2)}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} \right. \\
&+ (1 - \rho) \frac{h_2^2(v)(r_2 + t_2)}{h_2^2(v) \left[([2]_{p,q} - 1)^4 k^2 + (-[2]_{p,q}^4 + 6[2]_{p,q}^3 - 8[2]_{p,q}^2 - 2[3]_{p,q}^2 \right.} \\
&\quad \left. \left. + 2[2]_{p,q} + 4[3]_{p,q} - 1) k - 2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right]}{-2h_3(v) ((k - 1) [2]_{p,q} - k)^2 ([2]_{p,q} - 1)^2} \right] \\
&= h_2(v) \left[\frac{(r_2 - t_2)}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} + M(\rho, v)(r_2 + t_2) \right]
\end{aligned}$$

where

$$M(\rho, v) = \frac{(1 - \rho)h_2^2(v)}{h_2^2(v) \left[([2]_{p,q} - 1)^4 k^2 + (-[2]_{p,q}^4 + 6[2]_{p,q}^3 - 8[2]_{p,q}^2 - 2[3]_{p,q}^2 + 2[2]_{p,q} + 4[3]_{p,q} - 1) k \right.} \\
\left. - 2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right] - 2h_3(v) ((k - 1) [2]_{p,q} - k)^2 ([2]_{p,q} - 1)^2}$$

$$\begin{aligned}
a_3 - \rho a_2^2 &= h_2(v) \left[\frac{r_2}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} \right. \\
&\quad \left. - \frac{t_2}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} + M(\rho, v)r_2 + M(\rho, v)t_2 \right] \\
&= h_2(v) \left[\left(M(\rho, v) + \frac{r_2}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} \right) r_2 \right. \\
&\quad \left. \left(M(\rho, v) - \frac{t_2}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} \right) t_2 \right]
\end{aligned}$$

Hence, we have

$$|a_3 - \rho a_2^2| \leq \begin{cases} \frac{|h_2(v)|}{(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})}, & 0 \leq |M(\rho, v)| \leq \frac{1}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} \\ 2|h_2(v)||M(\rho, v)|, & |M(\rho, v)| \geq \frac{1}{2(-[3]_{p,q}^2 k + 2[3]_{p,q} k + [3]_{p,q}^2 - k - [3]_{p,q})} \end{cases}$$

This completes the proof of Theorem 4.1. \square

If we take $k = 1$ in Theorem 4.1, we obtain the following corollary,

Corollary 4.2. *If a function $\mathcal{F} \in \mathcal{A}$ and of the form (2.1) is in the class $BS_\Sigma(k, v)$ then*

$$|a_3 - \rho a_2^2| \leq \begin{cases} \frac{|h_2(v)|}{[3]_{p,q} - 1}, & |\rho - 1| \leq |T| \\ 2|h_2(v)||M(\rho, v)|, & |\rho - 1| \geq |T| \end{cases}$$

where

$$M(\rho, v) = \frac{(1 - \rho)h_2^2(v)}{h_2^2(v) \left\{ ([2]_{p,q} - 1)^4 - [2]_{p,q}^4 + 4[2]_{p,q}^3 - 6[2]_{p,q}^2 + 2[2]_{p,q} + 2[3]_{p,q} - 1 \right\} - 2h_3(v) ([2]_{p,q} - 1)^2}$$

and

$$T = \frac{b^2 v^2 \left\{ ([2]_{p,q} - 1)^4 - [2]_{p,q}^4 + 4[2]_{p,q}^3 - 6[2]_{p,q}^2 + 2[2]_{p,q} + 2[3]_{p,q} - 1 \right\} - 2(pbv^2 + qa) ([2]_{p,q} - 1)^2}{([3]_{p,q} - 1) b^2 v^2}$$

If we take $k = 0$ in Theorem 4.1, we obtain the following corollary:

Corollary 4.3. *If a function $\mathcal{F} \in \mathcal{A}$ and of the form (2.1) is in the class $BC_\Sigma(k, v)$ then*

$$|a_3 - \rho a_2^2| \leq \begin{cases} \frac{|h_2(v)|}{([3]_{p,q}^2 - [3]_{p,q})}, & |\rho - 1| \leq |T| \\ 2|h_2(v)||M(\rho, v)|, & |\rho - 1| \geq |T|. \end{cases}$$

where

$$M(\rho, v) = \frac{(1 - \rho)h_2^2(v)}{h_2^2(v) \left[-2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right] - 2h_3(v)[2]_{p,q}^2 ([2]_{p,q} - 1)^2}$$

and

$$T = \frac{b^2 v^2 \left[-2[2]_{p,q}^3 + 2[2]_{p,q}^2 + 2[3]_{p,q}^2 - 2[3]_{p,q} \right] - 2(pbv^2 + qa)[2]_{p,q}^2 ([2]_{p,q} - 1)^2}{([3]_{p,q}^2 - [3]_{p,q}) b^2 v^2}.$$

Remark 2. *Taking $p = 1$ and $q \rightarrow 1^-$ in Corollary 4.2, 4.3 we obtain Corollary 4.4, 4.5 respectively as stated below:*

Corollary 4.4. *Suppose that function $\mathcal{F} \in \mathcal{A}$ and of the form (2.1) is in the class $BS_\Sigma(k, v)$. Then*

$$|a_3 - \rho a_2^2| \leq \begin{cases} \frac{|h_2(v)|}{2}, & |\rho - 1| \leq |T| \\ 2|h_2(v)||M(\rho, v)|, & |\rho - 1| \geq |T| \end{cases}$$

where

$$M(\rho, v) = \frac{(1 - \rho)h_2^2(v)}{h_2^2(v) - 2h_3(v)} \quad \text{and} \quad T = \frac{b^2 v^2 - 2(pbv^2 + qa)}{2b^2 v^2}.$$

Corollary 4.5. *If a function $\mathcal{F} \in \mathcal{A}$ and of the form (2.1) is in the class $BC_\Sigma(k, v)$, then*

$$|a_3 - \rho a_2^2| \leq \begin{cases} \frac{|h_2(v)|}{6}, & |\rho - 1| \leq |T| \\ 2|h_2(v)||M(\rho, v)|, & |\rho - 1| \geq |T| \end{cases}$$

where

$$M(\rho, v) = \frac{(1 - \rho)h_2^2(v)}{4h_2^2(v) - 8h_3(v)} \quad \text{and} \quad T = \frac{4b^2 v^2 - 8(pbv^2 + qa)}{6b^2 v^2}.$$

Applications: The Horadam polynomial is a valuable mathematical sequence that is widely used in texture analysis and image processing. Its capabilities include filtering and resampling, making it a powerful tool in the fields of image processing and computer vision. The Horadam polynomial facilitates the creation and manipulation of scale-space image representations, which are essential for tasks such as edge detection, texture analysis, and conducting image analysis at various scales.

This might investigate the use of Horadam polynomials in texture analysis, emphasizing their impact on feature extraction, segmentation, and image denoising, while also presenting real-world cases where

these methods have enhanced image quality and analysis results. It allows for the assessment of statistical texture properties, including gray-level distribution and spatial arrangement. This article explores texture analysis using Canny images by convolving calculated coefficient values with image pixels through masking at eight distinct angles, ultimately presenting the optimal results.

Concluding Remarks: In conclusion, this study successfully introduces and examines a new generalized class of analytic and bi-univalent functions that are uniquely associated with Horadam polynomials. Here with the aid of subordination investigated coefficient bounds and derived Fekete-Szegő inequalities, providing valuable insights into the structural properties of functions within this class. The detailed observations and special cases not only validate our findings but also highlight the broader applicability and potential impact of this research in the field of geometric function theory. This work lay down the foundation for next idea into the intricate relationships between special polynomial sequences and subclasses of bi-univalent functions, potentially leading way of new discoveries and applications.

Acknowledgments

The authors would like to thank the Editor and the referees for their insightful comments and constructive suggestions, which have significantly improved the quality and clarity of this paper.

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