Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2025 (43) 1** : 1–8.
ISSN-0037-8712
doi:10.5269/bspm.78468

Certain Properties of Mathieu-Type Series Associated with Hypergeometric Functions

Harshal S. Gharat [1,2], B. V. Nathwani[1,*], P. Agarwal[3] *

ABSTRACT: The objective of current article is to investigate some properties of Mathieu-type series and with their alternating version as kernel Gauss hypergeometric function. Also we discussed some particular cases of the result obtained here.

Key Words: Hypergeometric function, Beta function, Integral representations, Mathieu type series, Cahen formula.

Contents

1. Introduction and Preliminaries

In last decade some researchers like Pogány, either alone and/or with his colleagues Saxena, Srivastava, Baricz, Butzer,and Tomovski [1,2,3,4,5,6] studied Mathieu–type series and their alternating variants with constitutive terms and many well known special functions, for example, Gauss hyper-geometric function(GHF), confluent hyper-geometric function (CHF), generalized hypergeometric function, Meijer G-functions and many more.

Many generalizations of GHF and other special function were studied in last few years by many scientists and researchers. The significance of these well known function is that they inherit most of the character and quality of the original functions.

In 2021, Jain et al. [7] have studied GHF, CHF type and also studied many properties, relations of these functions.

Generalized GHF and generalized confluent hypergeometric function are given as follows [7]:

$$F^{(s)}_{(v_1,v_2)}(p_0,p_1,p_2;z)=\sum_{l=0}^{\infty}\frac{B^{(s)}_{(v_1,v_2)}(p_1+l,p_2-p_1)}{B(p_1,p_2-p_1)}(p_0)_l\frac{z^l}{l!}, \tag{1.1}$$

where, $(p_0)_l$ is Pochhammar symbol defined in [7].

$$\Phi^{(s)}_{(v_1,v_2)}(p_1,p_2;z)=\sum_{l=0}^{\infty}\frac{B^{(s)}_{(v_1,v_2)}(p_1+l,p_2-p_1)}{B(p_1,p_2-p_1)}\frac{z^l}{l!}, \tag{1.2}$$

provided, $Re(p_2)>Re(p_1)>0$, $Re(v_1)>0, Re(v_2)>0$ and $s\geq 0$,and $|z|<1$;

* Corresponding author.
2010 *Mathematics Subject Classification:* 26A33.
Submitted August 18, 2025. Published December 19, 2025

where $B^{(s)}_{(v_1,v_2)}(w_1,w_2)$ is the generalized beta function [8] defined by

$$B^{(s)}_{(v_1,v_2)}(w_1,w_2) = \int_0^1 w^{w_1-1}(1-w)^{w_2-1}E_{v_1,v_2}\left(-s\left(w(1-w)\right)^{-1}\right)dw, \tag{1.3}$$

$\min\{Re(w_1), Re(w_2)\} > 0,\ Re(v_1) > 0, Re(v_2) > 0,\ s \geq 0.$

Here,

$$E_{v_1,v_2}(z) = \sum_{l=0}^{\infty} \frac{z^l}{\Gamma(lv_1+v_2)}, \qquad (Re(v_1) > 0, Re(v_2) > 0). \tag{1.4}$$

$E_{v_1,v_2}(z)$ is 2-parameter Mittag-Leffler function defined as [14,15,16,17,18].

Note that when $v_2 = 1$ in 1.1 and 1.2, we get extended GHF and extended CHF studied by Shadab et. al [10]

$$F_{s,v_1}(p_0,p_1,p_2;z) = \sum_{l=0}^{\infty} \frac{B^s_{v_1}(v_1+l, v_2-v_1)}{B(v_1,v_2-v_1)}(p_0)_l\frac{z^l}{l!}, \tag{1.5}$$

here, $Re(v_2) > Re(v_1) > 0,\ s \geq 0$ and $\mid z \mid < 1$;

$$\Phi_{s,v_1}(p_1,p_2;z) = \sum_{l=0}^{\infty} \frac{B^s_{v_1}(v_1+l, v_2-v_1)}{B(v_1,v_2-v_1)}\frac{z^l}{l!}, \tag{1.6}$$

here, $Re(v_2) > Re(v_1) > 0$, and $s \geq 0$.

Setting $v_1 = v_2 = 1$ in (1.1) and (1.2), we get extended GHF and CHF studied by Chaudhry et al. [11]

$$F_s(p_0,p_1,p_2;z) = \sum_{l=0}^{\infty} \frac{B(p_1+l, p_2-p_1;s)}{B(p_1,p_2-p_1)}(p_0)_l\frac{z^l}{l!}, \tag{1.7}$$

here, $s \geq 0,\ Re(p_2) > Re(p_1) > 0,\ \mid z \mid < 1$;

$$\Phi_s(p_1,p_2;z) = \sum_{l=0}^{\infty} \frac{B(p_1+l, p_2-p_1;s)}{B(p_1,p_2-p_1)}\frac{z^l}{l!}, \tag{1.8}$$

here, $s \geq 0$ and $Re(p_2) > Re(p_1) > 0$.

Yet another case, let $v_1 = v_2 = 1$ and $s = 0$ then (1.1) and (1.2) reduces to classical GHF and CHF defined in [12]

$$F(p_0,p_1,p_2;z) = \sum_{l=0}^{\infty} \frac{B(p_1+l, p_2-p_1)}{B(p_1,p_2-p_1)}(p_0)_l\frac{z^l}{l!}, \tag{1.9}$$

here, $Re(p_2) > Re(p_1) > 0$ and $\mid z \mid < 1$;

$$\Phi(p_1,p_2;z) = \sum_{l=0}^{\infty} \frac{B(p_1+l, p_2-p_1)}{B(p_1,p_2-p_1)}\frac{z^l}{l!}, \tag{1.10}$$

here, $Re(p_2) > Re(p_1) > 0$.

Now, by inserting the $F^{(s)}_{(v_1,v_2)}(p_0,p_1,p_2;z)$ as kernel in the definition of the Mathieu–type series in [1], we generalized the Mathieu–type a–series $\mathbf{M}_{m,n}$ and its alternating variant $\tilde{\mathbf{M}}_{m,n}$ in the form of series.

2. Main Results

Definition 2.1 *For $m, n, r > 0$; $\mathrm{Re}(p_2) > \mathrm{Re}(p_1) > 0$, $\mathrm{Re}(v_1) > 0, \mathrm{Re}(v_2) > 0$, $s \geq 0$ and $b = (b_k)_{k\geq 1}$ Mathieu-type series involving extended hypergeometric function defined as:*

$$\mathbf{M}_{m,n}[F^{(s)}_{(v_1,v_2)}; b; r] = \sum_{k\geq 1} \frac{F^{(s)}_{(v_1,v_2)}\left(m, p_2; p_3; -\frac{r^2}{b_k}\right)}{b_k^m (b_k + r^2)^n} \tag{2.1}$$

With same range of parameter, we also define its alternating variant.

Definition 2.2 *For given $m, n, r > 0$; $Re(p_2) > Re(p_1) > 0$, $Re(v_1) > 0, Re(v_2) > 0$, $s \geq 0$ and $b = (b_k)_{k\geq 1}$ alternating Mathieu–type series involving extended hypergeometric function defined as:*

$$\tilde{\boldsymbol{M}}_{m,n}[F^{(s)}_{(v_1,v_2)}; b; r] = \sum_{k\geq 1} \frac{(-1)^{k-1} F^{(s)}_{(v_1,v_2)}(m, p_2; p_3; \frac{-r^2}{b_k})}{{b_k}^m (b_k + r^2)^n} \tag{2.2}$$

where R and R^+ represent the sets of real and positive real numbers then the real sequence $b = (b_k)_{k\geq 1}$ is the restriction of an increasing function $b : R^+ \mapsto R^+$ such that $b(x)|_{x\in N} = b$.

To obtain our main results, we need to prove important Laplace transform of extended confluent hypergeometric function (1.2).

Theorem 2.1 *For $m > 0$; $Re(p_2) > Re(p_1) > 0$, $Re(v_1) > 0, Re(v_2) > 0$, $s \geq 0$, the following result holds true:*

$$\int_0^\infty e^{-zt} t^{m-1} \Phi^{(s)}_{(v_1,v_2)}(p_1, p_2; wt) dt = \frac{\Gamma(m)}{z^m} F^{(s)}_{(v_1,v_2)}(m, p_1, p_2; \frac{w}{z}) \tag{2.3}$$

Proof: To prove the result, consider left hand side and denote it by I,

$$I = \int_0^\infty e^{-zt} t^{m-1} \Phi^{(s)}_{(v_1,v_2)}(p_1, p_2; wt) dt. \tag{2.4}$$

From generalized confluent hypergeometric function (1.2), we have:

$$I = \int_0^\infty e^{-zt} t^{m-1} \left(\sum_{k=0}^{\infty} \frac{B^{(s)}_{(v_1,v_2)}(p_1 + k, p_2 - p_1)}{B(p_1, p_2 - p_1)} \frac{w^k t^k}{k!} \right) dt \tag{2.5}$$

On interchanging the integration and summation, we get

$$I = \sum_{k=0}^{\infty} \frac{B^{(s)}_{(v_1,v_2)}(p_1 + k, p_2 - p_1)}{B(p_1, p_2 - p_1)} \frac{w^k}{k!} \int_0^\infty e^{-zt} t^{m+k-1} dt \tag{2.6}$$

Then using the definition of Gamma function [13] and relation between Gamma function and pochhammer symbol $\frac{\Gamma(m+k)}{\Gamma(m)} = (m)_k$, we have

$$I = \frac{\Gamma(m)}{z^m} \sum_{k=0}^{\infty} \frac{B^{(s)}_{(v_1,v_2)}(p_1 + k, p_2 - p_1)}{B(p_1, p_2 - p_1)} (m)_k \frac{(\frac{w}{z})^k}{k!} \tag{2.7}$$

Then from equation (1.1), we get our desired result.

$$\int_0^\infty e^{-zt} t^{m-1} \Phi^{(s)}_{(v_1,v_2)}(p_1, p_2; wt) dt = \frac{\Gamma(m)}{z^m} F^{(s)}_{(v_1,v_2)}(m, p_1, p_2; \frac{w}{z}) \tag{2.8}$$

□

3. Integral Representation of $\mathbf{M}_{m,n}$ and $\tilde{\mathbf{M}}_{m,n}$

Here, we give integral form for the series $\mathbf{M}_{m,n}$ and $\tilde{\mathbf{M}}_{m,n}$ in the form of the linear combination of the two principal integrals.

Theorem 3.1 *Consider $m, n, r > 0; Re(v_1) > 0, Re(v_2) > 0$, $s \geq 0$ and the real sequence $b = (b_k)_{k\geq 1}$ monotone increasing and tends to ∞, we have:*

$$\boldsymbol{M}_{m,n}\left[F^{(s)}_{(v_1,v_2)}; b; r\right] = m\boldsymbol{H}^{s}_{v_1,v_2}(m+1, n, b_1) + n\boldsymbol{H}^{s}_{v_1,v_2}(m, n+1, b_1) \tag{3.1}$$

where, $\forall\ Re(p_2) > Re(p_1) > 0$,

$$\boldsymbol{H}^{s}_{v_1,v_2}(m, n, b_1) = \int_{b_1}^{\infty} \frac{F^{(s)}_{(v_1,v_2)}(m, p_2; p_3; \frac{-r^2}{x})[b^{-1}(x)]dx}{x^m(x+r^2)^n} \tag{3.2}$$

and $[b^{-1}]$ represent the integral part of the inverse of b.

Proof: Assume the Laplace transform of the function $t^{m-1}\Phi^{(s)}_{(v_1,v_2)}(p_1, p_2; wt)$ from the above Theorem (2.1) we have,

$$F^{(s)}_{(v_1,v_2)}(m, p_1, p_2; \frac{w}{z}) = \frac{z^m}{\Gamma(m)} \int_0^{\infty} e^{-zt}t^{m-1}\Phi^{(s)}_{(v_1,v_2)}(p_1, p_2; wt)dt \tag{3.3}$$

Then from definition of Gamma function we have,

$$\Gamma(n)g^{-n} = \int_0^{\infty} e^{-gt}t^{n-1}dt, \quad Re(g) > 0, Re(n) > 0. \tag{3.4}$$

Taking $g = b_k + r^2$ in above equation, we get

$$(b_k + r^2)^{-n} = \frac{1}{\Gamma(n)} \int_0^{\infty} e^{-(b_k+r^2)t}t^{n-1}dt. \tag{3.5}$$

Now, put $w = -r^2$, $z = b_k$ in the equation (3.3), we have

$$F^{(s)}_{(v_1,v_2)}(m, p_1, p_2; \frac{-r^2}{b_k}) = \frac{(b_k)^m}{\Gamma(m)} \int_0^{\infty} e^{-(b_k)t}t^{m-1}\Phi^{(s)}_{(v_1,v_2)}(p_1, p_2; -r^2t)dt \tag{3.6}$$

Then from definition of Mathieu series (2.1), we have

$$\mathbf{M}_{m,n}\left[F^{(s)}_{(v_1,v_2)}; b; r\right] = \sum_{k\geq 1} \frac{F^{(s)}_{(v_1,v_2)}(m, p_2; p_3; \frac{-r^2}{b_k})}{{b_k}^m(b_k + r^2)^n} \tag{3.7}$$

From equations (3.5), (3.6) and arranging the terms, we have

$$\mathbf{M}_{m,n}\left[F^{(s)}_{(v_1,v_2)}; b; r\right] = \int_0^{\infty}\int_0^{\infty} \frac{e^{-r^2u}t^{m-1}u^{n-1}}{\Gamma(m)\Gamma(n)} \left(\sum_{k\geq 1} e^{-b_k(t+u)}\right) \Phi^{(s)}_{(v_1,v_2)}(p_1, p_2; -r^2t)dtdu \tag{3.8}$$

By the Cahen formula [2] for summing up the Dirichlet series in the technique developed in [3], we conclude

$$F_b(t+u) = \sum_{k\geq 1} e^{-b_k(t+u)} = (t+u)\int_{b_1}^{\infty} e^{-(t+u)x}[b^{-1}(x)]dx \tag{3.9}$$

Which implies

$$\mathbf{M}_{m,n}\left[F^{(s)}_{(v_1,v_2)}; b; r\right] =$$
$$\frac{1}{\Gamma(m)\Gamma(n)} \int_0^{\infty}\int_0^{\infty}\int_{b_1}^{\infty} e^{(-r^2+x)u-tx}(t+u)t^{m-1}u^{n-1}[b^{-1}(x)]\Phi^{(s)}_{(v_1,v_2)}(p_1, p_2; -r^2t)dtdudx \tag{3.10}$$

Consider the RHS of the above equation and denote it by $I_t + I_u$

$$\mathbf{M}_{m,n}\left[F^{(s)}_{(v_1,v_2)};b;r\right] = I_t + I_u \tag{3.11}$$

where

$$I_t = \frac{1}{\Gamma(n)}\int_0^\infty\left(\int_{b_1}^\infty\left(\frac{e^{-xt}t^m\Phi^{(s)}_{(v_1,v_2)}(p_1,p_2;-r^2t)dt}{\Gamma(m)}\right)e^{-xu}[b^{-1}(x)]dx\right)e^{-r^2u}u^{n-1}du \tag{3.12}$$

Then using the above result (3.3) and re-arranging the terms , we have

$$I_t = m\int_{b_1}^\infty\left(\int_0^\infty\frac{u^{n-1}e^{-(x+r^2)u}du}{\Gamma(n)}\right)\frac{[b^{-1}(x)]}{x^{m+1}}F^{(s)}_{(v_1,v_2)}(m+1,p_2;p_3;\frac{-r^2}{x})dx \tag{3.13}$$

After using (3.5), we have

$$I_t = m\int_{b_1}^\infty F^{(s)}_{(v_1,v_2)}(m+1,p_2;p_3;\frac{-r^2}{x})\frac{[b^{-1}(x)]}{x^{m+1}(x+r^2)^n}dx \tag{3.14}$$

Assuming

$$\mathbf{H}^s_{v_1,v_2}(m,n,b_1) = \int_{b_1}^\infty\frac{F^{(s)}_{(v_1,v_2)}(m,p_2;p_3;\frac{-r^2}{x})[b^{-1}(x)]dx}{x^m(x+r^2)^n} \tag{3.15}$$

Then (3.14) becomes,

$$I_t = m\mathbf{H}^s_{v_1,v_2}(m+1,n,b_1) \tag{3.16}$$

Similarly we get,

$$I_u = n\int_{b_1}^\infty\frac{[b^{-1}(x)]}{(x+r^2)^{n+1}}\left(\int_0^\infty\frac{e^{-xt}t^{m-1}\Phi^{(s)}_{(v_1,v_2)}(p_1,p_2;-r^2t)dt}{\Gamma(m)}\right)dx \tag{3.17}$$

Then again using the (3.3) and re-arranging the terms we have,

$$I_u = n\int_{b_1}^\infty\frac{F^{(s)}_{(v_1,v_2)}(m,p_2;p_3;\frac{-r^2}{x})[b^{-1}(x)]dx}{x^m(x+r^2)^{n+1}} \tag{3.18}$$

From equation (3.15), we get

$$I_u = n\mathbf{H}^s_{v_1,v_2}(m,n+1,b_1) \tag{3.19}$$

Now, using equations (3.11),(3.16), (3.19) we get our desired result.

$$\mathbf{M}_{m,n}\left[F^{(s)}_{(v_1,v_2)};b;r\right] = m\mathbf{H}^s_{v_1,v_2}(m+1,n,b_1) + n\mathbf{H}^s_{v_1,v_2}(m,n+1,b_1) \tag{3.20}$$

□

Theorem 3.2 *Consider $m,n,r > 0$;$Re(v_1) > 0, Re(v_2) > 0$, $s \geq 0$ and the real sequence $b = (b_k)_{k\geq 1}$ monotone increasing and tends to ∞, we have:*

$$\tilde{\boldsymbol{M}}_{m,n}\left[F^{(s)}_{(v_1,v_2)};b;r\right] = m\tilde{\boldsymbol{H}}^s_{v_1,v_2}(m+1,n,b_1) + n\tilde{\boldsymbol{H}}^s_{v_1,v_2}(m,n+1,b_1) \tag{3.21}$$

where, $\forall\ Re(p_2) > Re(p_1) > 0$,

$$\tilde{\boldsymbol{H}}^s_{v_1,v_2}(m,n,b_1) = \int_{b_1}^\infty\frac{F^{(s)}_{(v_1,v_2)}(m,p_2;p_3;\frac{-r^2}{x})\sin^2\left(\frac{\pi}{2}[b^{-1}(x)]\right)dx}{x^m(x+r^2)^n} \tag{3.22}$$

and $[b^{-1}]$ denotes the integral part of the inverse of b.

Proof: The proof of Theorem (3.2) is done by similar method as used in Theorem (3.1). Dirichlet series $\tilde{\mathbf{F}}_b(x)$ integral form, keeping in mind the Cahen formula [2], we have

$$\tilde{\mathbf{F}}_b(x) = \sum_{k\geq 1}(-1)^{k-1}e^{-b_k(x)} = x\int_{b_1}^{\infty} e^{-xt}\tilde{\mathbf{B}}(\mathbf{t})dt, \tag{3.23}$$

and therefore,

$$\tilde{\mathbf{F}}_b(x) = x\int_{b_1}^{\infty} e^{-xt}\sin^2\left(\frac{\pi}{2}[b^{-1}(x)]\right)dt, \tag{3.24}$$

since the counting function reduces to

$$\tilde{\mathbf{B}}(\mathbf{t}) = \sum_{k:b_k\leq t}(-1)^{k-1} = \frac{1-(-1)^{[b^{-1}(t)]}}{2} = \sin^2\left(\frac{\pi}{2}[b^{-1}(t)]\right), \tag{3.25}$$

Hence, because

$$\tilde{\mathbf{F}}_b(t+u) = (t+u)\int_{b_1}^{\infty} e^{-(t+u)x}\sin^2\left(\frac{\pi}{2}[b^{-1}(t)]\right)dx, \tag{3.26}$$

we conclude proof of Theorem (3.2) by obtaining the similar remaining steps as Theorem (3.1). □

Particular case when $v_2 = 1$, Theorem (3.1), (3.2) reduces to the following corollary.

Corollary 3.1 *Consider $m, n, r > 0$;$Re(v_1) > 0$, $s \geq 0$ and the real sequence $b = (b_k)_{k\geq 1}$ monotone increasing and tends to ∞, we have:*

$$\boldsymbol{M}_{m,n}\left[F_{s,v_1}; b; r\right] = m\boldsymbol{H}_{s,v_1}(m+1, n, b_1) + n\boldsymbol{H}_{s,v_1}(m, n+1, b_1) \tag{3.27}$$

where, $\forall\ Re(p_2) > Re(p_1) > 0$,

$$\boldsymbol{H}_{s,v_1}(m, n, b_1) = \int_{b_1}^{\infty}\frac{F_{s,v_1}(m, p_2; p_3; \frac{-r^2}{x})[b^{-1}(x)]dx}{x^m(x+r^2)^n} \tag{3.28}$$

and $[b^{-1}]$ denotes the integral part of the inverse of b.

Corollary 3.2 *Consider $m, n, r > 0$;$Re(v_1) > 0$, $s \geq 0$ and the real sequence $b = (b_k)_{k\geq 1}$ monotone increasing and tends to ∞, we have:*

$$\tilde{\boldsymbol{M}}_{m,n}\left[F_{(s,v_1)}; b; r\right] = m\tilde{\boldsymbol{H}}_{s,v_1}(m+1, n, b_1) + n\tilde{\boldsymbol{H}}_{s,v_1}(m, n+1, b_1) \tag{3.29}$$

where, $\forall\ Re(p_2) > Re(p_1) > 0$,

$$\tilde{\boldsymbol{H}}_{s,v_1}(m, n, b_1) = \int_{b_1}^{\infty}\frac{F_{s,v_1}(m, p_2; p_3; \frac{-r^2}{x})\sin^2\left(\frac{\pi}{2}[b^{-1}(x)]\right)dx}{x^m(x+r^2)^n} \tag{3.30}$$

and $[b^{-1}]$ denotes the integral part of the inverse of b.

Case when $v_2 = 1$ and $v_1 = 1$ then Theorem (3.1), (3.2) reduces to the following corollary.

Corollary 3.3 *Consider $m, n, r > 0$; $s \geq 0$ and the real sequence $b = (b_k)_{k\geq 1}$ monotone increasing and tends to ∞, we have:*

$$\boldsymbol{M}_{m,n}\left[F_s; b; r\right] = m\boldsymbol{H}_s(m+1, n, b_1) + n\boldsymbol{H}_s(m, n+1, b_1) \tag{3.31}$$

where, $\forall\ Re(p_2) > Re(p_1) > 0$,

$$\boldsymbol{H}_s(m, n, b_1) = \int_{b_1}^{\infty}\frac{F_s(m, p_2; p_3; \frac{-r^2}{x})[b^{-1}(x)]dx}{x^m(x+r^2)^n} \tag{3.32}$$

and $[b^{-1}]$ denotes the integral part of the inverse of b.

Corollary 3.4 *Consider $m, n, r > 0$; $s \geq 0$ and the real sequence $b = (b_k)_{k\geq 1}$ monotone increasing and tends to ∞, we have:*

$$\tilde{\mathbf{M}}_{m,n}\left[F_s; b; r\right] = m\tilde{\mathbf{H}}_s(m+1, n, b_1) + n\tilde{\mathbf{H}}_s(m, n+1, b_1) \tag{3.33}$$

where, $\forall$ $\mathrm{Re}(p_2) > \mathrm{Re}(p_1) > 0$,

$$\tilde{\mathbf{H}}_s(m, n, b_1) = \int_{b_1}^{\infty} \frac{F_s\left(m, p_2; p_3; -\frac{r^2}{x}\right)\sin^2\left(\frac{\pi}{2}\left[b^{-1}(x)\right]\right)\, dx}{x^m(x+r^2)^n} \tag{3.34}$$

and $\left[b^{-1}\right]$ denotes the integral part of the inverse of b.

Remark 1 *The particular case of Theorem (3.1), (3.2) when $v_1 = v_2 = 1$ and $s = 0$ is immediately reduces to the GHF ${}_2F_1$ result in [1].*

4. Discussion

In this paper, firstly we have defined a Mathieu type series and its alternating variant involving extended gauss hypergeometric function. Then we have calculated closed integral form $\mathbf{M}_{m,n}$ and $\tilde{\mathbf{M}}_{m,n}$ in the form of the linear combination of the two principal integrals and also discussed some special cases of the result obtained here. An open problem can be posed concerning the existence of a generic (appropriately convergent) series instead of $F^{(s)}_{(v_1,v_2)}(p_0, p_1, p_2; z)$ in 2.1 and subsequently in 2.2 which could lead to general formulae similar to Theorems 2.1, 3.1.

References

1. Pogány, T.K., Integral representation of a series which includes the Mathieu a-series, J. Math. Anal. Appl.,2004, 296 , 309—313.
2. Cahen, E. , Sur la fonction $\zeta(s)$ de Riemann et sur des fontions analogues, Ann. Sci. l'École Norm. Sup. Sér. Math., 1894, 75-–164.
3. Pogány, T.K. and Tomovski, Ž, On Mathieu-type series which terms contain general- ized hypergeometric function pFq and Meijer's G-function, Math. Comput. Modell., 2008, 47, 9-10, 952—969.
4. Baricz, Á, Butzer, P.L. Butzer, Pogány T.K. , Alternating Mathieu series, Hilbert-Eisenstein series and their generalized Omega functions, in T. Rassias, G. V. Milovanović (Eds.), Analytic Number Theory, Approximation Theory, and Special Functions - In Honor of Hari M. Srivastava, 775, Springer, New York, 2014.
5. Pogány, T.K., Integral representation of Mathieu (a, λ)-series, Integral Transforms Spec. Funct.,2005, 16,8, 685—689.
6. Pogány,T.K.and H.M. Srivastava, Some Mathieu-type series associated with the Fox- Wright function, Comput. Math. Appl., 2009, 57, 1, 127-–140.
7. Jain, S.; Goyal, R.; Agarwal, P.; Lupica, A.; Cesarano, C. Some results of extended beta function and hypergeometric functions by using Wiman's function, Mathemtics 2021, 9(22), 2944.
8. Goyal, R.; Agarwal, P.; Momami, S.; Rassias, M.T. An Extension of Beta Function by Using wiman's function. Axioms, 2021,10, 187.
9. Wiman, A. U ber der Fundamentalsatz in der Theorie der Funktionen $E_\alpha(x)$. Acta Math. 1905, 29, 191-201.
10. Shadab, M.S.; Jabee, S.J.; Choi, J.C. An extended Beta function and its applications, Far East J. Math. Sci. (FJMS), 2018, 103, 235–251.
11. Chaudhry, M.A.; Qadir, A.; Srivastava, H.M.; Paris, R.B. Extended hypergeometric and confluent hypergeometric functions, Appl. Math. Comput., 2004, 159, 589–602.
12. Olver, W.J.F.; Lozier, W.D.; Boisvert, F.R.; Clark, W.C.,NIST Handbook of Mathematical Functions ; Cambridge University Press, New York, NY, USA, 2010.
13. . Rainville, E.D., Special Functions; Macmillan: New York, NY, USA, 1960.
14. Nathwani, B. V., Dave, B. I.; Generalized Mittag-Leffler function and its properties, The Mathematics student Journal, 2017, 86(1-2), 63–76.
15. Nathwani, B. V.; Inequalities involving Mittag-Leffler type q-Konhauser polynomial, Studia Universitatis Babes-Bolyai Mathematica, 2020, 65(3), 379–401.

16. Prajapati, J. C., Dave, B. I., Nathwani, B. V.; On a unification of generalized Mittag-Leffler function and family of Bessel functions, Advances in Pure Mathematics, 2013, 3, 127–137.

17. Prajapati, J. C., Nathwani, B. V.; Fractional calculus of a unified Mittag-Leffler function, Ukrainian Mathematical Journal, 2015, 66(8), 1267–1280.

18. Sanjhira, R. R., Nathwani, B. V., Dave, B. I.; Generalized Mittag-Leffler matrix function and associated matrix polynomials, The Journal of the Indian Mathematical Society, 2019, 86(1-2), 161–178.

[1] *Department of Mathematics,*
Amity School of Applied Sciences,
Amity University Maharashtra,
Panvel-410 206,
Maharashtra, INDIA.
E-mail address: bharti.nathwani@yahoo.com

and

[2] *Department of Basic Sciences and Humanities,*
Mukesh Patel School of Technology Management and Engineering,
SVKM's NMIMS Deemed to be University,
Mumbai-400 056,
Maharashtra, INDIA.
E-mail address: harshal21jan@gmail.com

and

[3] *Department of Mathematical Sciences,*
Saveetha School of Engineering, Chennai, Tamil Nadu, 602105, India
Department of Mathematics,
Anand International College of Engineering, Jaipur 303012, India.
E-mail address: goyal.praveen2011@gmail.com