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A New Approach on Triangulation of Triangular Fuzzy Random Variable

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ABSTRACT: The fuzzy random variable (FRV) concepts have been applied in the vague or unclear boundary and imprecision by using ($\alpha-$cut) membership grade. The triangular (μ, σ)concept in fuzzy random variable (FRV) is known as triangular fuzzy random variable ($TFRV$) with parameter (μ, σ), which is symmetric aboutμ, which is used to determine the acceptance area with respect to the significant value (α). If the triangular FRV concept is applied in triangulation, then we can find the acceptance region of all triangles of triangulation simultaneously for desired ($\alpha-cut$) significant values in vague situation. This article expose the successive iteration of triangulation which ends until the standard deviation of each triangle is tending to null. The above triangulation iterations are verified through few suitable examples.

Keywords: Fuzzy set, triangular fuzzy number, fuzzy trapezoidal, Sierpinski triangle, triangulation.

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1. Introduction

In vague and imprecise situations, we use the TFRV concepts to determine the area of acceptance based on the level of significance. Triangulation is a survey method that has more credibility and validity for research needs. If the triangulation is in a vague or unclear boundary situation, then we use the TFRV concepts; it raises the credibility and accuracy of the research needs. This triangulation is referred to as the triangulation of TFRV, which also exhibits symmetry with respect to certain parameters. In this paper, we discuss how triangulation in each subtriangle of previous triangulation is considered, and this process continues until the standard deviation of each subtriangle tends to zero. Here, the TFRV is called triangulation of iteration 0; the triangulation is called triangulation of iteration 1. After that, we create a triangulation for each sub-triangle within the triangulation of iteration 1, which is referred to as the triangulation of iteration 2. This process continues until the standard deviation of each sub-triangle tends to null. Some researchers provide a basic history of this article, which begins with Zadeh [13] illuminating the concept of fuzzy sets in 1965, and continues with the developments that led mathematician Kwakernaak to introduce fuzzy random variables in 1978. Focusing on probability space, the fuzzy random variables articulate ordinary real random factors, giving room for encrypting probability. An FRV is a function that maps from (S,B,P) to a group of fuzzy variables. A function that maps a pattern space to the real line (R) is known as a fuzzy variable. The FRV identified in this study by Kwakernaak [3, 4] (1978) is a fuzzy impression of a sharp but unobservable random variable.

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Kwakernaak can provide further explanations of the concepts related to fuzzy random variables (FRV). J.E.L. Priyakumar et al. [7] (2001) analyzed the relationship between fuzzy and probability uncertainty in Kwakernaak's fuzzy random variables. Taleshian et al. [12] (2011) examined the use of pointed trapezoidal fuzzy numbers in multiplication operations. Nagoor Gani et al. [5] (2012) presented triangular fuzzy numbers in a new modified method of subtraction and division. Javad Vahidi et al. [2] (2013) emphasized basic algebra concepts and Taylor's series to evaluate functions like exponential, square root, log, and inverse exponential for positive fuzzy numbers and reached a conclusion. They also discussed a method for determining the location points involved in the addition, subtraction, multiplication, and division of two trapezoidal fuzzy numbers. Md. Yasin et al. [6] (2016) reported on how to evaluate the multiplication of triangular fuzzy numbers using both the cut approach and the standard approximation method. Clement established the foundation for number theory. Joe Anand, M. et al., [1] (2017) provided information on fuzzy numbers and triangular fuzzy numbers. Senthil Murugan, C., et al., [9] (2019) presents symmetric trapezoidal FRV under arithmetic operations. The fractal and visually appealing The Sierpinski triangle [10] is a fixed set whose general form is an equilateral triangle split recursively into smaller equilateral triangles. In contrast to this concept of the Sierpinski triangle, we have maintained our view of subdividing into tiny ones. The number-theoretic aspect of fuzzy triangular numbers in the right-angle triangle and Sierpinski triangle has been established and studied, according to Sudha T et al. [11] (2019). In 2019, D. Rajan [8] et al. discussed the symmetric TFRV. Making timely, well-informed, and scientifically grounded decisions is crucial in today's competitive environment and can determine whether an organization succeeds or fails [14, 15]. Fuzzy theory is a useful tool for managing under uncertain settings and making optimal decisions, and decision makers have turned to it due to the complexity of the decision-making processes and the uncertainty they face [15, 17]. It was in 1965 that Lotfi A. Zadeh [13] first put forth this notion. An operational research technique that has been in widespread use for many years is linear programming. Although decision makers' opinions are necessary to set the parameters of fuzzy programming, in most circumstances, the calculation cannot be done precisely and decisively due to the constant ambiguity and uncertainty in expert and decision maker judgments. In the fuzzy phase, fuzzy linear programming can alternatively be viewed as a fully fuzzy problem, taking into account all of the variables and decision factors [18, 19]. Thus, a number of different approaches to solving fuzzy linear programming have been suggested for getting the best answer. In the first approach, Lotfi and others tried to solve the problems by getting the parameters as close as possible to the best fuzzy solution using symmetric triangular fuzzy numbers. They accomplished their goal by solving a multiobjective linear programming model. A fuzzy objective function was transformed into a specific state by Kumar et al. [21] using a linear ranking function, which ultimately led to an optimal solution for completely fuzzy situations. Other difficulties were handled by Ezzati et al. [22] utilizing the lexicography method in conjunction with a new algorithm to transform the problem to multiobjective linear programming. In fuzzy situations, Vijayabalan D et al. [23] explore a new approach to comparing the expectancies of stochastic models. Economic theory and actuarial science both rely on stochastic models. The key benefit of this research is that it helps us understand the new concepts of stochastic comparison among stochastic models based on the exponential order. Using the fuzzy mean inactive time order concept, we developed a new definition, solved the preservation properties and theorem, and implemented it. There are numerous uses for stochastic models. Vijayabalan D. et al. [24] developed a new technique for comparing stochastic model predictions in fuzzy environments, enhancing our understanding of actuarial science and economic modeling. They solved preservation properties, used a definition for fuzzy mean inactivity time order, and presented applications. The principles, characteristics, and uses of transforms are examined by Vijayabalan D [25]. Now let's explore one of the fundamental ideas of contemporary signal processing. Z-transforms and integrated Z-transforms will also be covered, with an emphasis on their functions in signal processing. Furthermore, we will provide a unique idea employing fuzzy random variables to illustrate the usefulness of transforms. This study looks at a novel method for employing triangular fuzzy numbers to solve FFLP issues with fuzzy decision parameters and variables. To identify the best totally fuzzy solution for real-world issues, a method based on modified triangular fuzzy numbers and alpha-cut theory is proposed. The new definition of the triangular fuzzy number is applied to solve the problem, which is regarded as a fully fuzzy problem, to maximize the objective function and decision variables. We use the middle object as the objective function and the upper and lower objects as constraints to solve the model. The model's parameters and variables are first written using the modified triangular fuzzy numbers. This approach yields ideal findings that are more precise and also reduces model uncertainty. 1.1 Main contributions of this study (i) The notion of integral, which takes into account every point in the two intervals, respectively, is used to establish a novel distance between intervals. Accordingly, the results will be more accurate and trustworthy the more information that is employed to reflect the distance. (ii) We propose a variety of novel distances between fuzzy numbers and verify their properties using the previous interval distance as a basis and a reduction function as a variable. Then, a novel fuzzy distance between each fuzzy number and the ideal fuzzy number is naturally proposed as a basis for ranking fuzzy numbers. (iii) The benefits of the recently suggested interval distance, fuzzy distance, and ranking algorithms are illustrated through a number of numerical evaluations and comparisons. This work's remaining sections are arranged as follows. The ideas surrounding distance and fuzzy numbers are reviewed in Section 2. In Section 3, the distance between intervals is first introduced using the integral definition. Then, a number of distances between fuzzy numbers are suggested based on the newly proposed interval distance and the general qualities are illustrated. A numerical analysis will be presented in Section 4. Lastly, a few conclusions are highlighted in Sect. 5.

2. Preliminaries

An impetuous on fuzzy random variables in relation to fuzzy set, membership grade, α-cut, beam of a fuzzy set, and fractal is studied.

Definition 2.1 (Kwakernaak Fuzzy random variable (*FRV*)) Let (S, A, P) be a probability space and $F(\mathbb{R})$ be the set of all fuzzy numbers on $\mathbb{R}$ with compact $\alpha-$ cut for $\alpha \in [0,1]$. A fuzzy number $X : R \to [0,1]$ can bebranded by its $\alpha-\text{cuts} : x_\alpha = \{x \in R/X(x) \geq \alpha,\ \text{if}\ \alpha \in (0,1]\}$. This represents the interval of values where the degree of membership is at least α. $X_\alpha = \text{cl}\{\sup X\}$, if $\alpha = 0$. This captures the closure of the supremum of the fuzzy set, indicating its spread at the lowest level of membership.

1. An FRV $X : S \to F(\mathbb{R})$ maps outcomes ω from the sample space S to fuzzy number in $F(\mathbb{R})$.

2. For each α, the following must hold:

Infimum: The real valued mapping $\inf\{X_\alpha : S \to \mathbb{R}\}$, satisfying $\inf X_\alpha(\omega) : \omega \in S = \inf\{(X(\omega))_\alpha\}$.
Supremum: $\sup\{X_\alpha : S \to \mathbb{R}\}$, satisfying $\sup X_\alpha(\omega) : \omega \in S = \sup\{(X(\omega))_\alpha\}$.

These relationships indicate that for a fixed α, the infimum and supremum of the fuzzy random variable over the sample space are themselves real-valued random variables.

Definition 2.2 (Triangular fuzzy random variable (*TFRV*)) Let $X \sim TFRV(\mu, \sigma)$, denotes a fuzzy set encapsulating a characteristic function and a elementary set of cardinal variables. A FRV X is a map $X : S \to F(\mathbb{R})$, fulfilling the following conditions:

1. The membership grade of triangular fuzzy number is

$$\mu_X(x) = \begin{cases} 0, & \text{for } x \leq (\mu-\sigma)\&(\mu+\sigma) \leq x \\ \left(\dfrac{X_\alpha^L - \mu + \sigma}{\sigma}\right), & \text{for } \mu-\sigma \leq x \leq \mu \\ \left(\dfrac{\mu + \sigma - X_\alpha^U}{\sigma}\right), & \text{for } \mu \leq x \leq \mu+\sigma \end{cases}$$

Here, $X_\alpha^L(x) = \inf\{x \in (\mu-\sigma, \mu) : \left(\dfrac{X_\alpha^L - \mu + \sigma}{\sigma}\right) \geq \alpha$ and $X_\alpha^U(x) = \sup\{x \in (\mu, \mu+\sigma) : \left(\dfrac{\mu + \sigma - X_\alpha^U}{\sigma}\right) \geq \alpha\}$, where $(0 < \alpha \leq 1)$ both are finite real - valued random variables defined on (S, A, P) such that the expectations $E(X_\alpha^L), E(X_\alpha^U)$ exists.

2. The α-cut of the triangular FRV is derived as below, it mentioned the boundaries $[X^L_\alpha, X^U_\alpha]$ and defined as,

$$\left(\frac{X^L_\alpha - \mu + \sigma}{\sigma}\right) \geq \alpha = (X^L_\alpha + (1-\alpha)\sigma - \mu) \geq 0 \ \&$$
$$\left(\frac{\mu + \sigma - X^U_\alpha}{\sigma}\right) \geq \alpha = (X^U_\alpha - (1-\alpha)\sigma - \mu) \leq 0$$

Thus, the membership grade of the triangular FRV is

$$P\left\{(X^L_\alpha + (1-\alpha)\sigma - \mu) \geq 0 \vee (X^U_\alpha - (1-\alpha)\sigma - \mu) \leq 0\right\}, \tag{2.1}$$

where $(0 < \alpha \leq 1)$ is a constant.

If $\alpha = 1$, then X is a normal triangular FRV; it is obvious that in a non-normal triangular FRV. If the membership grade $\mu_X(x)$ is continually linear, then X is referred to as a $TFRV$ and is usually marked as $X \sim TFRV(\mu, \sigma; \alpha)$. Here, among the three sides of the *TFRV*, X, two sides (i.e.,) the increasing $((X^L_\alpha + (1-\alpha)\sigma - \mu) \geq 0)$ and decreasing $((X^U_\alpha - (1-\alpha)\sigma - \mu) \leq 0)$ functions $(0 < \alpha \leq 1)$ are fuzzy and that the third side (support of X, $\alpha = 0$) is real. So that the $TFRV$ have both fuzzy and crisp set. Every $(\alpha > 0)$-cut values gives the sub-intervals (acceptance region) in support of X, remaining part of the support of X is called the rejection region (Note 3).

Note 1

Taking monotonous and continual functions $X^L_\alpha : [a, b] \to [0, \alpha]$ and $X^U_\alpha : [b, c] \to [0, \alpha]$, implying that their inverse functions also exhibit with a nature of the above mentioned conditions. Let $f^L_\alpha : [0, \alpha] \to [a, b]$ and $f^U_\alpha : [0, \alpha] \to [b, c]$ tends to be the inverse functions of X^L_α and X^U_α, respectively. Then $f^L_\alpha(y)$ and $f^U_\alpha(y)$ would integrate on the closed interval $[0, \alpha]$, establishing the conditionality's abide to inverse triangular FRV.

Note 2

The location points of triangular FRV, $(\mu - \sigma, \mu, \mu + \sigma)$ are in equidistance. The $\alpha -$cut membership grade is of strictly increasing function on $[\mu - \sigma, \mu]$ is $F(X) = P\left\{\left(X^L_\alpha + (1-\alpha)\sigma - \mu\right) \geq 0\right\} = 0.5$ and adhere to decrease function on $[\mu, \mu + \sigma]$ is

$$\overline{F(X)} = P\left\{\left(X^U_\alpha - (1-\alpha)\sigma - \mu\right) \leq 0\right\} = 0.5.$$

$\therefore F(X) + \overline{F(X)} = P\left\{\left(X^L_\alpha + (1-\alpha)\sigma - \mu\right) \geq 0 \vee \left(X^U_\alpha - (1-\alpha)\sigma - \mu\right) \leq 0\right\} = 1$

For this reason, the *TFRV* is said to be symmetric and it is denoted by $X \sim STFRV(\mu, \sigma; \alpha)$.

Put $\alpha = 1, P\left\{\left(X^L_\alpha - \mu_1\right) \geq 0 \vee \left(X^U_\alpha - \mu_1\right) \leq 0\right\}$, it presents, the mean of *TFRV* is μ_1.

Put $\alpha = 0, P\left\{\left(X^L_\alpha + \sigma_1 - \mu_1\right) \geq 0 \vee \left(X^U_\alpha - \sigma_1 - \mu_1\right) \leq 0\right\}$, it gives, the spreads (standard deviation) of triangular fuzzy random variable, both sides of its mean is σ_1.

Note 3

An endeavour is made to find out the area of acceptance and the level of significance $(\alpha -$ cut$)$ which directly to the rejection region thereto D. Rajan [8], the triangular fuzzy random variable (μ, σ) upholds a normal distribution with parameters (μ, σ) tending to archive all properties of normal distribution. It shows a way of finding out In all sub triangles in this methodology using triangular fuzzy random variable of iteration - from first onwards till standard deviation reading out to the status of nullification.

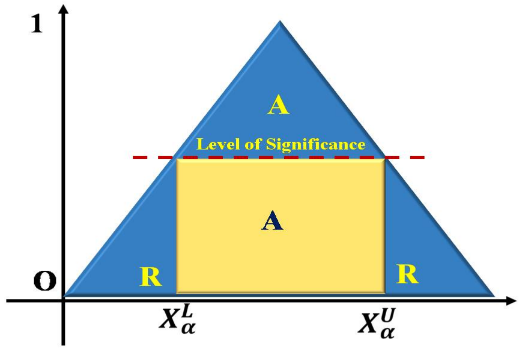

Figure 1: X_α^L and X_α^U - Lower and Upper critical values

A - Acceptance region, R - Rejection region and Level of significance (α)

The (α-cut) values indicate the lower and upper limit (α-cut) in increasing and decreasing function. This point is called a critical value or significant value and it is denoted by $\left(X_\alpha^L, X_\alpha^U\right)$, to draw the straight line from these critical values to the support of triangular fuzzy random variable. This region within the critical values $\left(X_\alpha^L, X_\alpha^U\right)$ is called an acceptance region and the remaining called as a rejection region.

3. Put $(0 < \alpha \leq 0.5)$ in Equation (2.1), we Grow the Symmetric Trapezoidal Fuzzy Random Variable (*STRFRV*).

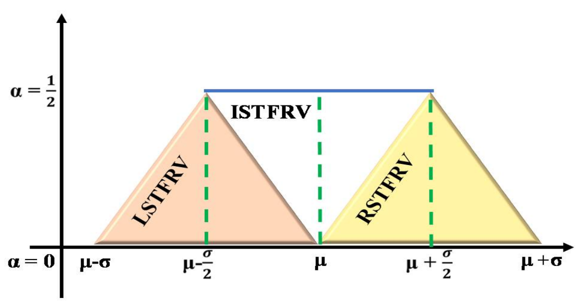

Figure 2: *STRFRV* = *LSTFRV*+*ISTFRV*+*RSTFRV*

The points $\left(\mu - \sigma, \mu - \dfrac{\sigma}{2}, \mu, \mu + \dfrac{\sigma}{2}, \mu + \sigma\right)_{\alpha=0.5}$, is a symmetric trapezoidal FRV with respect to μ. Further, noting that all are equidistant location points. One can construct the inverse $TFRV$ $\left(\mu - \dfrac{\sigma}{2}, \mu, \mu + \dfrac{\sigma}{2}\right)$ with parameters $(\mu, \dfrac{\sigma}{2})$ respectively, from the symmetric trapezoidal *FRV*. Residues are also TFRV. As is crystal clear that an integrated region of left, right and inverse symmetric triangular *FRV's* is a symmetric trapezoidal FRV establishing from above pictorial representation. From the above details, we can make sub-triangles in triangular fuzzy random variable. Here, also the increasing and decreasing sides of the sub-triangles are fuzzy and supports of the sub-triangles are crisp or real. So that the generation of inner sub-triangular FRV of the triangular FRV are possible, till the standard deviation of inner sub-triangle tends to zero.This concept can be used in the survey tool "triangulation" from the *TFRV*.

4. The Membership Gradeof Sub Triangles of Triangulation

The triangulation of iteration-1 has four triangles including one inverse triangle such as *USTFRV, ISTFRV, LSTFRV* and *RSTFRV* as given below:

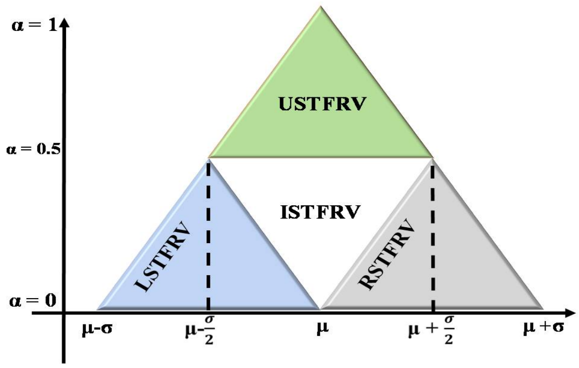

Figure 3: Triangulation of iteration - 1

Definition 4.1 (Symmetric Trapezoidal Fuzzy Random Variables (*STRFRV*))
A *FRV* $X : S \to F(R)$ up holding the below axioms:

1. The symmetric trapezoidal fuzz's membership grade is

$$\mu_A(X) = \begin{cases} 0, & \text{for } X \le \mu - \sigma \text{ and } \mu + \sigma \le X \\ \dfrac{X_\alpha^L - \mu + \sigma}{\frac{\sigma}{2}}, & \text{for } \mu - \sigma \le X \le \mu - \frac{\sigma}{2} \\ \alpha, & \text{for } \mu - \frac{\sigma}{2} \le X \le \mu + \frac{\sigma}{2} \\ \dfrac{\mu + \sigma - X_\alpha^U}{\frac{\sigma}{2}}, & \text{for } \mu + \frac{\sigma}{2} \le X \le \mu + \sigma \end{cases}$$

Here, $X_\alpha^L(x) = \inf\left\{ x \in \left(\mu - \sigma, \mu - \frac{\sigma}{2} z\right) : \dfrac{X_\alpha^L - \mu + \sigma}{\frac{\sigma}{2}} \ge \alpha \right\}$, $\dfrac{\left(\mu - \frac{\sigma}{2}\right) + \left(\mu + \frac{\sigma}{2}\right)}{2}$ is mean (Symmetric point)

and $X_\alpha^U(x) = \sup\left\{ x \in \left(\mu + \frac{\sigma}{2}, \mu + \sigma\right) : \dfrac{\mu + \sigma - X_\alpha^U}{\frac{\sigma}{2}} \ge \alpha \right\}$, where $(0 < \alpha \le 1)$ are finite real - valued random variables defined on such (S, A, P) that the expectations $E\left(X_\alpha^L\right), E\left(X_\alpha^U\right)$ comes into existence.

2. The derivative of α-cut of $\left[X_\alpha^L, X_\alpha^U\right]$ is

$$\frac{X_\alpha^L - \mu + \sigma}{\frac{\sigma}{2}} \ge \alpha \Rightarrow \left(X_\alpha^L + \left(1 - \frac{\alpha}{2}\right)\sigma - \mu\right) \ge 0 \ \&$$

$$\frac{\mu + \sigma - X_\alpha^U}{\frac{\sigma}{2}} \ge \alpha \Rightarrow \left(X_\alpha^U - \left(1 - \frac{\alpha}{2}\right)\sigma - \mu\right) \le 01$$

Therefore, the α-cut of membership grade of the trapezoidal *FRV* is

$$P\left\{\left(X_\alpha^L + \left(1 - \frac{\alpha}{2}\right)\sigma - \mu\right) \ge 0 \vee \left(X_\alpha^U - \left(1 - \frac{\alpha}{2}\right)\sigma - \mu\right) \le 0\right\}, \tag{2.2}$$

where $0 < \alpha \leq 1$ is a standardised.

If $\alpha = 1$, then X is a normal trapezoidal FRV; otherwise X is said to be a non-normal trapezoidal FRV. If the membership grade $\mu_X(x)$ is piecewise linear, then X is referred to as a trapezoidal FRV, it is also called symmetric trapezoidal FRV and is denoted by $X \sim STRFRV(\mu, \sigma; \alpha)$.

Definition 4.2 (Left Symmetric Triangular Fuzzy Random Variable (*LSTFRV*))
Let $X \sim LSTFRV\left(\mu-\sigma, \mu-\frac{\sigma}{2}, \mu\right)$ is moreover a fuzzy set with a membership gradeand a fundamental collection of underpinning variables. A FRV $X : S \to F(R)$ meeting the requirements listed below:

1. The characteristic function of LST fuzzy number is derived as follows:

$$\mu(X) = \begin{cases} 0, & \text{for } X \leq \mu-\sigma \text{ and } \mu \leq X \\ \dfrac{X_\alpha^L - (\mu-\sigma)}{\frac{\sigma}{2}}, & \text{for } \mu-\sigma \leq X \leq \mu - \dfrac{\sigma}{2} \\ \dfrac{\mu - X_\alpha^U}{\frac{\sigma}{2}}, & \textbf{for } \mu - \dfrac{\sigma}{2} \leq X \leq \mu \end{cases}$$

where, $X_\alpha^L(x) = \inf\left\{x \in \left(\mu-\sigma, \mu-\frac{\sigma}{2}\right) : \dfrac{X_\alpha^L - (\mu-\sigma)}{\frac{\sigma}{2}} \geq \alpha\right\}$

and $X_\alpha^U(x) = \sup\left\{x \in \left(\mu-\frac{\sigma}{2}, \mu\right) : \dfrac{\mu - X_\alpha^U}{\frac{\sigma}{2}} \geq \alpha\right\}$, here $0 < \alpha \leq 1$, are finite real valued random variables defined on such (S, A, P) that the mathematical expectations $E\left(X_\alpha^L\right), E\left(X_\alpha^U\right)$ exists.

2. The α-cut of $\left[X_\alpha^L, X_\alpha^U\right]$ is derived as follows

$$\frac{X_\alpha^L - (\mu-\sigma)}{\frac{\sigma}{2}} \geq \alpha \Rightarrow \left(X_\alpha^L - (\mu-\sigma) - \frac{\alpha\sigma}{2}\right) \geq 0 \;\&$$

$$\frac{\mu - X_\alpha^U}{\frac{\sigma}{2}} \geq \alpha \Rightarrow \left(X_\alpha^U - \mu + \frac{\alpha\sigma}{2}\right) \leq 0$$

Hence, the α-cut of membership grade of $LSTFRV$ is

$$P\left\{\left(X_\alpha^L - (\mu-\sigma) - \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X_\alpha^U - \mu + \frac{\alpha\sigma}{2}\right) \leq 0\right\},$$

where $0 < \alpha \leq 1$ is a constant.

If $\alpha = 1, \Rightarrow P\left\{\left(X_\alpha^L \geq \mu - \frac{\sigma}{2}\right) \vee \left(X_\alpha^U \leq \mu - \frac{\sigma}{2}\right)\right\}$

If $\alpha = 0, \Rightarrow P\left\{X_\alpha^L \geq \mu-\sigma \vee X_\alpha^U \leq \mu\right\}$

Hence, from α-values, the location points of $LSTFRV$ is $\left(\mu-\sigma, \mu-\frac{\sigma}{2}, \mu\right)$.

Definition 4.3 (Right Symmetric Triangular Fuzzy Random Variable (*RSTFRV*))
Let $X \sim RSTFRV\left(\mu, \mu+\frac{\sigma}{2}, \mu+\sigma\right)$ is also a fuzzy set which contains a membership grade and a elementary set of underlying variables. A FRV, X is a map $X : S \to F(R)$ sustaining the following axioms:

1. The membership grade of RST fuzzy number is

$$\mu(X) = \begin{cases} 0, & \text{for } X \le \mu \text{and } \mu + \sigma \le X \\ \dfrac{x-\mu}{\frac{\sigma}{2}}, & \text{for } \mu \le X \le \mu + \dfrac{\sigma}{2} \\ \dfrac{(\mu+\sigma)-x}{\frac{\sigma}{2}}, & \text{for } \mu + \dfrac{\sigma}{2} \le X \le \mu + \sigma \end{cases}$$

where,

$$X_\alpha^L(x) = \inf\left\{ x \in \left(\mu, \mu + \frac{\sigma}{2}\right) : \frac{X_\alpha^L - \mu}{\frac{\sigma}{2}} \ge \alpha \right\} \ \&$$
$$X_\alpha^U(x) = \sup\left\{ x \in \left(\mu + \frac{\sigma}{2}, \mu + \sigma\right) : \frac{(\mu+\sigma) - X_\alpha^U}{\frac{\sigma}{2}} \ge \alpha \right\},$$

where $0 < \alpha \le 1$ are finite real - valued random variables defined on such (S, A, P) that the mathematical expectations $E\left(X_\alpha^L\right), E\left(X_\alpha^U\right)$ exists.

2. The α-cut of $\left[X_\alpha^L, X_\alpha^U\right]$ is derived as follows

$$\frac{X_\alpha^L - \mu}{\frac{\sigma}{2}} \ge \alpha \Rightarrow \left(X_\alpha^L - \mu - \frac{\alpha\sigma}{2}\right) \ge 0 \ \&$$
$$\frac{(\mu+\sigma) - X_\alpha^U}{\frac{\sigma}{2}} \ge \alpha \Rightarrow \left(X_\alpha^U - (\mu+\sigma) + \frac{\alpha\sigma}{2}\right) \le 0$$

Hence, the α-cut of membership grade of $RSTFRV$ is

$$P\left\{\left(X_\alpha^L - \mu - \frac{\alpha\sigma}{2}\right) \ge 0 \vee \left(X_\alpha^U - (\mu+\sigma) + \frac{\alpha\sigma}{2}\right) \le 0\right\}$$

If $\alpha = 1 \Rightarrow P\left\{X_\alpha^L \ge \mu + \frac{\sigma}{2} \vee X_\alpha^U \le \mu + \frac{\sigma}{2}\right\}$ &
If $\alpha = 0 \Rightarrow P\left\{\left(X_\alpha^L \ge \mu\right) \vee \left(X_\alpha^U \le \mu + \sigma\right)\right\}$

Hence, from α-values, the position points of $RSTFRV$ is $(\mu, \mu + \frac{\sigma}{2}, \mu + \sigma)$.

Definition 4.4 (Inverse Symmetric Triangular Fuzzy Random Variable ($ISTFRV$))
Let $X \sim ISTFRV\left(\mu - \frac{\sigma}{2}, \mu, \mu + \frac{\sigma}{2}\right)$ is moreover a fuzzy set with a membership grade and a fundamental collection of underpinning variables. A $FRV\ X : S \to F(R)$ meeting the requirements listed below:

1. The membership grade of IST fuzzy number is

$$\mu(X) = \begin{cases} 0, & \text{for } X \le \left(\mu - \dfrac{\sigma}{2}\right) \text{ and } \left(\mu + \dfrac{\sigma}{2}\right) \le X \\ \dfrac{X_\alpha^L - \left(\mu - \frac{\sigma}{2}\right)}{\frac{\sigma}{2}}, & \text{for } \left(\mu - \dfrac{\sigma}{2}\right) \le X \le \mu \\ \dfrac{\left(\mu + \frac{\sigma}{2}\right) - X_\alpha^U}{\frac{\sigma}{2}}, & \text{for } \mu \le X \le \left(\mu + \dfrac{\sigma}{2}\right) \end{cases}$$

Here,

$$X_\alpha^L(x) = \inf\left\{x \in \left(\mu - \frac{\sigma}{2}, \mu\right) : \frac{X_\alpha^L - \left(\mu - \frac{\sigma}{2}\right)}{\frac{\sigma}{2}} \geq \alpha\right\} \ \&$$

$$X_\alpha^U(x) = \sup\left\{x \in \left(\mu, \mu + \frac{\sigma}{2}\right) : \frac{\left(\mu + \frac{\sigma}{2}\right) - X_\alpha^U}{\frac{\sigma}{2}} \geq \alpha\right\}.$$

Here $0 < \alpha \leq 1$, are defined on (S, A, P) such that the expectations $E\left(X_\alpha^L\right), E\left(X_\alpha^U\right)$ exist, and are finite real - valued random variables.

2. The α-cut of $\left[X_\alpha^L, X_\alpha^U\right]$ is derived as follows

$$\frac{X_\alpha^L - \left(\mu - \frac{\sigma}{2}\right)}{\frac{\sigma}{2}} \geq 1 - \alpha \Rightarrow \left(X_\alpha^L - \mu + \frac{\alpha\sigma}{2}\right) \geq 0 \ \&$$

$$\frac{\left(\mu + \frac{\sigma}{2}\right) - X_\alpha^U}{\frac{\sigma}{2}} \geq 1 - \alpha \Rightarrow X_\alpha^U - \left(\mu + \frac{\sigma}{2}\right) + \frac{(1-\alpha)\sigma}{2} \leq 0$$

Hence the $\alpha-cut$ of membership grade of $ISTFRV$ is

$$P\left\{\left(X_\alpha^L - \mu + \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X_\alpha^U - \left(\mu + \frac{\sigma}{2}\right) + \frac{(1-\alpha)\sigma}{2}\right) \leq 0\right\},$$

where $0 < \alpha \leq 1$

If $\alpha = 1, P\left\{\left(X_\alpha^L \geq \mu - \frac{\sigma}{2}\right) \vee \left(X_\alpha^U \leq \mu + \frac{\sigma}{2}\right)\right\}$

If $\alpha = 0, P\left\{\left(X_\alpha^L \geq \mu\right) \vee \left(X_\alpha^U \leq \mu\right)\right\}$

Hence, from α-values, the location points of $ISTFRV$ is $\left(\mu - \frac{\sigma}{2}, \mu, \mu + \frac{\sigma}{2}\right)$.

Definition 4.5 (Upper Symmetric Triangular Fuzzy Random Variable ($\boldsymbol{USTFRV}$))
Let $X \sim USTFRV(\mu - \sigma, \mu, \mu + \sigma)$ is moreover a fuzzy set with a membership grade and a fundamental collection of underpinning variables. A $FRV\ X : S \to F(R)$ meeting the requirements listed below:

1. The membership grade of UST fuzzy number is

$$\mu(X) = \begin{cases} 0, & \text{for } X \leq \mu - \sigma \text{ and } \mu + \sigma \leq X \\ \frac{X_\alpha^L - (\mu - \sigma)}{\sigma}, & \text{for } \mu - \sigma \leq X \leq \mu \\ \frac{(\mu + \sigma) - X_\alpha^U}{\sigma}, & \text{for } \mu \leq X \leq \mu + \sigma \end{cases}$$

Here, $X_\alpha^L(x) = \inf\left\{x \in \left(\mu - \frac{\sigma}{2}, \mu\right) : \frac{X_\alpha^L - (\mu - \sigma)}{\sigma} \geq \frac{1}{2} + \alpha\right\}, \alpha \in [0, 0.5]$

and $X_\alpha^U(x) = \sup\left\{x \in \left(\mu, \mu + \frac{\sigma}{2}\right) : \frac{(\mu + \sigma) - X_\alpha^U}{\sigma} \geq \frac{1}{2} + \alpha\right\}, \alpha \in [0, 0.5]$ are finite real valued random variables defined on such (S, A, P) that mathematical expectations $E\left(X_\alpha^L\right), E\left(X_\alpha^U\right)$ exists.

2. The derivation of α-cut $\left[X_\alpha^L, X_\alpha^U\right]$ is

$$\frac{X_\alpha^L - (\mu - \sigma)}{\sigma} \geq \frac{1}{2} + \alpha \frac{(\mu + \sigma) - X_\alpha^U}{\sigma} \geq \frac{1}{2} + \alpha,$$

and we get

$$X_\alpha^L - \mu + \left(\frac{1}{2} - \alpha\right)\sigma \geq 0$$
$$X_\alpha^U - \mu - \left(\frac{1}{2} - \alpha\right)\sigma \leq 0$$

Hence the $\alpha-cut$ of membership grade of $USTFRV$ is

$$P\left\{\left(X_\alpha^L - \mu + \left(\frac{1}{2} - \alpha\right)\sigma\right) \geq 0 \vee \left(X_\alpha^U - \mu - \left(\frac{1}{2} - \alpha\right)\sigma\right) \leq 0\right\}, \alpha \in [0, 0.5]$$

If $\alpha = 0, P\left\{\left(X_\alpha^L \geq \mu - \frac{\sigma}{2}\right) \vee \left(X_\alpha^U \leq \mu + \frac{\sigma}{2}\right)\right\}$

If $\alpha = 0.5, P\left\{X_\alpha^L \geq \mu \vee X_\alpha^U \leq \mu\right\}$

Hence, from α-values, the position points of $USTFRV$ is $\left(\mu - \frac{\sigma}{2}, \mu, \mu + \frac{\sigma}{2}\right)$.

5. Relation among *STRFRV, USTFRV, LSTFRV, ISTFRV* and *RSTFRV*

Theorem 5.1 *If $X \sim STRFRV(\mu, \sigma)$, then prove that $STRFRV = LSTFRV + ISTFRV + RSTFRV$ (i.e.,)*

$$\begin{aligned}
&P\left\{\left(X_\alpha^L + \left(1 - \frac{\alpha}{2}\right)\sigma - \mu\right) \geq 0 \vee \left(X_\alpha^U - \left(1 - \frac{\alpha}{2}\right)\sigma - \mu\right) \leq 0\right\}, \alpha \in [0, 1] \\
&= P\left\{\left(X_\alpha^L - (\mu - \sigma) - \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X_\alpha^U - \mu + \frac{\alpha\sigma}{2}\right) \leq 0\right\}, \alpha \in [0, 1] \\
&+ P\left\{\left(X - \mu + \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X - \left(\mu + \frac{\sigma}{2}\right) + \frac{(1-\alpha)\sigma}{2}\right) \geq 0\right\}, \alpha \in [0, 1] \\
&+ P\left\{\left(X_\alpha^L - \mu - \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X_\alpha^U - (\mu + \sigma) + \frac{\alpha\sigma}{2}\right) \leq 0\right\}, \alpha \in [0, 1]
\end{aligned} \tag{3.1}$$

Proof:

Let us take LHS of (3.1),

$$\begin{aligned}
&= P\left\{\left(X_\alpha^L - (\mu - \sigma) - \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X_\alpha^U - \mu + \frac{\alpha\sigma}{2}\right) \leq 0\right\} \\
&+ P\left\{\left(X - \mu + \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X - \left(\mu + \frac{\sigma}{2}\right) + \frac{(1-\alpha)\sigma}{2}\right) \geq 0\right\} \\
&+ P\left\{\left(X_\alpha^L - \mu - \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X_\alpha^U - (\mu + \sigma) + \frac{\alpha\sigma}{2}\right) \leq 0\right\}
\end{aligned}$$

Put $\alpha = 0$, we get

$$\begin{aligned}
&P\left\{\left(X_\alpha^L - (\mu - \sigma)\right) \geq 0 \vee \left(X_\alpha^U - \mu\right) \leq 0\right\} \\
&\quad + P\left\{(X - \mu) \geq 0 \vee (X - \mu) \leq 0\right\} \\
&\quad + P\left\{\left(X_\alpha^L - \mu\right) \geq 0 \vee \left(X_\alpha^U - (\mu + \sigma)\right) \leq 0\right\} \\
&= P\left\{X_\alpha^L \geq \mu - \sigma \vee X_\alpha^U \leq \mu\right\} \\
&\quad + P\left\{X_\alpha^L \geq \mu \vee \left(X_\alpha^U \leq \mu\right)\right\} \\
&\quad + P\left\{X_\alpha^L \geq \mu \vee X_\alpha^U \leq \mu + \sigma\right\} \\
&= P\left\{X_\alpha^L \geq \mu - \sigma \vee X_\alpha^U \leq \mu + \sigma\right\}
\end{aligned}$$

Put $\alpha = 1$ in LHS of equation(3.1), we get

$$
\begin{aligned}
&= P\left\{\left(X_\alpha^L - \left(\mu - \frac{\sigma}{2}\right)\right) \geq 0 \vee \left(X_\alpha^U - \left(\mu - \frac{\sigma}{2}\right)\right) \leq 0\right\} \\
&+ P\left\{\left(X - \left(\mu - \frac{\sigma}{2}\right)\right) \geq 0 \vee \left(X - \left(\mu + \frac{\sigma}{2}\right)\right) \geq 0\right\} \\
&+ P\left\{\left(X_\alpha^L - \left(\mu + \frac{\sigma}{2}\right)\right) \geq 0 \vee \left(X_\alpha^U - \left(\mu + \frac{\sigma}{2}\right)\right) \leq 0\right\} \\
&= P\left\{X_\alpha^L \geq \mu - \frac{\sigma}{2} \vee X_\alpha^U \leq \mu - \frac{\sigma}{2}\right\} \\
&+ P\left\{X_\alpha^L \geq \mu - \frac{\sigma}{2} \vee X_\alpha^U \leq \mu + \frac{\sigma}{2}\right\} \\
&+ P\left\{X_\alpha^L \geq \mu + \frac{\sigma}{2} \vee X_\alpha^U \leq \mu + \frac{\sigma}{2}\right\} \\
&= P\left\{X_\alpha^L \geq \mu - \frac{\sigma}{2} \vee X_\alpha^U \leq \mu + \frac{\sigma}{2}\right\}
\end{aligned}
$$

Hence, from α-values, the location points are $\left(\mu - \sigma, \mu - \frac{\sigma}{2}, \mu + \frac{\sigma}{2}, \mu + \sigma\right)$.

The distance between location points within the first and last pairs is equal and is doubled within the mid pairs. It is implied from the above derivative (2.2) and (3.1), formerly noting that 2, 3 and 4 referred it as trapezoidal FRV also known as symmetric trapezoidal FRV and the location points $\left(\mu - \sigma, \mu - \frac{\sigma}{2}, \mu, \mu + \frac{\sigma}{2}, \mu + \sigma\right)$, are equidistance. Hence, the membership grade of $STRFRV$ is

$$
P\left\{\left(X_\alpha^L + \left(1 - \frac{\alpha}{2}\right)\sigma - \mu\right) \geq 0 \vee \left(X_\alpha^U - \left(1 - \frac{\alpha}{2}\right)\sigma - \mu\right) \leq 0\right\}.
$$

This is the RHS. Therefore, $STRFRV = LSTFRV + ISTFRV + RSTFRV$.

Hence, (3.1) is proved.

Theorem 5.2 *If* $X\sim STFRV(\mu, \sigma)$, *then prove that* $STFRV = LSTFRV + ISTFRV + RSTFRV + USTFRV$

$$
\begin{aligned}
&P\left\{\left(X_\alpha^L + (1-\alpha)\sigma - \mu\right) \geq 0 \vee \left(X_\alpha^U - (1-\alpha)\sigma - \mu\right) \leq 0\right\}, \alpha \in [0,1] \\
&= P\left\{\left(X_\alpha^L - (\mu - \sigma) - \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X_\alpha^U - \mu + \frac{\alpha\sigma}{2}\right) \leq 0\right\}, \alpha \in [0,1] \\
&+ P\left\{\left(X - \mu + \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X - \left(\mu + \frac{\sigma}{2}\right) + \frac{(1-\alpha)\,\sigma}{2}\right) \geq 0\right\}, \alpha \in [0,1] \\
&+ P\left\{\left(X_\alpha^L - \mu - \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X_\alpha^U - (\mu + \sigma) + \frac{\alpha\sigma}{2}\right) \leq 0\right\}, \alpha \in [0,1] \\
&+ P\left\{\left(X_\alpha^L - \mu + \left(\frac{1}{2} - \alpha\right)\sigma\right) \geq 0 \vee X_\alpha^U - \mu - \left(\frac{1}{2} - \alpha\right)\sigma \leq 0\right\}, \alpha \in [0,0.5]
\end{aligned}
\tag{3.2}
$$

Proof:

Let us take LHS of (3.2),

$$
\begin{aligned}
&= P\left\{\left(X_\alpha^L - (\mu - \sigma) - \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X_\alpha^U - \mu + \frac{\alpha\sigma}{2}\right) \leq 0\right\}, \alpha \in [0,1] \\
&+ P\left\{\left(X - \mu + \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X - \left(\mu + \frac{\sigma}{2}\right) + \frac{(1-\alpha)\,\sigma}{2}\right) \geq 0\right\}, \alpha \in [0,1] \\
&+ P\left\{\left(X_\alpha^L - \mu - \frac{\alpha\sigma}{2}\right) \geq 0 \vee \left(X_\alpha^U - (\mu + \sigma) + \frac{\alpha\sigma}{2}\right) \leq 0\right\}, \alpha \in [0,1] \\
&+ P\left\{\left(X_\alpha^L - \mu + \left(\frac{1}{2} - \alpha\right)\sigma\right) \geq 0 \vee X_\alpha^U - \mu - \left(\frac{1}{2} - \alpha\right)\sigma \leq 0\right\}, \alpha \in [0,0.5]
\end{aligned}
$$

Put $\alpha = 0$, we get

$$\begin{aligned}
&= P\left\{\left(X_\alpha^L - (\mu-\sigma)\right) \geq 0 \vee \left(X_\alpha^U - \mu\right) \leq 0\right\} + P\left\{(X-\mu) \geq 0 \vee (X-\mu) \leq 0\right\} \\
&+ P\left\{\left(X_\alpha^L - \mu\right) \geq 0 \vee \left(X_\alpha^U - (\mu+\sigma)\right) \leq 0\right\} + P\left\{X_\alpha^L - \mu + \frac{\sigma}{2} \geq 0 \vee X_\alpha^U - \mu - \frac{\sigma}{2} \leq 0\right\} \\
&= P\left\{X_\alpha^L \geq \mu - \sigma \vee X_\alpha^U \leq \mu\right\} + P\left\{X_\alpha^L \geq \mu \vee X_\alpha^U \leq \mu\right\} \\
&+ P\left\{X_\alpha^L \geq \mu \vee X_\alpha^U \leq \mu + \sigma\right\} + P\left\{X_\alpha^L \geq \mu - \frac{\sigma}{2} 0 \vee X_\alpha^U \leq \mu + \frac{\sigma}{2}\right\} \\
&= P\left\{X_\alpha^L \geq \mu - \sigma \vee X_\alpha^U \leq \mu + \sigma\right\}
\end{aligned}$$

Put $\alpha = 0.5$ and 1 in LHS of equation (3.2), we get

$$\begin{aligned}
&= P\left\{\left(X_\alpha^L - (\mu-\sigma) - \frac{\sigma}{2}\right) \geq 0 \vee \left(X_\alpha^U - \mu + \frac{\sigma}{2}\right) \leq 0\right\} \\
&+ P\left\{\left(X - \mu + \frac{\sigma}{2}\right) \geq 0 \vee \left(X - \left(\mu + \frac{\sigma}{2}\right)\right) \geq 0\right\} \\
&+ P\{(X_\alpha^L - \mu - \frac{\sigma}{2}) \geq 0 \vee (X_\alpha^U - (\mu+\sigma) + \frac{\sigma}{2}) \leq 0)\} \\
&+ P\left\{X_\alpha^L - \mu \geq 0 \vee X_\alpha^U - \mu \leq 0\right\}, \alpha = 0.5 \\
&= P\left\{X_\alpha^L \geq \mu - \frac{\sigma}{2} \vee X_\alpha^U \leq \mu - \frac{\sigma}{2})\right\} + P\left\{X_\alpha^L \geq \mu - \frac{\sigma}{2} \vee X_\alpha^U \leq \mu + \frac{\sigma}{2}\right\} \\
&+ P\left\{X_\alpha^L \geq \mu + \frac{\sigma}{2} \vee X_\alpha^U \leq \mu + \frac{\sigma}{2}\right\} + P\left\{X_\alpha^L \geq \mu \vee X_\alpha^U \leq \mu\right\}, \alpha = 0.5 \\
&= P\left\{X_\alpha^L \geq \mu \vee X_\alpha^U \leq \mu\right\}
\end{aligned}$$

Hence, from α-values, the location points are $(\mu - \sigma, \mu, \mu + \sigma)$.

The location points of 0-contrives triangulation or symmetric triangular FRV.

Hence, the characteristic function of $STFRV$ is

$$P\left\{\left(X_\alpha^L + (1-\alpha)\sigma - \mu\right) \geq 0 \vee \left(X_\alpha^U - (1-\alpha)\sigma - \mu\right) \leq 0\right\}, \alpha \in [0,1]$$

This is the RHS. Therefore, $STFRV = LSTFRV + ISTFRV + RSTFRV + USTFRV$

Hence, (3.2) is proved.

Solution: Given $\mu = 3, \sigma = 1$

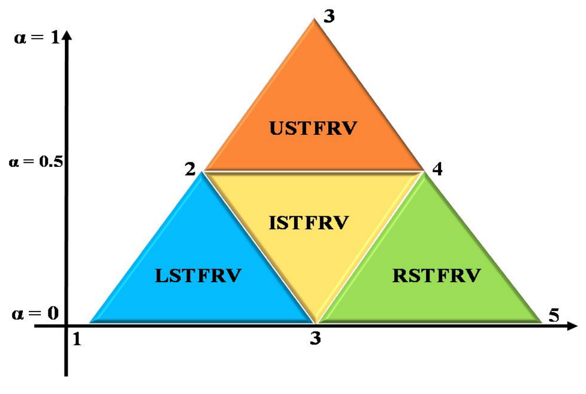

Figure 4: Triangulation of iteration − 1

It is affirmed from the above table that an integrated region of left, right, upper and an inverse symmetric $TFRV$ is a Triangulation of iteration - 1and its union is a symmetric $TFRV$. Furthering, an integrated region of left, right and inverse symmetric $TFRV$ is a symmetric trapezoidal FRV.

	α-cut of membership grade	$\alpha=0$	$\alpha=1$	TFN's
LSTFRV	$P\left\{\left(X_\alpha^L-(\mu-\sigma)-\frac{\alpha\sigma}{2}\right)\geq 0\vee\left(X_\alpha^U-\mu+\frac{\alpha\sigma}{2}\right)\leq 0\right\}$	(1, 3)	(2, 2)	(1, 2, 3)
ISTFRV	$P\left\{\left(X_\alpha^L-\mu+\frac{\alpha\sigma}{2}\right)\geq 0\vee\left(X_\alpha^U-\left(\mu+\frac{\sigma}{2}\right)+\frac{(1-\alpha)\,\sigma}{2}\right)\leq 0\right\}$	(3, 3)	(2, 4)	(2, 3, 4)
RSTFRV	$P\left\{\left(X_\alpha^L-\mu-\frac{\alpha\sigma}{2}\right)\geq 0\vee\left(X_\alpha^U-(\mu+\sigma)+\frac{\alpha\sigma}{2})\right)\leq 0\right\}$	(3, 5)	(4, 4)	(3, 4, 5)
USTFRV	$P\left\{\left(X_\alpha^L-\mu+\left(\frac{1}{2}-\alpha\right)\sigma\right)\geq 0\vee X_\alpha^U-\mu-\left(\frac{1}{2}-\alpha\right)\sigma\leq 0\right\}$	(2, 4)	(3, 3)	(2, 3, 4)
STFRV	$P\left\{\left(X_\alpha^L+(1-\alpha)\sigma-\mu\right)\geq 0\vee\left(X_\alpha^U-(1-\alpha)\sigma-\mu\right)\leq 0\right\}$	(1, 5)	(3, 3)	(1, 3, 5)
STRFRV	$P\left\{\left(X_\alpha^L+\left(1-\frac{\alpha}{2}\right)\sigma-\mu\right)\geq 0\vee\left(X_\alpha^U-\left(1-\frac{\alpha}{2}\right)\sigma-\mu\right)\leq 0\right\}$	(1,3,5)	(2, 4)	(1, 2, 3, 4, 5)

Table 1: Triangulation of iteration - 1

6. Triangulation of Fuzzy Random Variables of Iteration-2

This section, analysing triangulation of iteration - 2 and it can be generated from *LSTFRV, ISTFRV, RSTFRV* and *USTFRV*. First, we show that the *LSTFRV* is a triangulation of triangular *FRV*, by using similar way of definition 3.1 - 3.5 within the interval of *LSTFRV* $\left(\mu-\sigma,\mu-\frac{\sigma}{2},\mu;\alpha\right)$ and *LSTFRV, ISTFRV, RSTFRV* and *USTFRV* in the Left symmetric triangle are now denoted by *L-STRFRV, L-LSTFRV, L-ISTFRV, L-RSTFRV* and *L-USTFRV*.
The α-cut of membership grade of:

1. Left - Symmetric $TFRV$ (*L-STFRV*) is

$$P\left\{\left(X_\alpha^L+\left(1-\frac{\alpha}{2}\right)\sigma-\mu\right)\geq 0\vee\left(X_\alpha^U+\frac{\sigma\alpha}{2}-\mu\right)\leq 0\right\},\alpha\in[0,1],$$

its position points are$\left(\mu-\sigma,\mu-\frac{\sigma}{2},\mu\right)$.

2. Left - Symmetric $TFRV$ (*L-STRFRV*) is

$$P\left\{\left(X_\alpha^L+\left(1-\frac{\alpha}{4}\right)\sigma-\mu\right)\geq 0\vee\left(X_\alpha^U+\frac{\sigma\alpha}{4}-\mu\right)\leq 0\right\},\alpha\in[0,1],$$

its position points are $\left(\mu-\sigma,\mu-\frac{3\sigma}{4},\mu-\frac{\sigma}{2},\mu-\frac{\sigma}{4},\mu\right)$.

3. Left - Left Symmetric *TFRV* (*L-LSTFRV*) is

$$P\left\{\left(X_\alpha^L+\left(1-\frac{\alpha}{4}\right)\sigma-\mu\right)\geq 0\vee\left(X_\alpha^U+\left(1+\frac{\alpha}{2}\right)\frac{\sigma}{2}-\mu\right)\leq 0\right\},$$

$\alpha\in[0,1]$, its position points are $\left(\mu-\sigma,\mu-\frac{3\sigma}{4},\mu-\frac{\sigma}{2}\right)$.

4. Left - Right Symmetric $TFRV$ (*L-RSTFRV*) is

$$P\left\{\left(X_\alpha^L+\left(1-\frac{\alpha}{2}\right)\frac{\sigma}{2}-\mu\right)\geq 0\vee\left(X_\alpha^U+\frac{\alpha\sigma}{4}-\mu\right)\leq 0\right\},\alpha\in[0,1],$$

its position points are $\left(\mu-\frac{\sigma}{2},\mu-\frac{\sigma}{4},\mu\right)$.

5. Left - Inverse Symmetric $TFRV$ (L-$ISTFRV$) is

$$P\left\{\left(X_\alpha^L+\left(1+\frac{\alpha}{2}\right)\frac{\sigma}{2}-\mu\right)\geq 0\vee\left(X_\alpha^U+\left(1-\frac{\alpha}{2}\right)\frac{\sigma}{2}-\mu\right)\leq 0\right\},\alpha\in[0,1],$$

its position points are $(\mu-\frac{3\sigma}{4},\mu-\frac{\sigma}{2},\mu-\frac{\sigma}{4})$.

6. Left - Upper Symmetric $TFRV$ (L-$USTFRV$) is

$$P\left\{\left(X_\alpha^L+\left(\frac{3}{2}-\alpha\right)\frac{\sigma}{2}-\mu\right)\geq 0\vee X_\alpha^U+\left(\frac{1}{2}+\alpha\right)\frac{\sigma}{2}-\mu\leq 0\right\},\alpha\in[0,0.5],$$

its position points are $\left(\mu-\frac{3\sigma}{4},\mu-\frac{\sigma}{2},\mu-\frac{\sigma}{4}\right)$.

Solution:
Given $\mu=3,\sigma=1$

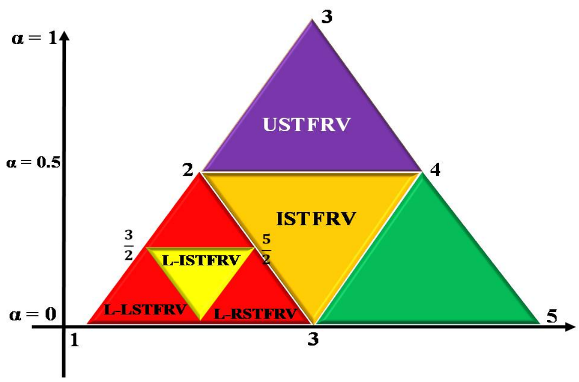

Figure 5: Triangulation of Iteration - 2 in $LSTFRV$

	α-cut of membership grade	$\alpha=0$	$\alpha=1$	TFN's
L-LSTFRV	$P\left\{\left(X_\alpha^L+\left(1-\frac{\alpha}{4}\right)\sigma-\mu\right)\geq 0\vee\left(X_\alpha^U+\left(1+\frac{\alpha}{2}\right)\frac{\sigma}{2}-\mu\right)\leq 0\right\}$	$(1,2)$	$\frac{3}{2}$	$\left(1,\frac{3}{2},2\right)$
L-ISTFRV	$P\left\{\left(X_\alpha^L+\left(1+\frac{\alpha}{2}\right)\frac{\sigma}{2}-\mu\right)\geq 0\vee\left(X_\alpha^U+\left(1-\frac{\alpha}{2}\right)\frac{\sigma}{2}-\mu\right)\leq 0\right\}$	$(2,2)$	$\left(\frac{3}{2},\frac{5}{2}\right)$	$\left(\frac{3}{2},2,\frac{5}{2}\right)$
L-RSTFRV	$P\left\{\left(X_\alpha^L+\left(1-\frac{\alpha}{2}\right)\frac{\sigma}{2}-\mu\right)\geq 0\vee\left(X_\alpha^U+\frac{\alpha\sigma}{4}-\mu\right)\leq 0)\right\}$	$(2,3)$	$\frac{5}{2}$	$\left(2,\frac{5}{2},3\right)$
L-USTFRV	$P\left\{\left(X_\alpha^L+\left(\frac{3}{2}-\alpha\right)\frac{\sigma}{2}-\mu\right)\geq 0\vee X_\alpha^U+\left(\frac{1}{2}+\alpha\right)\frac{\sigma}{2}-\mu\leq 0\right\}$, $\alpha\in[0,0.5]$	$\left(\frac{3}{2},\frac{5}{2}\right)$	2	$\left(\frac{3}{2},2,\frac{5}{2}\right)$
L-STRFRV	$P\left\{\left(X_\alpha^L+\left(1-\frac{\alpha}{4}\right)\sigma-\mu\right)\geq 0\vee\left(X_\alpha^U+\frac{\sigma\alpha}{4}-\mu\right)\leq 0\right\}$	$(1,2)$	$\left(\frac{3}{2},\frac{5}{2}\right)$	$\left(1,\frac{3}{2},\frac{5}{2},3\right)$
L-STFRV	$P\left\{\left(X_\alpha^L+\left(1-\frac{\alpha}{2}\right)\sigma-\mu\right)\geq 0\vee\left(X_\alpha^U+\frac{\sigma\alpha}{2}-\mu\right)\leq 0\right\}$, $\alpha\in[0,1]$	$(1,3)$	2	$(1,2,3)$

Table 2: Triangulation of iteration - 2

It seems an integrated region of L-left, L-right, L-upper and L-inverse of symmetric triangular FRV is iteration - 2 of triangulation and its union is a Left symmetric triangular FRV. More importantly, an integrated region of L-left, L-right and L-inverse symmetric triangular FRV is a Left symmetric trapezoidal FRV. Each iteration of $(n = 1, 2, 3 \cdots)$ triangulation has 4 sub - symmetric $TFRV$ so that the formula for the number of triangles in $n^{\mathbf{th}}$ iteration of triangulation is $4^n, n = 1, 2, 3 \ldots$

Iteration	**Location Points**	**STFRV**	**Numberof triangles**
0	(1, 3, 5)	(1, 3, 5)	$4^0 = 1$
1	*STFRV* (1, 3, 5)	$L = (1, 2, 3), I = (2, 3, 4), R = (3, 4, 5), U = (2, 3, 4)$	$4^1 = 4$
2	*L-LSTFRV* (1, 2, 3)	$L - L = (1, 3/2, 2), L - I = (3/2, 2, 5/2)$, $L - R = (2, 5/2, 3) L - U = (3/2, 2, 5/2)$	$4^2 = 16$
	L-RSTFRV (3, 4, 5)	R - L = (3, 7/2, 4), R - I = (7/2, 4, 9/2),R - R = (4, 9/2, 5), R - U = (7/2, 4, 9/2)	
	L-ISTFRV (2, 3, 4)	I - L = (2, 5/2, 3), I - I = (5/2, 3, 7/2), I - R = (3, 7/2, 4), I - U = (5/2, 3, 7/2)	
	L-USTFRV (2, 3, 4)	U - L = (2, 5/2, 3), U - I = (5/2, 3, 7/2),U - R = (3, 7/2, 4), U - U = (5/2, 3, 7/2)	
. . .	. . .	. . .	. . .

Table 3: Iteration - n of Triangulation triangular fuzzy random variable

Here, L - Left, I - Inverse, R - Right and U - Upper

Adding that it is shown that *ISTFRV, RSTFRV, LSTFRV* and *USTFRV* is a triangulation of iteration - 1. Moreover, the triangulation in four sub triangles of iteration - 1 are together called triangulation of iteration - 2. In the above similar manner, it is maintained that the succeeding iterations of triangulation till the standard deviation of the each $TFRV$ tends to nullification.

7. Conclusion

Triangulation of fuzzy random variables is relevant because it has made significant contributions to the development of symmetric trapezoidal, triangular, and triangulation fuzzy random variables. The membership grade is applicable to note the sub-interval from the support of (α= 0) and the corresponding triangulation of triangular fuzzy random variable in any iteration, for required ($0 < \alpha \leq 1$) accuracy. The basic important factor behind is triangulation of triangle in sighting into more number of similar shapes and showcasing attribute of methodology in triangulation. The triangulation concept is used to finding the validity and credibility of same concepts in multiple locations.

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