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The Band Generalized Fibonacci Difference Operator of Poisson Binomial Like Matrix on Rough Statistical Convergence on Triple Sequences and Its Rate

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ABSTRACT: In this paper, the definition of new rough statistical convergence with band generalized Fibonacci difference operator of Poisson binomial like matrix is given and some general properties of rough statistical convergence are examined. Furthermore, approximation theory worked as a rate of the rough statistical convergence. The results obtained in this study will be a more general expression of previous studies.

Keywords: Fibonacci difference operator.

Contents

1. Introduction and Preliminaries

In the theory of numbers, there are many different definitions of density. It is well known that the most popular of these definitions is asymptotic density. But, asymptotic density does not exist for all sequences. Now densities have been defined to fill those gaps and to serve different purposes. The asymptotic density is one of the possibilities to measure how large a subset of the set of natural number. We know intuitively that positive integers are much more than perfect squares. Because converge every perfect square is positive and many other positive integers exist besides. However, the set of positive integers is not infact larger than the set of perfect squares, both sets are infinite and countable and can therefore be put in 1-1 correspondence. Nevertheless, if one goes through the natural numbers, the squares be become increasingly scarce. It is precisely in this case, rational density help us and makes this intuition precise.

Let α be a subset of positive integers. We consider the interval $[1, n]$ and select an integer in this interval randomly. Then, the ratio of the number of elements of $\alpha \in [1, n]$ to the total number of elements in $[1, n]$ is being to α,probably, for $n \to \infty$, if this probability exists, that is this probability tends to some limit, then this limit is used as the asymptotic density of the set α. The mentions as that the asymptotic density is a kind of probability of choosing a number from the set α.

The set of positive integers will be denoted by $\mathbb{Z}^+$. Let α and β be subsets of $\mathbb{Z}^+$. If the symmetric difference $\alpha \triangle \beta$ is finite, then we can say α is asymptotically equal to β and denote $\alpha \approx \beta$. Freedman and Sember have introduced the concept of a lower asymptotic density and defined a concept of convergence of density.

The idea of statistical convergence was introduced by [7] and also independently by for real or complex sequences. Statistical convergence is a generalization of the usual notion of convergence which parallels the theory of or finally convergence.

A triple sequence (real or complex) can be defined as a function $X : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{R}(\mathbb{C})$,where $\mathbb{N}, \mathbb{R}$ and $\mathbb{C}$ denote the set of natural numbers, real numbers and complex numbers respectively. The different

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types of notions of triple sequences was introduced and investigated at the initial by [2, 3.4, 5, 6, 11, 12] and many others.

Let K be a subset of the set $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ and let us denote the set $\{(m, n, k) \in K : m \leq u, n \leq v, k \leq w\}$ by K_{uvw}. Then the natural density of K is given by

$$\delta(K) = \lim_{u,v,w \to \infty} \frac{|K_{uvw}|}{uvw},$$

where $|K_{uvw}|$ denotes the number of elements in K_{uvw}. Clearly, a finite subset has natural density zero, and we have $\delta(K^c) = 1 - \delta(K)$where $K^c = \mathbb{N} \diagdown K$ is the complement of k. If $K_1 \subseteq K_2$, then $\delta(K_1) \leq \delta(K_2)$.

Consider a triple sequence $x = (x_{mnk})$, such that $x_{mnk} \in \mathbb{R}$, $m, n, k \in \mathbb{N}$. A triple sequence $x = (x_{mnk})$ is said to be statistically convergent to $0 \in \mathbb{R}$, written as $St - \lim_{m,n,k \to \infty} x = 0$, provided that the set $\{(m, n, k) \in \mathbb{N}^3 : |x_{mnk} - 0| \geq \epsilon\}$has natural density zero for any $\epsilon > 0$. In this case, 0 is called the statistical limit of the convergence x.

If a triple sequence is statistically convergent, then for every $\epsilon > 0$, infinitely many terms of the sequence may remain outside the ϵ-neighborhood of the statistical limit, provided that the natural density of the set consisting of the indices of these terms is zero. This is an important property that distinguishes statistical convergence from ordinary convergence. Because the natural density of a finite set is zero, we can say that every ordinary convergent sequence is statistically convergent.

If a triple sequence $x = (x_{mnk})$ satisfies some property P for all m,n,k except a set of natural density zero, then we say that the triple sequence x satisfies P for " almost all (m, n, k)" and we abbreviate this by "a. a. (m, n, k)".

Let (x_{m_i,n_j,k_l}) be a sequence of $x = (x_{mnk})$. If the natural density of the set

$$K = \{(m_i, n_j, k_l) \in \mathbb{N}^3 : (i, j, k) \in \mathbb{N}^3\}$$

is different from zero, then (x_{m_i,n_j,k_l}) is called a non-thin subsequence of a triple sequence $x.c \in \mathbb{R}$ is called a statistical cluster point of a triple sequence $x = (x_{mnk})$ provided that the natural density of the set $\{(m, n, k) \in \mathbb{N}^3 : |x_{mnk} - c| < \epsilon\}$ is different from zero for every $\epsilon > 0$. We denote the set of all statistical cluster points of the sequence x by Γ_x.
A triple sequence $x = (x_{mnk})$ is said to be statistically analytic if there exists a positive number M such that

$$\delta\left(\left\{(m, n, k) \in \mathbb{N}^3 : |x_{mnk}|^{\frac{1}{m+n+k}} \geq M\right\}\right) = 0.$$

The theory of statistical convergence has been discussed in trigonometric series, summability theory, measure theory, turnpike theory, approximation theory, fuzzy set theory and so on.
Throughout $\Gamma(n, \mu)$ to denote the incomplete gamma function. It is defined by

$$\Gamma(n, \mu) = \int_{\mu}^{\infty} e^{-x} x^{n-1} \, dx. \tag{1.1}$$

Here $e = 2.71828\ldots$ is the Euler number. For $\mu > 0$, the RHS of (1) converges for all real n, clearly $\Gamma(n, 0) = \Gamma(n)$, where $\Gamma(n)$ will denote the gamma function of all real numbers $n \notin \{0, -1, -2, \ldots\}$. $\Gamma(n)$ can be expressed as an improper integral given by $\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} \, dx$. We state some properties of gamma function which are used throughout the text:

1. For $n \in \mathbb{N}$, $\Gamma(n + 1) = n!$.

2. For any real number $n \notin \{0, -1, -2, \ldots\}$, $\Gamma(n + 1) = n\Gamma(n)$.

3. For particular cases, we have $\Gamma(1) = \Gamma(2) = 1$, $\Gamma(3) = 2!$, $\Gamma(4) = 4!$. Here some of the basic properties of $\Gamma(n, \mu)$.

4. **Lemma 1:**

$$\Gamma(111,\mu) = e^{-\mu}$$
$$\Gamma(222,\mu) = e^{-\mu}[1+\mu^3]$$
$$\Gamma(333,\mu) = e^{-\mu}\Big[1+\mu^3+\frac{\mu^6}{8}\Big]$$
$$\vdots$$

In general

$$\Gamma(u+1,v+1,w+1,\mu) = u!v!w!\ e^{-\mu}\sum_{m=0}^{u}\sum_{n=0}^{v}\sum_{k=0}^{w}\frac{\mu^{m+n+k}}{m!n!k!}, u,v,w\in\mathbb{N}.$$

Lemma 2:

$$\Gamma(uvw+1,\mu) = uvw\ \Gamma(uvw)+\mu^{u+v+w}e^{-\mu}.$$

Lemma 3:

$$\lim_{u,v,w\to\infty}\frac{\Gamma(u+1\ v+1\ w+1,\mu)}{u!\,v!\,w!} = 1.$$

Let $\mu>0$ be fixed. We define a new poisson like three dimensional matrix $\mathscr{P}(\mu) = (p_{uvw}^{mnk})^{\mu}$ by

$$\left(p_{uvw}^{mnk}\right)^{\mu} = \begin{cases}\frac{u!v!w!}{\Gamma(u+1\ v+1\ w+1,\mu)}\frac{e^{-\mu}.\ \mu^{m+n+k}}{m!\,n!\,k!}, & \text{if } m\le u,\ n\le v,\ k\le w\\ 0, & \text{if } m\ge u,\ n\ge v,\ k\ge w\end{cases} \tag{1.2}$$

We can observe that

$$\sum_{m=0}^{u}\sum_{n=0}^{v}\sum_{k=0}^{w}\frac{u!v!w!}{\Gamma(u+1\ v+1\ w+1,\mu)}\frac{e^{-\mu}.\ \mu^{m+n+k}}{m!\,n!\,k!} = \frac{e^{-\mu}}{e^{-\mu}} = 1.$$

Denote

$$Pro(X=x) = \frac{u!v!w!}{\Gamma(u+1\ v+1\ w+1,\mu)}\frac{e^{-\mu}.\ \mu^{m+n+k}}{m!\,n!\,k!};\ m,n,k = 0,1,2,3\cdots$$

The probability of getting $(m,n,k)^{th}$ section successes in $[u,v,w]^{th}$ section in dependent trials. By taking $u,v,w\to\infty$ and applying lemma 3 we get,

$$Pro(X=x) = \frac{e^{-\mu}\mu^{m+n+k}}{m!\,n!\,k!};\ m,n,k = 0,1,2,3,\ldots$$

for fixed $\mu>0$. Hence, we may term the matrix (2) as the poisson "like" matrix or almost poisson matrix. Now, the matrix defined in the equation (2) may be rewritten as

$$\mathscr{P}(\mu) = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & \frac{e^{-\mu}}{\mu(222,\mu)}\\ \frac{e^{-\mu}}{\mu(222,\mu)} & \frac{\mu^3e^{-\mu}}{\mu(222,\mu)} & 0 & 0 & \cdots & \frac{8e^{-\mu}}{\mu(333,\mu)}\\ \frac{8e^{-\mu}}{\mu(333,\mu)} & \frac{8e^{-\mu}\mu^3}{\mu(333,\mu)} & \frac{e^{-\mu}\mu^6}{\mu(333,\mu)} & 0 & \cdots & \frac{216e^{-\mu}}{\mu(444,\mu)}\\ \frac{216e^{-\mu}}{\mu(444,\mu)} & \frac{216e^{-\mu}\mu^3}{\mu(444,\mu)} & \frac{7e^{-\mu}\mu^6}{\mu(444,\mu)} & \frac{\mu^9e^{-\mu}}{\mu(444,\mu)} & 0 & \cdots\\ 0 & 0 & 0 & 0 & \cdots & \vdots\\ \vdots & \vdots & \vdots & \vdots & & \end{bmatrix}.$$

Using Lemma 1, it is clear that the matrix $\mathscr{P}(\mu)$ is regular.

1.1. Difference Sequence Spaces

The triple difference sequence is defined as
$\Delta x_{mnk} = x_{mnk} - x_{mn+1k} - x_{mnk+1} + x_{mn+1k+1} - x_{m+1\ nk} + x_{m+1n+1k} + x_{m+1nk+1} - x_{m+1n+1k+1}$.
Now introducing the space $X(B(x,y,z))$ for $X \in \{\Gamma^3, \Lambda^3\}$ where $B(x,y,z) = \left\{ b^{rs}_{uvwmnk}(xyz) \right\}$ is a band triple difference matrix defined by

$$b^{rs}_{uvw,mnk} = \begin{cases} x & u = m,\ v = n,\ w = k \\ y & m = u-1;\ n = v-1;\ k = w-1 \\ z & m = u-2;\ n = v-2;\ k = w-2 \\ 0 & \text{otherwise} \end{cases}$$

where $x, y, z \in \mathbb{R} \setminus \{0\}$. Clearly $B(x,y,0) = B(x,y)$.

2. Fibonacci Sequence Spaces

Fibonacci numbers are also considered to be nature's numbers [13,14,19]. They can be found everywhere around us, from the leaf arrangements in plants, to the pattern of the florets of flowers; the bracts of pinecones, or the scales of pineapple. The number sequence $1, 1, 2, 3, 5, 8, \ldots$ is called the Fibonacci sequence. Note that any number in the sequence is the sum of the two numbers preceding it. Thus, if $\left(f_v\right)_{v=0}^{\infty} = 0$ is the sequence of Fibonacci numbers, then $f_0 = f_1 = 1, \quad f_v = f_{v-1} = f_{v-2}\ ; \quad v \geq 2$.

The ratio of successive terms in the Fibonacci sequence approaches an irrational number $\frac{1+\sqrt{5}}{2}$ which is called the golden ratio. This number has great application in the field of architecture, Science, and arts. Some more basic properties of Fibonacci numbers can be listed as follows:

$$\lim_{r\to\infty} \frac{f_{r+1}}{f_r} = \frac{1+\sqrt{5}}{2} (\text{Golden ratio}),$$
$$\sum_{v=0}^{r} f_r = f_{r+2} - 1 \quad (r \in \mathbb{N}),$$
$$\sum_{\gamma=0}^{\infty} \frac{1}{f_r} \quad \text{converges}, \quad f_{r-1}\ f_{r+1} - f_r^2 = (-1)^{r+1},\ r \geq 1 \quad (\text{Cassine formula}),$$

combining Fibonacci band matrix F and triple difference operator B, the product matrix

$$F(B) = \left(f(B) \right)_{uvw}^{mnk} \quad \text{is defined by}$$

$$\left(f(B) \right)_{uvw}^{mnk} = \begin{cases} \frac{x f_{uvw}}{f_{u+1v+1w+1}}; & m = u,\ n = v,\ k = w \\ -\frac{x f_{u+1v+1w+1}}{f_{uvw}} + \frac{y f_{uvw}}{f_{u+1v+1w+1}}; & m = u+1, n = v+1, k = w+1 \\ -\frac{y f_{u+1v+1w+1}}{f_{uvw}} + \frac{z f_{uvw}}{f_{u+1v+1w+1}}; & m = u+2, n = v+2, k = w+2 \\ -\frac{z f_{u+1v+1w+1}}{f_{uvw}}; & m = u+3, n = v+3, k = w+3 \end{cases}.$$

Equivalently, $F(B)$ can also be expressed as

$$F(B)=\begin{bmatrix} \frac{xf_{000}}{f_{111}} & 0 & 0 & 0 & 0 & \cdots \\ \frac{-xf_{222}}{f_{111}}+\frac{yf_{111}}{f_{222}} & \frac{xf_{111}}{f_{222}} & 0 & 0 & 0 & \cdots \\ \frac{-yf_{333}}{f_{222}}+\frac{zf_{222}}{f_{333}} & \frac{-xf_{333}}{f_{222}}+\frac{yf_{222}}{f_{333}} & \frac{xf_{222}}{f_{333}} & 0 & 0 & \cdots \\ \frac{-zf_{444}}{f_{333}} & \frac{-yf_{444}}{f_{333}}+\frac{zf_{333}}{f_{444}} & \frac{-xf_{444}}{f_{333}}+\frac{yf_{333}}{f_{444}} & \frac{xf_{333}}{f_{444}} & 0 & \cdots \\ 0 & \frac{-zf_{555}}{f_{444}} & \frac{-yf_{555}}{f_{444}}+\frac{zf_{444}}{f_{555}} & \frac{-xf_{555}}{f_{444}}-\frac{yf_{444}}{f_{555}} & \frac{xf_{444}}{f_{555}} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \cdots \end{bmatrix}.$$

One may clearly observe that $\big(F(B)\big)(1,0,0)=F$, $(F(B)(x,y,0))=(F(B))(x,y)$. Note that the terms with negative subscripts is considered to be zero. The triple sequence it is called $F(B)-$ transform of the sequence S. Now we define the spaces $\Gamma^3\left(\mathscr{P}^\mu(F(B))\right)$ and $\Lambda^3\left(\mathscr{P}^\mu(F(B))\right)$ by

$$\Gamma^3\left(\mathscr{P}^\mu(F(B))\right)=\left\{x=(x_{mnk})\in\omega^3:\mathscr{P}^\mu(F(B))x\in\Gamma^3\right\}$$

and

$$\Lambda^3\left(\mathscr{P}^\mu(F(B))\right)=\left\{x=(x_{mnk})\in\omega^3:\mathscr{P}^\mu(F(B))\;x\in\Lambda^3\right\}.$$

The spaces $\Gamma^3\left(\mathscr{P}^\mu(F(B))\right)$ and $\Lambda^3\left(\mathscr{P}^\mu(F(B))\right)$ may be redefined as
$\Gamma^3\left(\mathscr{P}^\mu(F(B))\right)=\left(\Gamma^3\right)_{\mathscr{P}^\mu(F(B))}$ and $\Lambda^3\left(\mathscr{P}^\mu(F(B))\right)=\left(\Lambda^3\right)_{P^\mu(F(B))}$.

We further emphasize that the spaces $\Gamma^3\left(\mathscr{P}^\mu(F(B))\right)$ and $\Lambda^3\left(\mathscr{P}^\mu(F(B))\right)$ are reduced to certain classes of sequence spaces in the special cases of $x,y,z\in\mathbb{R}$.

For $z=0$, the above sequence spaces reduces to the classes

$\Gamma^3\left(\mathscr{P}^\mu(F(B(x,y)))\right)=\left(\Gamma^3\right)_{\mathscr{P}^\mu(F(B(x,y))}$ and $\Lambda^3\left(\mathscr{P}^\mu(F(B(x,y)))\right)=\left(\Lambda^3\right)_{\mathscr{P}^\mu(F(B(x,y))}$.

2.1. Approximation Theory

Korovkin type approximation theorems are practical tools to check whether a given triple sequence $(\alpha_{mnk})_{m,n,k\geq1}$ of positive linear operators on $C[a,b]$ of all continuous functions on the real interval $[a,b]$ is an approximation process. That is, these theorems present a variety of test function which provide that the approximation property holds on the whole space if it holds for them such a property was determined by Korovkin in 1953 for the function $1,x,x^2$ in the space $C[a,b]$ as well as for the functions $1,\cos x$, and $\sin x$ in the space of all continuous 2π periodic functions on the real line.

3. The Band Fibonacci Difference Operator of Poisson Binomial Like of Rough Statistical Convergence

A sequence $x=(x_{mnk})$ is said to be triple analytic if $\sup_{m,n,k}|x_{mnk}|^{\frac{1}{m+n+k}}<\infty$. The vector space of all triple analytic sequences are usually denoted by Λ^3. A sequence $x=(x_{mnk})$ is called triple entire sequence if $|x_{mnk}|^{\frac{1}{m+n+k}}\to0$ as $m,n,k\to\infty$. The vector space of all triple entire sequences are usually denoted by Γ^3. The space Λ^3 and Γ^3 is a metric space with the metric

$$d(x,y)=\sup_{m,n,k}\left\{|x_{mnk}-y_{mnk}|^{\frac{1}{m+n+k}}:m,n,k=1,2,3\cdots\right\}$$

for all $x=(x_{mnk})$ and $y=(y_{mnk})$ in Γ^3. It is clear that Γ^3 and Λ^3 is a linear space. Then $\Gamma^3\left(\mathscr{P}^\mu(F(B))\right)$

and $\Lambda^3(\mathscr{P}^{\mu}(F(B)))$ is also a linear space.

If Γ^3 and Λ^3 is a complete metric space, then $\Gamma^3(\mathscr{P}^{\mu}(F(B)))$ and $\Lambda^3(\mathscr{P}^{\mu}(F(B)))$ is also a complete metric space with the metric

$$d(x,y)=\sup\left\{|\mathscr{P}^{\mu}(F(B))x_{mnk}-\mathscr{P}^{\mu}(F(B))y_{mnk}|:m,n,k=1,2,3,\ldots\right\}.$$

Lemma 2.1:

If $\Gamma^3_x\subset\Gamma^3_y$, then $\Gamma^3(\mathscr{P}^{\mu}(F(B))x)\subset\Gamma^3(\mathscr{P}^{\mu}(F(B)y))$.

Proof: It is obvious.

Theorem 2.2: Consider that Γ^3 is a complete metric space and α is a closed subset of Γ^3. Then $\alpha(\mathscr{P}^{\mu}(F(B)))$ is also closed in $\Gamma^3(\mathscr{P}^{\mu}(F(B)))$.

Proof: Since α is a closed subset of Γ^3 from Lemma 2.1, then we can write

$$\alpha(\mathscr{P}^{\mu}(F(B)))\subset\Gamma^3(\mathscr{P}^{\mu}(F(B))).$$

$\overline{\alpha(\mathscr{P}^{\mu}(F(B)))}$, $\bar{\alpha}$ denote the closures of $\alpha(\mathscr{P}^{\mu}(F(B)))$ and α respectively. It is enough to prove that

$$\overline{\alpha(\mathscr{P}^{\mu}(F(B)))}=\bar{\alpha}(\mathscr{P}^{\mu}(F(B))).$$

Firstly, we take $x\in\overline{\alpha(\mathscr{P}^{\mu}(F(B)))}$, there exists a sequence $(x^{uvw})\in\alpha(\mathscr{P}^{\mu}(F(B)))$ such that $|x^{uvw}-x|_{\mathscr{P}^{\mu}(F(B))}\to 0$ in $\alpha(\mathscr{P}^{\mu}(F(B)))$ for $u,v,w\to\infty$. Thus,

$$\left|x^{uvw}_{mnk}-x_{mnk}\right|_{\mathscr{P}^{\mu}(F(B))}\to 0.$$

as $u,v,w\to\infty$ in $x\in\alpha(\mathscr{P}^{\mu}(F(B)))$ so that

$$\sum_{r=1}^{i}\sum_{s=1}^{j}\sum_{t=1}^{l}|x^{uvw}_{rst}-x_{rst}|+\left|\mathscr{P}^{\mu}(F(B))x^{uvw}_{mnk}-\mathscr{P}^{\mu}(F(B))x_{mnk}\right|\to 0$$

for $(u,v,w)\to\infty$ in α. Therefore,

$$\mathscr{P}^{\mu}(F(B))x\in\bar{\alpha}\ \text{ and so }\ x\in(\mathscr{P}^{\mu}(F(B))).$$

Conversely, if we take $x\in\overline{\alpha(\mathscr{P}^{\mu}(F(B)))}$, then $x\in\alpha(\mathscr{P}^{\mu}(F(B)))$. Since α is closed. Then $\overline{\alpha(\mathscr{P}^{\mu}(F(B)))}=\bar{\alpha}(\mathscr{P}^{\mu}(F(B)))$. Hence, $\alpha(\mathscr{P}^{\mu}(F(B)))$ is a closed subset of $\Gamma^3(\mathscr{P}^{\mu}(F(B)))$.

Corollary 2.3: If Γ^3 is a separable space, then $\Gamma^3(\mathscr{P}^{\mu}(F(B)))$ is also a separable space.

Definition 2.4: A triple sequence $x=(x_{mnk})$ is said to be band Fibonacci difference operator of poisson binomial like matrix on rough statistically convergence if there is a number l, such that for every $\epsilon>0$ and r be a positive number, the set

$$k_{r+\epsilon}(\mathscr{P}^{\mu}(F(B))):=\{(m\le u,\ n\le v,\ k\le w):|\mathscr{P}^{\mu}(F(B))x_{mnk}-l|\ge r+\epsilon\}$$

has natural density zero, i.e., $d\left(K_{r+\epsilon}(\mathscr{P}^{\mu}(F(B)))\right)=0$. i.e.,

$$\lim_{u,v,w\to\infty}\frac{1}{uvw}\left|\{(m\le u,\ n\le v,\ k\le w):|\mathscr{P}^{\mu}(F(B))x_{mnk}-l|\ge r+\varepsilon\}\right|=0.$$

In this case, we write

$$d(\mathscr{P}^{\mu}(F(B)))-\lim_{m,n,k\to\infty}x=l\text{ or }x_{mnk}\to l\,(\mathscr{P}^{\mu}(F(B))).$$

The set of $\mathscr{P}^{\mu}(F(B))$ rough statistically convergent triple sequence space will be denoted by $rs(\mathscr{P}^{\mu}(F(B)))$. In this $l=0$, we will write it as $rs_0(\mathscr{P}^{\mu}(F(B)))$.

Definition 2.5: A triple sequence $x=(x_{mnk})$ is said to be band Fibonacci difference operator of poisson binomial like matrix on rough statistically Cauchy if there exists a number $N=N(r+\epsilon)$, such that for every $\epsilon>0$ and r be a positive number the set

$$\lim_{u,v,w\to\infty}\frac{1}{uvw}\left|\{(m\le u,\ n\le v,\ k\le w): |\mathscr{P}^{\mu}(F(B))x_{mnk}-\mathscr{P}^{\mu}(F(B))x_N|\ge r+\epsilon\}\right|=0.$$

Theorem 2.6: If a triple sequence space x is a band Fibonacci difference operator of poisson binomial like matrix on rough statistically convergent sequence, then x is a band Fibonacci difference operator of poisson binomial like matrix on rough statistically Cauchy sequence.
Proof: Let $\epsilon>0$ and r be a positive number. Assume that $(x_{mnk})\to l(rs(\mathscr{P}^{\mu}(F(B))))$. Then

$$|\mathscr{P}^{\mu}(F(B))x_{mnk}-l|<\frac{r+\varepsilon}{2}\ \text{ for almost all } m,n,k.$$

If N is chosen so that

$$|\mathscr{P}^{\mu}(F(B))x_N-l|<\frac{r+\varepsilon}{2},$$

then, we have

$$\begin{aligned}|\mathscr{P}^{\mu}(F(B))x_{mnk}-\mathscr{P}^{\mu}(F(B))x_N| &< |\mathscr{P}^{\mu}(F(B))x_{mnk}-l|+|\mathscr{P}^{\mu}(F(B))x_N-l|\\ &<\left(\frac{r+\varepsilon}{2}\right)+\left(\frac{r+\varepsilon}{2}\right)=r+\varepsilon\end{aligned}$$

for almost all m,n,k, which implies that x is band Fibonacci difference operator of poisson binomial like matrix on rough statistically Cauchy sequence.

Theorem 2.7: If x is a triple sequence for which there is a band Fibonacci difference operator of poisson binomial like matrix on rough statistically convergent sequence $y=(y_{mnk})$, such that $\mathscr{P}^{\mu}(F(B))x_{mnk}=\mathscr{P}^{\mu}(F(B))y_{mnk}$ for almost all m,n,k, then x is band Fibonacci difference operator of poisson binomial like matrix on rough statistically convergent sequence.

Proof: Suppose that $\mathscr{P}^{\mu}(F(B))x_{mnk}=\mathscr{P}^{\mu}(F(B))y_{mnk}$ almost all m,n,k and $(y_{mnk})\in l\,(rs(\mathscr{P}^{\mu}(F(B))))$. Then $\epsilon>0$ and r be a positive real number and for each u,v,w.

$$\begin{aligned}&\{(m\le u,\ n\le v,\ k\le w): |\mathscr{P}^{\mu}(F(B))x_{mnk}-l|\ge r+\epsilon\}\\ &\quad\subseteq\{(m\le u,\ n\le v,\ k\le w): |\mathscr{P}^{\mu}(F(B))x_{mnk}\ne\mathscr{P}^{\mu}(F(B))y_{mnk}|\ge r+\epsilon\}\\ &\quad\cup\{(m\le u,\ n\le v,\ k\le w): |\mathscr{P}^{\mu}(F(B))x_{mnk}-l|\le r+\epsilon\}\end{aligned}$$

Since $(y_{mnk})\to l\,(rs(\mathscr{P}^{\mu}(F(B))))$, the latter set contains a fixed number of integers, say $g=g(r+\epsilon)$. Therefore, for $\mathscr{P}^{\mu}(F(B))x_{mnk}=\mathscr{P}^{\mu}(F(B))y_{mnk}$ for almost all m,n,k.

$$\begin{aligned}&\lim_{u,v,w\to\infty}\frac{1}{uvw}\left|\{(m\le u,\ n\le v,\ k\le w): |\mathscr{P}^{\mu}(F(B))x_{mnk}-l|\ge r+\epsilon\}\right|\\ &\quad\le\lim_{u,v,w\to\infty}\frac{1}{uvw}\left|\{(m\le u,\ n\le v,\ k\le w): |\mathscr{P}^{\mu}(F(B))x_{mnk}\ne\mathscr{P}^{\mu}(F(B))y_{mnk}|\}\right|\\ &\quad+\lim_{u,v,w\to\infty}\frac{g(r+\epsilon)}{uvw}=0.\end{aligned}$$

Hence, $\left(x_{mnk}\right)\to l\,(rs(\mathscr{P}^{\mu}(F(B))))$.

4. Conclusion

In this paper, we study a new rough statistical convergence of the Poisson-like binomial matrix using the band-generalized Fibonacci difference operator, its general properties, and a rate of rough statistical convergence theory. The results provide a more general representation of previous studies.

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