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Extended Vertex Odd Mean Labeling in Certain Graphs

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ABSTRACT: Mean labeling was introduced and studied for certain classes of graphs by Somasundaram and Ponraj [8]. It is defined as an injective function

$$f : V \to \{0, 1, \ldots, q\},$$

and the induced edge-labeling function f^* assigns to each edge $uv \in E$ a label from $\{1, 2, \ldots, q\}$ by

$$f^*(uv) = \begin{cases} \dfrac{f(u)+f(v)}{2}, & \text{if } f(u)+f(v) \text{ is even,} \\ \dfrac{f(u)+f(v)+1}{2}, & \text{if } f(u)+f(v) \text{ is odd.} \end{cases}$$

The concept of *odd mean labeling* was introduced by K. Manickam and M. Marudai [3]. In this variant, vertex labels are taken from $\{0, 1, \ldots, 2q-1\}$ and edge labels are the odd integers $\{1, 3, \ldots, 2q-1\}$. N. Revathi later introduced *vertex odd mean* and *vertex even mean* labelings and proved that the umbrella graph, Mongolian tent, and $K_1 + C_n$ admit these labelings [5].

Motivated by these studies, in this paper we prove the existence of *extended vertex odd mean labeling* for the duplicate graphs of the path, comb, twig, star, and bistar graphs.

Key Words: Graph labeling, duplicate graph, path, comb, twig, ladder, star, bistar, Extended vertex odd mean labeling .

Contents

1. Introduction

In 1967 Rosa introduced graph labeling [4]. A graph labeling is a mapping that assigns integers to the elements of a graph: if the map assigns integers to vertices (respectively, edges) it is called a *vertex labeling* (respectively, an *edge labeling*), and if it assigns integers to both vertices and edges it is called a *total labeling*. Sampath Kumar E. introduced duplicate graphs in 1973 and proved several of their properties [6]. For a graph $G = (V, E)$ of order p and size q, the duplicate graph is denoted by G'. The duplicate graph G' contains $2p$ vertices: for each $x \in V$ let x' denote its duplicate, and for each edge $xy \in E$ the graph G' contains the edges xy' and yx'. Vijayakumar *et al.* proved that the extended duplicate graphs of the path, comb, and twig graphs admit Skolem difference mean labeling [13]. In 2015 N. Revathi introduced vertex odd mean and vertex even mean labelings and proved that the umbrella graph, Mongolian tent, and $K_1 + C_n$ admit these labelings [5]. The notions of vertex odd mean labeling (VOML) and vertex even mean labeling do not hold for graphs with $p = q + 1$. Therefore, in this paper we introduce the concept of *extended vertex odd mean labeling* (EVOML) and prove the existence of EVOML for paths, combs, twigs, stars, and bistars, as well as for their duplicate graphs.

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2. Preliminaries

2.1. Notations

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We use the following abbreviations through out this article.
$G(V,E)$ - "A simple graph with p vertices and q edges"
EVOML - "Extended vertex odd mean labeling"
$\mathcal{P}_m$ - "Extended duplicate graph of path graph"
$\mathcal{CB}_m$ - "Extended duplicate graph of comb graph"
$\mathcal{T}_m$ - "Extended duplicate graph of twig graph"
$\mathcal{S}_m$ - "Extended duplicate graph of star graph"
$\mathcal{BS}_{m,m}$ - "Extended duplicate graph of bistar graph"

Definition 2.1 *An injection $f : V \to \{t : 0 \le t \le q\}$ is called mean labeling of $G(V,E)$, if the induced function $f^*(uv) = \frac{f(u)+f(v)}{2}$ provides distinct labels from the set $\{t : 1 \le t \le q\}$ to every edge of G, also f^* is injective. A mean graph is a graph admitting mean labeling.*

Definition 2.2 *An injection $f :\to \{t : 0 \le t \le 2q-1\}$ is called an odd mean labeling of $G(V,E)$ if the induced function $f^*(uv)$ defined by*

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{whenever } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{whenever } f(u)+f(v) \text{ is odd} \end{cases} \tag{2.1}$$

is a bijection and allots distinct edge labels. A odd mean graph is a graph admitting odd mean labeling [8].

Definition 2.3 *A vertex odd mean labeling of a graph $G(V,E)$ is an injecton $f : V \to \{t : 1 \le t \le 2q-1\}$ such that every edge uv is distinctly labeled with the average of $f(u)$ and $f(v)$. A vertex odd mean graph is a graph admitting vertex odd mean labeling [5].*

Definition 2.4 *An EVOML of $G(V,E)$ is an injection $f : V \to \{2t-1 : 1 \le t \le q+1\}$ so that the edge labels are assigned by f^* defined by*

$$f^*(uv) = \frac{f(u)+f(v)}{2} \tag{2.2}$$

are distinct.

We refer [13], [12] and [11] for the construction of extended duplicate graphs.

3. Main Results

Theorem 3.1 *$\mathcal{P}_m$, $m \ge 3$ admits EVOML.*

Proof: Suppose v_t, v'_t for $1 \le t \le m$ be vertices and e_t $(1 \le t \le m)$, e'_t $(1 \le t \le m-1)$ be edges of $\mathcal{P}_m$.

Case (i): For odd m
$2m$ vertices of $\mathcal{P}_m$ are labeled as follows:
When $1 \le k \le \frac{m+1}{2}$,

$$v_{2k-1} \leftarrow 4k-3, \quad v'_{2k-1} \leftarrow 2q-4k+5.$$

When $1 \le k \le \frac{m-1}{2}$,

$$v_{2k} \leftarrow 2q-4k+3, \quad v'_{2k} \leftarrow 4k-1.$$

The edges of $\mathcal{P}_m$ are labeled using (2.2).

The $\dfrac{m-1}{2}$ edges e_{2t-1} $(1 \leq t \leq \dfrac{m-1}{2})$ receive labels $4t-2$ $\left(1 \leq t \leq \dfrac{q-1}{4}\right)$,
the $\dfrac{m-1}{2}$ edges e_{2t} $(1 \leq t \leq \dfrac{m-1}{2})$ are labeled with $2q-(4t-2)$ $\left(1 \leq t \leq \dfrac{q-1}{4}\right)$,
the $\dfrac{m-1}{2}$ edges e'_{2t-1} $(1 \leq t \leq \dfrac{m-1}{2})$ are labeled with $2q-(4t-4)$ $\left(1 \leq t \leq \dfrac{q}{4}\right)$,
the $\dfrac{m-1}{2}$ edges e'_{2t} $(1 \leq t \leq \dfrac{m-1}{2})$ are labeled with $4t$ $\left(1 \leq t \leq \dfrac{q-1}{4}\right)$.

and the edge e_m is labeled $q+1$. So all edge labels are distinct.

Case (ii): For even m

$2m$ vertices of $\mathcal{P}_m$ are labeled as follows:
For $1 \leq k \leq \frac{m}{2}$,

$$v_{2k-1} \leftarrow 4k-3, \quad v_{2k} \leftarrow 2q-4k+3, \quad v'_{2k-1} \leftarrow 2q-4k+5, \quad v'_{2k} \leftarrow 4k-1.$$

The edges of $\mathcal{P}_m$ are labeled using equation (2.2).

The $\dfrac{m}{2}$ edges e_{2t-1} $(1 \leq t \leq \dfrac{m}{2})$ receive labels $4t-2$ $\left(1 \leq t \leq \dfrac{q+1}{4}\right)$,
the $\dfrac{m-1}{2}$ edges e_{2t} $(1 \leq t \leq \dfrac{m-2}{2})$ receive labels $2q-(4t-2)$ $\left(1 \leq t \leq \dfrac{q-3}{4}\right)$,
the $\dfrac{m}{2}$ edges e'_{2t-1} $(1 \leq t \leq \dfrac{m}{2})$ are labeled with $2q-(2t-2)$ $\left(1 \leq t \leq \dfrac{q-1}{2}\right)$,
the $\dfrac{m-2}{2}$ edges e'_{2t} $(1 \leq t \leq \dfrac{m-2}{2})$ receive labels $4t$ $\left(1 \leq t \leq \dfrac{q+3}{4}\right)$.

and the edge e_m is labeled with $q+1$. So all edge labels are distinct.
Hence, $\mathcal{P}_m$, $m \geq 3$ admits EVOML.

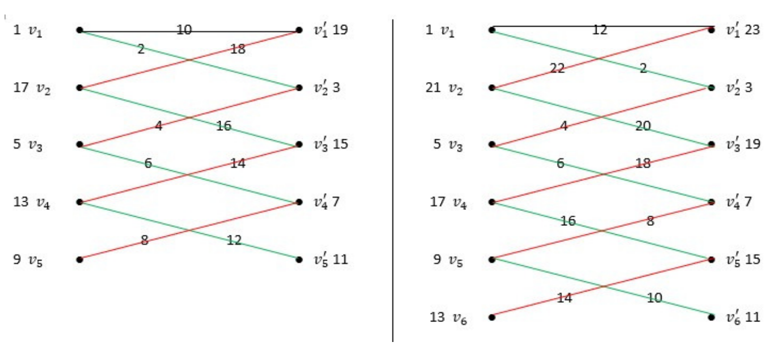

Figure 1: EVOML in $\mathcal{P}_5$ and $\mathcal{P}_6$

□

Theorem 3.2 *$\mathcal{CB}_{m,m}$, $m \geq 2$ admits EVOML.*

Proof: Suppose v_t $(1 \leq t \leq 2m)$, v'_t $(1 \leq t \leq 2m)$ be vertices and e_t $(1 \leq t \leq 2m)$, e'_t $(1 \leq t \leq 2m-1)$ be edges of $\mathcal{CB}_{m,m}$.

Case (i): For odd m
$4m$ vertices of $\mathcal{CB}_{m,m}$ are labeled as below:

$$v_1 \leftarrow 1, \quad v_{2m} \leftarrow q+2, \quad v'_1 \leftarrow 2q+1, \quad v'_{2m} \leftarrow q,$$

For $1 \leq k \leq \frac{m-1}{2}$,
$v_{4k-2} \leftarrow 2q-8k+7$ $v_{4k-1} \leftarrow 2q-8k+5$, $v_{4k} \leftarrow 8k-1$ $v_{4k+1} \leftarrow 8k+1$;

$$v'_{4k-2} \leftarrow 8k-5, \quad v'_{4k-1} \leftarrow 8k-3, \quad v'_{4k} \leftarrow 2q-8k+3, \quad v'_{4k+1} \leftarrow 2q-8k+1.$$

Making use of the function f^* defined in (2.2), the $m-1$ edges

$$\{e_{4t-3}, e_{4t-2} : 1 \leq t \leq \tfrac{2m-2}{4}\}$$

receive labels

$$\{8t-6, 8t-5 : 1 \leq t \leq \tfrac{q-3}{8}\},$$

the $m-1$ edges

$$\{e_{4t-1}, e_{4t} : 1 \leq t \leq \tfrac{m-1}{2}\}$$

receive labels

$$\{2q-(8t-4), 2q-(8t-3) : 1 \leq t \leq \tfrac{q-3}{8}\},$$

the $m-1$ edges

$$\{e'_{4t-3}, e'_{4t-2} : 1 \leq t \leq \tfrac{2m-2}{4}\}$$

receive labels

$$\{2q-(8t-8), 2q-(8t-6) : 1 \leq t \leq \tfrac{q-3}{8}\},$$

respectively, the $m-1$ edges

$$\{e'_{4t-1}, e'_{4t} : 1 \leq t \leq \tfrac{2m-2}{4}\}$$

receive labels

$$\{8t-2, 8t-1 : 1 \leq t \leq \tfrac{q-3}{8}\},$$

and the label $q-1$ is assigned to e'_{2m-1}, the label $q+1$ to e_{2m}, and the label $q+3$ to e_{2m-1}. Thus, all the edge labels of $\mathcal{CB}_{m,m}$ are distinct.

Case (ii): For even m
$4m$ vertices of $\mathcal{CB}_{m,m}$ are labeled as below.
Fix $v_1 \leftarrow 1$, $v_{2m} \leftarrow q$,

$$v'_1 \leftarrow 2q+1, \quad v'_{2m} \leftarrow q+2,$$

For $1 \leq k \leq \frac{m}{2}$:

$$v_{4k-2} \leftarrow 2q-8k+7, \quad v_{4k-1} \leftarrow 2q-8k+5;$$

$$v'_{4k-2} \leftarrow 8k-5, \quad v'_{4k-1} \leftarrow 8k-3.$$

For $1 \leq k \leq \frac{m-2}{2}$:

$$v_{4k} \leftarrow 8k-1, \quad v_{4k+1} \leftarrow 8k+1;$$

$$v'_{4k} \leftarrow 2q-8k+3, \quad v'_{4k+1} \leftarrow 2q-8k+1.$$

Making use of the function f^* defined in (2.2),
the m edges

$$\{e_{4t-3}, e_{4t-2} : 1 \leq t \leq \tfrac{2m}{4}\}$$

receive labels

$$\{8t-6, 8t-5 : 1 \leq t \leq \tfrac{q+1}{8}\},$$

respectively.

The $m-2$ edges

$$\{e_{4t-1}, e_{4t} : 1 \le t \le \tfrac{2m-4}{4}\}$$

receive labels

$$\{2q-(8t-4), 2q-(8t-3) : 1 \le t \le \tfrac{q-7}{8}\},$$

respectively.

The edges

$$\{e'_{4t-3}, e'_{4t-2} : 1 \le t \le \tfrac{2m}{4}\}$$

receive labels

$$\{2q-(8t-8), 2q-(8t-9) : 1 \le t \le \tfrac{q+3}{8}\},$$

and the $m-2$ edges

$$\{e'_{4t-1}, e'_{4t} : 1 \le t \le \tfrac{2m-4}{4}\}$$

receive labels

$$\{8t-2, 8t-1 : 1 \le t \le \tfrac{q-9}{8}\},$$

respectively.

Finally, the labels $q-1$, $q+1$, and $q+3$ are assigned to the edges e'_{2m-1}, e_{2m}, and e_{2m-1}, respectively.

Thus, all the edge labels are distinct.

We notice that in both cases $4m-1$ edges are labeled with distinct integers.

Hence, $\mathcal{CB}_{m,m}$, $m \ge 2$ admits EVOML.

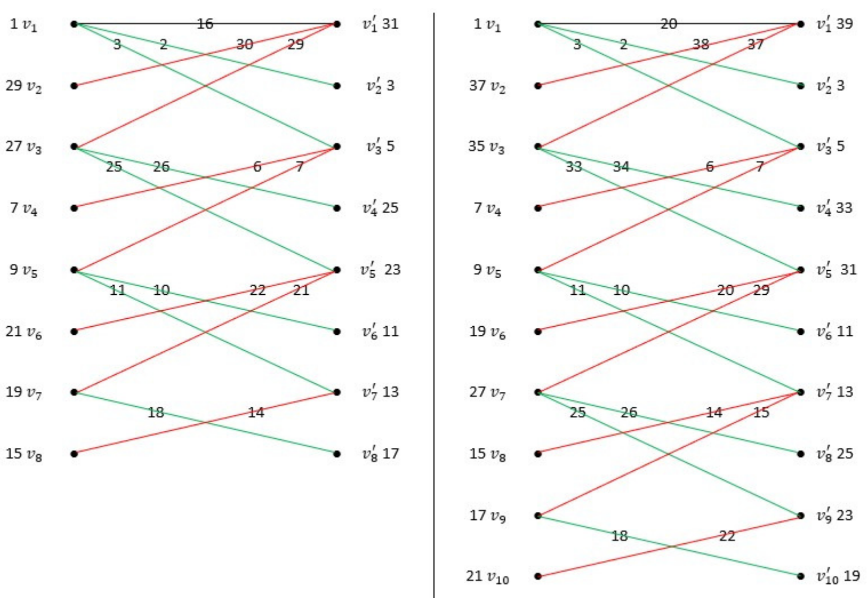

Figure 2: EVOML in $\mathcal{CB}_{4,4}$ and $\mathcal{CB}_{5,5}$

□

Theorem 3.3 *$\mathcal{T}_m$, $m \ge 2$ admits EVOML.*

Proof: Suppose v_t $(1 \le t \le 3m+2)$, v'_t $(1 \le t \le 3m+2)$ are vertices and e_t $(1 \le t \le 3m+2)$, e'_t $(1 \le t \le 3m+1)$ are edges of $\mathcal{T}_m$.

Case (i): For odd m
We label $6m+4$ vertices of $\mathcal{T}_m$ as follows:
Fix

$$v_1 \leftarrow q+2, \quad v_2 \leftarrow 1, \quad v'_1 \leftarrow 3, \quad v'_2 \leftarrow 2q+1.$$

For $1 \le k \le \frac{m+1}{2}$:

$$\begin{cases} v_{6k-3} \leftarrow 2q-12k+11, \\ v_{6k-2} \leftarrow 2q-12k+9, \\ v_{6k-1} \leftarrow 2q-12k+7; \end{cases} \qquad \begin{cases} v'_{6k-3} \leftarrow 12k-7, \\ v'_{6k-2} \leftarrow 12k-5, \\ v'_{6k-1} \leftarrow 12k-3. \end{cases}$$

For $1 \le k \le \frac{m-1}{2}$:

$$\begin{cases} v_{6k} \leftarrow 12k-1, \\ v_{6k+1} \leftarrow 12k+1, \\ v_{6k+2} \leftarrow 12k+3; \end{cases} \qquad \begin{cases} v'_{6k} \leftarrow 2q-12k+5, \\ v'_{6k+1} \leftarrow 2q-12k+3, \\ v'_{6k+2} \leftarrow 2q-12k+1. \end{cases}$$

The edges are labeled by the function f^* defined in (2.2) as below.
The $\frac{3m+3}{2}$ edges $\{e_{6t-4}, e_{6t-3}, e_{6t-2} : 1 \le t \le \frac{m+1}{2}\}$ receive labels 4, 5, 6, 16, 17, 18,..., $q-5$, $q-4$, $q-3$,
The $\frac{3m+3}{2}$ edges

$$\{e'_{6t-4}, e'_{6t-3}, e'_{6t-2} : 1 \le t \le \frac{m+1}{2}\}$$

are labeled with

$$2q,\ 2q-1,\ 2q-2,\ 2q-12,\ 2q-13,\ 2q-14, \ldots, q+9,\ q+8,\ q+7,$$

the $\frac{3m-3}{2}$ edges

$$\{e_{6t-1}, e_{6t}, e_{6t+1} : 1 \le t \le \frac{m-1}{2}\}$$

receive labels

$$2q-6,\ 2q-7,\ 2q-8,\ 2q-18,\ 2q-19,\ 2q-20, \ldots, q-5,\ q-4,\ q-3,$$

respectively.
The $\frac{3m-3}{2}$ edges

$$\{e'_{6t-1}, e'_{6t}, e'_{6t+1} : 1 \le t \le \frac{m-1}{2}\}$$

receive labels

$$10,\ 11,\ 12,\ 22,\ 23,\ 24, \ldots, q+9,\ q+8,\ q+7,$$

and for the three edges e'_1, e_{3m+2}, and e_1, the labels are

$$2, \quad q+2, \quad \frac{3(q+1)}{2},$$

respectively.

Case (ii): For even m
We label the $6m+4$ vertices of $\mathcal{T}_m$ as follows:
Fix

$$v_1 \leftarrow 3, \quad v_2 \leftarrow 1, \quad v'_1 \leftarrow q+2, \quad v'_2 \leftarrow 2q+1.$$

For $1 \le k \le \frac{m}{2}$:

$$\begin{cases} v_{6k-3} \leftarrow 2q-12k+11, \\ v_{6k-2} \leftarrow 2q-12k+9, \\ v_{6k-1} \leftarrow 2q-12k+7; \end{cases} \quad \begin{cases} v_{6k} \leftarrow 12k-1, \\ v_{6k+1} \leftarrow 12k+1, \\ v_{6k+2} \leftarrow 12k+3; \end{cases} \quad \begin{cases} v'_{6k-3} \leftarrow 12k-7, \\ v'_{6k-2} \leftarrow 12k-5, \\ v'_{6k-1} \leftarrow 12k-3. \end{cases}$$

$$v'_{6k} \leftarrow 2q-12k+5, \quad v'_{6k+1} \leftarrow 2q-12k+3, \quad v'_{6k+2} \leftarrow 2q-12k+1.$$

The edges are labeled by the function f^* defined in (2.2) as follows:
The $\frac{3m}{2}$ edges

$$\{e_{6t-4}, e_{6t-3}, e_{6t-2} : 1 \le t \le \frac{m}{2}\}$$

receive labels

$$4,5,6, \quad 16,17,18, \quad \ldots, \quad q-11, q-10, q-9,$$

the $\frac{3m}{2}$ edges

$$\{e'_{6t-4}, e'_{6t-3}, e'_{6t-2} : 1 \le t \le \frac{m}{2}\}$$

are labeled with

$$2q, 2q-1, 2q-2, \quad 2q-12, 2q-13, 2q-14, \quad \ldots, \quad q+15, q+14, q+13,$$

the $\frac{3m}{2}$ edges

$$\{e_{6t-1}, e_{6t}, e_{6t+1} : 1 \le t \le \frac{m}{2}\}$$

receive labels

$$2(q-3), 2q-7, 2(q-4), \quad 2(q-9), 2q-19, 2(q-10), \quad \ldots, \quad q+9, q+8, q+7,$$

the $\frac{3m}{2}$ edges

$$\{e'_{6t-1}, e'_{6t}, e'_{6t+1} : 1 \le t \le \frac{m}{2}\}$$

receive labels

$$10,11,12, \quad 22,23,24, \quad \ldots, \quad q-5, q-4, q-3,$$

and the three edges

$$e'_1, \quad e_1, \quad e_{3m+2}$$

are assigned the labels

$$\frac{q+1}{2}, \quad q+1, \quad q+2,$$

respectively.
So all edges are distinctly labeled. Hence $\mathcal{T}_m$, $m \ge 2$ admits EVOML.

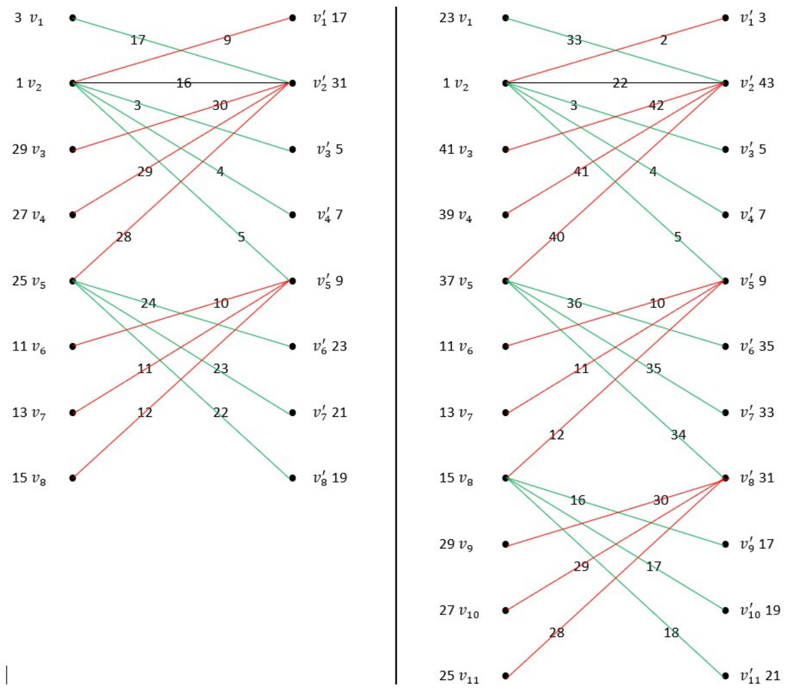

Figure 3: EVOML in $\mathcal{T}_2$ and $\mathcal{T}_3$

$\square$

Theorem 3.4 *$\mathcal{S}_m$, $m \geq 3$ admits EVOML.*

Proof: Let v_t $(1 \leq t \leq m)$ and v_t' $(1 \leq t \leq m)$ be the vertices, and e_t $(1 \leq t \leq m)$ and e_t' , $(1 \leq t \leq m-1)$ be the edges of $\mathcal{S}_m$.

The $2m$ vertices of $\mathcal{S}_m$ are labeled as follows:
Fix

$$v_1 \leftarrow 1, \quad v_1' \leftarrow q+2.$$

For $1 \leq k \leq m-1$,

$$v_{k+1} \leftarrow q + 2(k+1), \quad v_{k+1}' \leftarrow 2k+1.$$

The edges are labeled by the function f^* defined in (2.2) as below.

The m edges $\{e_t : 1 \leq t \leq m\}$ are labeled with

$$2, 3, \ldots, \frac{q+1}{2}, \frac{q+3}{2},$$

and the $m-1$ edges $\{e_t' : 1 \leq t \leq m-1\}$ are labeled with

$$q+4, q+5, q+6, \ldots, 2q+1.$$

Thus, all edge labels are distinct. Hence, $\mathcal{S}_m$, for $m \geq 4$, is an extended vertex odd mean graph.

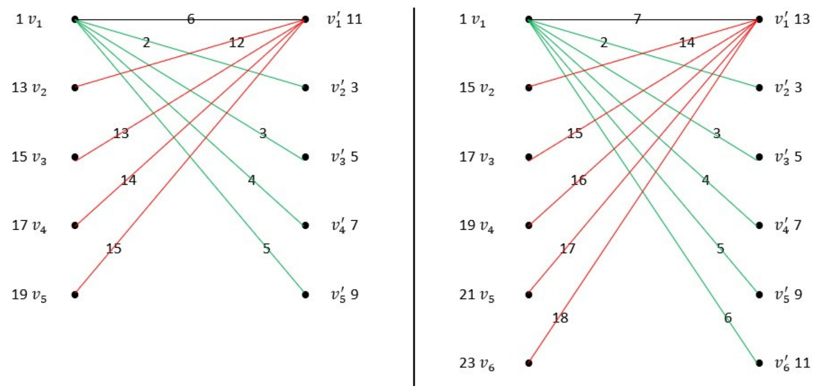

Figure 4: EVOML in $\mathcal{S}_5$ and $\mathcal{S}_6$

□

Theorem 3.5 *$\mathcal{BS}_{m,m}$, for $m \geq 2$ admits EVOML.*

Proof: Let v_t $(1 \leq t \leq 2m+2)$ and $v_t^{'}$ $(1 \leq t \leq 2m+2)$ be the vertices, and e_t $(1 \leq t \leq 2m+2)$ and $e_t^{'}$ $(1 \leq t \leq 2m+1)$ be the edges of $\mathcal{BS}_{m,m}$.

The $4m+4$ vertices of $\mathcal{BS}_{m,m}$ are labeled as below:

Fix

$$v_1 \leftarrow 1, \quad v_1^{'} \leftarrow 4m+5 (= q+2).$$

For $1 \leq k \leq m+1$,

$$v_{k+1} \leftarrow q+2k+2, \quad v_{k+1}^{'} \leftarrow 2k+1.$$

For $1 \leq k \leq m$,

(continue your labeling here...)

$$v_{m+k+2} \leftarrow \frac{q+1}{2} + 2k + 1, \quad v_{m+k+2}^{'} \leftarrow \frac{3q+9}{2} + 2(k-1).$$

The edges are labeled by the function f^* defined in (2.2) as below.

The $m+1$ edges $\{e_t : 1 \leq t \leq m+1\}$ are labeled with

$$2, 3, \ldots, m+2,$$

the m edges $\{e_{m+t}^{'} : 2 \leq t \leq m+1\}$ are assigned the labels

$$2m+4,\ 2m+5,\ \ldots,\ 4m-1,$$

the $m+1$ edges $\{e_t^{'} : 1 \leq t \leq m+1\}$ receive the labels

$$4m+6,\ 4m+7,\ 4m+8,\ \ldots,\ 5m+6,$$

the m edges $\{e_{m+t} : 2 \leq t \leq m\}$ are labeled with

$$6m + 8,\ 6m + 9,\ 6m + 10,\ \ldots,\ 7m + 7,$$

and the edge e_{2m+1} is labeled with

$$2m + 3.$$

Thus all edges are distinctly labeled. Hence $\mathcal{BS}_{m,m}$, for $m \geq 2$ admits admits EVOML.

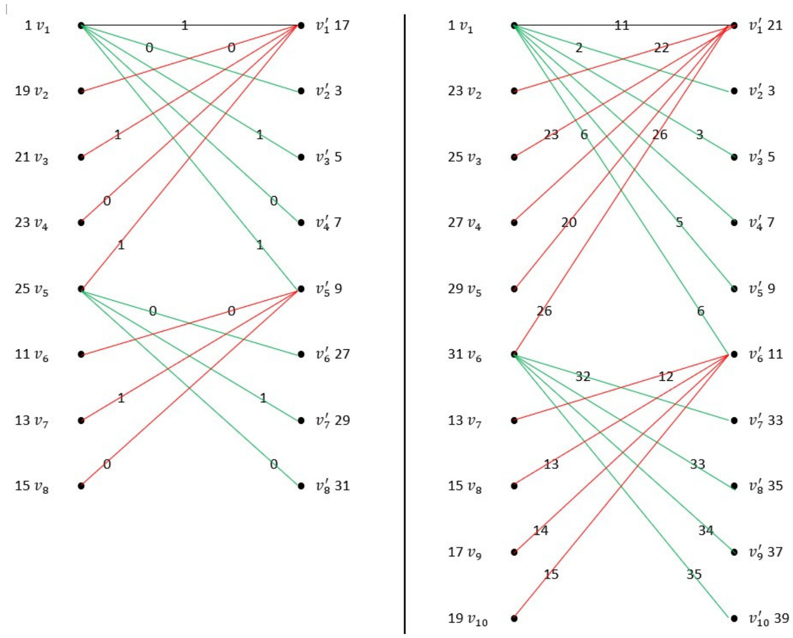

Figure 5: EVOML in $\mathcal{BS}_{3,3}$ and $\mathcal{BS}_{4,4}$

□

4. Conclusion

We have proved that the graphs $\mathcal{P}_m$, $\mathcal{CB}_{m,m}$, $\mathcal{T}_m$, $\mathcal{S}_m$, and $\mathcal{BS}_{m,m}$ admit EVOML.

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