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Impact of Constant Heat Source/Sink on Natural Convection Flow in a Vertical Channel: A Non-Linear Boussinesq Approximation Approach

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ABSTRACT: This study aims to investigate the impact of a constant source/sink on natural convection flow between two vertical walls within a porous medium. The problem is solved analytically for the steady fully developed Brinkman extended Darcy model. The influence of source/sink parameter (S), Darcy number (Da) and viscosities ratio (R_v) on fluid velocity as well as temperature is obtained in this research work. All the results are shown graphically for the variations of their relevant parameters such as Darcy number, heat source/sink parameter and ratio of viscosities. The result illustrates that the velocity profile is more in case of nonlinear density variation with temperature (NDT) than linear density variation with temperature (LDT). There is enhancement in temperature profile with the increment in source/sink parameter. The mathematical expressions for the skin friction are also obtained at both the walls. Mass flow rate in the channel is also derived analytically. Outcome of the study indicates that the nature of flow validates the previous researches done in the same area.

Key Words: Vertical channel, source/sink parameter, porous medium.

Contents

1. Introduction

Free/forced convection between a vertical channel filled with porous substances has been attracting researchers and scientists for the previous decades. Flow through porous has a wide range of applications in fields such as chemical engineering, filtration of water, and extraction of crude oil from oil reservoirs. There are additional applications worth mentioning from the rich literature related to cooling systems for electronic equipment and smoke stack dispersion in environmental engineering.

These days, many academicians are interested in investigating convection flow between two vertical parallel plates in which fluid is saturated with porous medium. The combined impact of viscous and Darcy resistances on the fully developed flow in a vertical porous stratum was examined analytically by Rudraiah and Nagraj [14]. The Darcy model breaks down for higher Darcy numbers, and inertia and viscous terms must be incorporated in the momentum transfer equation. Chang and Chang [11] have examined the convection that develops in a vertical tube partially filled with a porous medium. In their study, Chen et al. [12] examined non-Darcian mixed convection flow along non-isothermal vertical surfaces and reported heat transfer findings for the full mixed convection regime. By taking an abrupt shift in the boundary's temperature in contact with porous substrates while keeping the other boundary adiabatic, Al-Nimr and Alkam [13] numerically demonstrated the unsteady forced convection flow in concentric annuli partially filled with porous materials for the cases where the porous substrates are connected to either the inner cylinder (case I) or the outer cylinder (case II).

Mishra et al. [1] used the Brinkman-Extended-Darcy mathematical model to investigate the nature of the flow due to the temperature difference between porous medium-filled two parallel vertical walls. An analytical solution was obtained with the help of the perturbation technique. Mishra et al. [2] also

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studied the mixed convection in a vertical channel, in which analytical solutions are given for three different cases. Non-Darcy porous medium in a vertical channel with nonlinear convection flow is discussed by Partha [4]. Zueco and Ahmed [5] investigated a viscous incompressible fluid over a porous vertical infinite plate with a first-order chemical reaction. Mohammed Ibrahim and Suneetha [6] worked on the effects of the heat source and porosity of a vertical channel's mixed convection flow. Jha and Gwandu [9] studied MHD mixed convection past an infinite vertical plate. The flow affected by heat source/sink and chemical reaction was discussed by Singh and Singh [7], who investigated the free convective flow of a polar fluid within vertical concentric annuli under two separate source and sink conditions. Sandeep et al. [8] investigated the impact of radiation and magnetic fields on the flow of ferrofluid over a flat plate. They employed the `bvp5c` Matlab software for the numerical solution and similarity transformation for the analytical solution. Ermolaev and Zhbanov [3] analyzed the mixed convection flow in a vertical channel with two heat sources, where the solution was obtained using the finite element method. Natural convection in a vertical channel with the inclusion of temperature as well as concentration in the momentum equation was examined by Jha and Samaila [10], and solved analytically using the perturbation method.

In parallel to these fluid dynamics studies, several contributions have been made in graph theory. In particular, Shahzad et al. [15] computed the edge irregularity strength of star and banana trees using an algorithmic approach, while Shahzad et al. [16] extended this investigation to some classes of grid graphs. These works highlight the growing interest in algorithmic approaches for analyzing irregular labeling problems in discrete mathematics. Anurag et al. [17] investigated the flow of an annular porous zone with a heat source and sink. Dwivedi and Singh [18] examined fluid flow in a vertical cylinder due to a point line heat source/sink. Slip boundary conditions and the Boussinesq approximation were also considered in this situation. Heat source/sink which is temperature dependent in a rigid impermeable vertical cylinder is discussed by Dwivedi et al. [19]. The new time fractional-Fabrizio derived mathematical model used for natural convection flow with symmetric heat source/sink was studied analytically by Dumitru Vieru et al. [20]. The effect of heat source and MHD Casson fluid flow through a fluctuating plate was studied by B. Shankar Goud et al. [21], where the Galerkin element technique was used to solve the problem. The linear Boussinesq approximation (linear density variation with temperature [LDT]) is used in many earlier studies, such as Mishra et al. [1] and Jha & Samaila [10]. It is derived by truncating, following the second term, Taylor's series expansion of density with temperature. When there is very little temperature variation between the channel walls, this approximation is valid. However, the mathematical modeling of a linear density-temperature variation becomes imprecise when the heat variation within the walls is noticeably large. Goren [22] produced a similarity solution for the free convection flow boundary layer equations from a semi-infinite plate. Vajravelu and Sastri [23] used the NDT to study laminar free convection flow between two infinite vertical plates.

Numerous studies on free convection flow in a vertical channel with linear density variation with temperature (LDT) have been extended; however, nonlinear density variation with temperature (NDT) has a substantial impact on the flow's characteristics, which is illustrated graphically in this problem. Therefore, the novelty of this work is to consider a flow caused by natural convection between two parallel vertical walls with a heat source and sink filled with a porous material. This work also highlights the significance of the nonlinear Boussinesq approximation. This study examines the effects of the source/sink, Darcy number, and viscosity ratio.

2. Mathematical Analysis

Fully developed natural convection in a porous medium-filled vertical channel having distance L apart when a heat source or sink is present as shown in Figure 1. The y-axis is taken normal to one of the walls, and the x-axis is taken along the other wall. The temperature of the wall at $y' = 0$ is taken as T_h' whereas, at the other wall $y' = L$ it is T_0'. Here T_h' ($T_h' > T_0'$) is the cause of natural convection flow between the walls.

By extending Taylor's series to a second-degree polynomial, the mathematical model that captures the nonlinear Boussinesq approximation term is obtained as follows:

$$\rho = \rho_0 \left[1 - \beta_0(T - T_0) - \beta_1(T - T_0)^2\right]. \tag{2.1}$$

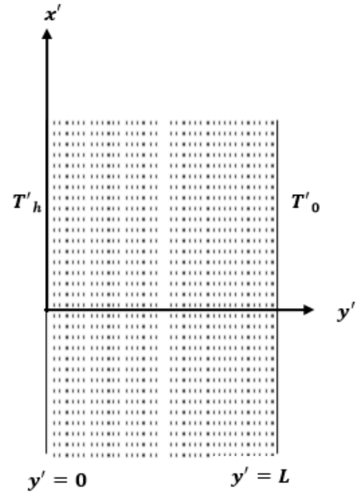

Figure 1: Physical configuration of the Model

The governing equations for free convective fluid flow of incompressible fluid with heat source/sink having a similar model as Jha and Samaila (2021) are obtained as under:

$$\frac{\mu_{\text{eff}}}{\mu_f}\frac{d^2u'}{dy'^2} - \frac{\mu_f u'}{K} + g\rho\left[1 - \beta_0(T' - T'_0) - \beta_1(T' - T'_0)^2\right] = 0 \tag{1}$$

$$\frac{d^2T'}{dy'^2} + \frac{Q_0}{k} = 0 \tag{2}$$

The boundary conditions are:

$$\begin{aligned} u' = 0, \quad T' = T'_h \quad \text{at } y' = 0 \\ u' = 0, \quad T' = T'_0 \quad \text{at } y' = L \end{aligned} \tag{3}$$

Following is the governing equation which is formed due to natural convection:

$$Rv\frac{d^2u}{dy^2} - \frac{u}{Da} + \theta + C\theta^2 = 0 \tag{4}$$

$$\frac{d^2\theta}{dy^2} + S = 0 \tag{5}$$

The following parameters are used in the process of non-dimensionalisation:

$$y = \frac{y'}{L}, \quad Da = \frac{K}{L^2}, \quad S = \frac{Q_0L^2}{k(T'_h - T'_0)}, \quad Rv = \frac{\mu_{\text{eff}}}{\mu_f},$$
$$\theta = \frac{T' - T'_0}{T'_h - T'_0}, \quad u = \frac{u'\mu_f}{g\beta L^2(T'_h - T'_0)}, \quad C = \frac{\beta_1}{\beta_0}(T'_h - T'_0).$$

The boundary conditions in dimensionless form are given as:

$$\begin{aligned} u = 0, \quad \theta = 1 \quad \text{at } y = 0 \\ u = 0, \quad \theta = 0 \quad \text{at } y = 1 \end{aligned} \tag{6}$$

Where: Da = Darcy Number, Rv = Ratio of viscosities, S = Source/Sink Parameter.
From equations (4), (5), and (6), expressions for the velocity and temperature profiles are as follows:

$$u = A_1 \sinh[m(y-1)] - A_4 \sinh(my) + A_5 y^4 + A_6 y^3 + (A_3 \sinh(m))y^2 + (A_2 \sinh(m))y + A_1 \sinh(m) \tag{7}$$

$$\theta = 1 + \left(\frac{S}{2} - 1\right) y - \frac{S}{2} y^2 \tag{8}$$

The skin frictions at both walls are:

$$\tau_1 = \left.\frac{du}{dy}\right|_{y=0} = mA_1 \cosh(m) - mA_4 + A_2 \sinh(m) \tag{9}$$

$$\tau_2 = \left.\frac{du}{dy}\right|_{y=1} = mA_1 - mA_4 \cosh(m) + 4A_5 + 3A_6 + (2A_3 + A_2) \sinh(m) \tag{10}$$

The mathematical term for mass flow rate between the walls is:

$$\int_0^1 u\,dy = \frac{1}{m}\left[(A_1 + A_4)(1 - \cosh(m))\right] + \frac{A_5}{5} + \frac{A_6}{4} + \left(\frac{A_3}{3} + \frac{A_2}{2} + A_1\right) \sinh(m) \tag{11}$$

Where:

$$m = \sqrt{\frac{1}{Da\,Rv}}, \quad A_0 = -\frac{S}{2} + \left(\left(\frac{S}{2} - 1\right)^2 - S\right) C,$$

$$A_1 = \left(1 + C + 2A_0 DaRv + 12Da^2 Rv S^2 C\right) Da,$$

$$A_2 = \left[\left(\frac{S}{2} - 1\right)(1 + 2C) - 6S\left(\frac{S}{2} - 1\right) CDaRv\right] Da,$$

$$A_3 = (A_0 + 12DaRv)Da, \quad A_4 = A_1 + A_2 + A_3 + S\left(1 - \frac{S}{2}\right) CDa + \frac{S^2}{4} CDa,$$

$$A_5 = \frac{S^2}{4} CDa \sinh(m), \quad A_6 = S\left(1 - \frac{S}{2}\right) CDa \sinh(m).$$

3. Result and Discussion

This section discusses how flow regulating parameters affect flow driven by natural convection of an incompressible fluid when the medium is porous between two parallel walls. The temperature and velocity profiles are shown graphically along with variations in the Darcy number (Da), ratio of viscosities (Rv), and heat source/sink parameter (S).

Figure 9 represents the velocity field with constant values of S and Da and the fluctuation of the ratio of viscosities (Rv). The graphical representation shows that the velocity is lower in the case of linear density variation with temperature (LDT) than in nonlinear density variation with temperature (NDT). In general, the velocity profile has a decreasing trend as Rv increases. Since porous media have a higher viscosity than fluid, velocity drops as the ratio of viscosities (Rv) increases.

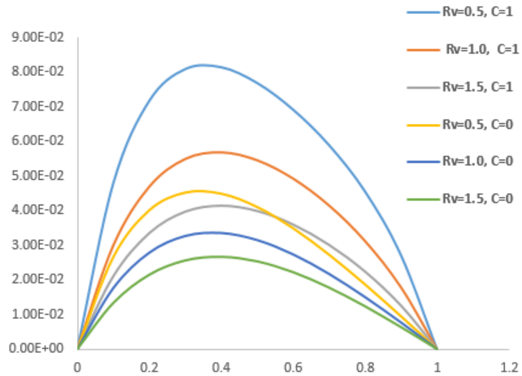

Figure 2: Velocity profile with variation of Rv for $S = 0.2$ and $Da = 0.1$.

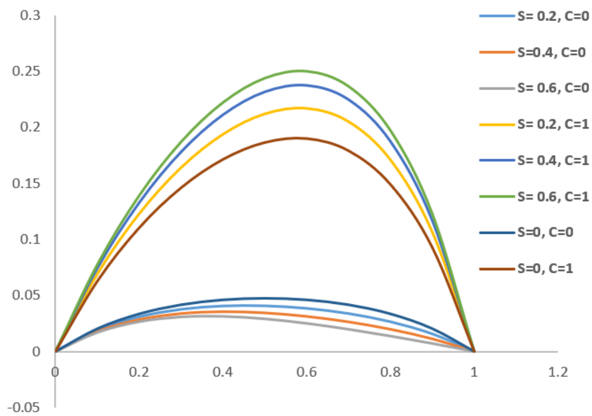

Figure 3: Velocity with the variation of Source (S), when $Da = 0.1$, $Rv = 1$

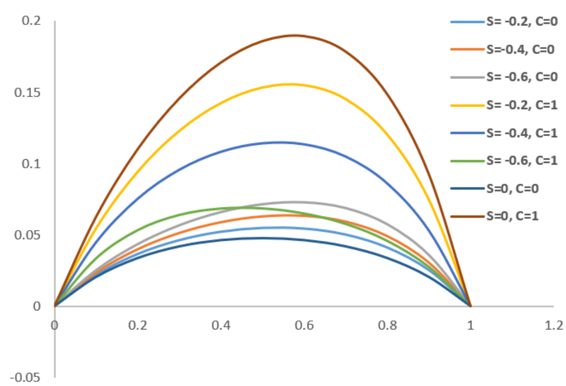

Figure 4: Velocity with the variation of Sink (S), when $Da = 0.1$, $Rv = 1$

The velocity fields for the variations of heat source (S) as well as heat sink $(-S)$ are given in Figure 3 and Figure 4, respectively, for the fixed values of (Da) and (Rv). These figures demonstrate that the velocity of the fluid improves with the elevation of the heat source, whereas it reduces in the case of the heat sink. This is quite obvious due to the fact that the heat source generates more heat energy, which causes the temperature to increase and consequently enhances the fluid velocity. Furthermore, the velocity shows an increasing trend in the case of NDT compared to LDT, which suggests that the velocity as well as the temperature would have been underestimated if the LDT model had been used to predict the flow formation.

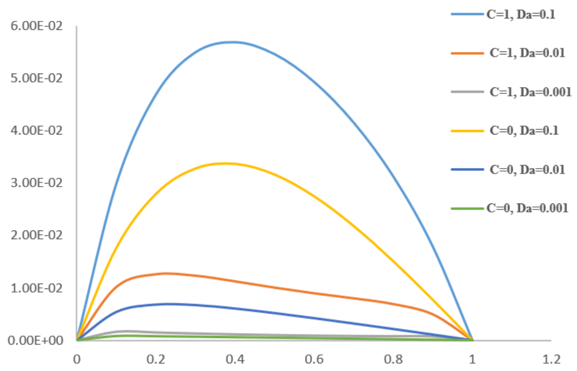

Figure 5: Velocity profile with the variation of Da

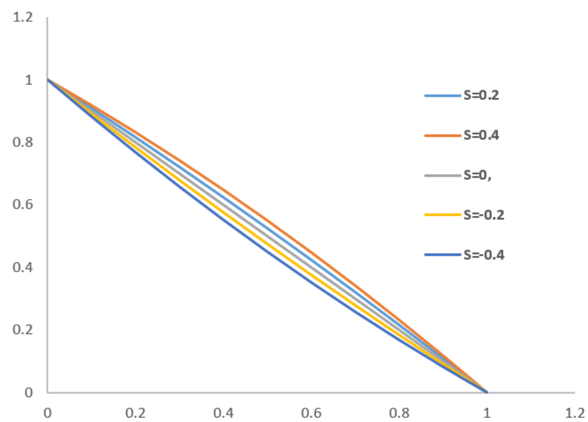

Figure 6: Temperature profile for the variation of heat source/sink

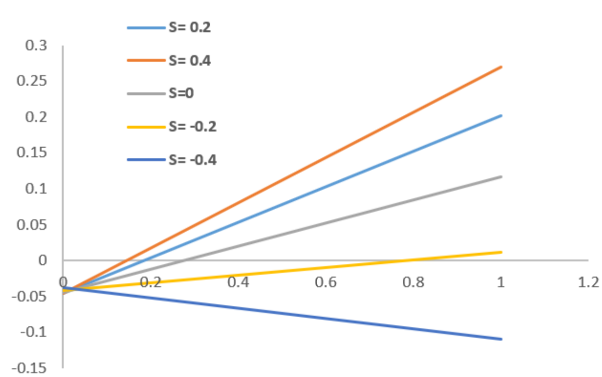

Figure 7: Mass flow rate with the variation of source/sink parameter

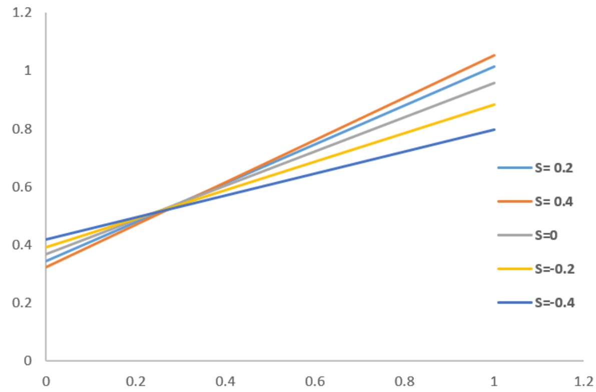

Figure 8: Skin frictions with the variation of source/sink parameter (S) at y=0

The velocity profile for the various Darcy number (Da) with the constant values of (Rv) and (S) is shown in Figure 5. The graph makes it clear that the velocity rises as the Darcy number grows. The physical interpretation of this property is that the bulk porous medium resistance decreases as K increases, and the Darcy number is directly proportional to the permeability K. Consequently, this raises the momentum development of the flow velocity.

Figure 6 displays the temperature curve for the various heat source (S) and heat sink ($-S$) parameter values. This figure illustrates that the temperature is higher in the case of a heat source, but lower in the case of a heat sink ($-S$). Additionally, the absence of a heat source/sink ($S = 0$) bifurcates the impact of the heat source and sink. The presence of a heat source provides an additional supply of thermal energy, which results in an increase in temperature.

Figure 7 reflects the mass flow rate with the changes in source as well as sink parameters. The mass flow rate elevates with the heat source (S), whereas it decreases with the heat sink ($-S$), which is a very natural phenomenon of heat transfer. It is also notable from the same figure that the mass flow rate shows an increasing trend as the NDT parameter increases.

Variations in the skin friction with source as well as sink are illustrated in Figure 8 and Figure 9 at the walls $y = 0$ and $y = 1$, respectively. The skin friction at wall $y = 0$ for the heat source is less than that for the heat sink in both cases, but the reversal trend can be observed in Figure 9 after $C = 0.5$.

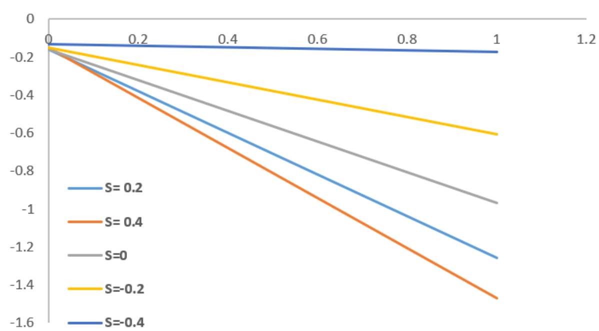

Figure 9: Velocity with the variation of Source (S), when $Da = 0.1$, $Rv = 1$

4. Conclusion

Natural convection flow in the presence of a heat source/sink and porous medium using the Darcy model, extended by the Brinkman formulation, is analyzed in this research work. The important outcomes are summarized pointwise as follows:

1. The velocity field as well as the temperature distribution in the case of NDL show an appreciating trend compared to LDL, which also validates the findings of previous research conducted by [24,25]. As expected, the velocity exhibits a diminishing nature with the elevation of the ratio of viscosities.
2. The heat source has a direct proportional relationship, which implies that the temperature rises with an increase in the source parameter, whereas it declines with an upward variation in the heat sink parameter.
3. The heat source parameter contributes positively to the mass flow rate component, while the heat sink parameter contributes negatively.
4. Skin friction illustrates opposite effects with respect to the heat source and sink parameters.

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