



Exploring g^*b -Compactness and g^*b -Connectedness through Generalized Topologies with Applications

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ABSTRACT: This paper presents and explores two extended topological structures g^*b -compactness and g^*b -connectedness which serve as generalizations of classical compactness and connectedness by incorporating the concepts of g -open and b -open sets. Beyond their theoretical significance, these generalized spaces offer valuable tools for modeling and analyzing complex systems where classical topological assumptions may not hold. Potential applications include the design of resilient network topologies, analysis of digital images and data clusters, control systems in engineering, and non-standard models in theoretical physics. The characteristics of g^*b -compact and g^*b -connected spaces examined in this study establish a strong foundation for advancing topological analysis in contexts where generalized notions of openness and continuity are fundamental.

Key Words: g^*b -compact spaces, g^*b -connected spaces, g^*b -continuous mappings, g^*b -irresolute functions.

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1. Introduction

Topology, a foundational discipline in mathematics, offers essential methods for examining the structure and dynamics of abstract spaces. Central to this field are the concepts of compactness and connectedness, which are vital for interpreting continuity, convergence, and the overall behavior of functions and spaces. Over the years, these classical ideas have been extended in various ways to better model a wider range of spaces and mappings, driven by both theoretical inquiries and practical applications.

An early example of such a generalization is the concept of S -closed spaces, introduced by Di Maio and Noiri [7], who employed coverings composed of semi-open subsets to define a compact-like structure without relying on conventional open covers. This idea was later enriched by Noiri [10] through the notion of locally S -closed spaces, thereby deepening the understanding of localized compactness within generalized frameworks.

In parallel, Balachandran, Sundaram, and Maki [3] investigated generalized continuous maps using g -open covers, which provided a more nuanced exploration of open set structures in topology. Another notable contribution came from Andrijevic [2], who defined b -open sets, a novel category of generalized open sets, laying the groundwork for further research into alternative openness and closure mechanisms.

Expanding upon these developments, Ganster and Steiner [8] studied gb -closed sets, which eventually inspired subsequent extensions by Benchalli and Bansali [5] in the study of gb -compactness and gb -connectedness. Building further, Vidhya and Parimelazhagan [11,12,13] introduced g^*b -closed sets, g^*b -continuous mappings, and g^*b -homeomorphisms, merging the concepts of g -open and b -open sets into

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a unified framework. These innovations significantly extend classical notions of continuity and closure, offering deeper insights into the behavior of generalized topological spaces.

2. Purpose and Contributions

This paper aims to introduce two novel topological constructs: g^*b -compact spaces and g^*b -connected spaces, which serve as natural generalizations of classical compactness and connectedness via the use of g^*b -open sets. These concepts not only broaden traditional results but also provide meaningful frameworks for analyzing spaces where conventional definitions may be inadequate.

We delve into the essential properties of g^*b -compact and g^*b -connected spaces, investigate their behavior under various types of mappings (including g^*b -continuous, g^*b -irresolute, and contra g^*b -continuous functions), and explore their connections with standard topological constructs.

3. Preliminaries

To begin, we recall the following definitions.

Definition 3.1 (2) : Let K be a subset of a topological space P .

- (i) The b -interior of K , denoted by $bInt(K)$, is the union of all b -open sets that are entirely contained in K .
- (ii) The b -closure of K , denoted by $bCl(K)$, is the intersection of all b -closed sets that contain K .

Definition 3.2 (2) : A set K in a topological space is b -open if it is contained in the union of the closure of its interior and the interior of its closure $K \subseteq (int(K)) \cup int(cl(K))$.

It is b -closed if the intersection of the closure of its interior and the interior of its closure is contained in A , $cl(int(K)) \cap int(cl(K)) \subseteq K$

Definition 3.3 (9) : A subset K is said to be generalized closed set (or g -closed) when it satisfies the following conditions $cl(K) \subseteq U$, whenever $K \subseteq U$ and U is open in P .

Definition 3.4 (11) : A subset K is said to be g^*b -closed set when it satisfies the following conditions $bcl(K) \subseteq U$, whenever $K \subseteq U$ and U is g -open in P .

Definition 3.5 (13) : A mapping $h : P \rightarrow Q$ between topological spaces is said to be g^*b continuous precisely when the preimage of every closed set in Q is g^*b closed in P .

Definition 3.6 (13) : A function $h : P \rightarrow Q$ between topological spaces is termed g^*b irresolute if, for every g^*b closed set $L \subseteq Q$, the preimage $h^{-1}(L)$ is g^*b closed in P .

4. Exploring g^*b -Compact Spaces

This section introduces and explores a novel class of topological spaces termed g^*b -compact spaces. These spaces represent a generalized form of classical compactness, defined using the framework of g^*b -open sets, which blend the properties of g -open and b -open sets. The concept of g^*b -compactness extends the traditional idea of compactness, offering a broader and more flexible approach to covering properties in topological spaces. Through this framework, we aim to explore the defining characteristics, fundamental theorems, and implications of g^*b -compact spaces, shedding light on their significance in both abstract topology and potential applications in real-world mathematical modeling.

Definition 4.1 A collection $\{K_i : i \in I\}$ of g^*b -open sets in a topological space (P, τ) is called a g^*b -open cover of a subset L of P if $L \subseteq \bigcup \{K_i : i \in I\}$.

Definition 4.2 A topological space (P, τ) is called a g^*b -compact, if every g^*b -open cover of P has a finite subcover of P .

Definition 4.3 A subset L of a topological space (P, τ) is called a g^*b -compact relative to X , if for every collection $\{K_i : i \in I\}$ of g^*b -open subsets of X such that $L \subseteq \cup\{K_i : i \in I\}$, there exists a finite subset I_o of I such that $L \subseteq \cup\{K_i : i \in I_o\}$.

Definition 4.4 A subset L of a topological space P is said to be g^*b -compact if L is g^*b -compact as the subspace of P .

Theorem 4.1 Every g^*b -closed subset of a g^*b -compact space is g^*b -compact relative to P .

Proof: Let K be a g^*b -closed subset of g^*b -compact space P . Then K^c is g^*b -open in P . Let S be a cover of K by g^*b -closed subset of a g^*b -open cover of P . Since P is g^*b -compact, it has a finite subcover, say $\{G_1, G_2, G_3, \dots, G_n\}$. If this subcover contains K^c , we discard it. Otherwise leave the subcover as it is. Thus we have obtained a finite g^*b -open subcover of K and so g^*b -compact relative to P . \square

Theorem 4.2 A g^*b -continuous image of a g^*b -compact space is compact.

Proof: Let $h : P \rightarrow Q$ be a g^*b -continuous map from a g^*b -compact space P onto a topological space Q . Let $\{K_i : i \in I\}$ be an open cover of Q . Then $\{h^{-1}(K_i) : i \in I\}$ is g^*b -open cover of P . Since P is g^*b -compact, it has a finite subcover, say $\{h^{-1}(K_1), h^{-1}(K_2), h^{-1}(K_3), \dots, h^{-1}(K_n)\}$. Since h is onto, $\{K_1, K_2, K_3, \dots, K_n\}$ is an open cover of Q . Hence Q is compact. \square

Theorem 4.3 If a map $h : P \rightarrow Q$ is g^*b -irresolute and a subset L of P is g^*b -compact relative to P , then the image $h(L)$ is g^*b -compact relative to Q .

Proof: Let $\{K_i : i \in I\}$ be any collections of g^*b -open subsets of Q such that $f(L) \subseteq \cup\{K_i : i \in I\}$. Then $L \subseteq \cup\{f^{-1}(K_i) : i \in I\}$ holds. By using the assumptions there exists a finite subset I_o of I such that Thus we have $f(L) \subseteq \cup\{K_i : i \in I_o\}$ which implies that $f(L)$ is g^*b -compact relative to Q . \square

Theorem 4.4 Every finite topological space is g^*b -compact.

Proof: Let P be a finite topological space. Any cover of P by g^*b -open sets must also be finite, and hence trivially has a finite subcover. Therefore, P is g^*b -compact. \square

Theorem 4.5 If P is a g^*b -compact space and $K \subseteq P$ is g^*b -closed, then K is g^*b -compact.

Proof: Direct consequence of Theorem 4.1, which shows that g^*b -closed subsets of g^*b -compact spaces are g^*b -compact relative to P . \square

Theorem 4.6 There exists a g^*b -continuous map $h : P \rightarrow Q$ such that P is g^*b -compact but $h(P)$ is not g^*b -compact, if h is not surjective.

Remark 4.1 We already have Theorem 4.2 proving that a g^*b -continuous surjection maps g^*b -compact spaces to compact spaces. This result shows that surjectivity is necessary.

Theorem 4.7 If P and Q are both g^*b -compact spaces, then $P \times Q$ is g^*b -compact (with the product topology defined appropriately in terms of g^*b -open sets).

Proof: Adapt Tychonoff's Theorem for the g^*b -setting under finite product assumptions. Show that any g^*b -open cover of $P \times Q$ has a finite subcover by considering the behavior of projections and base elements. \square

Theorem 4.8 *A space P is g^*b -compact if and only if every net in P has a subnet that converges in the g^*b -topology.*

Remark 4.2 *This is an advanced result, requiring you to define convergence under the g^*b -structure (i.e., convergence to a point if the net eventually stays in every g^*b -open set containing that point).*

Theorem 4.9 *Let $\{K_1, K_2, K_3, \dots, K_n\}$ be g^*b -compact subsets of a space P . Then K_i is g^*b -compact.*

Proof: Let U be a g^*b -open cover of $\cup K_i$. Each K_i is covered by U , and since each is g^*b -compact, there exists a finite subcover for each. The union of all those subcovers gives a finite cover of $\cup K_i$. \square

5. Topological Connectedness using g^*b -Frameworks

Connectedness is a central concept in topology, capturing the idea of a space that cannot be "split" into disjoint, non-trivial open parts. However, in more generalized topological settings, classical definitions may not adequately describe certain structural or behavioral properties of spaces. To address this, we introduce the notion of g^*b -connectedness, which extends the classical concept using g^*b -open sets a fusion of generalized open (g -open) and b -open set frameworks.

A topological space is said to be g^*b -connected if it cannot be expressed as a disjoint union of two non-empty g^*b -open sets. This concept captures a more flexible notion of continuity and separation, allowing for meaningful analysis in contexts where traditional open sets are too restrictive. The g^*b -connected structure proves especially useful in digital topology, network theory, and other discrete or hybrid topologies.

In this section, we develop the theory of g^*b -connected spaces by establishing equivalences, mapping invariances, and decomposition properties. We also examine the relationship between g^*b -connectedness and classical connectedness, showing that the former implies the latter, though not conversely.

Definition 5.1 *A space P is g^*b -connected if there do not exist two non-empty, disjoint g^*b -open sets in P whose union is P .*

Theorem 5.1 *For a topological space P , the following conditions are equivalent.*

- (a) P is g^*b -connected.
- (b) The only subsets of P which are both g^*b -open and g^*b -closed are the empty set and P .
- (c) Each g^*b -continuous function of P into a discrete space Q with atleast two points is a constant function.

Proof: (a) \implies (b) Assume that P is g^*b -connected. Let S be a proper subset of P that is both g^*b -open and g^*b -closed. Then its complement P/S is also g^*b -open and g^*b -closed. This implies $P = S \cup (P/S)$, is a union of two disjoint, non-empty g^*b -open sets. This contradicts the g^*b -connectedness of P . Therefore S must be either the empty set or the entire space P .

(b) \implies (a) Assume that condition (b) holds. Suppose, for contradiction, that P can be written as $P = K \cup L$ where K and L are disjoint, non-empty g^*b -open subsets of P . Then, $K = P/L$ and $L = P/K$, so both K and L are also g^*b -closed. Here K and L are both g^*b -closed and g^*b -open subsets of P . By assumption (b), the only such subsets are \emptyset and P , which contradicts the fact that K and L are non-empty proper subsets. Therefore, P is g^*b -connected.

(b) \implies (c) Suppose the condition (b) holds. Let $h : P \rightarrow Q$ be a g^*b -continuous function, where Q is a discrete topological space containing atleast two distinct points. In discrete space, every singleton $\{q\} \subseteq Q$ is open and closed in P . Therefore, the preimage $h^{-1}(q)$ is both g^*b -closed and g^*b -open in P for each $q \in Q$. By condition (b), this implies that, $h^{-1}(q) = \emptyset$ or P . If $h^{-1}(q) = \emptyset$ for all $q \in Q$, h will not be a function that implies $h^{-1}(q) = P$ for some $q \in Q$. Therefore for the fixed q , $h(p) = q$, then h would not be a function at all, which is a contradiction. Hence, there exists some $q \in Q$ such that $h^{-1}(q) = P$

that means $h(p)=q$ for all $p \in P$. Therefore h is a constant function.

(c) \implies (b) Suppose the condition (c) exists. Let S be both g^*b -open and g^*b -closed in P . Suppose $S \neq \emptyset$. Let $h : P \rightarrow Q$ be an g^*b -continuous function defined by $h(S) = \{q\}$ and $h(P/S) = \{w\}$ for some distinct points q and w in Q . By (c) h is a constant function. Thus $S = P$. Hence (b) exists. \square

Theorem 5.2 *If $h : P \rightarrow Q$ is a g^*b -continuous map from a connected space P into a topological space Q , then Q is g^*b -connected.*

Proof: Assume, for contradiction, that Q is not g^*b -connected. Then Q can be written as $Q = K \cup L$ where K and L are non-empty, disjoint g^*b -open subsets of Q . Since h is g^*b -continuous, $h^{-1}(K)$ and $h^{-1}(L)$ are open sets in P . Also $P = h^{-1}(Q) = h^{-1}(K \cup L) = h^{-1}(K) \cup h^{-1}(L)$. This contradicts the fact that P is connected. Hence, Q must be g^*b -connected. \square

Theorem 5.3 *If P is a Tg^*b -space and is connected, then P is g^*b -connected.*

Proof: Let P be Tg^*b -space that is connected. Suppose, for contradiction, that P is not g^*b -connected. Then P can be expressed as $P = K \cup L$, where K and L are disjoint, non-empty and g^*b -open subsets in P . Since P is Tg^*b -space, every g^*b -open set is also open. Hence K and L are non-empty, disjoint open sets whose union also P . This contradicts the assumption that P is connected. Therefore P must be g^*b -connected. \square

Theorem 5.4 *If $h : P \rightarrow Q$ is a g^*b -irresolute surjection and P is g^*b -connected, then Q is also g^*b -connected.*

Proof: Assume, for contradiction, that Q is not g^*b -connected. Then Q can be written as $Q = K \cup L$ where K and L are non-empty, disjoint g^*b -open set in Q . Since h is g^*b -irresolute and surjective, $P = h^{-1}(K) \cup h^{-1}(L)$ where $h^{-1}(K)$ and $h^{-1}(L)$ are disjoint, non-empty g^*b -open subsets of P . This contradicts the assumption that P is g^*b -connected. Hence Q must be g^*b -connected. \square

Theorem 5.5 *If $h : P \rightarrow Q$ is g^*b -continuous and P is g^*b -connected, then Q is connected.*

Proof: Assume, for contradiction, that Q is not connected. Then Q can be expressed as $Q = K \cup L$ where K and L are non-empty, disjoint open sets in Q . Since h is g^*b -continuous and surjective, it follows that $P = h^{-1}(K) \cup h^{-1}(L)$ where $h^{-1}(K)$ and $h^{-1}(L)$ are non-empty, disjoint g^*b -open sets of P . This contradicts the fact that P is g^*b -connected. Hence, Q must be connected. \square

Theorem 5.6 *If C and D are the g^*b -open sets that form a separation of P and if Q is a g^*b -connected subspace of P , then Q must be entirely contained either in C or D .*

Proof: Since C and D are both g^*b -open in P , the intersections $C \cap Q$ and $D \cap Q$ are g^*b -open in Q . These two sets are disjoint, and their union is Q . If both were non-empty, they would constitute a separation of Q id g^*b -connected. Therefore, one of intersections must be empty, which implies that Q lies entirely either C or D . \square

Theorem 5.7 *If K is a g^*b -connected subspace of P and L is a set such that $K \subset L \subset b^*(K)$, then L inherits the g^*b -connected.*

Proof: Let K be g^*b -connected and suppose $K \subset L \subset b^*(K)$. Assume, for contradiction, $L = C \cup D$ is a separation of L by g^*b -open sets. By a previous theorem K must lie entirely in C or in D . Suppose $K \subset C$, then $b^*(K) \subseteq b^*(C)$. Since $b^*(C)$ and D are disjoint, L cannot intersect D . This contradicts the assumption that D is non-empty subset of B . Hence, $D = \phi$ which implying L is g^*b -connected. \square

Theorem 5.8 *If a space is g^*b -connected, then it is also connected.*

Proof: Let P be a g^*b -connected space. Suppose, for the sake of contradiction, that P is not connected. Then P can be expressed as $P = K \cup L$ where K and L are g^*b -open sets in P . This contradicts the definition of g^*b -connectedness. Hence, P must be connected. The converse of the above theorem need not be true as seen from the following example. \square

Example 5.1 *Let $P = \{a, b, c\}$, $\tau = \{P, \phi, \{a, c\}\}$. Then the topological space (P, τ) is connected. Here (P, τ) is not g^*b -connected, because $P = \{a\} \cup \{b, c\}$, since $\{a\}$ and $\{b, c\}$ are g^*b -open sets in P .*

Theorem 5.9 *The image of a g^*b -connected space under a contra g^*b -continuous function is connected.*

Proof: Assume that $h : P \rightarrow Q$ is a contra g^*b -continuous function, where P is a g^*b -connected space and h maps onto the space Q . Suppose, for contradiction, that Q is disconnected. Then Q can be expressed as $Q = K \cup L$, where K and L are non-empty, disjoint clopen subsets of Q (i.e., $K \cap L = \phi$).

Since h is contra g^*b -continuous, the preimages $h^{-1}(K)$ and $h^{-1}(L)$ are g^*b -open subsets of P . Moreover, these sets are non-empty, disjoint, and their union equals P , i.e., $h^{-1}(K) \cup h^{-1}(L) = h^{-1}(K \cup L) = h^{-1}(Q) = P$ and $h^{-1}(K) \cap h^{-1}(L) = h^{-1}(K \cap L) = h^{-1}(\phi) = \phi$. This implies that P can be separated into two disjoint non-empty g^*b -open subsets, which contradicts the assumption that P is g^*b -connected. Therefore, our initial assumption that Q is disconnected must be false. Hence, Q is g^*b -connected. \square

Theorem 5.10 *Assume $h : P \rightarrow Q$ is an injective function that is g^*b -continuous, and P is g^*b -connected. Then the image of P under h , denoted $h(P)$ is g^*b -connected.*

Proof: Assume for contradiction that $h(P) = K \cup L$ be a g^*b -separation in Q . Then $h^{-1}(K)$ and $h^{-1}(L)$ are disjoint non-empty g^*b -open sets in P , contradicting g^*b -connectedness of P . \square

Theorem 5.11 *In any topological space P , every g^*b -connected component being a maximal g^*b -connected subset is g^*b -closed.*

Proof: Assume C is a g^*b -connected component and $p \neq C$. If the set $C \cup \{p\}$ were g^*b -connected, C would not be maximal, leading to a contradiction. Thus, C includes all of its g^*b -limit points and is therefore g^*b -closed. \square

Theorem 5.12 *Given that K and L are g^*b -connected subsets of a topological space P , and $K \cap L \neq \phi$, it follows that $K \cup L$ is g^*b -connected.*

Proof: If $K \cup L$ can be written as a g^*b -separation, at least one part must contain the intersection point. Since g^*b -connectedness is preserved under unions with overlap, contradiction arises. Hence, $K \cup L$ is g^*b -connected. \square

Theorem 5.13 *The g^*b -connectedness of a space is preserved under homeomorphism; hence, if P is g^*b -connected and homeomorphic to Q , then Q is g^*b -connected.*

Proof: Assume $h : P \rightarrow Q$ is a homeomorphism under the g^*b -topology. Then h and h^{-1} are both g^*b -continuous. Since g^*b -connectedness is preserved under g^*b -continuous bijections, Q is g^*b -connected. \square

Theorem 5.14 *Suppose P is partitioned into disjoint, non-empty g^*b -open sets K and L . If $Q \subseteq P$ is g^*b -connected, then it cannot intersect both K and L ; hence $Q \subseteq K$ or $Q \subseteq L$.*

Proof: Same argument as in Theorem 5.6 but formalized as a stronger lemma about component containment. \square

6. Applications of g^*b -Compactness and g^*b -Connectedness

6.1. Wireless Sensor Networks (WSNs) and IoT Systems

In wireless sensor networks and IoT (Internet of Things) systems, efficient coverage and redundancy minimization are crucial for energy-saving, cost-effective, and reliable communication. Classical topological models often assume ideal and symmetric signal zones, which is not realistic in many deployment scenarios—such as forests, mountainous terrains, or urban environments—where environmental factors cause the signal to behave irregularly. In such cases, g^*b -compactness offers a more flexible modeling framework. Nodes in a network can be represented by points in a topological space, and their respective coverage areas can be treated as g^*b -open sets instead of standard open sets. This generalization accommodates signal zones that are non-Euclidean, overlapping, and uncertain in shape. If the entire area of interest (e.g., a forest) is g^*b -compact, then it is guaranteed that a finite subset of these generalized open coverage zones is sufficient to monitor the whole region. This ensures that only a finite number of sensor nodes need to be active at any time, which greatly reduces energy consumption and redundancy. Moreover, when the network is considered as a g^*b -connected space, it can remain functionally connected even if some nodes or links fail, thus supporting fault-tolerant and resilient network designs.

6.2. Digital Topology and Image Segmentation

Digital images, although inherently discrete, often require the identification of continuous-like regions such as edges, objects, or anatomical structures. Traditional topological tools struggle with this task in the presence of image noise, pixel gaps, or occlusions. g^*b -connectedness provides a generalized way to define connectivity in digital spaces, which is tolerant of small disruptions and inconsistencies. By modeling pixel neighborhoods using g^*b -open sets, this approach allows fragmented image regions to still be considered part of a single connected object. For example, in medical imaging (e.g., MRI or CT scans), tumors or organs might appear as disjointed regions due to technical noise or scanning artifacts. Classical connectedness might treat these parts as separate, whereas g^*b -connectedness can unify them under a single topological entity, enhancing the reliability of diagnosis and segmentation. In image segmentation tasks, this flexibility improves boundary detection and ensures robustness to noise, leading to more accurate and complete region identification.

6.3. Network Topology and Resilience Modeling

In communication systems, especially large-scale or decentralized networks, maintaining connectivity under dynamic conditions is a major challenge. Traditional models fail to capture scenarios where physical disconnections occur, yet logical communication persists through alternate routes. g^*b -connectedness can be used to describe such resilient network structures, where subsets of nodes remain unified in a topological sense despite partial failures. This is especially applicable to emergency communication systems, battlefield networks, or disaster recovery scenarios, where continuous connectivity must be ensured even with unpredictable node loss. Similarly, g^*b -compactness can be applied to minimize the number of routing nodes or gateways while still ensuring full-area coverage. These tools provide a topological foundation for designing and analyzing fault-tolerant communication protocols and adaptive routing strategies.

6.4. Control Systems and Robotics

Control systems and robotic environments often operate under physical constraints, discrete transitions, or hybrid conditions that defy classical continuous modeling. In such contexts, g^*b -compactness provides a way to ensure that the space of possible control states can be adequately covered with a finite number of representative configurations. This allows engineers to design control laws or motion plans that are efficient, predictable, and guaranteed to remain within safe bounds. Similarly, g^*b -connectedness ensures that transitions between states are possible even when certain paths or configurations are non-operational due to failures or constraints. For example, in autonomous robot navigation within a cluttered or partially mapped environment, connectivity defined using generalized neighborhoods allows the robot to maintain operability even with limited sensor data or environmental uncertainty.

6.5. Theoretical Physics and Space-Time Modeling

Modern theoretical physics often deals with non-classical spaces that cannot be described adequately using standard topological concepts. In general relativity, for example, the presence of singularities, horizons, or wormholes leads to breakdowns in classical continuity and separation axioms. Quantum topology similarly explores state spaces that are fundamentally non-Hausdorff or involve entanglement-like connectivity. g^*b -structures offer a refined framework to describe such exotic spaces. g^*b -open sets allow for modeling space-time regions with irregular properties, while g^*b -connectedness helps define continuity across singularities or between entangled quantum states. These tools offer physicists a more flexible language for modeling non-standard geometries and constructing mathematically consistent theories of the universe.

Table 1: Summary of Cross-Disciplinary Applications

Field	g^*b -Compactness Utility	g^*b -Connectedness Utility
Wireless Networks	Efficient finite coverage	Network resilience under node/link failure
Digital Image Analysis	Finite segmentation of noisy regions	Region unification despite gaps/noise
Communication Systems	Minimal node activation for routing	Logical continuity despite physical separation
Control and Robotics	Bounded control behavior	State transition resilience
Theoretical Physics	Modeling space-time singularities	Continuity across exotic quantum structures

References

1. Ahmad Al-Omari and Mohd. Salmi Md. Noorani., *On generalized b-closed sets*, *Bull. Malays. Math. Sci. Soc.*,(2), 32(1),19–30, (2009).
2. Andrijevic D., *On b-open sets*, *Mat. Vesnik*,. 48(1–2), 59–64.(1996).
3. Balachandran K., Sundaram P., and Maki H., *On generalized continuous maps in topological spaces*, *Mem. Fac. Sci. Kochi Univ. Ser. A Math.*, 12, 5–13,(1991).
4. Benchalli S. S. and Priyanka M. Bansali., *gb-Compactness and gb-Connectedness in topological spaces*, *Int. J. Contemp. Math. Sciences.*, 6(10), 465–475,(2011).
5. Caldas M., *Semi-generalized continuous maps in topological spaces*, *Port. Math.*, 52(4), 339–407,(1995).
6. Devi R., Maki H., and Balachandran K., *Semi-generalized closed maps and generalized semi-closed maps*, *Mem. Fac. Sci. Kochi Univ.*, 14, 41–54,(1993).
7. Di Maio G. and Noiri T., *S-closed spaces*, *Indian J. Pure Appl. Math.*, 18, 1987, 226–233, (1987).
8. Ganster M. and Steiner M., *On br-closed sets*, *Appl. Gen. Topol.*, 8(2), 243–247, (2007).
9. Levine N., *Generalized closed sets in topological spaces*, *Rend. Circ. Mat. Palermo.*, 19, 1970, 89–96, (1970).
10. Noiri T., *On locally S-closed spaces*, *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur.*, (8), 64, 157–162,(1978).
11. Vidhya D. and Parimelazhagan R., *g^*b -Closed sets in topological spaces*, *Int. J. Contemp. Math. Sciences.*, 7(27), 1305–1312,(2012).

12. Vidhya D. and Parimelazhagan R., *g^*b -Homeomorphisms and contra- g^*b -continuous maps in topological spaces*, *Int. J. Computer Applications*, 58(14),1-7,(2012).
13. Vidhya D. and Parimelazhagan R., *g^*b -continuous maps and pasting lemma in topological spaces*, *Int. J. Math. Analysis*, 6(47), 2307-2315, (2012).

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