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Conformable Fractional Inverse Rayleigh Distribution

Kanak Modi, Rahul Sharma and Yudhveer Singh*

ABSTRACT: The conformable fractional Inverse Rayleigh distribution is examined in this study. The conformable cumulative distribution function and the hazard rate function are among the primary functions derived for this novel distribution. We also obtain a precise flalign for the new distribution's mean, variance, and r^{th} moment. Additionally, the quantile function and mode associated with this distribution are derived. Tsallis entropy and Shannon entropy are two examples of entropy metrics that are derived. Additionally, the density of the b^{th} order statistic for the new distribution is calculated, and we present the order statistics of a fractional random variable.

Keywords: Conformable fractional derivative, fractional probability distribution, inverse Rayleigh distribution, entropy, order Ssatistics.

Contents

1. Introduction

Fractional calculus is a branch of mathematics that deals with fractional derivatives and integrals, and it has numerous applications in a variety of scientific and engineering disciplines.
A current framework for describing non-local and memory-dependent complex processes is offered by fractional derivatives. One of the intriguing developments in this field is the idea of a "conformable fractional derivative", introduced by [9]. Integer order derivatives, which are frequently used in classical calculus, have some limitations, when applied to phenomena with non-differentiable or non-smooth behaviour.
Fractional derivatives, however, overcome these limitations by taking into account non-integer orders, which better capture a variety of natural and artificial systems utilizing the notion of conformable fractional derivative. This new concept provides a different perspective on fractional derivatives and their applications [7,8]. The conformable fractional derivative, while retaining the advantages of fractional calculus, offers a fascinating framework for modelling complex phenomena.
The Conformable Fractional Inverse Rayleigh Distribution (CFIRD), which offers an advanced framework for simulating various real-world phenomena, has emerged as a promising statistical tool. The development of sophisticated tools to model complex phenomena and derive meaningful insights is made

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* Corresponding author.
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possible by advancements in probability theory and statistical analysis. The CFIRD stands out among these tools as an effective and pioneering idea, providing a new viewpoint on probability distributions and their applications. The CFIRD and its numerous applications in various fields will be thoroughly explored in this research paper.
The inverse Rayleigh distribution was first presented by [11] as a model for examining survival and reliability data. [12] later investigated the model further and found that the inverse Rayleigh distribution could accurately represent the lifetime distributions of different experimental units. Voda also investigated its characteristics and offered a scale parameter maximum likelihood (ML) estimator. When examining the survival duration of particular diseases, the Inverse Rayleigh (IR) model may be a viable alternative to the log-normal distribution. A probability distribution is extended to a new generalised distribution based on fractional derivatives in the relatively recent field of fractional probability distribution. A method for defining a fractional probability distribution using a conformable fractional derivative was presented by [6], with the following definition:

Definition 1 *Let $\alpha \in (0,1]$ and $f : E \subseteq (0,\infty] \to \mathbb{R}$. For $t \in E$, let*

$$f^{(\alpha)}(t) = \lim_{\varepsilon \to 0} \frac{f\left(t + \varepsilon t^{1-\alpha}\right) - f(t)}{\varepsilon}.$$

If the limit exists, $f^{(\alpha)}(0) = \lim_{t \to 0^+} f^{(\alpha)}(t)$ at t=0 , is called the α-conformable fractional derivative of f at t. Also, $f^{(\alpha)}(t) = f'(t)\, t^{1-\alpha}$. for all $t > 0$, $\alpha \in (0,1]$, where $f'(t)$ is the usual first derivative and satisfies all the classical properties. We may refer to [7] and [1] for more information on conformable fractional derivative.

Remark 1 *The integral departs from the standard Riemann integral and instead involves an erroneous version. The parameter α takes values in the interval (0, 1). The conformable derivative, while maintaining all the conventional characteristics of the ordinary first derivative, is under consideration. Furthermore, the derivative gives rise to correct propositions according to the context presented.*

- $D_\alpha\left(t^\kappa\right) = \kappa t^{\kappa-\alpha}, \qquad for\, all\, \kappa \in \mathbb{R}$
- $D_\alpha\left(\sin {t^\alpha}/{\alpha}\right) = \cos {t^\alpha}/{\alpha}$
- $D_\alpha\left(\cos {t^\alpha}/{\alpha}\right) = -\sin {t^\alpha}/{\alpha}$
- $D_\alpha\left(e^{t^\alpha/\alpha}\right) = e^{t^\alpha/\alpha}$

[9] developed the relatively new concept of conformable fractional derivative. For certain fractional distributions, such as fractional chi-square, Rayleigh, gamma, beta, inverse gamma, and Lomax distributions, [6] presented a conformable probability distribution function (CPDF) using a fractional differential flalign. The fractional inverse gamma distribution was examined and some of its characteristics were provided by [5]. The conformable cumulative distribution function (CCDF), mode, moments, certain entropy measures, and the densities of the order statistics are among the primary characteristics of the conformable fractional Inverse Rayleigh distribution (CFIRD) that we examine in this study. We need the following Lemma to prove our results:

Lemma 1.1 *From ([3], eq. (3.381.4), Pg 346). For $Re(p) > 0$ and $Re(c) > 0$:*

$$\int_0^\infty x^{c-1} e^{-px}\, dx = \frac{\Gamma(c)}{p^c}.$$

Lemma 1.2 *On page 25, eq (1.110), given in [3]. For $\Upsilon > 0$ and $|z| \leq 1$, then by binomial series expansion, we have:*

$$(1-z)^{\Upsilon-1} = \sum_{p=0}^{\infty} (-1)^p \binom{\Upsilon-1}{p} z^p.$$

2. Fractional Inverse Rayleigh Distribution

The Inverse Rayleigh (IR) distribution was considered by [12]. He examines certain characteristics of the maximum likelihood estimator of the scale parameter of the inverse Rayleigh distribution, which is also utilised in lifetime experiments. The probability density function (pdf) of T with an IR distribution looks like this:

The Cumulative Distribution Function (CDF) of the Inverse Rayleigh distribution is given by

$$F\left(t\right)=e^{-\frac{\omega^2}{t^2}},\quad \text{where}\,\omega>0\,\text{is a scale parameter.} \tag{2.1}$$

The Probability Density Function (PDF) of the Inverse Rayleigh distribution is defined as:

$$f\left(t\right)=\frac{2\omega^2}{t^3}e^{-\frac{\omega^2}{t^2}}. \tag{2.2}$$

In this section, we will introduce the CFIRD using the following conformable differential equation defined as follows:

$$\begin{aligned} t^{3\alpha}D^{\alpha}y+\left[3t^{2\alpha}-2\omega^2\right]y&=0\\ \implies \frac{y'}{y}&=-\frac{3}{t}+\frac{2\omega^2}{t^{2\alpha+1}}\\ \implies \ln y&=-3\ln t-\frac{\omega^2t^{-2\alpha}}{\alpha}+\ln A\\ \implies y&=t^{-3}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}A=f_{\alpha}\left(t\right). \end{aligned}$$

Using pdf property, we can write:

$$\int_0^{\infty} f_{\alpha}\left(t\right)d^{\alpha}t=1.$$

Hence the normalizing factor is:

$$A=\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}.$$

Thus, the CFPDF of a random variable T with support (0,∞) is given by:

$$f_{\alpha}\left(t\right)=t^{-3}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}. \tag{2.3}$$

It is interesting to note that: $\lim\limits_{\alpha\to1}\left(f_{\alpha}\left(t\right)\right)=2\omega^2t^{-3}e^{-\frac{\omega^2}{t^2}}$ provided that $\omega>0$, which is a probability distribution function of Inverse Rayleigh distribution with parameter ω. Consequently, the CFCDF of a random variable T can be derived as:

$$F_{\alpha}\left(t\right)=\int_0^t f_{\alpha}\left(y\right)d^{\alpha}y,$$

$$F_{\alpha}\left(t\right)=\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\int_0^t y^{\alpha-4}e^{-\frac{\omega^2}{\alpha y^{2\alpha}}}dy,$$

put $y^{-2\alpha}=p. \Rightarrow -2\alpha y^{-2\alpha-1}dy=dp$ and $y=p^{-1/2\alpha}$.

$$F_{\alpha}\left(t\right)=1-\frac{\gamma\left(\frac{3}{2\alpha}-\frac{1}{2},\frac{\omega^2}{\alpha t^{2\alpha}}\right)}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}. \tag{2.4}$$

3. Survival and Hazard Rate Function

Let T be a random variable of the CFIR distribution with Parameter ω. Using equation (2.3) and equation (2.4), the survival function and hazard rate of the CFIR distribution can be expressed as:

$$S_\alpha(t) = \frac{\gamma\left(\frac{3}{2\alpha} - \frac{1}{2}, \frac{\omega^2}{\alpha t^{2\alpha}}\right)}{\Gamma\left(\frac{3}{2\alpha} - \frac{1}{2}\right)}, \tag{3.1}$$

and

$$h_\alpha(t) = t^{-3} e^{-\frac{\omega^2}{\alpha t^{2\alpha}}} \frac{2\alpha \left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha} - \frac{1}{2}\right)}}{\gamma\left(\frac{3}{2\alpha} - \frac{1}{2}, \frac{\omega^2}{\alpha t^{2\alpha}}\right)}. \tag{3.2}$$

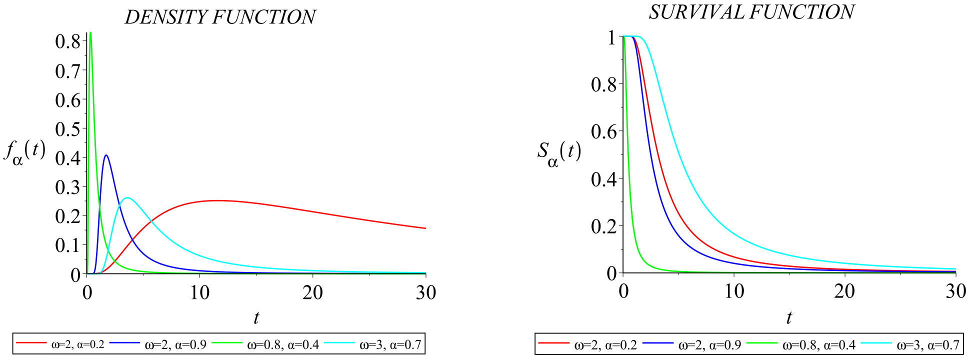

Figure 1: Graph of Density function (Left) and survival function (Right) the conformable fractional Inverse Rayleigh distribution for various choices of its parameter ω, α.

4. Mode

Theorem 4.1 *The mode of the CFIRD is given by* $t_0 = \left(\frac{2\omega^2}{3}\right)^{1/2\alpha}$.

proof: We know that

$$\log f_\alpha(t) = -3\log t - \frac{\omega^2}{\alpha t^{2\alpha}} + \log 2 + \log\alpha + \left(\frac{3}{2\alpha} - \frac{1}{2}\right)\log\left(\frac{\omega^2}{\alpha}\right) - \log\left(\Gamma\left(\frac{3}{2\alpha} - \frac{1}{2}\right)\right). \tag{4.1}$$

The mode of the CFIRD is given by $\log(f_\alpha(t))$ as:

$$D^\alpha(\log f_\alpha(t)) = \frac{2\omega^2 - 3t^{2\alpha}}{t^{3\alpha}}. \tag{4.2}$$

Equating equation (4.2) to zero, we conclude that $t_0 = \left(\frac{2\omega^2}{3}\right)^{1/2\alpha}$ is the mode of the CFIRD.

5. Quantile Function

The quantile function q can be obtained by solving the following equations:

$$F_\alpha(q) = p.$$

$$\gamma\left(\frac{3}{2\alpha} - \frac{1}{2}, \frac{\omega^2}{\alpha q^{2\alpha}}\right) = \Gamma\left(\frac{3}{2\alpha} - \frac{1}{2}\right)(1-p). \tag{5.1}$$

6. Moments

The conformable fractional r^{th} moment of a random variable T defined by [6], with CPDF $f_\alpha(t)$ can be presented as :
On integration, we obtain

$$E_\alpha\left(t^r\right)=\int_0^\infty t^r f_\alpha(t)\,d^\alpha t. \tag{6.1}$$

Using the pdf of CFIRD defined in equation (2.3), we obtain:

$$E_\alpha\left(t^r\right)=\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\int_0^\infty t^{r+\alpha-4}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}dt,$$

$$E_\alpha\left(t^r\right)=\frac{\left(\frac{\omega^2}{\alpha}\right)^{\frac{r}{2\alpha}}\Gamma\left(\frac{3}{2\alpha}-\frac{r}{2\alpha}-\frac{1}{2}\right)}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}. \tag{6.2}$$

Thus the Mean and Variance of CFIRD can be calculated as:

$$E_\alpha(t)=\frac{\left(\frac{\omega^2}{\alpha}\right)^{\frac{1}{2\alpha}}\Gamma\left(\frac{1}{\alpha}-\frac{1}{2}\right)}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}. \tag{6.3}$$

$$E_\alpha\left(t^2\right)=\frac{\left(\frac{\omega^2}{\alpha}\right)^{\frac{1}{\alpha}}\Gamma\left(\frac{1}{2\alpha}-\frac{1}{2}\right)}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}.$$

$$V_\alpha(t)=\frac{\left(\frac{\omega^2}{\alpha}\right)^{\frac{1}{\alpha}}\Gamma\left(\frac{1}{2\alpha}-\frac{1}{2}\right)}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}-\left(\frac{\omega^2}{\alpha}\right)^{\frac{1}{\alpha}}\left\{\frac{\Gamma\left(\frac{1}{\alpha}-\frac{1}{2}\right)}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\right\}^2. \tag{6.4}$$

7. Conformable Fractional Entropy

A well-known idea from information theory, entropy gives a probability distribution's degree of uncertainty. The Shannon and Tsallis entropies for the CFIRD are obtained in this section.

7.1. Shannon Entropy

The conformable fractional Shannon entropy of a random variable T for the CFIRD can be calculated as follows:

Theorem 7.1

$$SH_\alpha(t)=-\ln\left(\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\right)+\frac{2\alpha\,\Gamma\left(\frac{3}{2\alpha}+\frac{1}{2}\right)}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}-\frac{3}{2\alpha}\left[\psi\left(\frac{3}{2\alpha}-\frac{1}{2}\right)-\ln\left(\frac{\omega^2}{\alpha}\right)\right]. \tag{7.1}$$

Proof: Let T is a random variable then the conformable fractional [10] entropy is defined as follows:

$$SH_\alpha(t)=-E_\alpha\left(\ln f_\alpha(t)\right)=-\int_0^\infty f_\alpha(t)\ln f_\alpha(t)\,d^\alpha t.$$

Using pdf defined in equation (2.3), we obtain

$$SH_\alpha(t)=-\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\int_0^\infty t^{-3}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}\ln\left(\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}t^{-3}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}\right)d^\alpha t$$

$$= -\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\int\limits_0^\infty t^{-3}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}\left\{\ln\left(\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\right)-3\ln t-\frac{\omega^2}{\alpha t^{2\alpha}}\right\}d^\alpha t$$

$$= -\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\ln\left(\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\right)\int\limits_0^\infty t^{-3}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}d^\alpha t$$
$$+\frac{6\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\int\limits_0^\infty t^{-3}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}\ln t\, d^\alpha t+\frac{2\omega^2\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\int\limits_0^\infty t^{-3-2\alpha}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}d^\alpha t$$

$$= -\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\ln\left(\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\right)\int\limits_0^\infty t^{\alpha-4}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}dt$$
$$+\frac{6\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\int\limits_0^\infty t^{\alpha-4}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}\ln t\, dt+\frac{2\omega^2\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\int\limits_0^\infty t^{-\alpha-4}e^{-\frac{\omega^2}{\alpha t^{2\alpha}}}dt,$$

put $t^{-2\alpha}=p \Rightarrow -2\alpha t^{-2\alpha-1}dt=dp$ and $t=p^{-1/2\alpha}$

$$= -\frac{\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\ln\left(\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\right)\frac{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}{\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}+\frac{2\omega^2\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\frac{\Gamma\left(\frac{3}{2\alpha}+\frac{1}{2}\right)}{\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}+\frac{1}{2}\right)}}$$
$$-\frac{3\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{2\alpha\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\frac{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}{\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}\left[\psi\left(\frac{3}{2\alpha}-\frac{1}{2}\right)-\ln\left(\frac{\omega^2}{\alpha}\right)\right], \tag{7.2}$$

simplifying the above expression, we can obtain expression (7.1).

7.2. Tsallis Entropy

The conformable fractional Tsallis entropy of a random variable t for the CFIRD is defined as follows:

Theorem 7.2 *The conformable fractional Tsallis entropy of a random variable t for the CFIRD is defined as follows:*

$$T_\iota(t)=\frac{1}{2\alpha(1-\iota)}\left(\frac{2\alpha\left(\frac{\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\right)^{\iota}\frac{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}{\left(\frac{\iota\omega^2}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}-\frac{1}{(1-\iota)}. \tag{7.3}$$

Proof: Let t be a random variable then the conformable fractional Tsallis entropy is defined as follows:

$$T_\iota(t)=\frac{1}{(1-\iota)}\left[E_\alpha\left(f_\alpha(t)\right)^{\iota-1}-1\right]$$
$$=\frac{1}{(1-\iota)}\left[\int\limits_0^\infty\left(f_\alpha(t)\right)^{\iota-1}f_\alpha(t)\,d^\alpha t-1\right]$$
$$=\frac{1}{(1-\iota)}\left[\int\limits_0^\infty\left(f_\alpha(t)\right)^{\iota}d^\alpha t-1\right].$$

Using equation (2.3), we obtain

$$T_{\iota}\left(t\right)=\frac{1}{\left(1-\iota\right)}\left(\frac{2\alpha\left(\frac{\omega^{2}}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\right)^{\iota}\left[\int\limits_{0}^{\infty}e^{-\frac{\omega^{2}\iota}{\alpha t^{2\alpha}}}t^{\alpha-3\iota-1}dt-1\right],$$

put $t^{-2\alpha}=p\Rightarrow-2\alpha t^{-2\alpha-1}dt=dp$ and $t=p^{-1/2\alpha}$

$$T_{\iota}\left(t\right)=\frac{1}{2\alpha\left(1-\iota\right)}\left(\frac{2\alpha\left(\frac{\omega^{2}}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\right)^{\iota}\int\limits_{0}^{\infty}e^{-\frac{\omega^{2}\iota p}{\alpha}}p^{\left(\frac{3\iota}{2\alpha}-\frac{3}{2}\right)}dp-\frac{1}{\left(1-\iota\right)}.$$

Solving the above integral, we can obtain the above expression (7.3).

8. Order Statistics

This section computes the CFIRD of the b^{th} order statistic Y_b and introduces the order statistics of fractional distributions.

Theorem 8.1 *The density function of order statistics for CFIRD can be derived as follows:*

$$g_{b}\left(t\right)=\frac{n!}{(b-1)!(n-b)!}\sum_{j=0}^{n-b}\sum_{l=0}^{b+j-1}\left(-1\right)^{l+j}\binom{n-b}{j}\binom{b+j-1}{l}\frac{e^{-\frac{\omega^{2}}{\alpha t^{2\alpha}}}}{t^{3}}\times$$
$$\frac{2\alpha\left(\gamma\left(\frac{3}{2\alpha}-\frac{1}{2},\frac{\omega^{2}}{\alpha t^{2\alpha}}\right)\right)^{l}\left(\frac{\omega^{2}}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\left(\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)\right)^{l+1}}. \qquad (8.1)$$

Proof: Consider $t_1, t_2, \ldots, t_n$ be a random sample of size n drawn using CFIRD from a fractional distribution $f_{\alpha}\left(t\right)$ and $F_{\alpha}\left(t\right)$. The conformable PDF of the b^{th} order statistic Y_b, $1\leqslant b\leqslant n$ with $Y_1, Y_2, \ldots, Y_n$ denotes the order statistics corresponding to this sample, can be defined as:

$$\begin{aligned}g_{b}\left(t\right)&=\frac{n!}{(b-1)!(n-b)!}\left(F_{\alpha}\left(t\right)\right)^{b-1}\left(1-F_{\alpha}\left(t\right)\right)^{n-b}f_{\alpha}\left(t\right)\\&=\frac{n!}{(b-1)!(n-b)!}\sum_{j=0}^{n-b}\binom{n-b}{j}\left(-1\right)^{j}\left(F_{\alpha}\left(t\right)\right)^{b+j-1}f_{\alpha}\left(t\right) \qquad (8.2)\end{aligned}$$

$$g_{b}\left(t\right)=\frac{n!}{(b-1)!(n-b)!}\sum_{j=0}^{n-b}\binom{n-b}{j}\left(-1\right)^{j}\left(1-\frac{\gamma\left(\frac{3}{2\alpha}-\frac{1}{2},\frac{\omega^{2}}{\alpha t^{2\alpha}}\right)}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}\right)^{b+j-1}\frac{e^{-\frac{\omega^{2}}{\alpha t^{2\alpha}}}}{t^{3}}\frac{2\alpha\left(\frac{\omega^{2}}{\alpha}\right)^{\left(\frac{3}{2\alpha}-\frac{1}{2}\right)}}{\Gamma\left(\frac{3}{2\alpha}-\frac{1}{2}\right)},$$

using Lemma 1.2, we can obtain the required result.

Conclusion

The article provides important insights into the behaviour of the Conformable Fractional Inverse Rayleigh Distribution (CFIRD) and its possible uses in risk and reliability analysis by defining its Cumulative Distribution Function (CDF), survival function, and hazard function. Its basic tendency can be better understood by introducing statistical measurements like the mean, variance, r^{th} moments, and expected values as conformable fractional features. Furthermore, this study identifies the conformable fractional counterparts of well-known entropy metrics, such as the Shannon and Tsallis entropies, offering practical instruments for measuring randomness and uncertainty. The expression for the CFIRD's b^{th} order statistic has been determined.

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Kanak Modi,
Amity School of Applied Sciences,
Amity University Rajasthan, Jaipur-303002,
India.
E-mail address: `mangalkanak@gmail.com`

and

Rahul Sharma,
IcfaiTech,
The ICFAI University, Jaipur-302031,
India.
E-mail address: `rahulshrm297@gmail.com`

and

Yudhveer Singh,
Amity Institute of Information Technology,
Amity University Rajasthan, Jaipur-303002,
India.
E-mail address: `yudhvir.chahal@gmail.com`