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Degree Sum $(1/3)^{rd}$ Index of Certain Graphs

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ABSTRACT: In this paper, we have computed the degree sum $(1/3)^{rd}$ index of some double graphs, subdivision graphs and complement of standard graphs.Topological indices are numerical quantities linked to a graph structure which are frequently employed in computer science, mathematical biology, and theoretical chemistry. Degree based topological indices, such as degree sum $(1/3)^{rd}$ index offer important insights into the stability, reactivity, and branching of molecules. These indices are widely used to predict the physical, chemical, and biological characteristics of chemical compounds in the research of structure–property and structure–activity relationships (QSAR/QSPR). They are also helpful in network analysis, drug design, and nanostructure characterization.

Key Words: Degree sum index, double graphs, subdivision graphs, k-complement, k(i)-complement.

Contents

1. Introduction

Let G be a graph with n vertices $\{v_1, v_2, \dots, v_n\}$ and m edges. In this graph we have considered simple and undirected graphs. A topological index is a molecules numerical value that is calculated from its structural graph. Topological indices are employed in chemistry to model the physical, pharmacological, biological, and other characteristics of chemical substances. One of these topological indices is degree sum $(1/3)^{rd}$ index which is defined in terms of the vertex degrees as follows:

Definition 1.1 Degree sum $(1/3)^{rd}$ index $= \sum_{uv\in E(G)} \left(d_u^{1/3} + d_v^{1/3} + (d_u d_v)^{1/3}\right)$

Definition 1.2 [7] A graph G complement is another graph $\bar{G}$ with the same set of vertices in which two different vertices are adjacent if and only if they are not neighboring in G.

Definition 1.3 [7] Consider the graph G and its vertex set V as a partition, with $P_k = \{V_1, V_2, \dots, V_n\}$. The k-complement of G is defined similarly to the complement of a graph: remove the edges between V_i and V_j for every V_i and V_j in P_k with $i \neq j$, and then add the edges between the vertices of V_i and V_j that are not in G. The resulting graph is denoted by $\bar{G_k}$.

Definition 1.4 [7] Consider the graph G and its vertex set V as a partition, with $P_k = \{V_1, V_2, \dots, V_n\}$. Next, we obtain another type of graph complement, the k(i)-complement of G, as follows: For every partition set V_r in P_k, eliminate the G edges that connect the V_r vertices,then add the edges of G (complement of G), which is represented by $\bar{G_{k(i)}}$, to the vertices of V_r.

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The new graph created by appending an additional vertex to each edge of a basic graph G is known as its subdivision graph, or subgraph, $S(G)$. In the literature, the subgraphs have been examined

When a graph G has the vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$, we take another copy of G and name its vertices $\{v_1, v_2, \ldots, v_n\}$, with v_i for each i. When v_i is connected to its neighbors for every i, a new graph known as the double graph of G and denoted by $D(G)$ was studied in [4],[5],[6],[10].

2. Degree sum $(1/3)^{rd}$ index of some standard graphs

For these common graphs, numerous graph theorists calculated the various topological indices. It could be useful to generalize the link between various topological indices in the future. Hence we have computed the degree sum $(1/3)^{rd}$ index for few standard graphs.

Theorem 2.1 *The degree sum $(1/3)^{rd}$ index of some standard graphs are as follows.*

$$\textit{Degree sum } (1/3)^{rd} \textit{ index } = \begin{cases} 6.627(n) + 5.694(n)^{4/3} & \textit{if } G = F_n^3 \\ 4.107(n) - 5.6946 & \textit{if } G = P_n \\ 2(n-1)^{4/3} + (n-1) & \textit{if } G = S_n^0 \\ 2(n-1)^{4/3} + (n-1) & \textit{if } G = K_{1,n-1} \\ 4.1072(n) & \textit{if } G = C_n \\ 2(n-1)^{4/3} + (n-1)^{5/3} & \textit{if } G = K_n \\ n + 2(n)^{4/3} & \textit{if } G = K_1, n \\ 4.1072(n) - 12.9704 & \textit{if } G = T_{(p,q)} \\ 6.4068(n) - 6.406 + (n-1)^{4/3}(2.4422) & \textit{if } G = W_n \end{cases}$$

Proof: Let G be a Friendship graph F_n^3. It has 2n vertices of degree 2 and 2n in its centers. The vertices of degree 2 are connected by n edges. The central vertex and the other vertices of degree 2 are connected by 2n edges.

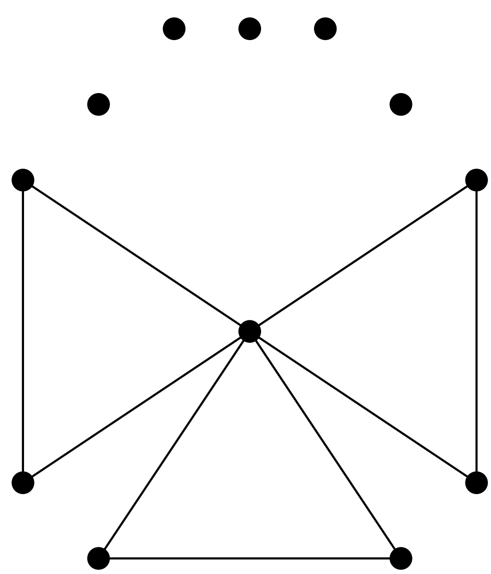

Figure 1: G: F_n^3

According to the definition of degree sum $(1/3)^{rd}$ index,

$$\begin{aligned}
\text{Degree sum } (1/3)^{rd} \text{ Index}(F_n^3) &= n\left(2^{1/3}+2^{1/3}+4^{1/3}\right)+\left[(2n)^{1/3}+2^{1/3}+(4n)^{1/3}\right](2n)\\
&= n\left(2^{1/3}+2^{1/3}+4^{1/3}\right)+\left[2^{1/3}n^{1/3}+2^{1/3}+4^{1/3}n^{1/3}\right](2n)\\
&= n\left(2^{1/3}+2^{1/3}+4^{1/3}\right)+2n\left[2^{1/3}n^{1/3}+2^{1/3}+4^{1/3}n^{1/3}\right]\\
&= n\left[2^{4/3}+2^{2/3}+2^{4/3}n^{1/3}+2^{4/3}+2^{5/3}n^{1/3}\right]\\
&= n\left[2^{4/3}+2^{2/3}+2^{4/3}+n^{1/3}(2^{4/3}+2^{5/3})\right]\\
&= n\left[2^{7/3}+2^{2/3}+n^{1/3}(2^{4/3}+2^{5/3})\right]\\
&= 6.627085252\,n+5.694644204\,n^{4/3}
\end{aligned}$$

Similarly degree sum $(1/3)^{rd}$ index of other graphs can be calculated. □

3. Degree sum $(1/3)^{rd}$ index of some subdivision graphs

Theorem 3.1 *The degree sum $(1/3)^{rd}$ index of subdivision graph of some standard graphs are as follows.*

$$\textit{Degree sum } (1/3)^{rd} \textit{ index} = \begin{cases} 7.077715+(4.1072)2(p+q-2) & \textit{if } G=T_{(p,q)}\\ [(n-1)^{1/3}(2.2599)+1.2599](3n) & \textit{if } G=C_n\\ 8.214486304(n) & \textit{if } G=K_n\\ 8.2144(n)-1.1748 & \textit{if } G=P_n\\ 14.817n+2.259(n-1)^{1/3}-14.8177 & \textit{if } G=W_n\\ 18.94(n)+5.694(n)^{4/3} & \textit{if } G=F_n^3\\ 1.7797(n)+2.2599(n-1)^{4/3} & \textit{if } G=S_n^0 \end{cases}$$

Proof: Let G_1 be a subdivision graph of K_n. In this case, the subdivision graph has $n(n+1)/2$ vertices and n(n-1)edges. Of these n original vertices have degree $(n-1)$ and $nC2$ new vertices have degree 2 each.

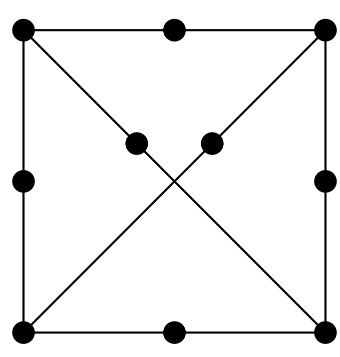

Figure 2: G_1: Subdivision graph of K_n

According to the definition of degree sum $(1/3)^{rd}$ index,

$$\begin{aligned}
\text{Degree sum } (1/3)^{rd} \text{ Index}(K_n) &= \left(2^{1/3} + 2^{1/3} + 4^{1/3}\right)(2n) \\
&= \left(2 \cdot 2^{1/3} + 2^{2/3}\right)(2n) \\
&= (2.5198421 + 1.5874011)(2n) \\
&= 4.1072432 \cdot (2n) \\
&= 8.2144864 \cdot n
\end{aligned}$$

Similarly degree sum $(1/3)^{rd}$ index of other subdivision graphs can be calculated.

□

4. Degree sum $(1/3)^{rd}$ index of some double graphs

Theorem 4.1 *The degree sum $(1/3)^{rd}$ index of double graph of some standard graphs are as follows.*

$$\textit{Degree sum } (1/3)^{rd} \textit{ index } = \begin{cases} 22.7785(n) - 29.5598 & \textit{if } G = P_n \\ 22.77857(n) & \textit{if } G = C_n \\ 2[(n^2-6n+13)^{1/3} + (n^2-6n+13)^{2/3}](n^2-31n+60) & \textit{if } G = K_n \\ 8.21446(n) - 3.1748 & \textit{if } G = S_n^0 \end{cases}$$

Proof: Let G_2 be a subdivision graph of P_n. Double graph of P_n has $4(n-3)$ vertices of degree 4 and 8 vertices of degree 2.

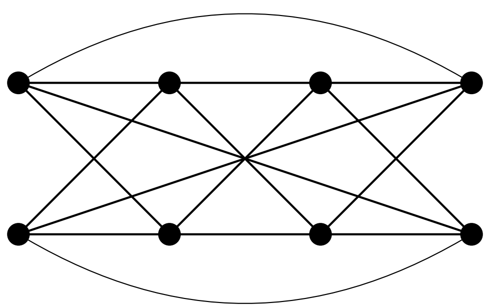

Figure 3: G_2 :Double graph of the path

According to the definition of degree $(1/3)^{rd}$ index,

$$\begin{aligned}
\text{Degree Sum } (1/3)^{rd} \text{ of } P_n &= \left(2^{1/3} + 4^{1/3} + 8^{1/3}\right) \cdot 8 + \left(4^{1/3} + 4^{1/3} + 16^{1/3}\right) \cdot 4(n-3) \\
&= [1.26 + 1.5874 + 2.0] \cdot 8 + [1.5874 + 1.5874 + 2.5198] \cdot 4(n-3) \\
&= 4.8474 \cdot 8 + 5.6946 \cdot 4(n-3) \\
&= 38.7792 + 22.7784(n-3) \\
&= 38.7792 + 22.7784n - 68.3352 \\
&= 22.7784n - 29.556
\end{aligned}$$

Similarly degree sum $(1/3)^{rd}$ index of other double graphs can be calculated.

□

5. Degree sum $(1/3)^{rd}$ index of complement of some graphs

Theorem 5.1 *The degree sum $(1/3)^{rd}$ index of complement graph of some standard graphs are as follows.*

$$\textit{Degree sum } (1/3)^{rd} \textit{ index } = \begin{cases} [2^{1/3} + 2^{1/3} + 4^{1/3}](n) & \textit{if } G = C_n \\ 3^{n-3} \cdot [2(n-1)^{1/3} + (n-1)^{2/3}] & \textit{if } G = S_n^0 \\ 6n & \textit{if } G = F_n^3 \end{cases}$$

Proof: Let G_3 be a Complement of the graph of S_n. In this case, the complement graph has 3^{n-1} vertices of degree $(n-1)$ and an isolated vertex.

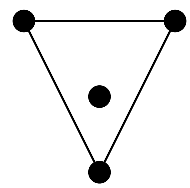

Figure 4: G_3 : Complement of S_n graph

According to the definition of degree sum $(1/3)^{rd}$ index,

$$\begin{aligned} \text{Degree sum } (1/3)^{rd} \text{ Index} &= 3^{n-3}\left[(n-1)^{1/3} + (n-1)^{1/3} + \left((n-1)^2\right)^{1/3}\right] \\ &= 3^{n-3}\left[2(n-1)^{1/3} + (n-1)^{2/3}\right] \\ &= 3^{n-3} \cdot (n-1)^{1/3}\left[2 + (n-1)^{1/3}\right] \\ &= 3^{n-3} \cdot [2(n-1)^{1/3} + (n-1)^{2/3}] \end{aligned}$$

Similarly degree sum $(1/3)^{rd}$ index of other double graphs can be calculated. □

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References

1. Das, K. C., Akgunes, N., Togan, M., Yurttas, A., Cangul, I. N., and Cevik, A. S. (2016). On the first Zagreb index and multiplicative Zagreb coindices of graphs. *Analele Stiintifice ale Universitatii Ovidius Constanta*, **24**(1), 153–176.
2. Hosamani, S. M., Lokesha, V., Cangul, I. N., and Devendraiah, K. M. (2016). On certain topological indices of the derived graphs of subdivision graphs.
3. Lokesha, V., Shetty, S. B., Ranjini, P. S., and Cangul, I. N. (2015). Computing ABC, GA, Randic and Zagreb indices. *Enlightenments of Pure and Applied Mathematics (EPAM)*, **1**(1), 17–28.
4. Ranjini, P. S., Lokesha, V., and Cangul, I. N. (2011). On the Zagreb indices of the line graphs of the subdivision graphs. *Applied Mathematics and Computation*, **218**, 699–702.
5. Prakasha, K. N., Kiran, K., and Rakshith, S. (2019). Randić Type SDI Index of Certain Graphs. *TWMS Journal of Applied and Engineering Mathematics*, **9**(4), 894–900.
6. Ranjini, P. S., Rajan, M. A., and Lokesha, V. (2010). On Zagreb Indices of the Subdivision Graphs. *International Journal of Mathematical Sciences & Engineering Applications (IJMSEA)*, **4**, 221–228.
7. E. Sampathkumar, L. Pushpalatha, C. V. Venkatachalam and Pradeep Bhat, *Generalized complements of a graph*, Indian J. Pure Appl. Math, 29(6) (1998), 625-639.
8. Togan, M., Yurttas, A., Cangul, I. N., and Cevik, A. S. Zagreb indices and multiplicative Zagreb indices of double graphs of subdivision graphs. *Preprint*.

9. Togan, M., Yurttas, A., and Cangul, I. N. Zagreb and multiplicative Zagreb indices of r-subdivision graphs of double graphs. *Preprint.*

10. Togan, M., Yurttas, A., and Cangul, I. N. (2015). Some formulae and inequalities on several Zagreb indices of r-subdivision graphs. *Enlightenments of Pure and Applied Mathematics (EPAM)*, **1**(1), 29–45.

11. Togan, M., Yurttas, A., and Cangul, I. N. (2016). All versions of Zagreb indices and coindices of subdivision graphs of certain graph types. *Advanced Studies in Contemporary Mathematics*, **26**(1), 227–236.

12. Vukicevic, D., Gutman, I., Furtula, B., Andova, V., and Dimitrov, D. (2011). Some observations on comparing Zagreb indices. *MATCH Communications in Mathematical and in Computer Chemistry*, **66**, 627–645.

13. Yurttas, A., Togan, M., and Cangul, I. N. (2016). Zagreb indices and multiplicative Zagreb indices of subdivision graphs of double graphs. *Advanced Studies in Contemporary Mathematics*, **26**(3), 407–416.

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