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Study of some unified integrals associated with extended k-generalized Mittag-Leffler function

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ABSTRACT: Many authors established a large range of integral formulas including various special functions throughout the years.We evolve certain integrals formulas involving extended k-generalized Mittag-Leffler function using the extension of beta function in this paper.Some special cases also discussed for our main results..

Key Words: Extended Mittag-Leffler function, extended k-Mittag-Leffler function,extended k-Beta function, extended k-Gamma function, k-Pochhammer symbol.

Contents

1. Introduction

During the past few decades special function and fractional calculus become active research area being diverse applications in physics and engineering. Special functions play a important role to solve both integral and differential equations of fractional order and Mittag-Leffler function is one of the tool which is used frequently by authors [4,10,11,16,26,14,15,16,17,27,19,20,21,22,23,24,25,26,27].It has enlarge applications in the field of Applied Sciences,Biology and many other areas.

The elementary Mittag-Leffler function is defined and represented as

$$E_r(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(nr+1)}, (r > 0, z \in \mathbb{C}) \tag{1.1}$$

let $r = 2$ and $r = 4$ then equation (1.1) become

$$E_2(z) = \cosh(\sqrt{z}), (z \in \mathbb{C}) \tag{1.2}$$

$$E_4(z) = \frac{1}{2}\left[\cos(z^{\frac{1}{4}}) + \cosh(z^{\frac{1}{4}})\right], (z \in \mathbb{C}) \tag{1.3}$$

.

Its generalization was given by Wiman [1]

$$E_{r,s}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(nr+s)}, (r, s, z \in \mathbb{C}, \Re(r) > 0, \Re(s) > 0) \tag{1.4}$$

After Wiman,Prabhakar [2] introduced generalization of Mittag-Leffler function as

$$E^{\rho}_{r,s}(z) = \sum_{n=0}^{\infty} \frac{(\rho)_n}{\Gamma(nr+s)} \frac{z^n}{n!}, (r, s, z, \rho \in \mathbb{C}; \Re(r) > 0, \Re(s) > 0) \tag{1.5}$$

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In 2007 Shukla and Prajapati [3] extended the definition of Prabhakar function as

$$E_{r,s}^{\rho,\sigma}(z)=\sum_{n=0}^{\infty}\frac{(\rho)_{n\sigma}}{\Gamma(nr+s)}\frac{z^n}{n!},(r,s,z,\rho\in\mathbb{C};\Re(r)>0,\Re(s)>0,\Re(\rho)>0,\sigma\in(0,1)\cup\mathfrak{N}) \tag{1.6}$$

G.A. Dorrego et.al. [4] in 2012, use the concept of k-Gamma function to define Mittag-Leffler function as

$$E_{k,r,s}^{\rho}(z)=\sum_{n=0}^{\infty}\frac{(\rho)_{n,k}}{\Gamma_k(nr+s)}\frac{z^n}{n!},(r,s,z,\rho\in\mathbb{C};k>0;\Re(r)>0,\Re(s)>0,\Re(\rho)>0) \tag{1.7}$$

where $(\rho)_{n,k}$ and $\Gamma_k(z)$ are well known k-Pochhammer symbol, and k-Gamma function.

The above k-Mittag Leffler function is further extended and generalized by Gehlot et.al. [5,10,21]

$$GE_{k,r,s}^{\rho,\sigma}(z)=\sum_{n=0}^{\infty}\frac{(\rho)_{n\sigma,k}}{\Gamma_k(nr+s)}\frac{z^n}{n!},(k\in\mathbb{R};z,r,s,\rho\in\mathbb{C};\Re(r),\Re(s),\Re(\rho)>0,\sigma\in(0,1)\cup\mathfrak{N}) \tag{1.8}$$

In 2018,Rahman et.al. [6] discussed on extension of k-Mittag leffler function as

$$E_{k,r,s}^{\rho;c}(z;p)=\sum_{n=0}^{\infty}\frac{B_k(\rho+nk,c-\rho;p)(c)_{n,k}}{B_k(\rho,c-\rho)\Gamma_k(nr+s)}\frac{z^n}{n!} \tag{1.9}$$

For

$$\frac{(\rho)_{n,k}}{(c)_{n,k}}=\frac{B_k(\rho+nk,c-\rho)}{B_k(\rho,c-\rho)} \tag{1.10}$$

where $B_k(r,s;p)$ is an extension of the k-Beta function given as

$$B_k(r,s)=\frac{1}{k}\int_0^1 u^{\frac{r}{k}-1}(1-u)^{\frac{s}{k}-1}e^{-}\frac{p^k}{ku(1-u)}du. \tag{1.11}$$

Shilpi jain et.al. [7] 2023 define extended k-generalized Mittag-Leffler function represents as follows:

$$E_{k,r,s}^{\rho,\sigma;c}(z;p)=\sum_{n=0}^{\infty}\frac{B_k(\rho+n\sigma k,c-\rho;p)}{B_k(\rho,c-\rho)}\frac{(c)_{n\sigma,k}}{\Gamma_k(nr+s)}\frac{z^n}{n!} \tag{1.12}$$

where $r,s,z,\rho\in\mathfrak{C};k>0;\Re(r)>0,\Re(s)>0,\Re(\rho)>0$ and $\sigma\in(0,1)\cup\mathfrak{N};p\geq 0$.

Fox and Wright [8], describe asymptotic expansion of wright hyper-geometric function such as generalized hypergeometric series ${}_pF_q$ as follows

$${}_p\Psi_q\left[\begin{array}{c}(l_1,L_1),\ldots,(l_p,L_p);\\(m_1,M_1),\ldots,(m_q,M_q);\end{array} z\right]=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}\Gamma(l_j+L_jn)}{\prod_{j=1}^{q}\Gamma(m_j+M_jn)}\frac{z^n}{n!} \tag{1.13}$$

where the cofficients $L_1,...,L_p$ and $M_1,....,M_q$ are positive real numbers such that
$(i) 1+\sum_{j=1}^{q}M_j-\sum_{j=1}^{p}M_j>0$ and $0<|z|<\infty;\ z\neq 0$
$(ii) 1+\sum_{j=1}^{q}M_j-\sum_{j=1}^{p}L_j=0$ and $0<|z|<L_1^{-L_1}\cdots L_p^{-L_p}M_1^{-B_1}\cdots M_q^{-M_q}$

The Wright function is a special case of (1.13)

$${}_p\Psi_q\left[\begin{array}{c}(l_1,1),\ldots,(l_p,1);\\(m_1,1),\ldots,(m_q,1);\end{array} z\right]=\frac{\prod_{j=1}^{p}\Gamma(l_j)}{\prod_{j=1}^{q}\Gamma(m_j)}{}_pF_q\left[\begin{array}{c}l_1,\ldots,l_p;\\m_1,\ldots,m_q;\end{array} z\right], \tag{1.14}$$

where ${}_pF_q$ is the generalized hypergeometric function defined by

$${}_pF_q\left[\begin{array}{c}l_1,\ldots,l_p;\\m_1,\ldots,m_q;\end{array} z\right]=\sum_{n=0}^{\infty}\frac{(l_1)_n\cdots(l_p)_n}{(m_1)_n\cdots(m_q)_n}\frac{z^n}{n!}={}_pF_q(l_1,\ldots,l_p;m_1,\ldots,m_q;z) \tag{1.15}$$

Lavoie and Trottier integral formula [9,22,25] will be required for our present investigation .

$$\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} d\lambda = \left(\frac{2}{3}\right)^{2\xi}\frac{\Gamma(\xi)\Gamma(\eta)}{\Gamma(\xi+\eta)} \tag{1.16}$$

where $\Re(\xi) > 0$ and $\Re(\eta) > 0$

2. Integral formulas for Extended k-Generalized Mittag-Leffler function

Theorem 1 *For $k > 0; z, r, s, \rho \in \mathfrak{C}; \Re(r) > 0, \Re(s) > 0, \Re(\rho) > 0, \Re(\xi) > 0, \Re(\eta) > 0$ and $\sigma \in (0,1)\cup\mathfrak{N}; p \geq 0$ following integral formula holds true*

$$\begin{aligned}\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} E_{k,r,s}^{\rho,\sigma;c}\left[y\lambda\left(1-\frac{\lambda}{3}\right)^2\right]d\lambda \\ = \left(\frac{2}{3}\right)^{2\xi} B(\xi,\eta)\,{}_2\psi_2\left[\begin{array}{c}(\rho,\sigma k),(\xi,1); \\ (s,r),(\xi+\eta,1);\end{array}\ \frac{4y}{9}\right]\end{aligned} \tag{2.1}$$

Proof: *using equ. (1.12) in the left hand side of equ. (2.1)*

$\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} E_{k,r,s}^{\rho,\sigma;c}\left[y\lambda\left(1-\frac{\lambda}{3}\right)^2\right]d\lambda$

$= \int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1}\sum_{n=0}^{\infty}\frac{B_k(\rho+n\sigma k,c-\rho)}{B_k(\rho,c-\rho)}\frac{(c)_{n\sigma,k}}{\Gamma_k(nr+s)}\frac{y^n\lambda^n\left(1-\frac{\lambda}{3}\right)^{2n}}{n!}d\lambda$

changing the order of integration and summation

$= \sum_{n=0}^{\infty}\frac{B_k(\rho+n\sigma k,c-\rho)}{B_k(\rho,c-\rho)}\frac{(c)_{n\sigma,k}}{\Gamma_k(nr+s)}\frac{y^n}{n!}\int_0^1 \lambda^{n+\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2n+2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1}d\lambda$

Now using (1.16) in above equation

$= \sum_{n=0}^{\infty}\frac{B_k(\rho+n\sigma k,c-\rho)}{B_k(\rho,c-\rho)}\frac{(c)_{n\sigma,k}}{\Gamma_k(nr+s)}\frac{y^n}{n!}\left(\frac{2}{3}\right)^{2n+2\xi}\frac{\Gamma(n+\xi)\Gamma(\eta)}{\Gamma(n+\xi+\eta)}$

$= \left(\frac{2}{3}\right)^{2\xi}\Gamma(\eta)\sum_{n=0}^{\infty}\frac{B_k(\rho+n\sigma k,c-\rho)}{B_k(\rho,c-\rho)}\frac{(c)_{n\sigma,k}}{\Gamma_k(nr+s)}\frac{y^n}{n!}\frac{\Gamma(n+\xi)}{\Gamma(n+\xi+\eta)}\left(\frac{2}{3}\right)^{2n}$

using equ. (1.10) in above

$= \left(\frac{2}{3}\right)^{2\xi}B(\xi,\eta)\sum_{n=0}^{\infty}\frac{(\rho)_{n\sigma,k}}{\Gamma_k(nr+s)}\frac{(\xi)_n}{(\xi+\eta)_n}\frac{(4y/9)^n}{n!}$

thus we obtain the required result (2.1)

Theorem 2 *For $k > 0; z, r, s, \rho \in \mathfrak{C}; \Re(r) > 0, \Re(s) > 0, \Re(\rho) > 0, \Re(\xi) > 0, \Re(\eta) > 0$ and $\sigma \in (0,1)\cup\mathfrak{N}; p \geq 0$ following integral formula holds true*

$$\begin{aligned}\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} E_{k,r,s}^{\rho,\sigma;c}\left[y(1-\lambda)^2\left(1-\frac{\lambda}{4}\right)\right]d\lambda \\ = \left(\frac{2}{3}\right)^{2\xi} B(\xi,\eta)\,{}_2\psi_2\left[\begin{array}{c}(\rho,\sigma k),(\eta,1); \\ (s,r),(\xi+\eta,1);\end{array}\ y\right]\end{aligned} \tag{2.2}$$

Proof: *using equ. (1.12) in left hand side of equ. (2.2)*

$\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} E_{k,r,s}^{\rho,\sigma;c}\left[y\lambda\left(1-\frac{\lambda}{3}\right)^2\right]d\lambda$

$= \int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1}\sum_{n=0}^{\infty}\frac{B_k(\rho+n\sigma k,c-\rho)}{B_k(\rho,c-\rho)}\frac{(c)_{n\sigma,k}}{\Gamma_k(nr+s)}\frac{y^n(1-\lambda)^{2n}\left(1-\frac{\lambda}{4}\right)^n}{n!}d\lambda$

changing the order of integration and summation

$= \sum_{n=0}^{\infty} \frac{B_k(\rho+n\sigma k, c-\rho)}{B_k(\rho, c-\rho)} \frac{(c)_{n\sigma,k}}{\Gamma_k(nr+s)} \frac{y^n}{n!} \int_0^1 \lambda^{\xi-1}(1-\lambda)^{2n+2\eta-1} \left(1-\frac{\lambda}{3}\right)^{2\xi-1} \left(1-\frac{\lambda}{4}\right)^{n+\eta-1} d\lambda$

Now using (1.16) *in above equation*

$= \sum_{n=0}^{\infty} \frac{B_k(\rho+n\sigma k, c-\rho)}{B_k(\rho, c-\rho)} \frac{(c)_{n\sigma,k}}{\Gamma_k(nr+s)} \frac{y^n}{n!} \left(\frac{2}{3}\right)^{2\xi} \frac{\Gamma(\xi)\Gamma(n+\eta)}{\Gamma(n+\xi+\eta)}$

$= \left(\frac{2}{3}\right)^{2\xi} \Gamma(\xi) \sum_{n=0}^{\infty} \frac{B_k(\rho+n\sigma k, c-\rho)}{B_k(\rho, c-\rho)} \frac{(c)_{n\sigma,k}}{\Gamma_k(nr+s)} \frac{y^n}{n!} \frac{\Gamma(n+\eta)}{\Gamma(n+\xi+\eta)}$

using equ. (1.10) *in above*

$= \left(\frac{2}{3}\right)^{2\xi} B(\xi,\eta) \sum_{n=0}^{\infty} \frac{(\rho)_{n\sigma,k}}{\Gamma_k(nr+s)} \frac{(\eta)_n}{(\xi+\eta)_n} \frac{(y)^n}{n!}$

thus we obtain the required result (2.2)

3. Special Cases

Corollary 1 *If we put* $c = 1$ *in equation* (2.1) *and use equation* (1.8), *we get*

$$\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1} \left(1-\frac{\lambda}{3}\right)^{2\xi-1} \left(1-\frac{\lambda}{4}\right)^{\eta-1} E^{\rho,\sigma}_{k,r,s}\left[y\lambda\left(1-\frac{\lambda}{3}\right)^2\right] d\lambda = \left(\frac{2}{3}\right)^{2\xi} B(\xi,\eta)\, {}_2\psi_2 \left[\begin{array}{c} (\rho,\sigma k),(\xi,1); \\ (s,r),(\xi+\eta,1); \end{array} \frac{4y}{9}\right] \tag{3.1}$$

Corollary 2 *If we put* $c = \sigma = 1$ *in equation* (2.1) *and use equation* (1.7), *we get*

$$\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1} \left(1-\frac{\lambda}{3}\right)^{2\xi-1} \left(1-\frac{\lambda}{4}\right)^{\eta-1} E^{\rho}_{k,r,s}\left[y\lambda\left(1-\frac{\lambda}{3}\right)^2\right] d\lambda = \left(\frac{2}{3}\right)^{2\xi} B(\xi,\eta)\, {}_2\psi_2 \left[\begin{array}{c} (\rho,k),(\xi,1); \\ (s,r),(\xi+\eta,1); \end{array} \frac{4y}{9}\right] \tag{3.2}$$

Corollary 3 *If we put* $c = \sigma = k = 1$ *in equation* (2.1) *and use equation* (1.5), *we get*

$$\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1} \left(1-\frac{\lambda}{3}\right)^{2\xi-1} \left(1-\frac{\lambda}{4}\right)^{\eta-1} E^{\rho}_{r,s}\left[y\lambda\left(1-\frac{\lambda}{3}\right)^2\right] d\lambda = \left(\frac{2}{3}\right)^{2\xi} B(\xi,\eta)\, {}_2\psi_2 \left[\begin{array}{c} (\rho,1),(\xi,1); \\ (s,r),(\xi+\eta,1); \end{array} \frac{4y}{9}\right] \tag{3.3}$$

Corollary 4 *If we put* $c = \sigma = k = \rho = 1$ *in equation* (2.1) *and use equation* (1.4), *we get*

$$\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1} \left(1-\frac{\lambda}{3}\right)^{2\xi-1} \left(1-\frac{\lambda}{4}\right)^{\eta-1} E_{r,s}\left[y\lambda\left(1-\frac{\lambda}{3}\right)^2\right] d\lambda = \left(\frac{2}{3}\right)^{2\xi} B(\xi,\eta)\, {}_2\psi_2 \left[\begin{array}{c} (1,1),(\xi,1); \\ (s,r),(\xi+\eta,1); \end{array} \frac{4y}{9}\right] \tag{3.4}$$

Corollary 5 *If we put* $c = \sigma = k = \rho = s = 1$ *in equation* (2.1) *and use equation* (1.1), *we get*

$$\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1} \left(1-\frac{\lambda}{3}\right)^{2\xi-1} \left(1-\frac{\lambda}{4}\right)^{\eta-1} E_{r}\left[y\lambda\left(1-\frac{\lambda}{3}\right)^2\right] d\lambda = \left(\frac{2}{3}\right)^{2\xi} B(\xi,\eta)\, {}_2\psi_2 \left[\begin{array}{c} (1,1),(\xi,1); \\ (1,r),(\xi+\eta,1); \end{array} \frac{4y}{9}\right] \tag{3.5}$$

Corollary 6 *If we put $c=\sigma=k=\rho=s=1$ and $r=2$ in equation (2.1) and use equation (1.2), we get*

$$\begin{aligned}\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} cosh\left[\sqrt{y\lambda}\left(1-\frac{\lambda}{3}\right)^{2}\right]d\lambda \\ =\left(\frac{2}{3}\right)^{2\xi}B(\xi,\eta){}_2\psi_2\left[\begin{array}{c}(1,1),(\xi,1);\\(1,2),(\xi+\eta,1);\end{array}\frac{4y}{9}\right]\end{aligned} \tag{3.6}$$

Corollary 7 *If we put $c=\sigma=k=\rho=s=1$ and $r=4$ in equation (2.1) and use equation (1.3), we get*

$$\begin{aligned}\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1}\left\{cos\left(\lambda y\left(1-\tfrac{\lambda}{3}\right)^{2}\right)^{\frac{1}{4}}+cosh\left(\lambda y\left(1-\tfrac{\lambda}{3}\right)^{2}\right)^{\frac{1}{4}}\right\}d\lambda \\ =2\left(\frac{2}{3}\right)^{2\xi}B(\xi,\eta){}_2\psi_2\left[\begin{array}{c}(1,1),(\xi,1);\\(1,4),(\xi+\eta,1);\end{array}\frac{4y}{9}\right]\end{aligned} \tag{3.7}$$

Corollary 8 *If we put $c=1$ in equation (2.2) and use equ (1.7), we get*

$$\begin{aligned}\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} E_{k,r,s}^{\rho,\sigma}\left[y(1-\lambda)^2\left(1-\frac{\lambda}{4}\right)\right]d\lambda \\ =\left(\frac{2}{3}\right)^{2\xi}B(\xi,\eta){}_2\psi_2\left[\begin{array}{c}(\rho,\sigma k),(\eta,1);\\(s,r),(\xi+\eta,1);\end{array}y\right]\end{aligned} \tag{3.8}$$

Corollary 9 *If we put $c=\sigma=1$ in equation (2.2) and use equ (1.6), we get*

$$\begin{aligned}\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} E_{k,r,s}^{\rho}\left[y(1-\lambda)^2\left(1-\frac{\lambda}{4}\right)\right]d\lambda \\ =\left(\frac{2}{3}\right)^{2\xi}B(\xi,\eta){}_2\psi_2\left[\begin{array}{c}(\rho,k),(\eta,1);\\(s,r),(\xi+\eta,1);\end{array}y\right]\end{aligned} \tag{3.9}$$

Corollary 10 *If we put $c=\sigma=k=1$ in equation (2.2) and use equ (1.5), we get*

$$\begin{aligned}\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} E_{r,s}^{\rho}\left[y(1-\lambda)^2\left(1-\frac{\lambda}{4}\right)\right]d\lambda \\ =\left(\frac{2}{3}\right)^{2\xi}B(\xi,\eta){}_2\psi_2\left[\begin{array}{c}(\rho,1),(\eta,1);\\(s,r),(\xi+\eta,1);\end{array}y\right]\end{aligned} \tag{3.10}$$

Corollary 11 *If we put $c=\sigma=k=\rho=1$ in equation (2.2) and use equ (1.4), we get*

$$\begin{aligned}\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} E_{r,s}\left[y(1-\lambda)^2\left(1-\frac{\lambda}{4}\right)\right]d\lambda \\ =\left(\frac{2}{3}\right)^{2\xi}B(\xi,\eta){}_2\psi_2\left[\begin{array}{c}(1,1),(\eta,1);\\(s,r),(\xi+\eta,1);\end{array}y\right]\end{aligned} \tag{3.11}$$

Corollary 12 *If we put $c=\sigma=k=\rho=s=1$ in equation (2.2) and use equ (1.1), we get*

$$\begin{aligned}\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} E_{r}\left[y(1-\lambda)^2\left(1-\frac{\lambda}{4}\right)\right]d\lambda \\ =\left(\frac{2}{3}\right)^{2\xi}B(\xi,\eta){}_2\psi_2\left[\begin{array}{c}(1,1),(\eta,1);\\(1,r),(\xi+\eta,1);\end{array}y\right]\end{aligned} \tag{3.12}$$

Corollary 13 *If we put $c=\sigma=k=\rho=s=1$and $r=2$ in equation (2.2) and use equ (1.2), we get*

$$\begin{aligned}\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1} cosh\left[\sqrt{y\left(1-\frac{\lambda}{4}\right)}(1-\lambda)\right]d\lambda \\ =\left(\frac{2}{3}\right)^{2\xi}B(\xi,\eta){}_2\psi_2\left[\begin{array}{c}(1,1),(\eta,1);\\(1,2),(\xi+\eta,1);\end{array}y\right]\end{aligned} \tag{3.13}$$

Corollary 14 *If we put* $c = \sigma = k = \rho = s = 1$*and* $r = 4$ *in equation* (2.2) *and use equ* (1.3), *we get*

$$\int_0^1 \lambda^{\xi-1}(1-\lambda)^{2\eta-1}\left(1-\frac{\lambda}{3}\right)^{2\xi-1}\left(1-\frac{\lambda}{4}\right)^{\eta-1}\left\{cos\left[y(1-\lambda)^2\left(1-\frac{\lambda}{4}\right)\right]^{\frac{1}{4}}+cosh\left[y(1-\lambda)^2\left(1-\frac{\lambda}{4}\right)\right]^{\frac{1}{4}}\right\}d\lambda$$
$$= 2\left(\frac{2}{3}\right)^{2\xi} B(\xi,\eta)\,{}_2\psi_2\left[\begin{array}{c}(1,1),(\eta,1);\\(1,4),(\xi+\eta,1);\end{array}\; y\right] \tag{3.14}$$

4. Conclusion

By involving extended k-generalized Mittag-Leffler function,two unified integrals established in the form of generalized Wright hyper-geometric function. Also discussed certain cases which are converted into the elementary Mittag-Leffler function.

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