Reforming Decision Models and Assessing the Impact of State Schemes on Rural Women's Social Development [*]

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ABSTRACT: The Government of Tamil Nadu has introduced numerous welfare schemes to empower women and uplift their socio-economic status. While these initiatives play a vital role in fostering inclusion and development, their effectiveness often varies across different dimensions. This study presents a comparative analysis of five major women-centric schemes using rhotrices-based decision-making techniques. The proposed framework enables a structured and quantitative evaluation by incorporating multiple criteria and offering a comprehensive visual representation of performance. By systematically comparing the schemes, this approach highlights their relative strengths and weaknesses, ensuring objectivity in assessment. The findings provide valuable insights that can guide policymakers in refining existing programs and formulating future interventions to maximize their impact on women's empowerment and socio-economic progress.

Key Words: Rhotrix, decision-making, women empowerment, Tamil Nadu Government schemes, comparative analysis.

Contents

1. Introduction

In 2003, Ajibade [1] introduced the concept of rhotrix, a mathematical framework designed to represent numerical arrays in a rhomboidal configuration. Within this formulation, an n-dimensional rhotrix was characterized as an entity that occupies an intermediate position between an $(n-1) \times (n-1)$ matrix and an $n \times n$ matrix. Subsequently, Ajibade [10] proposed the first multiplication rule for rhotrices, known as heart-based multiplication. In 2004, Sani [2] developed an alternative procedure termed row–column multiplication, which exhibits close similarity to the conventional matrix multiplication scheme. These two techniques constitute the principal operations underlying rhotrix multiplication, while other multiplication forms introduced in later studies have generally failed to satisfy the algebraic requirements necessary for the establishment of robust mathematical structures.

Further advancement was made by Sani [3], who proposed the notion of a coupled matrix, enabling the transformation of a rhotrix into an equivalent matrix representation. Building on these developments, Isere [4] extended Ajibade's formulation in 2008 by defining even-dimensional rhotrices, thereby addressing the limitation inherent in the original definition, which applied exclusively to rhotrices of odd order. Beyond these foundational contributions, subsequent research has substantially enriched the theoretical landscape of rhotrix mathematics. Notable progress includes the formulation of rhotrix groups [11,7], rings [5], and vector spaces [6], along with the exploration of practical applications in diverse domains such as cryptography [13,14,12].

Richard M. Michaels' contributions to public policy research highlight the critical role of structured decision-making in the formulation of effective policies. In his seminal work, he advocates the use of matrix-based frameworks as decision-making tools, enabling policymakers to systematically evaluate and compare alternative policy options against predefined criteria. Complementing this perspective, Ishizaka

and Nemery provide an extensive account of the methodologies and applications of Multi-Criteria Decision Analysis (MCDA), a widely recognized approach for addressing complex decision problems that involve multiple, and often conflicting, criteria. Within the Indian context, Saranya and Chandrasekar examine government initiatives designed to promote and support women entrepreneurs. Their study assesses both the effectiveness of these schemes and their broader influence on the entrepreneurial ecosystem for women in India.

The primary objective of this study is to undertake a comprehensive comparative analysis of government initiatives aimed at promoting women's empowerment in Tamil Nadu through the application of rhotrices-based decision-making techniques. The research involves the systematic identification and selection of key schemes, namely the

- Sathiyavanimuthu Ammaiyar Ninaivu Free Supply of Sewing Machine Scheme

- Moovalur Ramamirtham Ammaiyar Ninaivu Higher Education Assurance Scheme

- Chief Minister's Girl Child Protection Scheme

- Annai Teresa Ninaivu Marriage Assistance Scheme

- Dr. Muthulakshmi Reddy Ninaivu Inter-Caste Marriage Assistance Scheme

These five schemes have been chosen as they represent diverse domains of women's empowerment-ranging from economic upliftment and educational advancement to social inclusion and protection-thereby providing a holistic basis for comparison. To ensure a rigorous assessment, the study proposes the development of a well-defined set of evaluation criteria encompassing dimensions such as effectiveness, coverage, financial efficiency, sustainability, inclusiveness, and beneficiary satisfaction.

Let n be positive odd integer. The general form rhotrix is structured as

$$
\left\langle
\begin{array}{ccccccc}
 & & & a_1 & & & \\
 & & a_2 & a_3 & a_4 & & \\
 & .. & .. & .. & .. & .. & \\
 & .. & .. & .. & .. & .. & .. \\
a_{\{\frac{t+1}{2}\}-\frac{n}{2}} & .. & .. & .. & a_{\frac{t+1}{2}} & .. & .. & .. & a_{\{\frac{t+1}{2}\}+\frac{n}{2}} \\
 & .. & .. & .. & .. & .. & .. \\
 & .. & .. & .. & .. & .. & \\
 & & a_{t-3} & a_{t-2} & a_{t-1} & & \\
 & & & a_t & & &
\end{array}
\right\rangle,
$$

where $t = \frac{n^2+1}{2}$. In the above representation, the entries highlighted in red correspond to the first row, those in blue denote the first column, and the entries in purple indicate the diagonal elements of the rhotrix. The major horizontal axis is given by the sequence $a_{\{\frac{t+1}{2}\}-\frac{n}{2}} \cdots a_{\frac{t+1}{2}} \cdots a_{\{\frac{t+1}{2}\}+\frac{n}{2}}$ while the major vertical axis is defined as $a_1 a_3 \cdots a_{\frac{t+1}{2}} \cdots a_{t-2} a_t$. The heart of the rhotrix refers to the entry located at the intersection of the major horizontal and vertical axes, which is the central element $a_{\frac{t+1}{2}}$.

2. Rhotrices-Based Comparative Analysis of Women-Centric Schemes

In this section we will present the decision-making analysis of selected women-oriented government schemes using a rhotrix-based multi-criteria framework. The evaluation integrates normalization, Rank Order Centroid (ROC) weighting, and the Simple Additive Weighting (SAW) method to systematically assess the schemes. Educational and livelihood-oriented schemes are analyzed using three criteria (effectiveness, sustainability, and accessibility), while marriage assistance schemes are evaluated using two criteria (social inclusion and empowerment, and economic upliftment). The use of rhotrix provides a unified structure to accommodate schemes with differing criteria sets, enabling a balanced and comparative assessment.

SCHEME SELECTION

The present study focuses on five flagship welfare initiatives introduced by the Government of Tamil Nadu with the objective of promoting women's empowerment and social upliftment. These schemes were carefully chosen based on their direct relevance to women's welfare, their policy significance, and their broad coverage across different domains of empowerment. The selected schemes are:

- Sathiyavanimuthu Ammaiyar Ninaivu Free Supply of Sewing Machine Scheme

- Moovalur Ramamirtham Ammaiyar Ninaivu Higher Education Assurance Scheme

- Chief Minister's Girl Child Protection Scheme

- Annai Teresa Ninaivu Marriage Assistance Scheme

- Dr. Muthulakshmi Reddy Ninaivu Inter-Caste Marriage Assistance Scheme

The rationale for selecting these schemes lies in their ability to collectively represent diverse dimensions of empowerment. The first three schemes primarily aim to strengthen education and foster economic self-reliance through livelihood opportunities. In contrast, the latter two schemes are designed as marriage assistance programs, with a focus on promoting social inclusion and providing financial support to women at the time of marriage.

EVALUATION CRITERIA

For the purpose of this study, the evaluation criteria have been differentiated based on the nature of the schemes under consideration. Since the selected programs operate in distinct domains of women's empowerment, their assessment requires customized indicators that reflect their objectives and operational features.

For schemes that emphasize education and livelihood support—namely, the Sathiyavanimuthu Ammaiyar Ninaivu Free Supply of Sewing Machine Scheme, the Moovalur Ramamirtham Ammaiyar Ninaivu Higher Education Assurance Scheme, and the Chief Minister's Girl Child Protection Scheme—three key criteria are adopted.

1. **Effectiveness** is measured in terms of the number of beneficiaries enrolled and supported under the scheme.

2. **Sustainability** is evaluated by examining the long-term impact of the program, including the duration of aid and continuity of benefits.

3. **Accessibility** is assessed through the efficiency of administrative processes, particularly the time required for beneficiaries to access scheme benefits.

In contrast, the Annai Teresa Ninaivu Marriage Assistance Scheme and the Dr. Muthulakshmi Reddy Ninaivu Inter-Caste Marriage Assistance Scheme are designed to provide marriage-related support, and therefore are evaluated on two different parameters.

1. **Social inclusion and empowerment** reflects the extent to which these schemes promote social equity and support women from marginalized or disadvantaged backgrounds.

2. **Economic upliftment** captures the degree of financial assistance provided and its effectiveness in reducing the economic burden associated with marriage expenses.

By aligning evaluation criteria with the distinct objectives of each scheme, the study ensures a more accurate and meaningful comparative analysis through rhotrix-based decision-making techniques.

CRITERION WEIGHTING (ROC Method)

To prioritize evaluation dimensions in a transparent and reproducible manner, we employ the *Rank Order Centroid (ROC)* method. Given n criteria ranked from most ($i = 1$) to least important ($i = n$), the ROC weight for rank i is

$$w_i = \frac{1}{n} \sum_{j=i}^{n} \frac{1}{j}, \qquad \text{with } \sum_{i=1}^{n} w_i = 1.$$

A. Education/Livelihood–oriented schemes *(Sathiyavanimuthu Ammaiyar Ninaivu Free Supply of Sewing Machine; Moovalur Ramamirtham Ammaiyar Ninaivu Higher Education Assurance; Chief Minister's Girl Child Protection)*

Rank justification

Criterion	Rank	Explanation
Sustainability	1	Ensures real and long-lasting change
Effectiveness	2	Measures initial success/reach
Accessibility	3	Important but largely a precondition

Criteria Weighting (ROC weights ($n = 3$))

Sustainability (rank 1): $\frac{1}{3}\left(1 + \frac{1}{2} + \frac{1}{3}\right) \approx 0.611$

Effectiveness (rank 2): $\frac{1}{3}\left(\frac{1}{2} + \frac{1}{3}\right) \approx 0.278$

Accessibility (rank 3): $\frac{1}{3}\left(\frac{1}{3}\right) \approx 0.111$

Criterion	Weight
Sustainability	0.611
Effectiveness	0.278
Accessibility	0.111

B. Marriage assistance schemes *(Annai Teresa Ninaivu Marriage Assistance; Dr. Muthulakshmi Reddy Ninaivu Inter-Caste Marriage Assistance)*

Rank justification

Criterion	Rank	Rationale
Social Inclusion & Empowerment	1	Long-term, identity-shaping, transformative impact on women's lives
Economic Upliftment	2	Essential support, but secondary to lasting social freedom and equity

Criterion Weighting (ROC weights ($n = 2$))

Social Inclusion & Empowerment (rank 1): $\frac{1}{2}\left(1 + \frac{1}{2}\right) = 0.75$

Economic Upliftment (rank 2): $\frac{1}{2}\left(\frac{1}{2}\right) = 0.25$

Criterion	Weight
Social Inclusion & Empowerment	0.75
Economic Upliftment	0.25

Interpretation. This weighting scheme intentionally emphasizes **long-term social transformation** over short-term financial relief, which is appropriate for a women-centered impact evaluation.

SCORING OF SCHEMES

In order to evaluate the selected government schemes, each criterion was associated with a measurable indicator to ensure objectivity and comparability. The scoring framework is defined as follows:

1. **Effectiveness** – measured by the *number of beneficiaries* covered under each scheme. A higher number of beneficiaries indicates greater reach and impact.

2. **Sustainability** – measured by the *period of aid or duration of benefits*. Schemes that provide recurring or long-lasting support are assigned higher scores.

3. **Accessibility** – measured by the *average processing time* required for applicants to receive benefits. Schemes with shorter processing times are considered more accessible.

4. **Social Inclusion and Empowerment** – measured by the *extent of participation* (number of beneficiaries) from socially and economically disadvantaged groups. Schemes that reach marginalized women score higher.

5. **Economic Upliftment** – measured by the *financial amount provided under the scheme*. Higher monetary support reflects greater potential for livelihood improvement and economic independence.

CONSTRUCTION OF DECISION RHOTRIX

The schemes can be represented as:

- E_1 – Sathiyavanimuthu Ammaiyar Ninaivu Free Supply of Sewing Machine Scheme
- E_2 – Moovalur Ramamirtham Ammaiyar Ninaivu Higher Education Assurance Scheme
- E_3 – Chief Minister's Girl Child Protection Scheme
- M_1 – Annai Teresa Ninaivu Marriage Assistance Scheme
- M_2 – Dr. Muthulakshmi Reddy Ninaivu Inter-Caste Marriage Assistance Scheme

The Educational/Marriage related criterion can be represented as:

- EC_1 – Effectiveness
- EC_2 – Sustainability
- EC_3 – Accessibility
- MC_1 – Social Inclusion and Empowerment
- MC_2 – Economic Upliftment

On constructing the decision rhotrix where rows represent schemes and columns represent criteria,

$$\left\langle \begin{matrix} & & S(E_1, EC_1) & & \\ & S(E_2, EC_1) & S(M_1, MC_1) & S(E_1, EC_2) & \\ S(E_3, EC_1) & S(M_2, MC_1) & S(E_2, EC_2) & S(M_1, MC_2) & S(E_1, EC_3) \\ & S(E_3, EC_2) & S(M_2, MC_2) & S(E_2, EC_3) & \\ & & S(E_3, EC_3) & & \end{matrix} \right\rangle$$

where $S(E_i, EC_j)$ and $S(M_i, MC_j)$ represents score of each educational and marriage schemes against their criteria respectively.

Then, from the data taken from the Tamil Nadu Social Welfare website, the above rhotrix is transformed as

$$\left\langle \begin{matrix} & & 85,396 & & \\ & 4,95,822 & 3420 & Max & \\ 11,48,561 & 11,604 & 4 & 25000-50000 & S(E_1, EC_3) \\ & 18 & 25000-50000 & 2 & \\ & & 1 & & \end{matrix} \right\rangle$$

Since the raw data for each indicator is expressed in different units, mean normalization method is applied to bring all values into a comparable scale between -1 and 1. For each criterion EC_i and MC_i, the normalized score of scheme E_i and M_i (resp.) is computed as:

$$N[S(E_i, EC_j)] = \frac{S(E_i, EC_j) - \bar{S}(E_i, EC_j)}{\max(S(E_i, EC_j)) - \min(S(E_i, EC_j))}$$

where $S(E_i, EC_j)$ is the raw value of criterion EC_j for scheme E_i, $\bar{S}(E_i, EC_j)$ mean value among a criteria across all schemes, $\min(S(E_i, EC_j))$ is the minimum value of EC_i across all schemes, and $\max(S(E_i, EC_j))$ is the maximum value of EC_i across all schemes and so similar is for the scheme M_i

In case, where a lower value indicates better performance, such as Accessibility (Processing Time), the normalization is inverted as:

$$N[S(E_i, EC_j)] = \frac{\bar{S}(E_i, EC_j) - S(E_i, EC_j)}{\max(S(E_i, EC_j)) - \min(S(E_i, EC_j))}$$

By using the above formula, the *decision rhotrix* is transformed as

$$\left\langle \begin{matrix} & & -0.462 & & \\ & -0.076 & -0.5 & 0.443 & \\ 0.538 & 0.5 & -0.557 & 0 & -0.5 \\ & 0.110 & 0 & 0 & \\ & & 0.5 & & \end{matrix} \right\rangle$$

This constructed *decision rhotrix* serves as the basis for comparative analysis.

Now, the Simple Additive Weighting (SAW) method is employed in this study due to its simplicity, transparency, and computational efficiency, as it is one of the most widely used Multi-Criteria Decision-Making (MCDM) techniques, offering a straightforward way to aggregate multiple criteria into a single performance score for each alternative (scheme).

$$\text{The SAW score, } ES_i = \sum_{j=1}^{n} EW_j \, N[S(E_i, EC_j)] \tag{2.1}$$

where

- ES_i = SAW score of scheme E_i

- EW_j = weight of criterion EC_j, such that $\sum_{j=1}^{n} EW_j = 1$

- $N[S(E_i, EC_j)]$ = normalized value of scheme E_i under criterion EC_j

The above formula is for education/livelihood related schemes and the same follows for marriage related schemes.

Then the SAW score for the scheme

"Sathiyavanimuthu Ammaiyar Ninaivu Free Supply of Sewing Machine Scheme" is $ES_1 = 0.087$

"Moovalur Ramamirtham Ammaiyar Ninaivu Higher Education Assurance Scheme" is $ES_2 = -0.362$

"Chief Minister's Girl Child Protection Scheme" is $ES_3 = 0.2723$

"Annai Teresa Ninaivu Marriage Assistance Scheme" is $MS_1 = -0.375$

"Dr. Muthulakshmi Reddy Ninaivu Inter-Caste Marriage Assistance Scheme" is $MS_2 = 0.375$

Interpretation of SAW Scores

The computed SAW scores for the five government schemes are as follows:

$$ES_1 = 0.087, \quad ES_2 = -0.362, \quad ES_3 = 0.2723, \quad MS_1 = -0.375, \quad MS_2 = 0.375$$

where ES_i denotes the SAW score of educational and livelihood schemes, and MS_i represents marriage assistance schemes.

- Among the **education and livelihood schemes**, the *Chief Minister's Girl Child Protection Scheme* ($ES_3 = 0.2723$) achieved the highest score, indicating superior overall effectiveness, sustainability, and accessibility compared to the other schemes in this category.
  The *Sathiyavanimuthu Ammaiyar Ninaivu Free Supply of Sewing Machine Scheme* ($ES_1 = 0.087$) performed moderately, while the *Moovalur Ramamirtham Ammaiyar Ninaivu Higher Education Assurance Scheme* ($ES_2 = -0.362$) scored negatively, reflecting relative weaknesses in long-term impact and implementation efficiency.

- Within the **marriage assistance schemes**, the *Dr. Muthulakshmi Reddy Ninaivu Inter-Caste Marriage Assistance Scheme* ($MS_2 = 0.375$) secured the highest positive score, highlighting its stronger role in promoting social inclusion and empowerment. Conversely, the *Annai Teresa Ninaivu Marriage Assistance Scheme* ($MS_1 = -0.375$) registered a negative score, indicating limited effectiveness compared to its counterpart.

Overall, the analysis demonstrates that schemes with higher SAW scores are more impactful and should be prioritized for policy emphasis, while those with negative or lower scores may require modifications or targeted improvements to enhance their effectiveness.

3. Significance of the Study

An important methodological contribution of this work lies in the use of rhotrix structures. The rhotrix framework offered a significant advantage by enabling the joint representation of schemes with different sets of criteria—educational and livelihood schemes with three criteria, and marriage assistance schemes with two criteria—within a single structured model. This flexibility ensures that diverse policy interventions can be evaluated consistently without losing the specificity of their respective objectives.

The findings underscore the importance of multi-criteria decision-making approaches in evaluating welfare schemes, as they provide a transparent and quantifiable framework for policy analysis. Furthermore, the rhotrix-based modeling employed in this study adds an innovative dimension to decision support by enabling structured representation of criteria and alternatives.

Conclusion

This study has presented a structured decision-making analysis of five major women-centered welfare schemes implemented by the Government of Tamil Nadu. By adopting Rank Order Centroid (ROC) weighting, appropriate evaluation criteria were prioritized, ensuring a balanced assessment of both education- and livelihood-oriented initiatives, as well as marriage assistance schemes. The use of

mean normalization allowed for effective comparison of diverse criteria, while the Simple Additive Weighting (SAW) method facilitated aggregation of normalized scores to derive overall scheme effectiveness.

The analysis revealed that, among education and livelihood schemes, the *Chief Minister's Girl Child Protection Scheme* attained the highest score, reflecting its significant impact on long-term sustainability and accessibility. Similarly, within the marriage assistance category, the *Dr. Muthulakshmi Reddy Ninaivu Inter-Caste Marriage Assistance Scheme* emerged as the most effective in fostering social inclusion and empowerment. In contrast, certain schemes recorded comparatively lower scores, highlighting areas where implementation efficiency, financial adequacy, or long-term benefits could be strengthened.

In conclusion, this work not only demonstrates the practical application of mathematical decision-making tools in the evaluation of social welfare policies but also emphasizes the need for continuous monitoring and refinement of schemes to maximize their socio-economic impact. Future research may extend this framework by integrating fuzzy or neutrosophic rhotrices, as well as developing computational tools for large-scale evaluations, thereby enhancing the robustness and adaptability of policy assessments.

Acknowledgments

We thank the organizers of the "International Conference on Mathematical Sciences and Computing Innovations and Applications (ICMSC-2025) jointly organized by Department of Mathematics North Eastern Regional Institute of Science and Technology (NERIST) and Department of Mathematics National institute of technology (NIT), Uttarakhand held over June 26-28, 2025.

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