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Intelligent Energy Optimization in Smart Homes Using Cubic Fuzzy Frank Aggregation Operators

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ABSTRACT: Energy efficiency in smart homes is a complex challenge that requires intelligent decision-making under uncertainty. Fuzzy sets and interval-valued fuzzy sets (IFSs) provide effective mathematical frameworks for handling imprecise data, making them crucial for optimizing energy consumption. This paper introduces a novel Cubic Fuzzy Frank (CFF) methodology, integrating cubic fuzzy averaging and geometric aggregation operators to enhance decision-making for energy optimization in smart homes. We develop several new aggregation operators, including: Cubic Fuzzy Frank Weighted Averaging (CFFWA); Cubic Fuzzy Frank Ordered Weighted Averaging (CFFOWA); Cubic Fuzzy Frank Hybrid Averaging (CFFHA); Cubic Fuzzy Frank Weighted Geometric (CFFWG); Cubic Fuzzy Frank Ordered Weighted Geometric (CFFOWG) and Cubic Fuzzy Frank Hybrid Geometric (CFFHG). These operators, based on Frank t-norm and Frank t-conorm, enable more accurate and adaptive energy optimization by considering varying levels of uncertainty. Additionally, we introduce new score and precision functions to refine the decision-making process. A systematic step-by-step methodology is presented for applying the CFF approach to smart home energy management. To validate its effectiveness, we provide a numerical case study demonstrating its superior performance compared to existing techniques. The results highlight the efficiency, adaptability, and practicality of the proposed method, making it a powerful tool for optimizing energy consumption in intelligent home environments.

Key Words: multi-attribute decision making, cubic fuzzy set, frank

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1. Introduction

The drive for sustainable living and growing environmental concerns have increased the need for energy efficiency in buildings, particularly smart houses. The development of AI technology has opened up new avenues for energy consumption optimization. However, conventional optimization techniques are severely hampered by the complexity of building systems, which include varying occupancy levels and erratic weather.

Figure 1 is supply chain given as below.

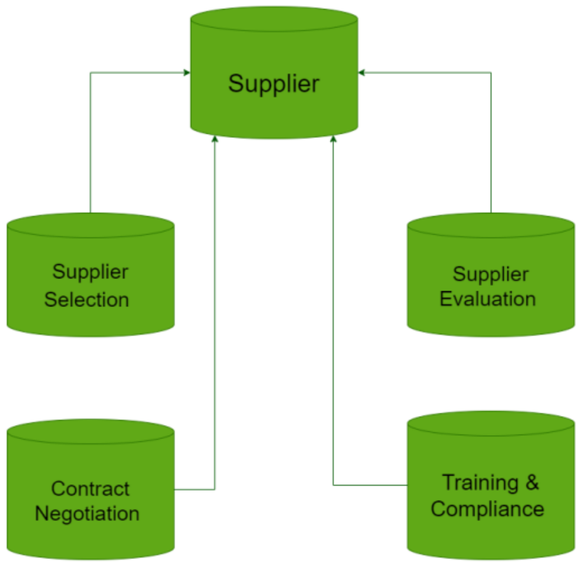

Figure 1, different supply selection.

Current methods, such as those based on static models or heuristic algorithms, frequently struggle to adjust to changes in the environment in real time, which restricts their capacity to achieve consistent and ideal energy use [40]. Fuzzy logic was the foundation of many AI-driven techniques for building energy optimization. Although these approaches were successful in controlling uncertainty, they usually depended on oversimplified models that were unable to fully represent the intricacy of smart home environments. Systems such as fuzzy-based decision frameworks for assessing HVAC system performance [42] and adaptive neuro-fuzzy inference system for load forecasting [41] had trouble managing uncertainty in real-time energy management, especially when taking dynamic factors like weather or human behavior into account.

Figure 2 is given as

The concept of a fuzzy set, which can address the issue of uncertainty in various contexts, was developed by Zadeh [1]. Sets with degrees of membership called FSs, and membership functions with values in the interval [0, 1] are allowed according to the FS theory. However, because there could be some reluctant degree, it might not always hold in real life that the degree of nonmembership function is equal to one minus membership function. The intuitionistic FS was initially proposed by Atanassov [2]. Truth and falsity grades are assigned to the constituents of IFSs. The degree of hesitancy about an element's truth and falsity grades within a set must be represented using IFS. IFSs are used in numerous real-world scenarios to solve issues. One example of this is when we toss a coin; there are two possible outcomes, head or tail. Expert opinions were represented using the Basic Uncertain Information (BUI) technique, and in a group decision-making context, these opinions were combined using aggregation operations [3]. The coin shows either way at a time, but not both ways at once. It developed the theory of Pythagorean fuzzy sets to address these types of issues [4]. Because PyFS and IFS have the same structure but different conditions,

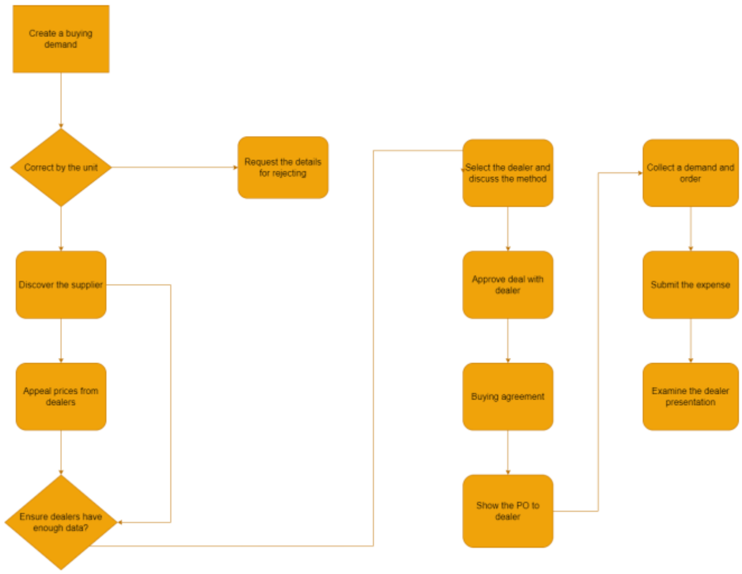

Figure 2 dealers.

PyFS was the improved version of IFSs that has overcome their limitations, Aggregation operators [5, 6], similarity measures [7, 8], decision-making approaches [9, 10], and various sorts of procedures [11, 12] are a few examples of uses of FS and its extensions. By clustering fuzzy c-numbers, Xu and Li [13] prophesied the reversion of a fuzzy time sequence. This study offers a class of fuzzy clustering procedures specifically tailored for processing fuzzy data. Fuzzy c-number clusterings are novel algorithms designed to handle different kinds of fuzzy data more efficiently, fuzzy number forms such as conventional, trapezoidal, LR-type, and triangular fuzzy numbers [14]. Akram et al. [15-17] introduced several PFS-imitable applications. A thorough case study on choosing medical subject experts was used to test an integrated MCDM algorithm that was created using the suggested operators and the combined criterion weight determination model [18]. As a result, it can represent the relative relevance of the supplied Pythagorean fuzzy argument and its ordered position. Several interval-valued Pythagorean fuzzy point operators were created by Peng et al. [19]. Furthermore, we provide some interval-valued Pythagorean fuzzy point weighted averaging operators that can modify the degree of the aggregated arguments with a parameter by combining the interval-valued Pythagorean fuzzy point operators with the IVPFWA operator. To address multi-attribute group decision-making under interval-valued Pythagorean fuzzy information, Rahman et al. [20] presented an operator. The value and compatibility of the discussed methodologies and decision support systems were examined [21]. The generalized IVPNFWG operator were presented by Yang et al. [22] to aggregate IVPNF information. A new concept of Pythagorean neutrosophic normal interval-valued weighted averaging, Pythagorean neutrosophic normal interval-valued weighted geometric and generalized Pythagorean neutrosophic normal interval-valued weighted averaging as well as generalized Pythagorean neutrosophic normal interval-valued weighted geometric were discussed by Palanikumar et al. [23]. Two novel approaches to multi-attribute decision-making in a fuzzy environment were presented by Seikh et al. [24]. Based on the consistency of the InPLPR, Wang et al. [25] created a decision-making method that includes estimating missing data, enhancing consistency, and evaluating the options. An incomplete probabilistic linguistic term set was presented by Liu et al. [26]. The built using decision-making-based 3WD techniques and the suggested multigranulation q-ROF probabilistic models by Zhang et al. [27]. Three iterations of MG q-ROF probabilistic rough sets (PRSs) were utilized by Zhang et al. [28]. Rough and neutrosophic rough sets, and soft and neutrosophic soft rough sets (SRNSs and NSRSs) are the terms used by Zhang et al. [29] to describe these sets. The distinction between absolute and relative knowledge distances in the structural characteristics of hierarchical clustering was

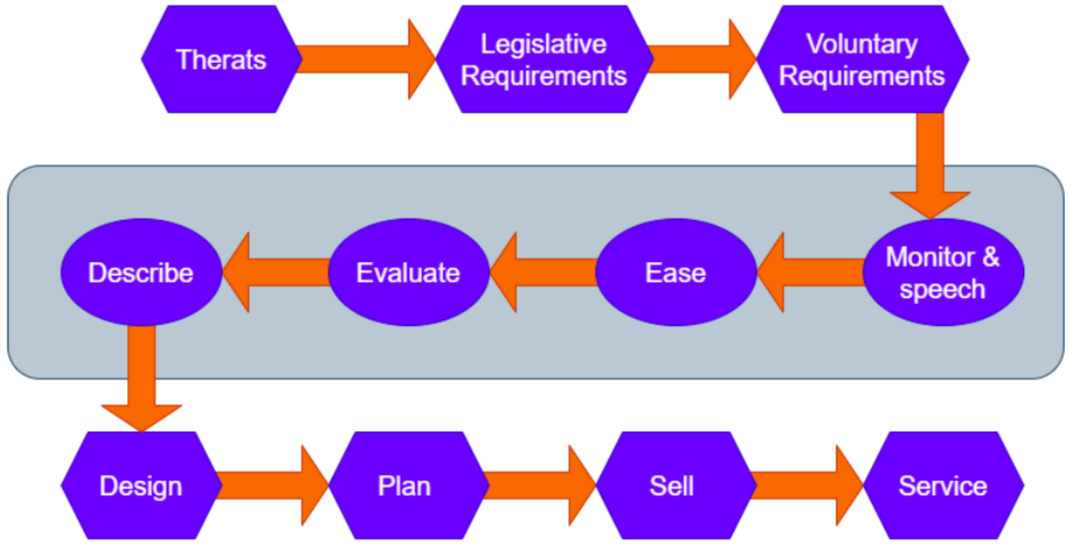

Figure 3 is different designed.

examined by Lian et al. [30]. Anusha et al. [31] covered the hybridizations in addition to providing an extension of the MSM operators and their requests based on q-rung probabilistic dual hesitant fuzzy sets. Depending on the evaluation values of each choice, it was frequently possible to examine many possibilities to arrive at a comprehensive assessment result, for example, by using MADM [32, 33]. A unique approach to selecting robotic systems for homogenous group DM was presented by Bairagi [34]. These operators were utilized to devise a method for handling group decision-making with CPF information. To demonstrate the usefulness and efficiency of the operators and method, a numerical example was presented [35]. Yahya et al. [36] introduced the Artificial Intelligence applied in DM to choose a maintenance approach and other research [37, 38, 39, 40, 41, 42].

Figure 3 is given as

This manuscript is structured as follows: Section 2 introduces the concept of CSs. Section 3 presents operational rules for CFSs, including algebraic and Frank operational laws. In Section 4, we propose the CFFWA, CFFOWA, CFFHWA, CFFWG CFFOWG and CFFHWG operators. Section 5 outlines an optimized MADM process for the CF model. Section 6 define the case study and a comparative analysis. Finally, Section 7 concludes the study.

1.1. Contribution of study

The application of Cubic Fuzzy Frank Aggregation Operators (CFFAO) has revolutionized intelligent energy management in smart homes by effectively handling uncertainty and optimizing energy consumption. The key contributions of this study are as follows:

(a) We define operational laws and Cubic Fuzzy sets (CFS) to establish a solid mathematical foundation for energy optimization in smart home environments.

(b) A new accuracy and scoring function is proposed to enhance the precision of decision-making in energy management systems.

(c) We introduce various CFS aggregation operators, such as CFFWA, CFFOWA, CFFWHA, CFFWG, CFFOWG and CFFHG operators, tailored for multi-criteria decision-making (MCDM) in smart home energy optimization. These operators improve adaptability in managing energy distribution and consumption under dynamic conditions.

(d) The CFFAO framework effectively integrates and processes inconsistent and varied energy data, leading to improved energy efficiency, demand forecasting, and cost reduction. This method is particularly beneficial in smart homes where energy consumption fluctuates based on user behavior and external factors.

(e) The CFFAO approach is highly effective in multi-dimensional smart home systems, where multiple energy sources, appliances, and renewable integrations must be managed efficiently. By identifying consumption patterns and optimizing energy allocation, this method minimizes waste, maximizes sustainability, and enhances overall energy efficiency.

This study presents a robust framework that significantly improves energy management in smart homes, offering a data-driven, intelligent, and adaptive approach to optimizing energy consumption.

1.2. Motivation

This study focuses on the challenge of energy optimization in smart homes in the development of intelligent energy management systems.

The Cubic Fuzzy Set (CFS) framework has been designed to allow energy experts to offer insights more easily. By covering a wider range of information uncertainty, CFS surpasses traditional fuzzy and interval-valued fuzzy sets, preventing the loss of critical data when converting qualitative energy usage patterns into quantitative information.

The case study presented can be adapted for use by smart home energy management teams, helping them enhance energy efficiency through structured decision-making processes. This method provides a roadmap from expert opinions to actionable energy optimization solutions.

Smart home energy data is often complex, uncertain, and incomplete due to varying consumption patterns and environmental factors. Conventional optimization methods struggle with such uncertainty. The Cubic Fuzzy Frank Aggregation Operators (CFFAO) effectively manage and integrate this data, significantly improving the reliability of energy optimization decisions and enhancing the overall efficiency of smart home energy systems.

1.3. Novelty

In this article, we aim to design the following:

i. To define advanced operational laws for Cubic Fuzzy Frank statistics that serve as a valuable extension to basic operational laws, specifically for optimizing energy management in smart homes, and to analyze their mathematical properties.

ii. To introduce novel aggregation operators, such as Cubic Fuzzy Frank Aggregation Operators, tailored for intelligent energy optimization in smart homes, improving decision-making in dynamic environments.

iii. To propose a Multi-Criteria Group Decision-Making (MCGDM) technique in the context of cubic fuzzy sets (CF), designed to optimize various energy-related criteria (e.g., consumption, cost, efficiency) in smart homes.

iv. To solve a real-world numerical problem related to smart home energy optimization, validating the effectiveness of the proposed methodology in practical scenarios.

v. To demonstrate the effectiveness and reliability of the proposed approach, a sensitivity analysis is carried out, assessing the impact of different factors on energy optimization in smart home systems.

The abbreviation of table 1 is written below.

Table 1 of abbreviations	
Abbreviations	Full Name
IFFNs	Intuitionistic fuzzy frank numbers
CFN	Cubic fuzzy number
CFFAO	Cubic fuzzy frank aggregation operator
CFFWA	Cubic fuzzy frank weighted average
CFFOWA	Cubic fuzzy frank ordered weighted average
CFFHWA	Cubic fuzzy frank hybrid weighted average
CFFWG	Cubic fuzzy frank weighted geometric
CFFOWG	Cubic fuzzy frank ordered weighted geometric
CFFHWG	Cubic fuzzy frank hyrid weighted geometric

2. Basic ideas

Definition 2.1 *[1]Considering that $\Phi \neq X$ and let is use a fuzzy set $\gamma = \left\{ \begin{array}{c} \langle x, \mu_{\gamma(x)} \rangle \\ : x \in X \end{array} \right\}$. An element x in X is represented by the membership function, $\mu_{\gamma(x)}$ is a mapping from X to $[0,1]$.*

Definition 2.2 *Let $a_1 = [L_1, \kappa_1]$ and $a_2 = [L_2, \kappa_2]$ be the IFAANs and $\lambda > 0$, , then*

$$a_1 \oplus a_2 = \left[1 - \log_\beta^{\left(1+\frac{(\beta^{1-L_1}-1)(\beta^{1-L_2}-1)}{\beta-1}\right)}, \log_\beta^{\left(1+\frac{(\beta^{k_2}-1)(\beta^{\kappa_2}-1)}{\beta-1}\right)} \right];$$

$$a_1 \otimes a_2 = \left[\log_\beta^{\left(1+\frac{(\beta^{L_1^-}-1)(\beta^{L_2}-1)}{\beta-1}\right)}, 1 - \log_\beta^{\left(1+\frac{(\beta^{1-K_1}-1)(\beta^{1-\kappa_2}-1)}{\beta-1}\right)} \right];$$

$$a_1^\lambda = \left[\log_\beta^{\left(1+\frac{(\beta^{L_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, 1 - \log_\beta^{\left(1+\frac{(\beta^{1-\kappa_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)} \right];$$

$$\lambda a_1 = \left[1 - \log_\beta^{\left(1+\frac{(\beta^{1-L_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, \log_\beta^{\left(1+\frac{(\beta^{1-\kappa_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)} \right]$$

Definition 2.3 *[39] Let $a = \{\varsigma, \chi\}$ be the IFSs, then the score function is $a = \varsigma_\alpha - \chi_\alpha$.*

Definition 2.4 *[39] Let $a = \{\varsigma, \chi\}$ be the IFSs, then the accuracy function is $a = \varsigma_\alpha + \chi_\alpha$.*

3. Operational laws on Frank

The section address the operational laws of the frank t-norm and t-conorm. The frank operational laws are a collection of axioms that control how operations in fuzzy logic, such as t-norms and t-conorms, behave. These rules offer a foundation for fuzzy set reasoning and are necessary to create dependable fuzzy logic systems.

Definition 3.1 *Let $a_1 = \langle [\kappa_1^-, \kappa_1^+], \kappa_1 \rangle$ and $a_2 = \langle [\kappa_2^-, \kappa_2^+], \kappa_2 \rangle$ be CFSs and $\lambda > 0$, then*

$$a_1 \oplus a_2 = \left\langle \left[\begin{array}{c} 1 - \log_\beta^{\left(1+\frac{(\beta^{1-\kappa_1^-}-1)(\beta^{1-\kappa_2^-}-1)}{\beta-1}\right)}, \\ 1 - \log_\beta^{\left(1+\frac{(\beta^{1-\kappa_1^+}-1)(\beta^{1-\kappa_2^+}-1)}{\beta-1}\right)} \end{array} \right], \log_\beta^{\left(1+\frac{(\beta^{\kappa_1}-1)(\beta^{\kappa_2}-1)}{\beta-1}\right)} \right\rangle;$$

$$a_1 \otimes a_2 = \left\langle \left[\log_\beta^{\left(1+\frac{(\beta^{\kappa_1^-}-1)(\beta^{\kappa_2^-}-1)}{\beta-1}\right)}, \log_\beta^{\left(1+\frac{(\beta^{\kappa_1^+}-1)(\beta^{\kappa_2^+}-1)}{\beta-1}\right)} \right], 1 - \log_\beta^{\left(1+\frac{(\beta^{1-\kappa_1}-1)(\beta^{1-\kappa_2}-1)}{\beta-1}\right)} \right\rangle;$$

$$a_1^\lambda = \left\langle \left[\log_\beta^{\left(1+\frac{(\beta^{\kappa_1^-}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, \log_\beta^{\left(1+\frac{(\beta^{\kappa_1^+}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)} \right], 1 - \log_\beta^{\left(1+\frac{(\beta^{1-\kappa_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)} \right\rangle;$$

$$\lambda a_1 = \left\langle \left[1 - \log_\beta^{\left(1+\frac{(\beta^{1-\kappa_1^-}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)}, 1 - \log_\beta^{\left(1+\frac{(\beta^{1-\kappa_1^+}-1)^\lambda}{(\beta-1)^{\lambda=1}}\right)} \right], \log_\beta^{\left(1+\frac{(\beta^{1-\kappa_1}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right)} \right\rangle$$

Definition 3.2 *The CFSs are* $a = \left\langle [\kappa_j^-, \kappa_j^+], \kappa_j \right\rangle$, *score function* I *is define as:*$I = \frac{\left\langle [\kappa_j^- - \kappa_j^+] + \kappa_j \right\rangle}{3}$.

Definition 3.3 *The CFSs are* $a = \left\langle [\kappa_j^-, \kappa_j^+], \kappa_j \right\rangle$, *accuracy function* L *is define as:*$L = \frac{\left\langle [\kappa_j^- + \kappa_j^+] + \kappa_j \right\rangle}{3}$.

4. CFSs based on Frank operators

This section presents the CFFWA, CFFOWA, CFFWHA, CFFWG, CFFOWG and CFFHG operators new methods for CFSs with several noteworthy characteristics, based on frank operators.

4.1. CFFWA operator

Definition 4.1 *Let* $d_j = \langle [\kappa^-, \kappa^+], \kappa \rangle$ *be the gathering of CFSs and* $u = (u_1, u_2, ..., u_m)^T$ *is the weight vector with* $u_j \in [0,1]$ *and* $\sum_{j=1}^{m} u_j = 1$. *Then* $CFFWA(d_1, d_2, ..., d_m) = \bigoplus_{j=1}^{m} d_j^{u_j}$ *is said CFFWA operator.*

Theorem 4.1 *The collection of CFSs are* $a_j = \langle [\kappa^-, \kappa^+], \kappa \rangle$ *and* $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ *is the weight vector with* $\lambda_j \in [0,1]$ *and* $\sum_{j=1}^{n} \lambda_j = 1$. *Then it is said CFFWA operator and* $CFFWA(a_1, a_2, ..., a_n) =$

$$\left\langle \begin{bmatrix} 1 - \log_\beta \left(1 + \frac{\prod_{j=1}^{n} (\beta^{1-\kappa_j^-} - 1)^{\lambda_j}}{(\beta - 1)^{\lambda_j - 1}} \right), \\ 1 - \log_\beta \left(1 + \frac{\prod_{j=1}^{n} (\beta^{1-\kappa_j^+} - 1)^{\lambda_j}}{(\beta - 1)^{\lambda_j - 1}} \right) \end{bmatrix}, \log_\beta \left(1 + \frac{\prod_{j=1}^{n} (\beta^{1-\kappa_j} - 1)^{\lambda_j}}{(\beta - 1)^{\lambda_j - 1}} \right) \right\rangle.$$

Theorem 4.2 ***(Idempotency):****If* $\widetilde{ZV} = \langle [\kappa^-, \kappa^+], \kappa \rangle$ *for all* $L = 1, 2, 3, ..., m$, *then* $CFFWA(\widetilde{ZV}, \widetilde{ZV}, \widetilde{ZV}, ..., \widetilde{ZV}) = \widetilde{ZV}$.

Theorem 4.3 ***(Commutativity):****If* $(d_1^{'}, d_2^{'}, ..., d_n^{'})$ *is any permutation of* $(d_1, d_2, ..., d_n)$, *then* $CFFWA(d_1^{'}, d_2^{'}, ..., d_n^{'}) = CFFWA(d_1, d_2, ..., d_n)$.

Theorem 4.4 ***(Boundedness):****If* $Y^- = min(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m), Y^+ = \max(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m)$, *then* $Y^- \leq CFFWA(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m) \leq Y^+$.

4.2. CFFOWA operator

Definition 4.2 *Let* $c_j = \langle [\kappa^-, \kappa^+], \kappa \rangle$ *be the gathering of CFNs and the weight vector is* $g = (g_1, g_2, ..., g_m)^T$ *with* $g_j \in [0,1]$ *and* $\sum_{j=1}^{m} g_j = 1$. *Then*

$$CFFOWA(c_1, c_2, ..., c_m) = \bigoplus_{j=1}^{m} c_j^{g_j}$$

is said CFFOWA operator.

Theorem 4.5 *Let $a_j = \langle [\kappa^-, \kappa^+], \kappa \rangle$ be the collection of CFSs and the weight vector is $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ with $\lambda_j \in [0,1]$ and $\sum_{j=1}^{n} \lambda_j = 1$. Then it is said CFFOWA operator and CFFOWA$(a_1, a_2, ..., a_n) =$*

$$\left\langle \left[1 - \log_\beta\left(1 + \frac{\prod_{j=1}^{n} (\beta^{1-\kappa_j^-} - 1)^{\lambda_j}}{(\beta-1)^{\lambda_j - 1}}\right), 1 - \log_\beta\left(1 + \frac{\prod_{j=1}^{n} (\beta^{1-\kappa_j^+} - 1)^{\lambda_j}}{(\beta-1)^{\lambda_j - 1}}\right) \right], \log_\beta\left(1 + \frac{\prod_{j=1}^{n} (\beta^{1-\kappa_j} - 1)^{\lambda_j}}{(\beta-1)^{\lambda_j - 1}}\right) \right\rangle.$$

Theorem 4.6 ***(Idempotency):****If $\widetilde{BU} = \langle [\kappa^-, \kappa^+], \kappa \rangle$ for all $L = 1, 2, 3, ..., m$, then CFFOWA$(BU, BU, BU, ..., BU) = BU$.*

Theorem 4.7 ***(Commutativity):****If $(b_1^{'}, b_2^{'}, ..., b_n^{'})$ is any permutation of $(b_1, b_2, ..., b_n)$, then CFFOWA$(b_1^{'}, b_2^{'}, ..., b_n^{'})$ =CFFOWA$(b_1, b_2, ..., b_n)$*

Theorem 4.8 ***(Boundedness):****If $Y^- = min(f_1, f_2, ..., f_m)$, $Y^+ = \max(f_1, f_2, ..., f_m)$, then $Y^- \leq$CFFOWA$(f_1, f_2, ..., f_m) \leq Y^+$.*

4.3. CFFHWA operator

Definition 4.3 *The gathering of CFSs are $c_j = \langle [\kappa^-, \kappa^+], \kappa \rangle$ and the weight vector is $u = (u_1, u_2, ..., u_n)^T$ with $u_j \in [0,1]$ and $\sum_{j=1}^{n} u_j = 1$, the associated vector is $u = (u_1, u_2, ..., u_n)^T$ with $u_j \in [0,1]$ and $\sum_{j=1}^{n} u_j = 1$. Then CFFHWA$(c_1, c_2, ..., c_n) = \bigoplus_{j=1}^{n} c_j^{u_j}$ is said CFFHWA operator.*

Theorem 4.9 *Let $a_j = \langle [\kappa^-, \kappa^+], \kappa \rangle$ be the collection of CFSs and the weight vector is $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ with $\lambda_j \in [0,1]$ and $\sum_{j=1}^{n} \lambda_j = 1$. Then it is said CFFHWA operator and CFFHWA$(a_1, a_2, ..., a_n) =$*

$$\left\langle \left[1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}\left(\beta^{1-\kappa_j^{-}}-1\right)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right), 1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}\left(\beta^{1-\kappa_j^{+}}-1\right)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \right], \log_{\beta}\left(1+\frac{\prod_{j=1}^{n}\left(\beta^{1-\kappa_j}-1\right)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \right\rangle.$$

Theorem 4.10 ***(Idempotency):****If* $\widetilde{c} = \langle[\kappa^{-},\kappa^{+}],\kappa\rangle$ *for all* $L = 1,2,3,...,m,$ *then* $CFFHWA(\widetilde{c},\widetilde{c},\widetilde{c},...,\widetilde{c}) = \widetilde{c}$.

Theorem 4.11 ***(Commutativity)*** ***:****If* $(p_1^{'},p_2^{'},...,p_n^{'})$ *is any permutation of* $(p_1,p_2,...,p_n)$, *then* $CFFHWA(p_1^{'},p_2^{'},...,p_n^{'}) = CFFHWA(p_1,p_2,...,p_n)$

Theorem 4.12 ***(Boundedness):****If* $Y^{-} = min(f_1,f_2,...,f_n)$, $Y^{+} = \max(f_1,f_2,...,f_n)$, *then* $Y^{-} \leq CFFHWA(f_1,f_2,...,f_n) \leq Y^{+}$.

4.4. CFFWG operator

Definition 4.4 *Let* $d_j = \langle[\kappa^{-},\kappa^{+}],\kappa\rangle$ *be the gathering of CFSs and* $u = (u_1,u_2,...,u_m)^T$ *is the weight vector with* $u_j \in [0,1]$ *and* $\sum_{j=1}^{m} u_j = 1$. *Then* $CFFWG(d_1,d_2,...,d_m) = \bigotimes_{j=1}^{m} d_j^{u_j}$ *is said CFFWG operator.*

Theorem 4.13 *The collection of CFSs are* $a_j = \langle[\kappa^{-},\kappa^{+}],\kappa\rangle$ *and* $\lambda = (\lambda_1,\lambda_2,...,\lambda_n)^T$ *is the weight vector with* $\lambda_j \in [0,1]$ *and* $\sum_{j=1}^{n}\lambda_j = 1$. *Then it is said CFFWG operator and* $CFFWG(a_1,a_2,...,a_n) =$

$$\left\langle \left[\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}\left(\beta^{\kappa_j^{-}}-1\right)^{\lambda}}{(\beta-1)^{\lambda-1}}\right), \log_{\beta}\left(1+\frac{\prod_{j=1}^{n}\left(\beta^{\kappa_j^{+}}-1\right)^{\lambda}}{(\beta-1)^{\lambda-1}}\right) \right], 1-\log_{\beta}\left(1+\frac{\prod_{j=1}^{n}\left(\beta^{1-\kappa_j}-1\right)^{\lambda}}{(\beta-1)^{\lambda-1}}\right) \right\rangle.$$

Theorem 4.14 ***(Idempotency):****If* $\widetilde{ZV} = \langle[\kappa^-, \kappa^+], \kappa\rangle$ *for all* $L = 1, 2, 3, ..., m,$ *then* *CFFWG*$(\widetilde{ZV}, \widetilde{ZV}, \widetilde{ZV}, ..., \widetilde{ZV}) = \widetilde{ZV}$.

Theorem 4.15 ***(Commutativity):****If* $(d_1^{'}, d_2^{'}, ..., d_n^{'})$ *is any permutation of* $(d_1, d_2, ..., d_n)$, *then* *CFFWG*$(d_1^{'}, d_2^{'}, ..., d_n^{'})$ =*CFFWG*$(d_1, d_2, ..., d_n)$.

Theorem 4.16 ***(Boundedness):****If* $Y^- = min(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m), Y^+ = \max(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m)$, *then* $Y^- \leq$*CFFWG*$(\widetilde{ZV}_1, \widetilde{ZV}_2, ..., \widetilde{ZV}_m) \leq Y^+$.

4.5. CFFOWG operator

Definition 4.5 *Let* $c_j = \langle[\kappa^-, \kappa^+], \kappa\rangle$ *be the gathering of CFSs and the weight vector is* $g = (g_1, g_2, ..., g_m)^T$ *with* $g_j \in [0, 1]$ *and* $\sum_{j=1}^{m} g_j = 1$. *Then*

$$CFFOWG\,(c_1, c_2, ..., c_m) = \bigotimes_{j=1}^{m} c_j^{g_j}$$

is said CFFOWG operator.

Theorem 4.17 *Let* $a_j = \langle[\kappa^-, \kappa^+], \kappa\rangle$ *be the collection of CFSs and the weight vector is* $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ *with* $\lambda_j \in [0, 1]$ *and* $\sum_{j=1}^{n} \lambda_j = 1$. *Then it is said CFFOWG operator and* *CFFOWG*$(a_1, a_2, ..., a_n) =$

$$\left\langle \begin{bmatrix} \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{\kappa_j^-}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right), \\ \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{\kappa_j^+}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right) \end{bmatrix}, \; 1-\log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-\kappa_j}-1)^\lambda}{(\beta-1)^{\lambda-1}}\right) \right\rangle.$$

Theorem 4.18 ***(Idempotency):****If* $\widetilde{BU} = \langle[\kappa^-, \kappa^+], \kappa\rangle$ *for all* $L = 1, 2, 3, ..., m,$ *then* *CFFOWG*$(BU, BU, BU, ..., BU) = BU$.

Theorem 4.19 ***(Commutativity):****If* $(b_1^{'}, b_2^{'}, ..., b_n^{'})$ *is any permutation of* $(b_1, b_2, ..., b_n)$, *then* *CFFOWG*$(b_1^{'}, b_2^{'}, ..., b_n^{'})$ =*CFFOWG*$(b_1, b_2, ..., b_n)$

Theorem 4.20 ***(Boundedness):****If* $Y^- = min(f_1, f_2, ..., f_m)$, $Y^+ = \max(f_1, f_2, ..., f_m)$, *then* $Y^- \leq$*CFFOWG*$(f_1, f_2, ..., f_m) \leq Y^+$.

4.6. CFFHWG operator

Definition 4.6 *The gathering of CFSs are* $c_j = \langle [\kappa^-, \kappa^+], \kappa \rangle$ *and the weight vector is* $u = (u_1, u_2, ..., u_n)^T$ *with* $u_j \in [0, 1]$ *and* $\sum_{j=1}^{n} u_j = 1$*, the associated vector is* $u = (u_1, u_2, ..., u_n)^T$ *with* $u_j \in [0, 1]$ *and* $\sum_{j=1}^{n} u_j = 1$*. Then* $CFFHWG(c_1, c_2, ..., c_n) = \bigotimes_{j=1}^{m} c_j^{u_j}$ *is said CFFHWG operator.*

Theorem 4.21 *Let* $a_j = \langle [\kappa^-, \kappa^+], \kappa \rangle$ *be the collection of CFSs and the weight vector is* $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)^T$ *with* $\lambda_j \in [0, 1]$ *and* $\sum_{j=1}^{n} \lambda_j = 1$*. Then it is said CFFHWG operator and* $CFFHWG(a_1, a_2, ..., a_n) =$

$$\left\langle \begin{bmatrix} \log_\beta \left(1 + \frac{\prod_{j=1}^{n} (\beta^{\kappa_j^-} - 1)^{\lambda}}{(\beta - 1)^{\lambda - 1}} \right), \\ \log_\beta \left(1 + \frac{\prod_{j=1}^{n} (\beta^{\kappa_j^+} - 1)^{\lambda}}{(\beta - 1)^{\lambda - 1}} \right) \end{bmatrix}, \\ 1 - \log_\beta \left(1 + \frac{\prod_{j=1}^{n} (\beta^{1-\kappa_j} - 1)^{\lambda}}{(\beta - 1)^{\lambda - 1}} \right) \right\rangle.$$

Theorem 4.22 ***(Idempotency):****If* $\widetilde{c} = \langle [\kappa^-, \kappa^+], \kappa \rangle$ *for all* $L = 1, 2, 3, ..., m$*, then* $CFFHWG(\widetilde{c}, \widetilde{c}, \widetilde{c}, ..., \widetilde{c}) = \widetilde{c}$.

Theorem 4.23 ***(Commutativity)*** *:If* $(p_1^{'}, p_2^{'}, ..., p_n^{'})$ *is any permutation of* $(p_1, p_2, ..., p_n)$*, then* $CFFHWG(p_1^{'}, p_2^{'}, ..., p_n^{'}) = CFFHWG(p_1, p_2, ..., p_n)$

Theorem 4.24 ***(Boundedness):****If* $Y^- = min(f_1, f_2, ..., f_n)$*,* $Y^+ = \max(f_1, f_2, ..., f_n)$*, then* $Y^- \leq CFFHWG(f_1, f_2, ..., f_n) \leq Y^+$.

5. Proposed technique based on CFAA operator fuzzy C-mean clustering algorithm

Fuzzy clustering algorithms are used to group people according to shared characteristics or habits, a process known as user profiling. Users can have their membership degrees as a measure of how much they belong to each cluster assigned to them, thanks to fuzzy clustering. This strategy is especially helpful in situations where users may simultaneously display traits from several categories.

This is a simple overview of how fuzzy clustering for user profiling could be used.

Compile pertinent user information that can be utilized for profiling. The demographic data, browsing history, purchasing patterns, and interactions with a website or application are some examples of this data.

Choose the characteristics or features that will be utilized to the clustering process. These attributes ought to accurately reflect the traits of the users and have a bearing on the process of profiling.

For the given problem, choose a fuzzy clustering algorithm that is suitable Fuzzy C-Means.

Step 1:Describe the CF decision matrix

Step 2:Describe the CFFWA operator and $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$.

$$\mathrm{CFFWA}(d_1, d_2, ..., d_n) = \left\langle \left[1 - \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-\kappa_j^-}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right), 1 - \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-\kappa_j^+}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \right], \log_\beta\left(1+\frac{\prod_{j=1}^{n}(\beta^{1-\kappa_j}-1)^{\lambda_j}}{(\beta-1)^{\lambda_j-1}}\right) \right\rangle$$

Step 3:The FCM algorithms work on CFFWA operator $U = [U_{ic}]_{i=1...n}^{c=1...c}$

Step 4:Using k-means clusting value is defined as cluster's parameters interval-valued fuzzy set and fuzzy set

Step 5:Calculate the cluster center to find the centroid

$$C_j = \frac{\left\langle \left[\sum_{j=1}^{n} (\kappa_j^-)^\alpha \xi_j, \sum_{j=1}^{n} (\kappa_j^+)^\alpha \delta_j \right], \sum_{j=1}^{n} (\kappa_j^-)^\alpha \upsilon_j \right\rangle}{\left\langle \left[\sum_{j=1}^{n} (\kappa_j^-)^\alpha, \sum_{j=1}^{n} (\kappa_j^+)^\alpha \right], \sum_{j=1}^{n} (\kappa_j^-)^\alpha \right\rangle}$$

Step 6:Find out the distance of each point from the centroid

$D_j = \langle [\|r^- - C_j\|, \|r^+ - C_j\|], \|r - C_j\| \rangle$

Step 7:Calculate the score function $\frac{\langle [D_j^- - D_j^+] + D_j \rangle}{3}$

Step 8:Find the ranking.

6. Case history

Recognize the goal of optimizing energy usage in smart homes. Effective energy optimization requires a well-structured review process, based on data-driven insights, and focusing on energy consumption trends (not just assumptions or guesswork). This requires proper planning and implementation of strategies to optimize energy resources. Instead of focusing solely on what isn't working, engage in a constructive conversation about how to improve. Prioritize development, sustainability, and cost-saving over simply criticizing the system. Share findings and recommendations for enhancing energy usage with the residents and suggest next steps for more efficient consumption. If any of these issues seem familiar, it's because these symptoms often arise in every smart home system, leading to increased costs, waste, or missed energy efficiency goals.

While technology in smart homes can help reduce energy waste, many systems struggle to identify and solve energy inefficiencies. These small inefficiencies, if ignored, often escalate into larger, harder-to-manage issues.

For example, if energy consumption spikes unexpectedly in certain areas of the house, after conducting a thorough analysis, it may be due to inefficient appliances or incorrect settings. In your role as an energy manager or homeowner, you might suggest replacing old appliances or adjusting energy settings to reduce consumption.

However, do you act on these issues early, or, like many, do you allow minor inefficiencies to snowball into bigger problems that could be harder to address later?

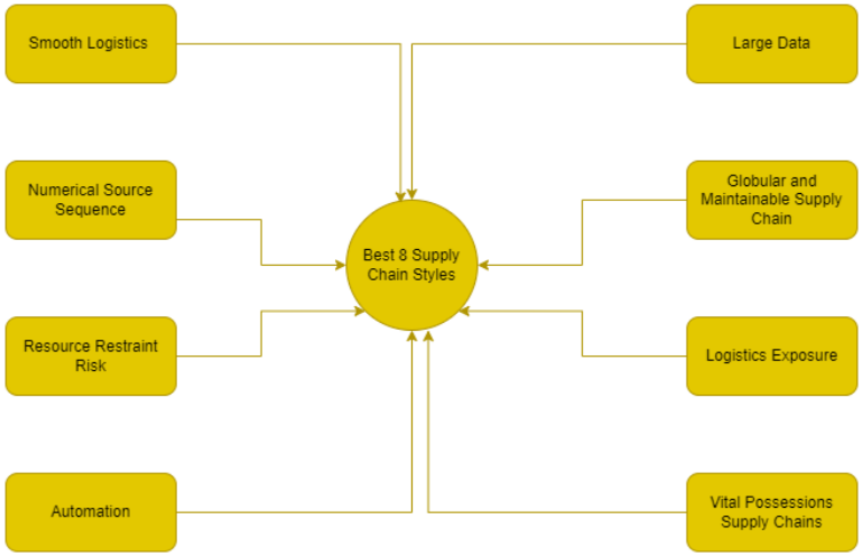

Figure 4 supply chain.

Just like medical symptoms serve as early warnings, energy inefficiencies in smart homes act as warning signs that need immediate attention. If not treated early, these inefficiencies can lead to higher energy costs, system overload, or even failures. Addressing issues early enables quick fixes and prevents significant consequences.

$PIPS_1$:Smart Home Energy Systems use advanced technologies like AI and machine learning to optimize energy usage by analyzing real-time data and predicting future energy consumption. These systems improve energy efficiency by monitoring energy-hungry devices and optimizing settings based on usage patterns.

$PIPS_2$: Integration of solar power, smart meters, energy storage, and electric vehicles within the smart home system introduces complex dynamics. By managing energy from these diverse sources, smart homes can balance consumption and enhance sustainability. Efficiently integrating these systems can reduce energy waste and increase self-sufficiency.

$PIPS_3$:Energy Monitoring and Analytics are key to identifying inefficiencies in smart home systems. With data-driven insights, homeowners can optimize energy use, pinpoint areas of waste, and make informed decisions about how to save energy. These insights empower users to adjust energy consumption and minimize waste.

$PIPS_4$:Predictive models and automation can further optimize energy consumption. Smart home systems can use AI to predict when energy demand will peak and adjust usage accordingly. Automated adjustments in heating, cooling, lighting, and appliance use can maximize energy savings and minimize waste.

Figure 4 is given as

Step 1:Explain the table 2 and 3 in CF decision matrix.

Table 2 of the CF decision matrix.

	Sur	Rad	Chem	Clin
PS_1	$\left\langle [0.1, 0.3], 0.2 \right\rangle$	$\left\langle [0.02, 0.05], 0.04 \right\rangle$	$\left\langle [0.11, 0.13], 0.1 \right\rangle$	$\left\langle [0.01, 0.14], 0.2 \right\rangle$
PS_2	$\left\langle [0.02, 0.05], 0.04 \right\rangle$	$\left\langle [0.1, 0.3], 0.2 \right\rangle$	$\left\langle [0.31, 0.37], 0.6 \right\rangle$	$\left\langle [0.21, 0.34], 0.4 \right\rangle$
PS_3	$\left\langle [0.01, 0.14], 0.2 \right\rangle$	$\left\langle [0.02, 0.05], 0.04 \right\rangle$	$\left\langle [0.1, 0.3], 0.2 \right\rangle$	$\left\langle [0.11, 0.13], 0.1 \right\rangle$
PS_4	$\left\langle [0.01, 0.14], 0.2 \right\rangle$	$\left\langle [0.21, 0.34], 0.4 \right\rangle$	$\left\langle [0.02, 0.05], 0.04 \right\rangle$	$\left\langle [0.2, 0.4], 0.3 \right\rangle$

CF decision matrix table 3.

	Sur	Rad	Chem	Clin
$PIPS_1$	$\left\langle [0.11, 0.13], 0.12 \right\rangle$	$\left\langle [0.1, 0.3], 0.1 \right\rangle$	$\left\langle [0.21, 0.23], 0.22 \right\rangle$	$\left\langle [0.31, 0.34], 0.22 \right\rangle$
$PIPS_2$	$\left\langle [0.31, 0.34], 0.22 \right\rangle$	$\left\langle [0.2, 0.4], 0.3 \right\rangle$	$\left\langle [0.31, 0.37], 0.6 \right\rangle$	$\left\langle [0.21, 0.34], 0.4 \right\rangle$
$PIPS_3$	$\left\langle [0.31, 0.37], 0.6 \right\rangle$	$\left\langle [0.1, 0.3], 0.1 \right\rangle$	$\left\langle [0.31, 0.37], 0.6 \right\rangle$	$\left\langle [0.21, 0.34], 0.4 \right\rangle$
$PIPS_4$	$\left\langle [0.3, 0.5], 0.2 \right\rangle$	$\left\langle [0.11, 0.14], 0.13 \right\rangle$	$\left\langle [0.37, 0.39], 0.29 \right\rangle$	$\left\langle [0.11, 0.22], 0.13 \right\rangle$

Step 2:Describe the CFFWA operator and $\xi = (0.26, 0.21, 0.25, 0.28)$.
CFFWA operator is in table 4.
CFFWA operator table 4.

	Sur	Rad	Chem	Clin
$PIPS_1$	$\left\langle [0.1001, 0.1983], 0.1092 \right\rangle$	$\left\langle [0.1452, 0.1675], 0.1124 \right\rangle$	$\left\langle [0.2561, 0.2343], 0.2032 \right\rangle$	$\left\langle [0.3091, 0.3344], 0.2092 \right\rangle$
$PIPS_2$	$\left\langle [0.3031, 0.3014], 0.2052 \right\rangle$	$\left\langle [0.1231, 0.3453], 0.9872 \right\rangle$	$\left\langle [0.0001, 0.1004], 0.2123 \right\rangle$	$\left\langle [0.1091, 0.1023], 0.1347 \right\rangle$
$PIPS_3$	$\left\langle [0.0131, 0.1134], 0.2359 \right\rangle$	$\left\langle [0.0212, 0.0556], 0.0411 \right\rangle$	$\left\langle [0.3051, 0.3084], 0.2082 \right\rangle$	$\left\langle [0.1091, 0.1973], 0.1123 \right\rangle$
$PIPS_4$	$\left\langle [0.3061, 0.3074], 0.2122 \right\rangle$	$\left\langle [0.1011, 0.1033], 0.117 \right\rangle$	$\left\langle [0.0122, 0.0045], 0.0054 \right\rangle$	$\left\langle [0.3001, 0.3944], 0.2342 \right\rangle$

Step 3:The FCM algorithms work on CFFWA operator $U = [U_{ic}]_{i=1...n}^{c=1...c}$.
FCM algorithms work on CFFWA operator in table 5.
FCM algorithms table 5.

	Sur	Rad	Chem	Clin
$PIPS_1$	0.4	0.2	0.1	0.3
$PIPS_2$	0.02	0.12	0.03	0.12
$PIPS_3$	0.01	0.23	0.01	0.13
$PIPS_4$	0.22	0.34	0.11	0.14

Step 4:Using k-means clusting value is defined as cluster's parameters interval-valued fuzzy set and fuzzy set in table 6.

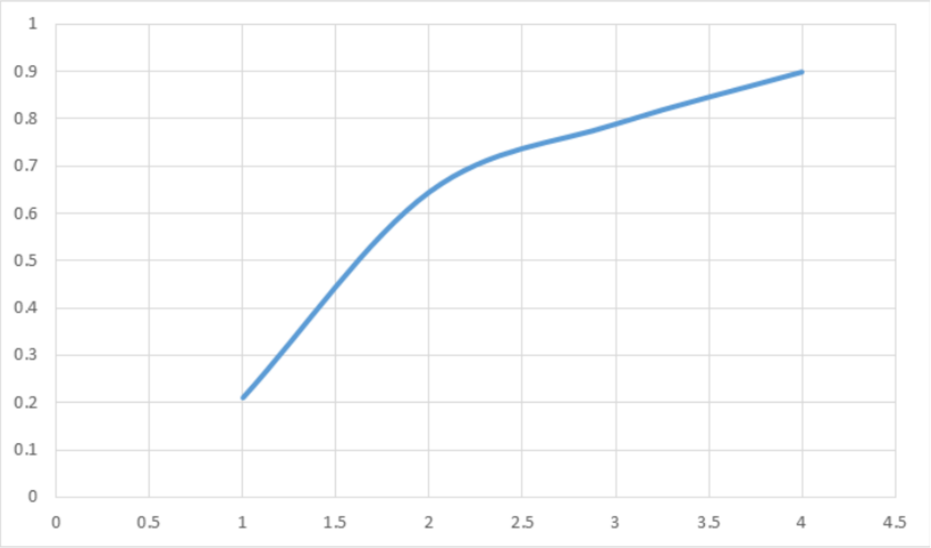

Figure 5, score function of different ranking.

k-means clusting table 6

	Sur	Rad	Chem	Clin
$PIPS_1$	$\langle [0.1232, 0.2345], 0.0987 \rangle$	$\langle [0.3087, 0.3987], 0.1245 \rangle$	$\langle [0.0342, 0.3054], 0.0234 \rangle$	$\langle [0.1209, 0.3567], 0.2098 \rangle$
$PIPS_2$	$\langle [0.1012, 0.4585], 0.6532 \rangle$	$\langle [0.4563, 0.9635], 0.2745 \rangle$	$\langle [0.1036, 0.6987], 0.7896 \rangle$	$\langle [0.2589, 0.3785], 0.7412 \rangle$
$PIPS_3$	$\langle [0.4585, 0.8965], 0.7025 \rangle$	$\langle [0.1212, 0.3635], 0.4563 \rangle$	$\langle [0.1298, 0.7852], 0.3405 \rangle$	$\langle [0.1231, 0.4569], 0.4669 \rangle$
$PIPS_4$	$\langle [0.3423, 0.3986], 0.0563 \rangle$	$\langle [0.1123, 0.3369], 0.4563 \rangle$	$\langle [0.1698, 0.3874], 0.4785 \rangle$	$\langle [0.0147, 0.0459], 0.0147 \rangle$

Step 5:Calculate the cluster center to find the centroid
$C_1 = 0.2345, C_2 = 0.1034, C_3 = 0.3456, C_{41} = 0.5645$.
Step 6:Find out the distance of each point from the centroid in table 7.

Centroid table 7	
$PIPS_1$	$\langle [0.1256, 0.5678], 0.6789 \rangle$
$PIPS_2$	$\langle [0.1567, 0.4987], 0.8909 \rangle$
$PIPS_3$	$\langle [0.1432, 0.6756], 0.6087 \rangle$
$PIPS_4$	$\langle [0.1876, 0.7876], 0.7865 \rangle$

Step 7:Calculate the score function $DH_1 = 0.2098, DH_2 = 0.6456, DH_3 = 0.7896, DH_4 = 0.8998$.
Step 8:Find the ranking.
$DH_4 > DH_3 > DH_2 > DH_1$ and DH_4 is the best.
Figure 5 is given below

6.1. Comparison technique with existing way

Different existing ways are written below Table 8.
Table 8 existing techniques

Methods	Average operator	Geometric operator
hamacher [6]	$\left\{ \begin{matrix} DH_4 > \\ DH_3 > \\ DH_2 > \\ DH_1 \end{matrix} \right\}$	$\left\{ \begin{matrix} DH_4 > \\ DH_3 > \\ DH_2 > \\ DH_1 \end{matrix} \right\}$
MCDM [7]	$\left\{ \begin{matrix} DH_4 > \\ DH_3 > \\ DH_2 > \\ DH_1 \end{matrix} \right\}$	$\left\{ \begin{matrix} DH_4 > \\ DH_3 > \\ DH_2 > \\ DH_1 \end{matrix} \right\}$
MCDM [8]	$\left\{ \begin{matrix} DH_4 > \\ DH_3 > \\ DH_2 > \\ DH_1 \end{matrix} \right\}$	$\left\{ \begin{matrix} DH_4 > \\ DH_3 > \\ DH_2 > \\ DH_1 \end{matrix} \right\}$

6.2. Validity way

Table 9 is provided below, and in this subsection, we present the proposed way in validity way. Validity of the aggregating operator in table 9.

Ways	Average operator	Geometric operator	hybrid
Proposed technique	✓	✓	✓
MCDM [37]	✓	✓	✓
MCDM [36]	✓	✓	✓
PDF [16]	✓	✓	✓
FH operators [4]	✓	✓	✓

6.3. Results and discussion

There are several reasons why the proposed method is superior to existing energy optimization approaches. Traditional models are often based on static data and fail to adapt to real-time environmental and behavioral changes.

To enhance the capabilities of current energy management systems and smart home technology, integrate CF aggregation operators to process complex and uncertain data, such as fluctuating occupancy, varying weather conditions, and appliance usage patterns.

Provide clear guidelines for energy management professionals on how to use systems based on CF aggregation operators. Highlight the importance of interpreting data accurately and emphasize the benefits of cubic fuzzy methods in optimizing energy use.

To ensure the reliability and effectiveness of CF aggregation operators in practical settings, conduct pilot tests in a variety of smart homes. Gather feedback and data during these tests to make necessary improvements before widespread implementation.

Collaborate with regulatory bodies to ensure that the use of CF aggregation operators in smart home energy management adheres to all relevant standards and regulations.

6.4. Advantages

There are numerous important benefits to using Cubic Fuzzy Frank Aggregation Operators in decision-making for intelligent energy optimization in smart homes.

• An instruction to demonstrate MCDM evidence in a non-verbal manner using the CF method. From then on, the CF method proved essential for clarifying uncertain and incomplete energy management results.

• CF aggregation operators successfully integrate imprecise and heterogeneous data, increasing the accuracy of energy consumption forecasts. This results in more precise energy usage predictions, particularly in complicated cases with varying occupancy and environmental conditions.

• CFFAO enables consistent decision-making among different energy optimization scenarios by combining energy consumption criteria in a more advanced way. As a result, energy efficiency reliability is increased, and variability in energy usage is minimized.

• MCDM can more easily and transparently comprehend the decision-making process due to CFFO's structured approach to data aggregation. This promotes more confident energy optimization decisions and makes the reasoning behind energy consumption easier to understand.

• Due to their proficiency in the energy optimization domain, their ability to make practical judgments, and the effectiveness of the evaluation processes, energy management specialists are frequently required for optimizing smart home systems. All of the methods currently in use rely on the expertise of professionals to interpret and optimize energy consumption.

• Optional approach: When used in typical MCDM scenarios, CF proved to be an accurate, collaborative system that overcomes numerous uncertainties in energy optimization. Its effectiveness in handling ambiguous data and achieving reliable outcomes is well-defined.

6.5. Sensitive study

We define the sensitive study in this subsection, which is represented in table 10 below.

Sensitive study in table 10

Technique	Score way	Order of preference
Interaction [12]	$\left\{ \begin{array}{c} DH_1 = 0.0107, \\ DH_2 = 0.8955, \\ DH_3 = 0.2345, \\ DH_4 = 0.4987 \end{array} \right\}$	$\left\{ \begin{array}{c} DH_2 > \\ DH_4 > \\ DH_3 > \\ DH_1 \end{array} \right\}$
MCDM [16]	$\left\{ \begin{array}{c} DH_1 = 0.0106, \\ DH_2 = 0.6565, \\ DH_3 = 0.4002, \\ DH_4 = 0.5678 \end{array} \right\}$	$\left\{ \begin{array}{c} DH_2 > \\ DH_4 > \\ DH_3 > \\ DH_1 \end{array} \right\}$

7. Conclusion

In this section, we introduce cubic fuzzy set-based Frank Aggregation Operators applied to intelligent energy optimization in smart homes. The operational laws are defined, along with the corresponding score and accuracy functions. This study uses the CFFWA, CFFOWA, CFFWHA, CFFWG, CFFOWG and CFFHG operators within the Multi-Criteria Decision-Making (MCDM) approach to tackle energy optimization issues in smart homes. It is demonstrated that these operators enable the MCDM technique to effectively differentiate between various energy management alternatives, offering flexibility in making decisions related to energy consumption and efficiency. These operators exhibit essential properties such as commutativity, idempotency, boundedness, associativity, and monotonicity. When combined with clustering techniques like the Fuzzy C-means algorithm, these aggregation operators enhance the accuracy and computational efficiency of the energy optimization process, making it possible to cluster energy consumption data effectively for more reliable decision-making.

In the future, we plan to incorporate artificial intelligence into smart home energy management. This includes utilizing neural networks, automation, data analysis, and virtual assistants to enhance decision-making processes related to energy consumption. Additionally, we aim to expand the current approach by incorporating Generalized Cubic Fuzzy Frank Aggregation Operators and Cubic Fuzzy Frank Geometric Aggregation Operators, allowing for more advanced, adaptive, and efficient energy management systems for smart homes.

Compliance with Ethical Standards

The authors declare that there is no conflict of interests regarding the publication of this paper. Compliance with Ethical Standards: This study is not supported by any source or any organizations.

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

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