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Modelling drug resistance and insecticide effects in infectious disease transmission with saturated incidence for control interventions

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ABSTRACT: The burden of vector-borne infections is a complex interplay of biological, environmental, and social factors. Dengue infection constitutes a substantial and multifaceted threat to both human health and the socio-economic aspects of impoverished regions. In order to address these challenges, a thorough comprehension of the complex dengue dynamics is necessary. In this study, we construct an epidemic model for dengue with saturated incidence in the framework of Caputo-Fabrizio derivative with drug resistance. Boundedness and positivity of the solution of the suggested model are examined. The endemic indicator, denoted by $\mathcal{R}_0$, is computed using the next-generation matrix technique. It is demonstrated that for $\mathcal{R}_0 < 1$, the system's infection-free steady-state is locally asymptotically stable. The fixed-point theorem is then used to examine the existence and uniqueness of the proposed system's solution. The time series analysis of the model has been presented to illustrate the influence of several parameters on the dengue infection system. The role of memory index has been conceptualized through numerical findings. Our findings anticipate the pivotal scenario within the system pertinent to the control and prevention of dengue.

Key Words: Vector-borne infection, fractional-calculus, dengue dynamics, fixed-point theory, numerical method, time series analysis.

Contents

1. Introduction

Vector-borne infections are diseases transmitted to animals and humans through vectors including sandflies, fleas, ticks and mosquitoes. These infections are significant global health issue in different regions of the globe, and can lead to widespread morbidity and mortality [1]. Prominent vector-borne diseases include Zika virus, dengue, malaria, Lyme disease, and leishmaniasis. The burden of vector-borne infections extends beyond health impacts, imposing economic costs on healthcare systems and affected communities [2]. Efforts to control these diseases include vector control measures, such as insecticide-treated nets, environmental management, and the development of vaccines. Dengue, a vector-borne infection transmitted primarily by Aedes mosquitoes, poses a significant global health threat [3]. It is worth noting that there are rare instances of vertical transmission of the virus occurring in both human hosts and mosquitoes [4]. There are numerous control strategies for the elimination of this vector borne infection. The goal of ongoing research is to combat dengue infection by developing vaccinations and antiviral drugs. Public health efforts to control dengue focus on vector control, community education,

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and surveillance to reduce the spread of the virus. It is well-known that there is currently no fully effective medication for treatment of this vector-borne infection. Moreover, various vaccines for dengue infection have been developed and are undergoing clinical testing, as reported in references [5,6]. Thus, additional preventative measures are crucial to manage and lower the incidence of dengue.

Mathematical models are useful tools for investigating infectious diseases and establishing efficient infection control plans [7,8]. These models identify and emphasize the most critical and influential factors contributing to the spread and management of the disease [9,10]. Several researchers have examined the dynamics of dengue using varying assumptions in the literature [11,12,13]. Lourdes Esteva developed an epidemic model for dengue fever, exploring its behavior within a variable human population [14,15]. In another study, a model with two-strain of dengue has been analyzed by the researchers [16]. Furthermore, some researchers constructed the dynamics of dengue and examined the stability of their models [17,18]. In [19], the authors proposed a model for dengue infection and offered quantitative and qualitative analysis of the model to highlight the key factors of dengue. In [20], the research uses a two-patch model of dengue transmission, incorporating human mobility between patches, to simulate the virus's spread. Recently, a novel model is introduced to examine the dynamics of dengue, with a particular emphasis on the disease's 2023 breakout [21]. The authors in this work used machine learning approach to investigate the behaviours and patterns of dengue in Bangladesh. In this work, we present the dynamics of dengue along with some control measures and the index of memory. Our main goal is to identify the most effective control strategy and to determine whether the index of memory can be used as a control parameter for this vector-borne infection.

Fractional calculus provides a sophisticated and nuanced framework to understanding and controlling infectious diseases [22,23]. By incorporating memory effects and anomalous diffusion, these models provide a more accurate and flexible framework for studying the complex nature of infections [24]. These models are particularly useful in capturing the intricate phenomena of diseases. Fractional models are more flexible and provide more accurate results for real data of the infection [25,26]. This flexibility makes them powerful tools for modeling complex biological systems. These models can evaluate the effectiveness of control measures like vaccination, quarantine, and treatment by incorporating the fractional order to simulate different intervention scenarios more realistically. Therefore, we propose modeling the dynamics of dengue within a fractional framework with saturated incidence to obtain more accurate results and provide effective control for the infection.

The layout of this work is as follows: Section 2 presents the fundamental results and definitions of fractional theory. In Section 3, we develop a mathematical model for dengue using fractional order derivatives. The basic reproduction number is calculated, and the equilibria are analyzed in Section 4. Section 5 explores the solution's existence and uniqueness with the help of fixed-point theory. In Section 6, we focus on the time series analysis of the proposed fractional system to numerically assess the system. Finally, Section 7 provides concluding remarks and summarizes the overall analysis.

2. Fundamental theory

Here, we will introduce the basic results of the recently formulated Caputo-Fabrizio (CF) operator to analyze the system. The outcomes of the CF derivative are outlined as follows:

Definition 2.1 The CF operator [29] for a function $g \in G^1(r_1, r_2)$ is defined as

$$D_t^{\xi}(g(t)) = \frac{\mathcal{M}(\xi)}{1-\xi}\int_a^t g'(y)\exp\Big[-\xi\frac{t-y}{1-\xi}\Big]dy, \tag{2.1}$$

in which the normality is indicated by $\mathcal{M}(\tau)$ and fractional order $\xi \in [0,1]$. Moreover, we have r_1 smaller than r_2. In the case, if $g \notin G^1(r_1, r_2)$, then we get

$$D_t^{\xi}(g(t)) = \frac{\xi U(\xi)}{1-\xi}\int_{r_1}^t (g(t)-g(y))\exp\Big[-\xi\frac{t-y}{1-\xi}\Big]dy. \tag{2.2}$$

Remark 2.1 In the case if $\beta = \frac{1-\xi}{\xi} \in [0,\infty)$ and $\xi = \frac{1}{1+\beta} \in [0,1]$, then the above (2.2) becomes as

$$D_t^{\xi}(g(t)) = \frac{N(\beta)}{\beta}\int_{r_1}^t g'(y)e^{[-\frac{t-y}{\beta}]}dy, \;\; N(0) = N(\infty) = 1. \tag{2.3}$$

Furthermore, there is

$$\lim_{\beta \longrightarrow 0} \frac{1}{\beta} \exp\Big[-\frac{t-y}{\beta}\Big] = \delta(y-t). \tag{2.4}$$

Definition 2.2 [30], Let us take g, then the integral of CF operator for g is as follows

$$I_t^{\xi}(g(t)) = \frac{2(1-\xi)}{(2-\xi)U(\xi)} g(t) + \frac{2\xi}{(2-\xi)U(\xi)} \int_0^t g(v)dv, \quad t \geq 0. \tag{2.5}$$

Remark 2.2 Here, the upper mentioned 2.2 leads to the below

$$\frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)} + \frac{2\xi}{(2-\xi)\mathcal{M}(\xi)} = 1. \tag{2.6}$$

The above give us $\mathcal{M}(\xi) = \frac{2}{2-\xi}, \ 0 < \xi < 1$. In [30], the below definition is given

$$D_t^{\xi}(g(t)) = \frac{1}{1-\xi} \int_0^t g'(y) \exp\Big[\xi \frac{t-y}{1-\xi}\Big] dy. \tag{2.7}$$

3. Formulation of the model

Let the entire population of vectors and hosts be denoted by $\mathscr{N}_v$ and $\mathscr{N}_h$, respectively. The population of host is classified into $(\mathscr{S}_h)$ susceptible, $(\mathscr{V}_h)$ vaccinated, $(\mathscr{I}_h)$ infected , and $(\mathscr{R}_h)$ recovered classes while the overall population of female mosquitoes $\mathscr{N}_v$ is grouped into $(\mathscr{S}_v)$ susceptible and $(\mathscr{I}_v)$ infected classes. Here, μ_v and μ_h are considered to be the natural birth and death rates for hosts and vectors, respectively. Furthermore, rare cases of dengue death has been noticed so we did not assume it here. We assumed nonlinear forces of infection represented by $\frac{b\beta_1}{1+\alpha_h \mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v$, $\frac{b\beta_2}{1+\alpha_h \mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v$, and $\frac{\beta_3 b}{1+\alpha_v \mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h$. Also, a proportion p of the $\mathscr{S}_h$ moves to $\mathscr{V}_h$ after vaccination and a fraction ν of the recovered class $(\mathscr{R}_h)$ loses immunity and becomes susceptible again. The dynamics of dengue infection are consequently described as

$$\begin{cases} \frac{d\mathscr{S}_h}{dt} &= \Lambda_h - \frac{b\beta_1}{1+\alpha_h \mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v - \mu_h\mathscr{S}_h - p\mathscr{S}_h + \upsilon\mathscr{R}_h, \\ \frac{d\mathscr{V}_h}{dt} &= p\mathscr{S}_h - \frac{b\beta_2}{1+\alpha_h \mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v - \mu_h\mathscr{V}_h, \\ \frac{d\mathscr{I}_h}{dt} &= \frac{b\beta_1}{1+\alpha_h \mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v + \frac{b\beta_2}{1+\alpha_h \mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v - (\mu_h+\gamma_h)\mathscr{I}_h - \xi_1\theta_1(1-q_1)\mathscr{I}_h, \\ \frac{d\mathscr{R}_h}{dt} &= \gamma_h\mathscr{I}_h + \xi_1\theta_1(1-q_1)\mathscr{I}_h - \upsilon\mathscr{R}_h - \mu_h\mathscr{R}_h, \\ \frac{d\mathscr{S}_v}{dt} &= \Lambda_v - \frac{\beta_3 b}{1+\alpha_v \mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h - \mu_v\mathscr{S}_v - \xi_2\theta_2(1-q_2)\mathscr{S}_v, \\ \frac{d\mathscr{I}_v}{dt} &= \frac{b\beta_3}{1+\alpha_v \mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h - \mu_v\mathscr{I}_v - \xi_2\theta_2(1-q_2)\mathscr{I}_v, \end{cases} \tag{3.1}$$

with the following

$$0 \leq \mathscr{S}_v(0), 0 \leq \mathscr{I}_v(0), 0 \leq \mathscr{S}_h(0), 0 \leq \mathscr{V}_h(0), 0 \leq \mathscr{I}_h(0), 0 \leq \mathscr{R}_h(0),$$

where, $\alpha_h \in [0,1]$ denotes the antibody response rate when subjected to antigens generated by vectors, while $\alpha_v \in [0,1]$ indicates the antibody production rate against antigens encountered from infectious hosts. The drug was administered to the infected patients at $\xi_1\theta_1$ in which θ_1 reflects the drug's contribution to recovery rate and ξ_1 indicates the efficacy of drugs. In addition, $\xi_1\theta_1 q_1\mathscr{I}_h$ in which $q_1 \in [0,1]$ represents the medication's resistance acquisition ratio, provides the number of infected people resistant to the drug. As a result, the word $\xi_1\theta_1(1-q_1)\mathscr{I}_h$ denotes the percentage of people that are drug-sensitive. The effect of insecticides reduces the vector population at a rate of $\xi_2\theta_2$, where θ_2 is the mosquito population death caused by pesticides and ξ_2 is the insecticide efficacy. The resistance acquisition ratio to the insecticides is represented by $q_2 \in [0,1]$, and the mosquitoes resistant to the pesticides is $\xi_2\theta_2 q_2$. Because of this, the formula $\xi_2\theta_2(1-q_2)$ indicates the percentage of mosquitoes that are insecticide-sensitive.

Fractional calculus enhances the capability of epidemic models by incorporating memory effects, non-local interactions, and better fitting to empirical data. It provides a more flexible and accurate framework for visualization and predicting the spread of infectious diseases. As the understanding and computational tools for fractional calculus continue to develop, its application in epidemiology is likely to grow, offering deeper insights and more effective strategies for disease control and prevention. Therefore, we employ the CF-derivative to represent the dynamics of dengue fever, aiming for a more precise understanding of the transmission phenomena as follows

$$\begin{cases} {}_0^{CF}D_t^{\xi}\mathscr{S}_h = \Lambda_h - \frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v - \mu_h\mathscr{S}_h - p\mathscr{S}_h + \upsilon\mathscr{R}_h, \\ {}_0^{CF}D_t^{\xi}\mathscr{V}_h = p\mathscr{S}_h - \frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v - \mu_h\mathscr{V}_h, \\ {}_0^{CF}D_t^{\xi}\mathscr{I}_h = \frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v + \frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v - (\mu_h+\gamma_h)\mathscr{I}_h - \xi_1\theta_1(1-q_1)\mathscr{I}_h, \\ {}_0^{CF}D_t^{\xi}\mathscr{R}_h = \gamma_h\mathscr{I}_h + \xi_1\theta_1(1-q_1)\mathscr{I}_h - \upsilon\mathscr{R}_h - \mu_h\mathscr{R}_h, \\ {}_0^{CF}D_t^{\xi}\mathscr{S}_v = \Lambda_v - \frac{\beta_3 b}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h - \mu_v\mathscr{S}_v - \xi_2\theta_2(1-q_2)\mathscr{S}_v, \\ {}_0^{CF}D_t^{\xi}I_v = \frac{b\beta_3}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h - \mu_v\mathscr{I}_v - \xi_2\theta_2(1-q_2)\mathscr{I}_v, \end{cases} \tag{3.2}$$

in which ${}_0^{CF}D_t^{\xi}$ is the Caputo-Fabrizio derivative and ξ is the order of CF derivative with the considtion $0 \le \xi \le 1$.

Theorem 3.1 *The suggested model (3.2) of the vector-borne infection has positive and bounded solutions $(\mathscr{S}_h, \mathscr{V}_h, \mathscr{I}_h, \mathscr{R}_h, \mathscr{S}_v, \mathscr{I}_v)$ for suitable initial conditions.*

4. Threshold parameter

Equilibrium points in both classical and fractional epidemic models are determined by setting the derivatives to zero. Identifying these points, namely the disease-free equilibrium (DFE) and the endemic equilibrium, provides crucial insight into the potential long-term behavior of an epidemic. We take ${}_0^{CF}D_t^{\xi}\mathscr{S}_h, {}_0^{CF}D_t^{\xi}\mathscr{V}_h, {}_0^{CF}D_t^{\xi}\mathscr{I}_h, {}_0^{CF}D_t^{\xi}\mathscr{R}_h, {}_0^{CF}D_t^{\xi}\mathscr{S}_v$, and ${}_0^{CF}D_t^{\xi}\mathscr{I}_v$ equal to zero for steady-states. For DFE, we put the infected classes equal to zero and get the following

$$\mathcal{E}_0(\mathscr{S}_h^0, \mathscr{V}_h^0, \mathscr{I}_h^0, \mathscr{R}_h^0, \mathscr{S}_v^0, \mathscr{I}_v^0) = \left(\frac{\Lambda_h}{p+\mu_h}, \frac{\Lambda_h p}{(\mu_h+p)\mu_h}, 0, 0, \frac{\Lambda_v}{\mu_v + \xi_2\theta_2(1-q_2)}, 0\right).$$

The details of the basic reproduction number is provided in [27,28], typically represented by $\mathcal{R}_0$, and its calculated as

$$\mathcal{F} = \begin{bmatrix} \frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v + \frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v \\ \frac{b\beta_3}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h \end{bmatrix} \mathit{and} \ \ \mathcal{V} = \begin{bmatrix} (\mu_h+\gamma_h)\mathscr{I}_h + \xi_1\theta_1(1-q_1)\mathscr{I}_h \\ \mu_v\mathscr{I}_v + \xi_2\theta_2(1-q_2)\mathscr{I}_v \end{bmatrix},$$

evaluating the jacobian of the above matrices at $\mathcal{E}_0$, yields the following

$$F = \begin{bmatrix} 0 & \frac{\mu_h b\beta_1\Lambda_h + b\beta_2\Lambda_h p}{(\mu_h+p)\mu_h} \\ \frac{b\beta_3\Lambda_v}{\xi_2\theta_2(1-q_2)+\mu_v} & 0 \end{bmatrix} \mathit{and} \ \ V = \begin{bmatrix} \xi_1\theta_1(1-q_1)+\mu_h+\gamma_h & 0 \\ 0 & \mu_v + \xi_2\theta_2(1-q_2) \end{bmatrix},$$

which gives

$$FV^{-1} = \begin{bmatrix} 0 & \frac{\mu_h b\beta_1\Lambda_h + b\beta_2\Lambda_h p}{(\xi_2\theta_2(1-q_2)+\mu_v)(\mu_h+p)\mu_h} \\ \frac{b\beta_3\Lambda_v}{(\xi_1\theta_1(1-q_1)+\mu_h+\gamma_h)(\xi_2\theta_2(1-q_2)+\mu_v)} & 0 \end{bmatrix}.$$

Through next generation matrix we get the $\mathcal{R}_0$ of the fractional order dengue model (3.2) as

$$\rho(FV^{-1}) = \frac{b}{(\xi_2\theta_2(1-q_2)+\mu_v)}\sqrt{\frac{\beta_3\Lambda_h\Lambda_v(p\beta_2+\beta_1\mu_h)}{\mu_h(p+\mu_h)(\xi_2\theta_2(1-q_2)+\mu_v)(\xi_1\theta_1(1-q_1)+\mu_h+\gamma_h)}},$$

$$\Rightarrow \mathcal{R}_0 = \frac{b}{(\xi_2\theta_2(1-q_2)+\mu_v)}\sqrt{\frac{\beta_3\Lambda_h\Lambda_v(p\beta_2+\beta_1\mu_h)}{\mu_h(p+\mu_h)(\xi_2\theta_2(1-q_2)+\mu_v)(\xi_1\theta_1(1-q_1)+\mu_h+\gamma_h)}}.$$

Theorem 4.1 *If $\mathcal{R}_0 < 1$, then $\mathcal{E}_0$ of the suggested model (3.2) of vector-borne infection is locally asymptomatically stable and unstable in other cases.*

Proof: For the proof of this theorem, we will take the Jacobian of system (3.2) at $\mathcal{E}_0$ as

$$\mathcal{J} = \begin{bmatrix} -(\mu_h+p) & 0 & 0 & \upsilon & 0 & \frac{-b\beta_1\Lambda_h}{\mu_h+p} \\ p & -\mu_h & 0 & 0 & 0 & \frac{-b\beta_2\Lambda_h p}{\mu_h(\mu_h+p)} \\ 0 & 0 & -\mathcal{A} & 0 & 0 & \frac{b\beta_1\Lambda_h\mu_h+b\beta_2\Lambda_h p}{\mu_h(\mu_h+p)} \\ 0 & 0 & \mathcal{C} & -(\upsilon+\mu_h) & 0 & 0 \\ 0 & 0 & -\frac{b\beta_3\Lambda_v}{\mathcal{D}} & 0 & -\mathcal{D} & 0 \\ 0 & 0 & \frac{b\beta_3\Lambda_v}{\mathcal{D}} & 0 & 0 & -\mathcal{B} \end{bmatrix}, \tag{4.1}$$

where $\mathcal{A} = (\xi_1\theta_1(1-q_1)+\mu_h+\gamma_h)$, $\mathcal{B} = (\xi_2\theta_2(1-q_2)+\mu_v)$, $\mathcal{C} = (\xi_1\theta_1(1-q_1)+\gamma_h)$ and $\mathcal{D} = (\xi_2\theta_2(1-q_2)+\mu_v)$. Here, the first eigenvalue is $\lambda_1 = -\mu_h$ with the following Jacobian

$$\mathcal{J}_1 = \begin{bmatrix} -(\mu_h+p) & 0 & \upsilon & 0 & \frac{-b\beta_1\Lambda_h}{\mu_h+p} \\ 0 & -\mathcal{A} & 0 & 0 & \frac{b\beta_1\Lambda_h\mu_h+b\beta_2\Lambda_h p}{\mu_h(\mu_h+p)} \\ 0 & \mathcal{C} & -(\upsilon+\mu_h) & 0 & 0 \\ 0 & -\frac{b\beta_3\Lambda_v}{\mathcal{D}} & 0 & -\mathcal{D} & 0 \\ 0 & \frac{b\beta_3\Lambda_v}{\mathcal{D}} & 0 & 0 & -\mathcal{B} \end{bmatrix}, \tag{4.2}$$

the second eigenvalue of the system is $\lambda_2 = -(\mu_h+p)$ with the following Jacobian

$$\mathcal{J}_2 = \begin{bmatrix} -\mathcal{A} & 0 & 0 & \frac{b\beta_1\Lambda_h\mu_h+b\beta_2\Lambda_h p}{\mu_h(\mu_h+p)} \\ \mathcal{C} & -(\upsilon+\mu_h) & 0 & 0 \\ -\frac{b\beta_3\Lambda_v}{\mathcal{D}} & 0 & -\mathcal{D} & 0 \\ \frac{b\beta_3\Lambda_v}{\mathcal{D}} & 0 & 0 & -\mathcal{B} \end{bmatrix}, \tag{4.3}$$

the third eigenvalue of the system is $\lambda_3 = -(\upsilon+\mu_h)$ with the following Jacobian

$$\mathcal{J}_3 = \begin{bmatrix} -\mathcal{A} & 0 & \frac{b\beta_1\Lambda_h\mu_h+b\beta_2\Lambda_h p}{\mu_h(\mu_h+p)} \\ -\frac{b\beta_3\Lambda_v}{\mathcal{D}} & -\mathcal{D} & 0 \\ \frac{b\beta_3\Lambda_v}{\mathcal{D}} & 0 & -\mathcal{B} \end{bmatrix}, \tag{4.4}$$

the fourth eigenvalue of the system is $\lambda_4 = -\mathcal{D}$ with the below Jacobian

$$\mathcal{J}_4 = \begin{bmatrix} -\mathcal{A} & \frac{b\beta_1\Lambda_h\mu_h+b\beta_2\Lambda_h p}{\mu_h(\mu_h+p)} \\ \frac{b\beta_3\Lambda_v}{\mathcal{D}} & -\mathcal{B} \end{bmatrix}. \tag{4.5}$$

Here, we will show that the other two eigenvalues of the $\mathcal{J}_4$ are negative. Equivalently, we will show that $Det(\mathcal{J}_4) > 0$ and $Tr(\mathcal{J}_4) < 0$ for $\mathcal{R}_0 < 1$. The $Det(\mathcal{J}_4) = \mathcal{AB} - \frac{b\beta_1\Lambda_h\mu_h+b\beta_2\Lambda_h p}{\mu_h(\mu_h+p)}\frac{b\beta_3\Lambda_v}{\mathcal{D}}$, this implies that $Det(\mathcal{J}_4) = \mathcal{AB}\Big[1 - \frac{b\beta_1\Lambda_h\mu_h+b\beta_2\Lambda_h p}{\mathcal{AB}\mu_h(\mu_h+p)}\frac{b\beta_3\Lambda_v}{\mathcal{D}}\Big]$. This shows that $Det(\mathcal{J}_4) > 0$ for $\mathcal{R}_0 < 1$. Clearly, $Tr(\mathcal{J}_4) < 0$, thus the remaining eigenvalues are negative for $\mathcal{R}_0 < 1$. As a result, $\mathcal{E}_0$ of the system is LAS for $\mathcal{R}_0 < 1$ and unstable in other cases. ☐

$$\left\{ \begin{array}{rcl} \frac{d\mathscr{S}_h}{dt} &=& \Lambda_h - \frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v - \mu_h\mathscr{S}_h - p\mathscr{S}_h + \upsilon\mathscr{R}_h, \\ \frac{d\mathscr{V}_h}{dt} &=& p\mathscr{S}_h - \frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v - \mu_h\mathscr{V}_h, \\ \frac{d\mathscr{I}_h}{dt} &=& \frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v + \frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v - (\mu_h+\gamma_h)\mathscr{I}_h - \xi_1\theta_1(1-q_1)\mathscr{I}_h, \\ \frac{d\mathscr{R}_h}{dt} &=& \gamma_h\mathscr{I}_h + \xi_1\theta_1(1-q_1)\mathscr{I}_h - \upsilon\mathscr{R}_h - \mu_h\mathscr{R}_h, \\ \frac{d\mathscr{S}_v}{dt} &=& \Lambda_v - \frac{\beta_3 b}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h - \xi_2\theta_2(1-q_2)\mathscr{S}_v - \mu_v\mathscr{S}_v, \\ \frac{d\mathscr{I}_v}{dt} &=& \frac{b\beta_3}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h - \xi_2\theta_2(1-q_2)\mathscr{I}_v - \mu_v\mathscr{I}_v, \end{array} \right. \tag{4.6}$$

5. Existence theory

In order to verify the existence and uniqueness of the solution of the system (3.2) of a vector-borne disease, we will employ fixed point theory. We express our system (3.2) as follows:

$$\begin{cases} \mathscr{S}_h(t)-\mathscr{S}_h(0) &= {}_0^{CF}I_t^{\xi}\Big\{\Lambda_h-\frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v-\mu_h\mathscr{S}_h-p\mathscr{S}_h+\upsilon\mathscr{R}_h\Big\},\\ \mathscr{V}_h(t)-\mathscr{V}_h(0) &= {}_0^{CF}I_t^{\xi}\Big\{p\mathscr{S}_h-\frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v-\mu_h\mathscr{V}_h\Big\},\\ \mathscr{I}_h(t)-\mathscr{I}_h(0) &= {}_0^{CF}I_t^{\xi}\Big\{\frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v+\frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v-\xi_1\theta_1(1-q_1)\mathscr{I}_h-(\mu_h+\gamma_h)\mathscr{I}_h\Big\},\\ \mathscr{R}_h(t)-\mathscr{R}_h(0) &= {}_0^{CF}I_t^{\xi}\Big\{\xi_1\theta_1(1-q_1)\mathscr{I}_h+\gamma_h\mathscr{I}_h-\upsilon\mathscr{R}_h-\mu_h\mathscr{R}_h\Big\},\\ \mathscr{S}_v(t)-\mathscr{S}_v(0) &= {}_0^{CF}I_t^{\xi}\Big\{\Lambda_v-\frac{\beta_3 b}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h-\xi_2\theta_2(1-q_2)\mathscr{S}_v-\mu_v\mathscr{S}_v\Big\},\\ \mathscr{I}_v(t)-\mathscr{I}_v(0) &= {}_0^{CF}I_t^{\xi}\Big\{\frac{b\beta_3}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h-\xi_2\theta_2(1-q_2)\mathscr{I}_v-\mu_v\mathscr{I}_v\Big\}. \end{cases} \tag{5.1}$$

Through [30], the below is obtained

$$\begin{aligned}
\mathscr{S}_h(t)-\mathscr{S}_h(0) &= \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Big\{\Lambda_h-\frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v-\mu_h\mathscr{S}_h-p\mathscr{S}_h+\upsilon\mathscr{R}_h\Big\}\\
&\quad+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t\Big\{\Lambda_h-\frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v-\mu_h\mathscr{S}_h-p\mathscr{S}_h+\upsilon\mathscr{R}_h\Big\}dy,\\
\mathscr{V}_h(t)-\mathscr{V}_h(0) &= \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Big\{p\mathscr{S}_h-\frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v-\mu_h\mathscr{V}_h\Big\}\\
&\quad+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t\Big\{p\mathscr{S}_h-\frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v-\mu_h\mathscr{V}_h\Big\}dy,\\
\mathscr{I}_h(t)-\mathscr{I}_h(0) &= \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Big\{\frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v+\frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v-(\mu_h+\gamma_h)\mathscr{I}_h-\xi_1\theta_1(1-q_1)\mathscr{I}_h\Big\}\\
&\quad+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t\Big\{\frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v+\frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v-(\mu_h+\gamma_h)\mathscr{I}_h-\xi_1\theta_1(1-q_1)\mathscr{I}_h\Big\}dy,\\
\mathscr{R}_h(t)-\mathscr{R}_h(0) &= \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Big\{\xi_1\theta_1(1-q_1)\mathscr{I}_h+\gamma_h\mathscr{I}_h-\upsilon\mathscr{R}_h-\mu_h\mathscr{R}_h\Big\}\\
&\quad+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t\Big\{\xi_1\theta_1(1-q_1)\mathscr{I}_h+\gamma_h\mathscr{I}_h-\upsilon\mathscr{R}_h-\mu_h\mathscr{R}_h\Big\}dy,\\
\mathscr{S}_v(t)-\mathscr{S}_v(0) &= \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Big\{\Lambda_v-\frac{\beta_3 b}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h-\xi_2\theta_2(1-q_2)\mathscr{S}_v-\mu_v\mathscr{S}_v\Big\}\\
&\quad+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t\Big\{\Lambda_v-\frac{\beta_3 b}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h-\xi_2\theta_2(1-q_2)\mathscr{S}_v-\mu_v\mathscr{S}_v\Big\},\\
\mathscr{I}_v(t)-\mathscr{I}_v(0) &= \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Big\{\frac{b\beta_3}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h-\xi_2\theta_2(1-q_2)\mathscr{I}_v-\mu_v\mathscr{I}_v\Big\}\\
&\quad+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t\Big\{\frac{b\beta_3}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h-\xi_2\theta_2(1-q_2)\mathscr{I}_v-\mu_v\mathscr{I}_v\Big\}.
\end{aligned} \tag{5.2}$$

Moreover, we have

$$\begin{cases} \mathcal{B}_1(t,\mathscr{S}_h) &= \Lambda_h-\frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v-\mu_h\mathscr{S}_h-p\mathscr{S}_h+\upsilon\mathscr{R}_h,\\ \mathcal{B}_2(t,\mathscr{V}_h) &= p\mathscr{S}_h-\frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v-\mu_h\mathscr{V}_h,\\ \mathcal{B}_3(t,\mathscr{I}_h) &= \frac{b\beta_1}{1+\alpha_h\mathscr{I}_v}\mathscr{S}_h\mathscr{I}_v+\frac{b\beta_2}{1+\alpha_h\mathscr{I}_v}\mathscr{V}_h\mathscr{I}_v-(\mu_h+\gamma_h)\mathscr{I}_h-\xi_1\theta_1(1-q_1)\mathscr{I}_h,\\ \mathcal{B}_4(t,\mathscr{R}_h) &= \gamma_h\mathscr{I}_h+\xi_1\theta_1(1-q_1)\mathscr{I}_h-\upsilon\mathscr{R}_h-\mu_h\mathscr{R}_h,\\ \mathcal{B}_5(t,\mathscr{S}_v) &= \Lambda_v-\frac{\beta_3 b}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h-\mu_v\mathscr{S}_v-\xi_2\theta_2(1-q_2)\mathscr{S}_v,\\ \mathcal{B}_6(t,\mathscr{I}_v) &= \frac{b\beta_3}{1+\alpha_v\mathscr{I}_h}\mathscr{S}_v\mathscr{I}_h-\mu_v\mathscr{I}_v-\xi_2\theta_2(1-q_2)\mathscr{I}_v. \end{cases} \tag{5.3}$$

Theorem 5.1 *If $0\le b\beta_1\mathscr{A}+\mu_h+p<1$ satisfies, then the kernels $\mathcal{B}_1,\mathcal{B}_2,\mathcal{B}_3,\mathcal{B}_4,\mathcal{B}_5$ and $\mathcal{B}_6$ fulfills the Lipschitz and contraction condition.*

Proof: To get the required result, we will take $\mathscr{S}_h$ and $\mathscr{S}_{h_1}$, and proceed as mentioned below:

$$\begin{aligned}\mathcal{B}_1(t,\mathscr{S}_h)-\mathcal{B}_1(t,\mathscr{S}_{h_1}) &= -\frac{b\beta_1 I_v}{1+\alpha_h I_v}\{\mathscr{S}_h(t)-\mathscr{S}_h(t_1)\}-\mu_h(t)\{\mathscr{S}_h(t)-\mathscr{S}_h(t_1)\}\\ &- p\{\mathscr{S}_h(t)-\mathscr{S}_h(t_1)\}.\end{aligned} \tag{5.4}$$

Taking norm on the both sides, yields that

$$\begin{aligned}\|\mathcal{B}_1(t,\mathscr{S}_h)-\mathcal{B}_1(t,\mathscr{S}_{h_1})\| &\leq b\beta_1\left\|\frac{I_v}{1+\alpha_h I_v}\right\|\|\{\mathscr{S}_h(t)-\mathscr{S}_h(t_1)\}\|+\mu_h\|\{\mathscr{S}_h(t)-\mathscr{S}_h(t_1)\}\|\\ &+ p\|\{\mathscr{S}_h(t)-\mathscr{S}_h(t_1)\}\|\\ &\leq b\beta_1\mathscr{A}\|\{\mathscr{S}_h(t)-\mathscr{S}_h(t_1)\}\|+\mu_h\|\{\mathscr{S}_h(t)-\mathscr{S}_h(t_1)\}\|\\ &+ p\|\{\mathscr{S}_h(t)-\mathscr{S}_h(t_1)\}\|\\ &\leq (b\beta_1\mathscr{A}+\mu_h+p)\|\{\mathscr{S}_h(t)-\mathscr{S}_h(t_1)\}\|,\end{aligned} \tag{5.5}$$

where $\left\|\frac{I_v}{1+\alpha_h I_v}\right\| \leq \mathscr{A}$. Taking $\Xi_1 = b\beta_1\mathscr{A}+\mu_h+p$, we get

$$||\mathcal{B}_1(t,\mathscr{S}_h)-\mathcal{B}_1(t,\mathscr{S}_{h_1})|| \leq \Xi_1||\mathscr{S}_h(t)-\mathscr{S}_h(t_1)||. \tag{5.6}$$

As a result, the condition of Lipschitz is proved for $\mathcal{B}_1$, also the contraction can be obtained from $0 \leq b\beta_1\mathscr{A}+\mu_h+p < 1$. We may also determine the remaining as

$$\begin{aligned}||\mathcal{B}_2(t,\mathscr{V}_h)-\mathcal{B}_2(t,\mathscr{V}_{h_1})|| &\leq \Xi_2||\mathscr{V}_h(t)-\mathscr{V}_h(t_1)||,\\ ||\mathcal{B}_3(t,I_h)-\mathcal{B}_3(t,\mathscr{I}_{h_1})|| &\leq \Xi_3||I_h(t)-I_h(t_1)||,\\ ||\mathcal{B}_4(t,\mathscr{R}_h)-\mathcal{B}_4(t,\mathscr{R}_{h_1})|| &\leq \Xi_4||\mathscr{R}_h(t)-\mathscr{R}_h(t_1)||,\\ ||\mathcal{B}_5(t,\mathscr{S}_v)-\mathcal{B}_5(t,\mathscr{S}_{v_1})|| &\leq \Xi_5||\mathscr{S}_v(t)-\mathscr{S}_v(t_1)||,\\ ||\mathcal{B}_6(t,w)-\mathcal{B}_6(t,\mathscr{I}_{v_1})|| &\leq \Xi_6||\mathscr{I}_v(t)-\mathscr{I}_v(t_1)||.\end{aligned} \tag{5.7}$$

From (5.2), we get the following

$$\begin{cases}\mathscr{S}_h(t) = \mathscr{S}_h(0)+\frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_1(t,\mathscr{S}_h)+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t(\mathcal{B}_1(z,\mathscr{S}_h))dz,\\ \mathscr{V}_h(t) = \mathscr{V}_h(0)+\frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_2(t,\mathscr{V}_h)+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t(\mathcal{B}_2(z,\mathscr{V}_h))dz,\\ \mathscr{I}_h(t) = \mathscr{I}_h(0)+\frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_3(t,\mathscr{I}_h)+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t(\mathcal{B}_3(z,\mathscr{I}_h))dz,\\ \mathscr{R}_h(t) = \mathscr{R}_h(0)+\frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_4(t,\mathscr{R}_h)+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t(\mathcal{B}_4(z,\mathscr{R}_h))dz,\\ \mathscr{S}_v(t) = \mathscr{S}_v(0)+\frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_5(t,\mathscr{S}_v)+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t(\mathcal{B}_5(z,\mathscr{S}_v))dz,\\ \mathscr{I}_v(t) = \mathscr{I}_v(0)+\frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_6(t,\mathscr{I}_v)+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t(\mathcal{B}_6(z,\mathscr{I}_v))dz.\end{cases} \tag{5.8}$$

Moreover, we obtain

$$\begin{cases}\mathscr{S}_{h_\wp}(t) = 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_1(t,\mathscr{S}_{h_{(\wp-1)}})+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t(\mathcal{B}_1(z,\mathscr{S}_{h_{(\wp-1)}}))dz,\\ \mathscr{V}_{h_\wp}(t) = 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_2(t,\mathscr{V}_{h_{(\wp-1)}})+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t(\mathcal{B}_2(z,\mathscr{V}_{h_{(\wp-1)}}))dz,\\ \mathscr{I}_{h_\wp}(t) = 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_3(t,\mathscr{I}_{h_{(\wp-1)}})+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t(\mathcal{B}_3(z,\mathscr{I}_{h_{(\wp-1)}}))dz,\\ \mathscr{R}_{h_\wp}(t) = 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_4(t,\mathscr{R}_{h_{(\wp-1)}})+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t(\mathcal{B}_4(z,\mathscr{R}_{h_{(\wp-1)}}))dz,\\ \mathscr{S}_{v_\wp}(t) = 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_5(t,\mathscr{S}_{v_{(\wp-1)}})+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\int_0^t(\mathcal{B}_5(z,\mathscr{S}_{v_{(\wp-1)}}))dz,\\ \mathscr{I}_{v_\wp}(t) = 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\mathcal{B}_6(t,\mathscr{I}_{v_{(\wp-1)}})+2\frac{\xi}{(2-\xi)\mathcal{U}(\xi)}\int_0^t(\mathcal{B}_6(z,\mathscr{I}_{v_{(\wp-1)}}))dz,\end{cases} \tag{5.9}$$

where

$$\mathscr{S}_h^0(t) = \mathscr{S}_h(0), \mathscr{V}_h^0(t) = \mathscr{V}_h(0), \mathscr{I}_h^0(t) = \mathscr{I}_h(0), \mathscr{R}_h^0(t) = \mathscr{R}_h(0), \mathscr{S}_v^0(t) = \mathscr{S}_v(0), \mathscr{I}_v^0(t) = \mathscr{I}_v(0).$$

We get the difference terms as

$$\begin{aligned}
\varrho_{1\wp}(t) &= \mathscr{S}_{h_\wp}(t) - \mathscr{S}_{h_{(\wp-1)}}(t) = \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}(\mathcal{B}_1(t, \mathscr{S}_{h_{(\wp-1)}}) - \mathcal{B}_1(t, \mathscr{S}_{h_{(\wp-2)}})) \\
&\quad +2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)} \int_0^t (\mathcal{B}_1(z, \mathscr{S}_{h_{(\wp-1)}}) - \mathcal{B}_1(z, \mathscr{S}_{h_{(\wp-2)}}))dz, \\
\varrho_{2\wp}(t) &= \mathscr{V}_{h_\wp}(t) - \mathscr{V}_{h_{(\wp-1)}}(t) = \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}(\mathcal{B}_1(t, \mathscr{V}_{h_{(\wp-1)}}) - \mathcal{B}_1(t, \mathscr{V}_{h_{(\wp-2)}})) \\
&\quad +2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)} \int_0^t (\mathcal{B}_1(z, \mathscr{V}_{h_{(\wp-1)}}) - \mathcal{B}_1(z, \mathscr{V}_{h_{(\wp-2)}}))dz, \\
\varrho_{3\wp}(t) &= \mathscr{I}_{h_\wp}(t) - \mathscr{I}_{h_{(\wp-1)}}(t) = \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}(\mathcal{B}_1(t, \mathscr{I}_{h_{(\wp-1)}}) - \mathcal{B}_1(t, \mathscr{I}_{h_{(\wp-2)}})) \\
&\quad +2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)} \int_0^t (\mathcal{B}_1(y, \mathscr{I}_{h_{(\wp-1)}}) - \mathcal{B}_1(z, \mathscr{I}_{h_{(\wp-2)}}))dz, \\
\varrho_{4\wp}(t) &= \mathscr{R}_{h_\wp}(t) - \mathscr{R}_{h_{(\wp-1)}}(t) = \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}(\mathcal{B}_1(t, \mathcal{R}_{h(\wp-1)}) - \mathcal{B}_1(t, \mathcal{R}_{h_{(\wp-2)}})) \\
&\quad +2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)} \int_0^t (\mathcal{B}_1(y, \mathscr{R}_{h_{(\wp-1)}}) - \mathcal{B}_1(z, \mathscr{R}_{h_{(\wp-2)}}))dz, \\
\varrho_{5\wp}(t) &= \mathscr{S}_{v_\wp}(t) - \mathscr{S}_{v_{(\wp-1)}}(t) = \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}(\mathcal{B}_1(t, \mathscr{S}_{v_{(\wp-1)}}) - \mathcal{B}_1(t, \mathscr{S}_{v_{(\wp-2)}})) \\
&\quad +2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)} \int_0^t (\mathcal{B}_1(z, \mathscr{S}_{v_{(\wp-1)}}) - \mathcal{B}_1(z, \mathscr{S}_{v_{(\wp-2)}}))dz, \\
\varrho_{6\wp}(t) &= \mathscr{I}_{v_\wp}(t) - \mathscr{I}_{v_{(\wp-1)}}(t) = \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}(\mathcal{B}_1(t, \mathscr{S}_{v_{(\wp-1)}}) - \mathcal{B}_1(t, \mathscr{I}_{v_{(\wp-2)}})) \\
&\quad +2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)} \int_0^t (\mathcal{B}_1(z, \mathscr{I}_{v_{(\wp-1)}}) - \mathcal{B}_1(z, \mathscr{I}_{v_{(\wp-2)}}))dz. \qquad (5.10)
\end{aligned}$$

Thus, the below is obtained

$$\begin{cases}
\mathscr{S}_\wp(t) = \sum_{\hbar=1}^{\wp} \varrho_{1\hbar}(t), \\
\mathscr{V}_{h_\wp}(t) = \sum_{\hbar=1}^{\wp} \varrho_{2\hbar}(t), \\
\mathscr{I}_{h_\wp}(t) = \sum_{\hbar=1}^{\wp} \varrho_{3\hbar}(t), \\
\mathscr{R}_{h_\wp}(t) = \sum_{\hbar=1}^{\wp} \varrho_{4\hbar}(t), \\
\mathscr{S}_{v_\wp}(t) = \sum_{\hbar=1}^{\wp} \varrho_{5\hbar}(t), \\
\mathscr{I}_{v_\wp}(t) = \sum_{\hbar=1}^{\wp} \varrho_{6\hbar}(t).
\end{cases} \qquad (5.11)$$

After simplification we get that

$$\begin{aligned}
||\varrho_{1\wp}(t)|| &= ||\mathscr{S}_{h_\wp}(t) - \mathscr{S}_{h_{(\wp-1)}}(t)|| = \Big\| 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}(\mathcal{B}_1(t, \mathscr{S}_{h_{(\wp-1)}}) - \mathcal{B}_1(t, \mathscr{S}_{h_{(\wp-2)}})) \\
&\quad +2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)} \int_0^t (\mathcal{B}_1(z, \mathscr{S}_{h_{(\wp-1)}}) - \mathcal{B}_1(z, \mathscr{S}_{h_{(\wp-2)}}))dz \Big\|. \qquad (5.12)
\end{aligned}$$

From (5.12), we get

$$\begin{aligned}\|\mathscr{S}_{h_\wp}(t)-\mathscr{S}_{h_{(\wp-1)}}(t)\| &\leq 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\|(\mathcal{B}_1(t,\mathscr{S}_{h_{(\wp-1)}})-\mathcal{B}_1(t,\mathscr{S}_{h_{(\wp-2)}}))\| \\ &\quad +2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\left\|\int_0^t(\mathcal{B}_1(z,\mathscr{S}_{h_{(\wp-1)}})-\mathcal{B}_1(z,\mathscr{S}_{h_{(\wp-2)}}))dz\right\|. \end{aligned} \tag{5.13}$$

Here, the following is obtained

$$\begin{aligned}\|\mathscr{S}_{h_\wp}(t)-\mathscr{S}_{h_{(\wp-1)}}(t)\| &\leq 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_1\|\mathscr{S}_{h_{(\wp-1)}}-\mathscr{S}_{h_{(\wp-2)}}\|+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\Xi_1 \\ &\quad \times\int_0^t\|\mathscr{S}_{h_{(\wp-1)}}-\mathscr{S}_{h_{(\wp-2)}}\|dz. \end{aligned} \tag{5.14}$$

Additionally

$$\|\varrho_{1\wp}(t)\| \leq 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_1\|\varrho_{1(\wp-1)}(t)\|+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\Xi_1\int_0^t\|\varrho_{1(\wp-1)}(z)\|dz. \tag{5.15}$$

Comparative analysis implies

$$\begin{aligned}\|\varrho_{2\wp}(t)\| &\leq 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_2\|\varrho_{2(\wp-1)}(t)\|+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\Xi_2\int_0^t\|\varrho_{2(\wp-1)}(z)\|dz,\\ \|\varrho_{3\wp}(t)\| &\leq 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_3\|\varrho_{3(\wp-1)}(t)\|+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\Xi_1\int_0^t\|\varrho_{3(\wp-1)}(z)\|dz,\\ \|\varrho_{4\wp}(t)\| &\leq \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_4\|\varrho_{4(\wp-1)}(t)\|+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\Xi_4\int_0^t\|\varrho_{4(\wp-1)}(z)\|dz,\\ \|\varrho_{5\wp}(t)\| &\leq 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_5\|\varrho_{5(\wp-1)}(t)\|+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\Xi_5\int_0^t\|\varrho_{5(\wp-1)}(z)\|dz,\\ \|\varrho_{6\wp}(t)\| &\leq 2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_6\|\varrho_{6(\wp-1)}(t)\|+2\frac{\xi}{(2-\xi)\mathcal{U}(\xi)}\Xi_6\int_0^t\|\varrho_{6(\wp-1)}(z)\|dz. \end{aligned} \tag{5.16}$$

□

Theorem 5.2 *If we get a t_0 such that the below satisfies*

$$2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_1+2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\Xi_1 t_0<1,$$

then, for system (3.2) we get an exact coupled-solution.

Proof: Since $\mathscr{S}_h(t)$, $\mathscr{V}_h(t)$, $\mathscr{I}_h(t), \mathscr{R}_h(t)$, $\mathscr{S}_v(t)$ and $\mathscr{I}_v(t)$ are bounded and the Lipschitz condition holds true. Then, from (5.15) and (5.16) we get the following:

$$\begin{aligned}\|\varrho_{1\wp}(t)\| &\leq ||S_{h_\wp}(0)||\Big[\Big(2\frac{(1-\xi)}{(2-\xi)\mathcal{U}(\xi)}\Xi_1\Big)+\Big(2\frac{\xi}{(2-\xi)\mathcal{U}(\xi)}\Xi_1 t\Big)\Big]^{\wp},\\ \|\varrho_{2\wp}(t)\| &\leq ||V_{h_\wp}(0)||\Big[\Big(2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_2\Big)+\Big(2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\Xi_2 t\Big)\Big]^{\wp},\\ \|\varrho_{3\wp}(t)\| &\leq ||I_{h_\wp}(0)||\Big[\Big(2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_3\Big)+\Big(2\frac{\xi}{(2-\xi)\mathcal{U}(\xi)}\Xi_3 t\Big)\Big]^{\wp},\\ \|\varrho_{4\wp}(t)\| &\leq ||R_{h_\wp}(0)||\Big[\Big(2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_4\Big)+\Big(2\frac{\xi}{(2-\xi)\mathcal{U}(\xi)}\Xi_4 t\Big)\Big]^{\wp}, \end{aligned}$$

$$\begin{aligned}
\|\varrho_{5\wp}(t)\| &\leq ||S_{v_\wp}(0)||\Big[\Big(2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_5\Big)+\Big(2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\Xi_5 t\Big)\Big]^{\wp}, \\
\|\varrho_{6\wp}(t)\| &\leq ||I_{v_\wp}(0)||\Big[\Big(2\frac{(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_6\Big)+\Big(2\frac{\xi}{(2-\xi)\mathcal{M}(\xi)}\Xi_6 t\Big)\Big]^{\wp}. \qquad (5.17)
\end{aligned}$$

Hence, the solutions' continuity and existence are obtained. For the solution model (3.2) of dengue, we will proceed in the following manner

$$\begin{aligned}
\mathscr{S}_h(t)-\mathscr{S}_h(0) &= S_{h_\wp}(t)-\mathscr{P}1_\wp(t), \\
\mathscr{V}_h(t)-\mathscr{V}_h(0) &= V_{h_\wp}(t)-\mathscr{P}2_\wp(t), \\
I_h(t)-I_h(0) &= I_{h_\wp}(t)-\mathscr{P}3_\wp(t), \\
R_h(t)-R_h(0) &= R_{h_\wp}(t)-\mathscr{P}4_\wp(t), \\
S_v(t)-S_v(0) &= S_{v_\wp}(t)-\mathscr{P}5_\wp(t), \\
I_v(t)-I_v(0) &= I_{v_\wp}(t)-\mathscr{P}6_\wp(t). \qquad (5.18)
\end{aligned}$$

Moreover, we have

$$\begin{aligned}
\|B_\wp(t)\| &= \Big|\Big|\frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}(\mathcal{B}_1(t,S_{h_\wp})-\mathcal{B}_1(t,S_{h_{(\wp-1)}}))+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\times \\
&\quad \int_0^t(\mathcal{B}_1(z,S_{h_\wp})-\mathcal{B}_1(z,S_{h_{(\wp-1)}}))dz\Big|\Big|, \\
&\leq \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\|(\mathcal{B}_1(t,S_{h_\wp})-(\mathcal{B}_1(t,S_{h_{(\wp-1)}}))\|+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\times \\
&\quad \int_0^t||(\mathcal{B}_1(z,\mathscr{S}_h)-\mathcal{B}_1(z,S_{h_{(\wp-1)}}))||dz, \\
&\leq \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}\Xi_1\|\mathscr{S}_h-S_{h_{(\wp-1)}}\|+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\Xi_1\|\mathscr{S}_h-S_{h_{(\wp-1)}}\|t, \qquad (5.19)
\end{aligned}$$

and

$$\|\mathscr{P}1_\wp(t)\| \leq \Big(\frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}t\Big)^{\wp+1}\Xi_1^{\wp+1}a. \qquad (5.20)$$

Here, at t_0 we have the following:

$$\|\mathscr{P}1_\wp(t)\| \leq \Big(\frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}+\frac{2\xi}{(2-\xi)\mathcal{U}(\xi)}t_0\Big)^{\wp+1}\Xi_1^{\wp+1}a. \qquad (5.21)$$

Similarly proceeding and utilizing (5.21), we obtained the following

$$\|\mathscr{P}1_\wp(t)\| \longrightarrow 0, \quad as \quad \wp\to\infty.$$

Similarly, one can get that $\mathscr{P}2_\wp(t), \mathscr{P}3_\wp(t), \mathscr{P}4_\wp(t), \mathscr{P}5_\wp(t), \mathscr{P}6_\wp(t) \Rightarrow 0$ as $\wp\Rightarrow\infty$. □

To show the uniqueness of the solution of our model, let $(\mathscr{S}_{h_1}(t),\ \mathscr{V}_{h_1}(t),\ \mathscr{I}_{h_1}(t),\ \mathscr{R}_{h_1}(t),\ \mathscr{S}_{v_1}(t),\ \mathscr{I}_{v_1}(t))$ is another solution of model (3.2):

$$\mathscr{S}_h(t)-\mathscr{S}_{h_1}(t) = \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)}(\mathcal{B}_1(t,\mathscr{S}_h)-\mathcal{B}_1(t,\mathscr{S}_{h_1}))+\frac{2\xi}{(2-\xi)\mathcal{M}(\xi)}\times$$

$$\int_0^t (\mathcal{B}_1(z, \mathscr{S}_h) - \mathcal{B}_1(z, \mathscr{S}_{h_1}))\, dz. \tag{5.22}$$

Simplification yields the following

$$\begin{aligned} \|\mathscr{S}_h(t) - \mathscr{S}_{h_1}(t)\| \quad &\leq \quad \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)} \|\mathcal{B}_1(t, \mathscr{S}_h) - \mathcal{B}_1(t, \mathscr{S}_{h_1})\| + \frac{2\ell}{(2-\ell)\mathcal{M}(\ell)} \times \\ &\int_0^t \|\mathcal{B}_1(z, \mathscr{S}_h) - \mathcal{B}_1(z, \mathscr{S}_{h_1})\| dz. \end{aligned} \tag{5.23}$$

Further, we get

$$\begin{aligned} \|\mathscr{S}_h(t) - \mathscr{S}_{h_1}(t)\| \quad &\leq \quad \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)} \Xi_1 \|\mathscr{S}_h(t) - \mathscr{S}_{h_1}(t)\| + \frac{2\xi}{(2-\xi)\mathcal{M}(\xi)} \times \\ &\int_0^t \Xi_1 t \|\mathscr{S}_h(t) - \mathscr{S}_{h_1}(t)\| dz. \end{aligned} \tag{5.24}$$

From (5.24), we get

$$\|\mathscr{S}_h(t) - \mathscr{S}_{h_1}(t)\| \Big(1 - \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)} \Xi_1 - \frac{2\xi}{(2-\ell)\mathcal{M}(\xi)} \Xi_1 t \Big) \leq 0. \tag{5.25}$$

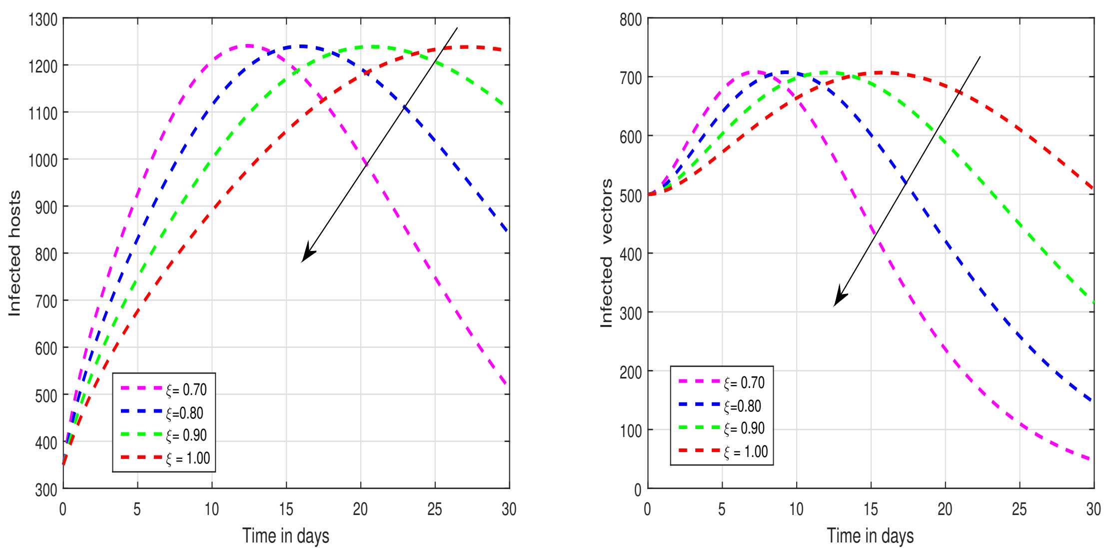

Figure 1: Time series analysis of infected hosts and infected vectors of our model (3.2) with various values of ξ, i.e., $\xi = 0.70, 0.80, 0.90, 1.00$ to conceptualize the impact of memory on the dynamics.

Theorem 5.3 *We can get a solution of system (3.2), if*

$$\Big(1 - \frac{2(1-\xi)}{(2-\xi)\mathcal{M}(\xi)} \Xi_1 - \frac{2\xi}{(2-\xi)\mathcal{M}(\xi)} \Xi_1 t \Big) > 0. \tag{5.26}$$

Proof: To show the existence of the solution, we get the below from the above is (5.26), when (5.25) is satisfied

$$\|\mathscr{S}_h(t) - \mathscr{S}_{h_1}(t)\| = 0. \tag{5.27}$$

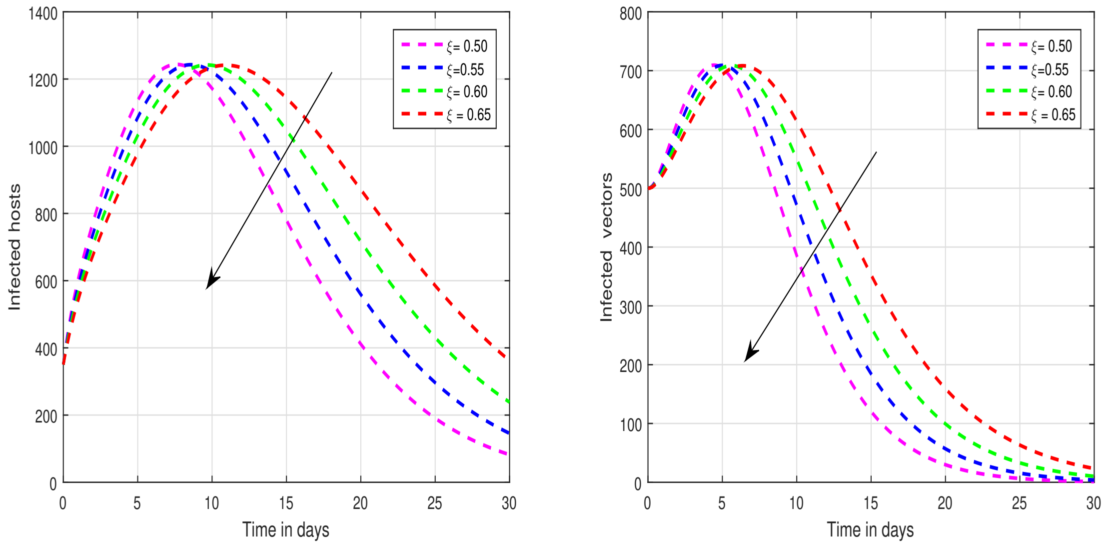

Figure 2: Time series analysis of infected hosts and infected vectors of our model (3.2) with various values of ξ, i.e., $\xi = 0.50, 0.55, 0.60, 0.65$ to conceptualize the impact of memory on the dynamics.

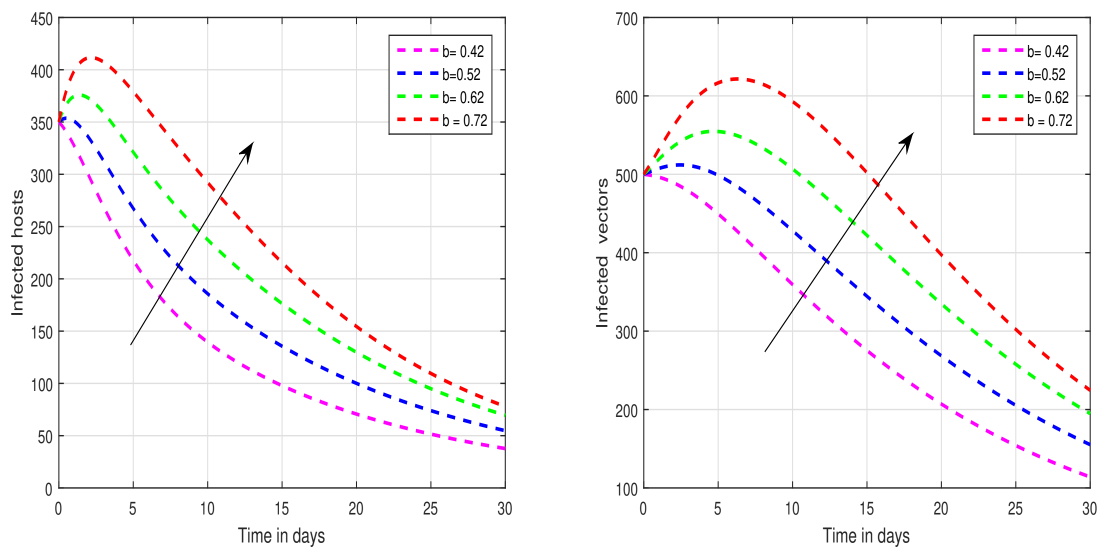

Figure 3: Plotting the solution pathways of infected hosts and infected vectors of our model (3.2) with various values of biting rate b, i.e., $b = 0.88, 0.92, 0.96, 1.000$.

Further, we get

$$\mathscr{S}_h(t) = \mathscr{S}_{h_1}(t). \tag{5.28}$$

Proceeding similarly, we get the following

$$\mathscr{V}_h(t) = \mathscr{V}_{h_1}(t), \mathscr{I}_h(t) = \mathscr{I}_{h_1}(t),$$

$$\mathscr{R}_h(t) = \mathscr{R}_{h_1}(t), \mathscr{S}_v(t) = \mathscr{S}_{v_1}(t), \mathscr{I}_v(t) = \mathscr{I}_{v_1}(t).$$

□

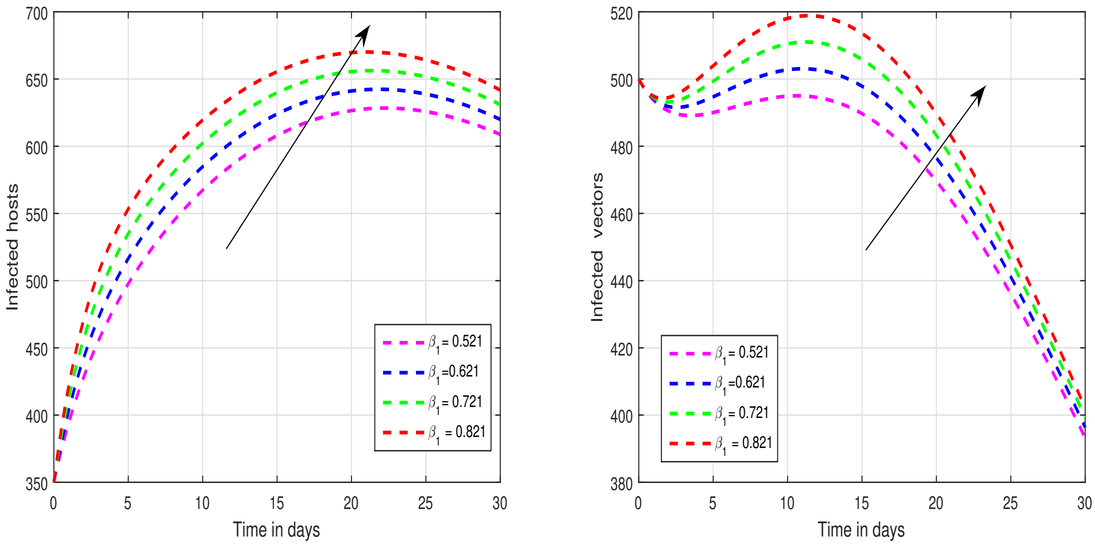

Figure 4: Plotting the solution pathways of infected hosts and infected vectors of our model (3.2) with various values of transmission rate β_1, i.e., $\beta_1 = 0.521, 0.621, 0.721, 0.821$.

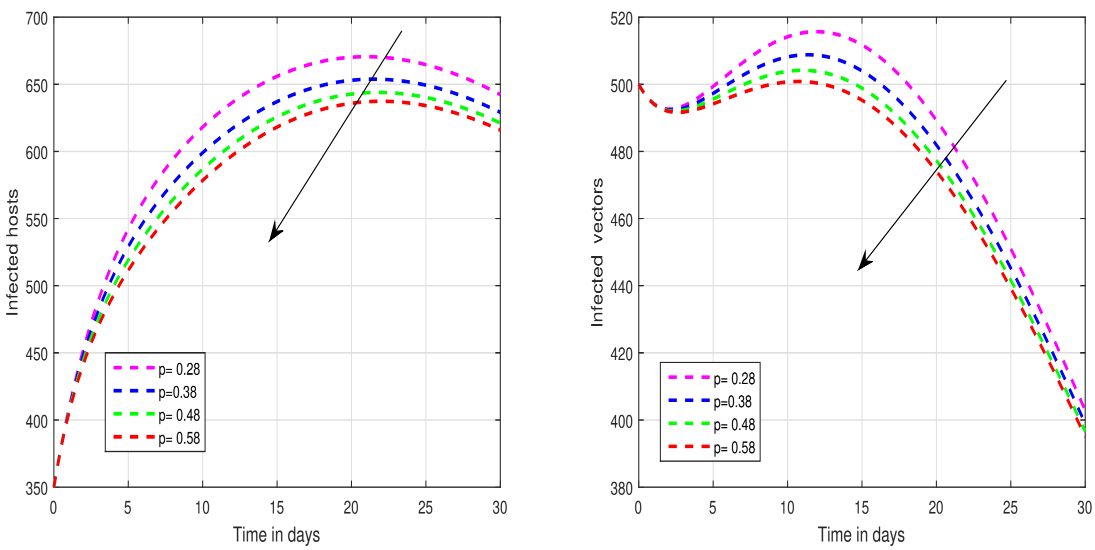

Figure 5: Illustration of the dynamical nature of infected hosts and infected vectors of the model (3.2) of the infection with the variation of p, i.e., $p = 0.28, 0.38, 0.48, 0.58$ to visualize the effect of vaccination on the system.

6. Result and discussion

The dynamical behavior of dengue infection within a population is complex and can be effectively described using mathematical models that incorporate various epidemiological factors. Factors such as population density, memory, human mobility, climate, and immunity levels contribute significantly in the transmission of this vector-borne infection. Mathematical analyses, numerical simulations, and sensitivity studies help in understanding the implications of these factors on the spread of the virus and can aid in designing effective public health interventions. Furthermore, the potential for severe outcomes, like DHF

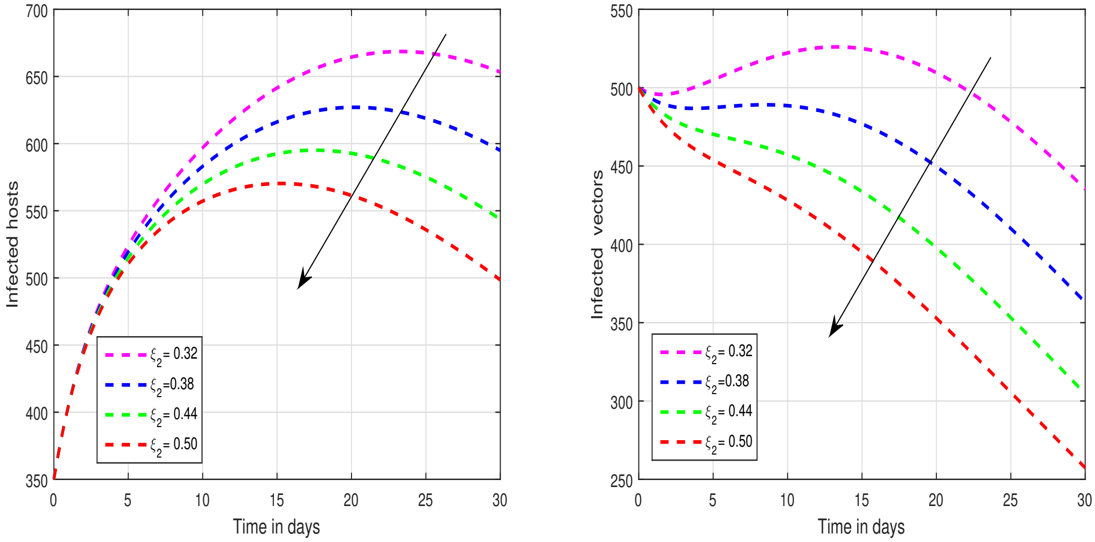

Figure 6: Illustration of the dynamical nature of infected hosts and infected vectors of the model (3.2) of the infection with the variation of ξ_2, i.e., $\xi_2 = 0.32, 0.38, 0.44, 0.50$.

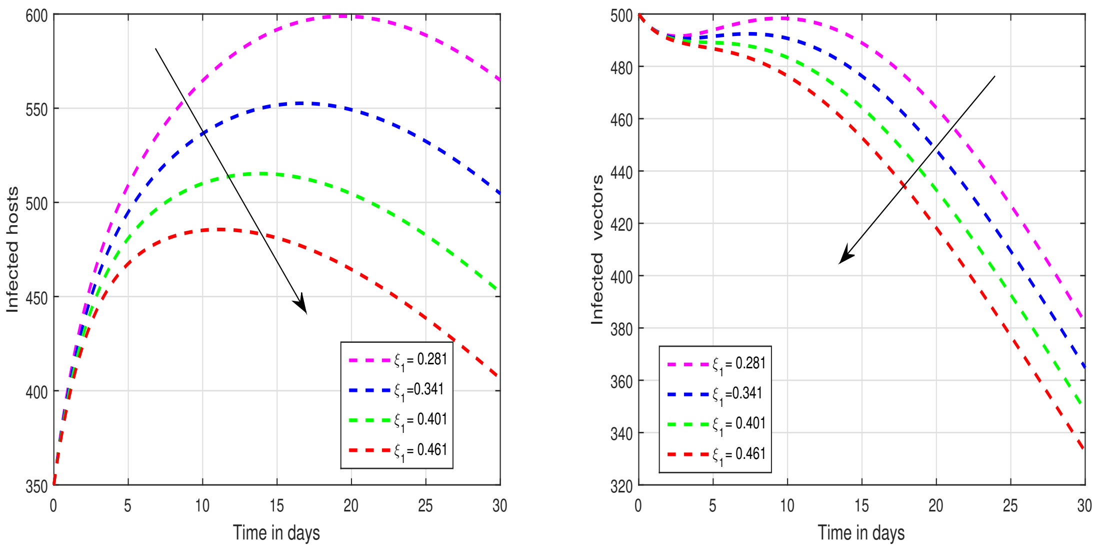

Figure 7: Solution pathways of infected hosts and infected vectors of the model (3.2) of the infection with the variation of ξ_1, i.e., $\xi_1 = 0.281, 0.341, 0.401, 0.461$.

and DSS, adds another layer of complexity to the dynamical behavior of dengue infection, necessitating comprehensive and adaptive strategies for disease control and prevention. Here, we will demonstrate how the various input elements of the suggested system affect the solution pathways of the infection.

We perform numerous simulations to visualize the variation in the endemic level of the infection. For simulation purposes, we assumed the values of state-variable and the parameters of the system. We illustrate our conceptual understanding of the effect of memory on infected hosts and vectors in Figures 1 and 2. We took the values of ξ to be $0.70.0.80, 0.90$, and 1.00 in Figure 1, and the values of ξ to be $0.55, 0.60, 0.65, 0.70$. A comparative study of integer and non-integer systems is also shown in the first

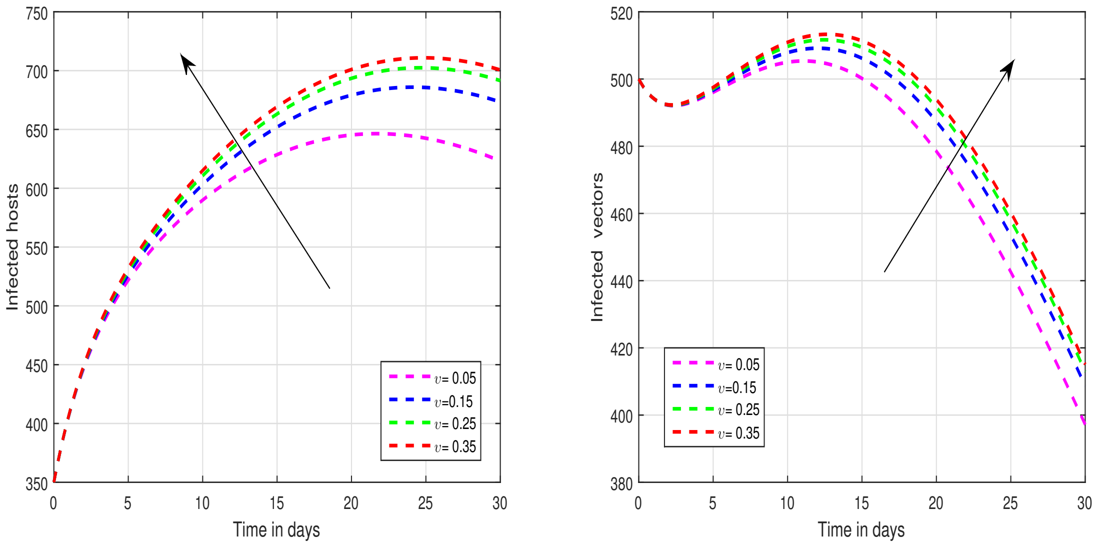

Figure 8: Visualization of the dynamics of the infected hosts and infected vectors of the model (3.2) with υ, i.e., $\upsilon = 0.05, 0.15, 0.25, 0.35$.

Figure, demonstrating the greater flexibility of non-integer systems over classical systems. It has been observed that the index of memory is a desirable feature that has the potential to lower the level of infection. For the purpose of infection control and prevention, we thus advise policymakers to implement it. We elucidate the effect of vector biting rate on the dynamics of infected individuals across both classes in Figure 3 with b equal to 0.42, 0.52, 0.62 and 0.72. Our findings underscore the critical role of the biting rate b as a pivotal element for the level of the infection. Global worming can increase the biting rate and hence raise the risk of this vector borne infection.

We emphasized the impact of transmission rate β_1 on the behavior of infected people in Figure 4. In this simulation, we took the values of β_1 to be 0.521, 0.621, 0.721, and 0.821. It has been observed that when β_1 grows, so does the infection level of both hosts and vectors. As a result, it is advised to public health professionals for improved management of infections. The impact of vaccination on the infected individuals of the system is depicted in Figure 5. We used the values of p to be 0.28, 0.38, 0.48, and 0.58 in this simulation. We have observed that immunization can successfully treat systemically ill persons. As a result, we recommended to the decision-makers that vaccine efficacy be increased for better control. In Figure 6 and Figure 7, we have shown the impact of the efficacy of insecticide and the efficacy of drugs, respectively. It can be observed that increased efficacy of insecticide reduce the level of infected vectors quickly which reduce the infection level of hosts while the increased efficacy of drugs can decrease the level of infected hosts quickly which decrease the level of infected vectors. These two policies are good and can be used to lover the level of infection in the community.

The loss of immunity in dengue is a complex process that varies based on several factors, including the specific serotype of the dengue virus, the host's immune response, and the time elapsed since the infection. In Figure 8, we illustrated the impact of losing rate of immunity on the dynamics of the infection. In this simulation, we assumed the values of υ to be 0.05, 0.15, 0.25 and 0.35. It is obvious that this factor poses a risk and makes the management of the infection more challenging. Our findings indicate that the memory index can decrease infection levels and serve as a control parameter for managing dengue infection. Conversely, the biting rate and transmission rate pose significant risks. Furthermore, our research highlights that the rate of immunity loss is a critical factor, complicating dengue control efforts. Additionally, we have demonstrated that drug efficacy and overall treatment effectiveness are promising control strategies. Our results helps in understanding the disease dynamics, evaluate control policies, assessing intervention impacts and supporting policy makers decisions.

7. Conclusion

Dengue infection constitute a significant component of the global disease burden, underscoring the imperative need for efficacious strategies to protect individuals from these potentially fatal diseases. In this work, we used Caputo-Fabrizio operator to formulate a novel model for dengue infection with the effect of vaccination and saturated incidence rate. We introduced the fundamental results of fractional theory associated with CF derivative for the analysis of proposed model. We investigated the steady-states and determined the endemic indicator, symbolized by $\mathcal{R}_0$ through the approach of next-generation matrix. It has been shown that the disease-free steady state is locally asymptotically stable if $\mathcal{R}_0 < 1$, otherwise unstable. The existence theory is introduced, moreover, we examined the uniqueness and existence of the solution of the recommended system. Time series analysis of the model was done with the help of numerical results to show the most effective factors of the dynamics. Our numerical findings highlighted essential factors of the dynamics for the control and management of the infection, providing valuable insights for public health interventions and policy considerations.

Declarations

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Not applicable.

Author Contributions

Zahir Shah: Formal analysis, investigations, resources and planning, software and writing original draft. Rashid Jan: Methodology, conceptualization, formal analysis and writing original draft. Mohammadi Begum Jeelani: Supervision, conceptualization, resources and planning, validation, review and editing. Nouf Abdulrahman Alqahtani: Methodology, validation, investigation, validation and writing original draft. Elisabeta Antonescu: Supervision, project administration, investigation, validation, visualizations, review and editing.

Data availability statement

No new data was used in this work.

Conflict of interest

There is no competing interest regarding this work.

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