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Several Parameters of Domination and Inverse Domination in Discrete Topological Graphs

Mohammed A. Abdlhusein[1], Haiffa M. Buite[2], Lamees K. Alzaki[2],
Zainab N. Jwair[2] and Jakkepalli Pavan Kumar[3]

ABSTRACT: Let $G_\tau = (V, E)$ be a topological graph where $V(G_\tau)$ is a set of all vertices of G_τ and each vertex is represent a set of the topology τ non-equal to $\emptyset$ or X. The set of all edges of G_τ is $E(G_\tau)$ such that there is an edge between any two vertices if no one of their topological sets are subset from the other. In this paper, many bounds and properties of domination are studied and applied on the topological graph. Three types of domination parameters are studied: co-independent domination, complementary tree domination and bi-domination. The co-independent domination number of G_τ, denoted by $\gamma_{coi}(G_\tau)$, the complementary tree domination number of G_τ denoted by $\gamma_{ctd}(G_\tau)$ and the bi-domination number of G_τ, denoted by $\gamma_{bi}(G_\tau)$ are proved and found. Also, the inverse dominating set and inverse domination number for these types is calculated.

Keywords: Topological graph, discrete topology, dominating set, domination number.

Contents

1. Introduction

$G = (V, E)$ is a graph with $V(G)$ is a set of all vertices and $E(G)$ is a set of all edges in G. The vertex u is adjacent with a vertex v if there is an edge incident between them. The number of all elements in $V(G)$ is called the order of a graph G, denoted by $|V(G)|$. The number of all elements in $E(G)$ is called the size of a graph G. The subgraph M of G is induced subgraph denoted by $G[M]$ formed by all vertices of $M \subseteq V(G)$ and all edges between vertices of M. A graph G is connected graph if every pair of vertices are joined by a path. A tree T is a connected graph and has no cycle, see [16, 30]. The subset D is called dominating set if for each vertex of $V - D$ is adjacent to one or more vertices of D. The domination number denoted by $\gamma(G)$ is the cardinality of the minimum dominating set [20]. The inverse dominating set in G denoted by D^{-1} is a minimum dominating set exist in $V - D$ The inverse domination number denoted by $\gamma^{-1}(G)$ is the cardinality of the minimum inverse dominating set [24,25]. The subset D is an co-independent dominating set if all vertices of $G[V - D]$ are isolated vertices. The co-independent domination number is denoted by $\gamma_{coi}(G)$. The subset D is a complementary tree dominating set if $G[V - D]$ is a tree. The complementary tree domination number is denoted by $\gamma_{ctd}(G)$. The subset D is a bi-dominating set if every vertex in D is adjacent to exactly two vertices in $V - D$. The bi-domination number is denoted by $\gamma_{bi}(G)$. For more information about domination see [1-15, 17-19, 26-29]. The discrete topology is denoted by (X, τ) such that X is a non-empty set and τ is a family of all subsets of X, where $\tau = P(X)$. There are many papers that include linking the graph to topology, seeing [21-23]. Here, many parameters of domination are applied on the topological graph with their inverse.

Theorem 1.1 [29]: Let G be a graph has co-independent dominating set. Then, $\gamma_{coi}(K_n) = n - 1$ for $n \geq 2$.

Theorem 1.2 [28]: Let G be any graph has complementary tree dominating set. Then, $\gamma_{ctd}(K_n) = n - 2$, $n \geq 3$.

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2. Some Results of the Topological Graph

In this section, several properties of the topological graph are proved with some examples and figures.

Definition 2.1: Let X be a non-empty set and τ be a discrete topology on X. The discrete topological graph denoted by $G_\tau = (V, E)$ is a graph of the vertex set $V = \{A; A \in \tau \text{ and } A \neq \emptyset, X\}$ and the edge set $E = \{AB; A \nsubseteq B \text{ and } B \nsubseteq A\}$.

Proposition 2.2: Let X be a non-empty set of order n and let τ be a discrete topology on X. If $n = 2$, then $G_\tau \cong K_2$.

Proof: Let $X = \{1, 2\}$, then $\tau = \{\emptyset, X, \{1\}, \{2\}\}$ so $V = \{\{1\}, \{2\}\}$. Since $\{1\}$ is not subset of $\{2\}$ and $\{2\}$ is not subset of $\{1\}$. Then, there is an edge between them. Therefore, the graph G_τ isomorphic to K_2. □

Proposition 2.3: Let X be a non-empty set of order n and let τ be a discrete topology on X. If $n = 3$, then $G_\tau \cong \overline{C_6}$.

Proof: Let $X = \{1, 2, 3\}$, then $\tau = \{\emptyset, X, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$ so $V = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$. Let $u = \{a\}$ and $v = \{b\}$ are two vertices of singleton elements. Since $\{a\}$ is not subset of $\{b\}$ and $\{b\}$ is not subset of $\{a\}$ for all vertices of singleton element. Thus, u is adjacent with v. Then, the vertices of singleton element $\{\{1\}, \{2\}, \{3\}\}$ form a complete induced subgraph K_3 of G_τ. In similar prove above the set $\{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$ also form a complete induced subgraph K_3 of G_τ. So that, since every vertex of two elements such as $\{a, b\}$ is adjacent to only one vertex has singleton element say $\{c\}$ if $c \neq a$ and $c \neq b$. Where $\{a, b\}$ is not subset of $\{c\}$ and $\{c\}$ is not subset of $\{a, b\}$. Therefore, $G_\tau \cong \overline{C_6}$. □

Proposition 2.4: Let $|X| = n$ and G_τ be a discrete topological graph. Then, the graph G_τ has $n-1$ complete induced subgraphs K_t such that $t \geq n$.

Proof: Let S' be a set of all vertices of singleton elements such that $|S'| = n$ let u, $v \in S'$. Since u is not subset of v and v is not subset of u for all elements of S', then u is adjacent to v. Hence, $G[S']$ be a complete induced subgraph of order n, so that $G[S'] = K_n$. Let S'' be a set of all vertices have two elements where $|S''| = \binom{n}{2}$, let u_1, $u_2 \in S''$. Since u_1 is not subset of u_2 and u_2 is not subset of u_1 for all elements of S''. Thus, u_1 is adjacent with u_2. Then, $G[S'']$ be a complete induced subgraph of order $\binom{n}{2}$, so $G[S''] = K_{\binom{n}{2}}$ and so on. Also, let S^{n-1} be a set of all vertices of $n-1$ elements and for any v_1, $v_2 \in S^{n-1}$. In similar prove above we get as $G[S^{n-1}] = K_{\binom{n}{n-1}} = K_n$. Therefore, the graph G_τ has $n-1$ complete induced subgraphs. □

Proposition 2.5: Let $|X| = n$, then the order of discrete topological graph G_τ is $2^n - 2$.

Corollary 2.6: Let $|X| = n$, then the order of the topological graph G_τ is $\Sigma_{i=1}^{n-1} \binom{n}{i}$.

Proof: Let S_i, $i = 1, 2, 3, \ldots, n-1$, be a set of all vertices of i elements where the number of all vertices of i elements successively is $\binom{n}{i}$, $i = 1, 2, 3, \ldots, n-1$. As a result, the number of all vertices in the graph G_τ is the order of it, where $|G_\tau| = \sum_{i=1}^{n-1} \binom{n}{i}$. □

3. Some Parameters of Domination in Topological Graph

In this section, different types of domination are studied on the topological graph such as: co-independent domination, complementary tree domination, and bi-domination. So that, the inverse domination for all these types are studied.

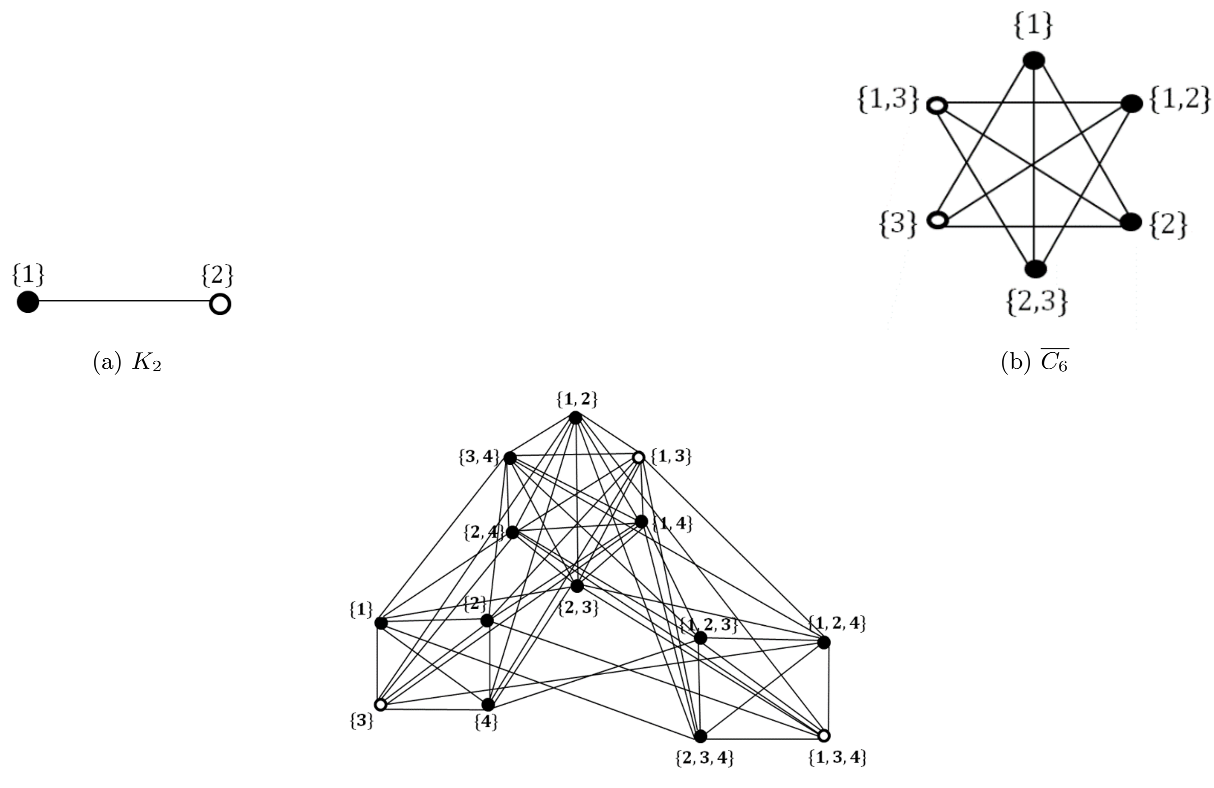
Observation 3.1: For any topological graph G_τ of order $2^n - 2$ has an co-independent domination. If $\gamma_{coi}(G_\tau) > \frac{2^n-2}{2}$, then G_τ has no inverse co-independent domination.

Observation 3.2: Let G_τ be a discrete topological graph of order $2^n - 2$ has complementary tree domination. If $\gamma_{ctd}(G_\tau) > \frac{2^n-2}{2}$ thus it has no inverse complementary tree domination.

Observation 3.3: Let G_τ be a discrete topological graph of order $2^n - 2$ has a bi-dominating set. If $\gamma_{bi}(G_\tau) > \frac{2^n-2}{2}$ thus it has no inverse bi-dominating set.

Theorem 3.4: Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has co-independent domination where $\gamma_{coi}(G_\tau) = \Sigma_{i=1}^{n-1}[\binom{n}{i} - 1]$.

Proof: Let S' be a set have all vertices of singleton element and have order $\binom{n}{1} = n$. Since $G[S'] = K_n$ was proofed of Proposition 2.4. Then, $\gamma_{coi}(S') = \binom{n}{1} - 1$, by Theorem 1.1. So, there is only one vertex from S' say u_1 belong to $V - D$. Again, let S'' be a set of all vertices have two elements and of order $\binom{n}{2}$. Since $G[S''] = K_{\binom{n}{2}}$ was proofed of Proposition 2.4. Then, $\gamma_{coi}(S'') = \binom{n}{2} - 1$, by Theorem 1.1. Thus, there is at least one vertex from S'' say u_2, and this vertex must be not adjacent with u_1 where $u_1 \subseteq u_2$. So, $u_2 \in V - D$, and so on. The last step if S^{n-1} be a set of all vertices have $n-1$ elements and of order $\binom{n}{n-1} = n$. In similar way to proof above, then $\gamma_{coi}(S^{n-1}) = \binom{n}{n-1} - 1$. Thus, there is at least one vertex from S^{n-1} say u_{n-1}. Where this vertex must be not adjacent with $u_1, u_2, u_3, \ldots, u_{n-2}$, this mean $u_1 \subseteq u_2 \subseteq u_3 \subseteq \cdots \subseteq u_{n-2} \subseteq u_{n-1}$. So that, $u_{n-1} \in V - D$. Hence, $V - D = \{u_1, u_2, u_3, \ldots, u_{n-1}\}$ and all vertices of it isolated. To prove D is a minimum dominating set let D' be a minimum dominating set such that $|D'| < |D|$. So, there is two or more vertices in $V - D$ are adjacent and this contrary with definition of co-independent domination. Therefore, D is a minimum co-independent dominating set where $\gamma_{coi}(G_\tau) = \Sigma_{i=1}^{n-1}[\binom{n}{i} - 1]$. As an example, see Figure 1. □



(a) K_2

(b) $\overline{C_6}$

(c)

Figure 1: The co-independent dominating set for $|X| = 2, 3, 4$

Corollary 3.5: Let G_τ be a discrete topological graph defined on a set X. Such that $|X| = n$, then G_τ has co-independent dominating set such as:

1. $\gamma_{coi}(G_\tau) = \Sigma_{i=1}^{n-1}\binom{n}{i} - n + 1$
2. $\gamma_{coi}(G_\tau) = 2^n - n - 1$.

Proof:

1. Since G_τ has $n-1$ a complete induced subgraph, by Proposition 2.4, and from proof of Theorem 3.4, each complete induced subgraph of it contains one vertex belong to $V - D$, and the remaining

vertices of it belong to D. So that, D has all vertices of G_τ unless $n-1$ vertices. Since the order of G_τ is $\Sigma_{i=1}^{n-1}\binom{n}{i}$ by Corollary 2.6. Then, $\gamma_{coi}(G_\tau) = \Sigma_{i=1}^{n-1}\binom{n}{i} - n + 1$.

2. From above proof since D has all vertices of G_τ unless $n-1$ vertices, and G_τ has order $2^n - 2$ by Proposition 2.5. Therefore, $\gamma_{coi}(G_\tau) = 2^n - n - 1$.

□

Proposition 3.6: Let $|X| = n(n > 2)$ and G_τ be a discrete topological graph. Then, G_τ has no inverse co-independent domination.

Proof: If $n = 2$, then $G_\tau \cong K_2$ by Proposition 2.2, and it is clear the inverse co-independent domination number of K_2 is one, where $\gamma_{coi}^{-1}(G_\tau) = 1$, as an example see Figure 2, and if $n > 2$, then G_τ has no inverse co-independent domination number. Since $\gamma_{coi}(G_\tau) > \frac{2^n-2}{2}$ according to Observation 3.1. □

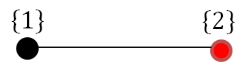


Figure 2: The inverse domination number of K_2. (Vertices {1} and {2} connected by an edge. One vertex is colored for the inverse dominating set.)

Proposition 3.7: Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has a complementary tree domination where:

$$\gamma_{ctd}(G_\tau) = \begin{cases} 1, & if\ n = 2 \\ 2, & if\ n = 3. \end{cases}$$

Proof: If $|X| = 2$, so $G_\tau \cong K_2$ by Proposition 2.2, and it is clear K_2 has complementary tree domination such that $\gamma_{ctd}(G_\tau) = 1$, as an example see Figure 1 (a). If $|X| = 3$, since the set $\{\{1\},\{2\},\{3\}\}$ form a complete induced subgraph K_3 from proof of Proposition 2.3, and $\gamma_{ctd}(K_3) = 1$ by Theorem 1.2. Thus, there is two vertices of it say $\{2\},\{3\}$ belong to $V - D$ and only one vertex of it say $\{1\}$ belong to D. Also, in similar way to prove above in a set $\{\{1,2\},\{1,3\},\{2,3\}\}$ such that there is two vertices of it belong to $V - D$, and only one vertex of it belong to D. So, to form a tree in $V - D$ Let $\{1,3\},\{2,3\}$ belong to $V - D$ and $\{1,2\} \in D$ or let $\{1,2\},\{2,3\}$ belong to $V - D$ and $\{1,3\} \in D$ Thus, in a set $V - D$ there is only one vertex of two elements say $\{1,3\}$ is adjacent with only one vertex of singleton element say $\{2\}$. Then, $V - D = \{\{2\},\{3\},\{1,3\},\{2,3\}\}$ form a tree, and $D = \{\{1\},\{1,2\}\}$ where the vertices of D are not adjacent, since $\{1\} \subseteq \{1,2\}$. Unlike this proof we will get $V - D$ is not form a tree. Hence, D is a minimum complementary tree dominating set and $\gamma_{ctd}(G_\tau) = 2$ As an example, see Figure 3 (a). □

Proposition 3.8: Let G_τ be a discrete topological graph defined on a set X of order n. Then, G_τ has an inverse complementary tree domination where:

$$\gamma_{ctd}^{-1}(G_\tau) = \begin{cases} 1, & if\ n = 2 \\ 2, & if\ n = 3. \end{cases}$$

Proof: By the same technique of proof of Proposition 3.7, then $\gamma_{ctd}^{-1}(G_\tau) = 1$ for $n = 2$, see Figure 2, and for $n = 3$ let $D^{-1} = \{\{3\},\{2,3\}\}$ so $V - D^{-1} = \{\{1\},\{2\},\{1,2\},\{1,3\}\}$. Where $V - D^{-1}$ form a tree and D^{-1} is a minimum inverse dominating. Then, $\gamma_{ctd}^{-1}(G_\tau) = 2$ As an example, see Figure 3 (b). □

Theorem 3.9: Let $|X| = n(n \geq 4)$ and G_τ be a discrete topological graph. Then, G_τ has a complementary tree domination where:

$$\gamma_{ctd}(G_\tau) = \Sigma_{i=1}^{n-1}\left[\binom{n}{i} - 2\right].$$

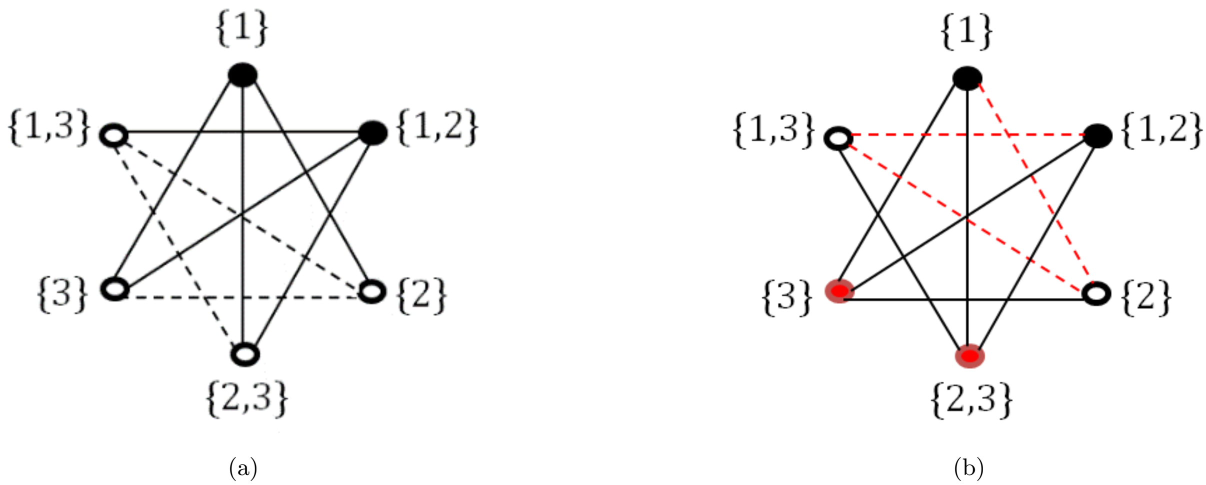


Figure 3: D and D^{-1} of complementary tree domination for $\overline{C_6}$. (a) $D = \{\{1\}, \{1,2\}\}$ is the complementary tree dominating set. (b) $D^{-1} = \{\{3\}, \{2,3\}\}$ is the inverse complementary tree dominating set.

Proof: Let S' be a set of all vertices of singleton element and have order $\binom{n}{1} = n$ Since $G[S'] = K_n$ from proof of Proposition 2.4. Then, $\gamma_{ctd}(S') = \binom{n}{1} - 2$, by Theorem 1.2. So, there is only two vertices from S' say u_1, $v_1 \in V - D$. Again, let S'' be a set of all vertices have two elements and have order $\binom{n}{2}$. Since $G[S''] = K_{\binom{n}{2}}$ from proof of Proposition 2.4. Then, $\gamma_{ctd}(S'') = \binom{n}{2} - 2$, by Theorem 1.2. Thus, there is only two vertices from S'' say u_2, $v_2 \in V - D$. Now, to consist the tree in $V - D$ We must choose only one vertex from a set $\{u_2, v_2\}$ say u_2 is adjacent with only one vertex from a set $\{u_1, v_1\}$ say u_1, where $u_1 \nsubseteq u_2 \wedge u_2 \nsubseteq u_1$. Also, let S''' be a set of all vertices of three elements and have order $\binom{n}{3}$, and in similar way to proof above there is only two vertices from S''' say u_3, $v_3 \in V - D$, and to consist the tree in $V - D$ We must choose only one vertex from a set $\{u_3, v_3\}$ say u_3 is adjacent with only one vertex from a set $\{u_2, v_2\}$ say u_2, where $u_2 \nsubseteq u_3 \wedge u_3 \nsubseteq u_2$ and so on. The last step assume that S^{n-1} is a set of all vertices of $n-1$ elements and have order $\binom{n}{n-1} = n$. Also, in similar way to proof above, there is only two vertices from S^{n-1} say u_{n-1}, v_{n-1} belong to $V - D$ Now, by the same technique of proof above let u_{n-2}, v_{n-2} be two vertices have $n-2$ elements and belong to $V - D$ and u_{n-2} is adjacent with v_{n-2}. So, to consist the tree in $V - D$. We must choose only one vertex from a set $\{u_{n-1}, v_{n-1}\}$ say u_{n-1} is adjacent with only one vertex from a set $\{u_{n-2}, v_{n-2}\}$ say u_{n-2} where $u_{n-2} \nsubseteq u_{n-1} \wedge u_{n-1} \nsubseteq u_{n-2}$. Then, $V - D = \{u_1, v_1, u_2, v_2, u_3, v_3, \ldots, u_{n-2}, v_{n-2}, u_{n-1}, v_{n-1}\}$ form a tree. To proof D is a minimum dominating set let D_1 is a minimum dominating set where $|D_1| < |D|$. Thus, there is three or more vertices are adjacent in $V - D$ and it will contain a cycle and this contrary with definition of tree. Then, $\gamma_{ctd}(G_\tau) = \Sigma_{i=1}^{n-1}[\binom{n}{i} - 2]$. As an example, see Figure 4 and Figure 5. □

Corollary 3.10: Let G_τ be a discrete topological graph defined on a set X, and $|X| = n (n \geq 4)$, then G_τ has a complementary tree domination such as:

1. $\gamma_{ctd}(G_\tau) = \Sigma_{i=1}^{n-1}\binom{n}{i} - 2n + 2$.

2. $\gamma_{ctd}(G_\tau) = 2^n - 2n$.

Proof:

1. By Proposition 2.4 since G_τ has $n-1$ complete induced subgraphs. Where each complete induced subgraph of it contains two vertices belong to $V - D$ and the remaining vertices of it belong to D from proof of Theorem 3.9. Thus, D has all vertices of G_τ unless $2(n-1)$ vertices. Since the order of G_τ is $\Sigma_{i=1}^{n-1}\binom{n}{i}$ by Corollary 2.6. Hence, $\gamma_{ctd}(G_\tau) = \Sigma_{i=1}^{n-1}\binom{n}{i} - 2n + 2$.

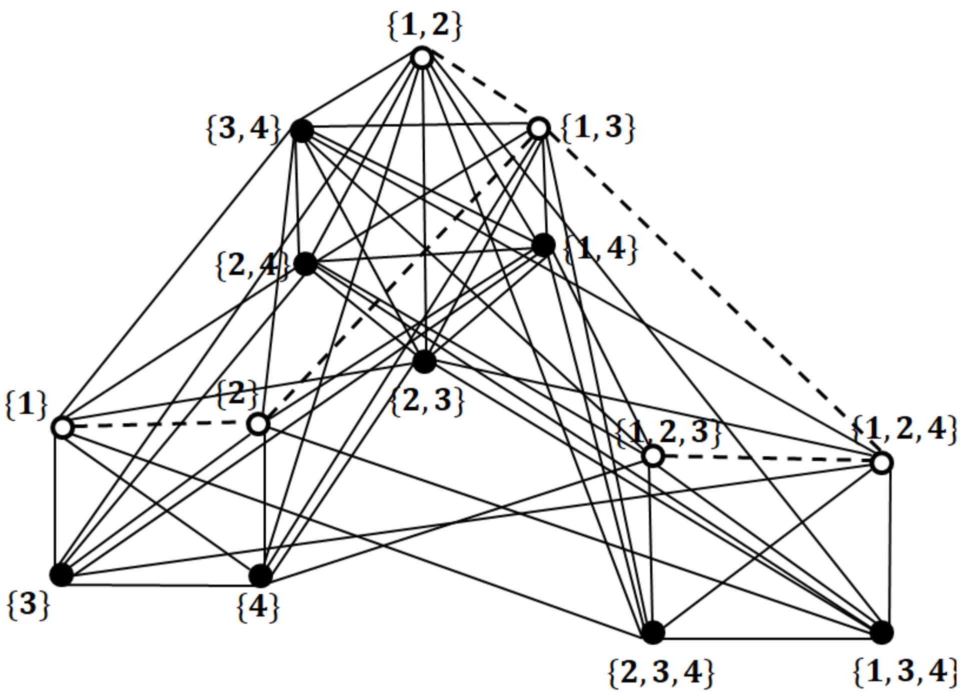


Figure 4: The complementary tree domination in G_τ, for $|X| = 4$. (Note: Black nodes form D, white nodes form $V - D$, and dashed edges are in $V - D$.)

2. From above proof since D has all vertices of G_τ unless $2(n-1)$ vertices, and the order of G_τ is $2^n - 2$, by Proposition 2.5. Then, $\gamma_{ctd}(G_\tau) = 2^n - 2n$.

□

Proposition 3.11: Let $|X| = n (n \geq 4)$ and G_τ be a discrete topological graph. Then, G_τ has no inverse complementary tree domination.

Proof: Since the order is $2^n - 2$ by Proposition 2.5, and $\gamma_{ctd}(G_\tau) = 2^n - 2n > \frac{2^n-2}{2}$. Then, it has no inverse complementary tree dominating set according to Observation 3.2. □

Proposition 3.12: Let $|X| = n$ and G_τ be a discrete topological graph. Then, G_τ has a bi-dominating set and $\gamma_{bi}(G_\tau) = 2$.

Proof: If $|X| = 2$, then $G_\tau \cong K_2$ by Proposition 2.2, let $D = \{1\}$ where $\{1\}$ dominates only one vertex. Hence, G_τ has no bi-dominating set, and for $|X| = 3$ let S' be a set of all vertices of singleton element. Then, from proof of Proposition 2.4, $G[S'] = K_3$. So, if we let any vertex of singleton element say $\{1\} \in D$. Then, this vertex dominates two vertices of them. Since each vertex of singleton element adjacent with only one vertex have two elements from proof of Proposition 2.3. So, $\{1\}$ is adjacent to one vertex have two elements and this vertex is only $\{2,3\}$, where $\{1\} \not\subseteq \{2,3\}$ and $\{2,3\} \not\subseteq \{1\}$. Thus, to achieve definition of bi-domination we must choose $\{1\}, \{2\} \in D$. Then, both $\{1\}$ and $\{2\}$ are dominate two different vertices. Therefore, $D = \{\{1\}, \{2\}\}$ and $\gamma_{bi}(G_\tau) = 2$. As an example, see Figure 6 (a). □

Proposition 3.13: Let $|X| = n$, then G_τ has inverse bi-dominating set and $\gamma_{bi}^{-1}(G_\tau) = 2$.

Proof: If $n = 2$, then it is clear G_τ has no inverse bi-domination. If $n = 3$, by the same technique of proof of Proposition 3.12, let $D^{-1} = \{\{1,3\}, \{2\}\}$. Where both $\{1,3\}$ and $\{2\}$ are dominate two different vertices in $V - D^{-1}$. Then, $D^{-1} = \{\{1,3\}, \{2\}\}$ and $\gamma_{bi}^{-1}(G_\tau) = 2$. As an example, see Figure 6 (b). □

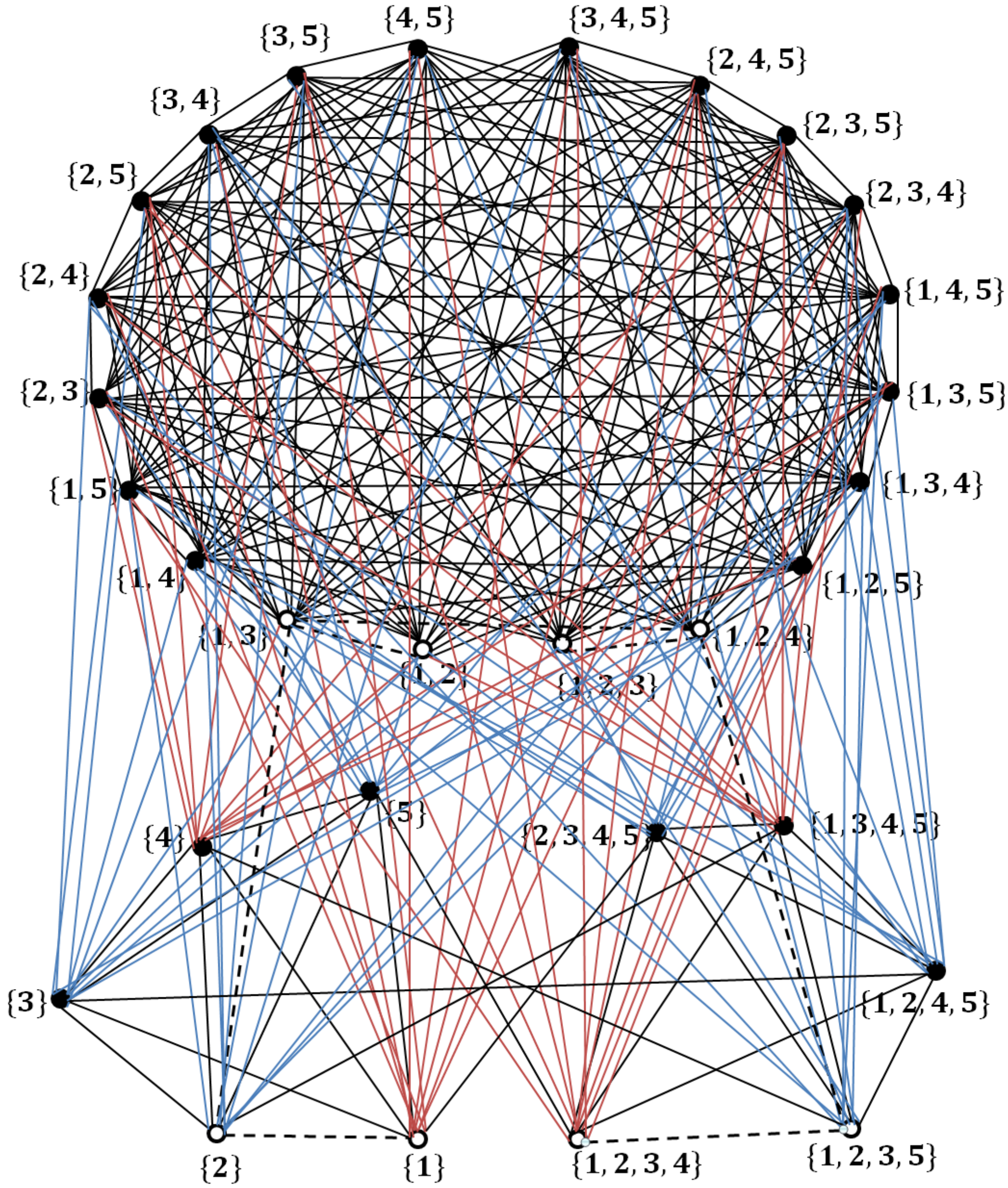


Figure 5: The complementary tree domination in G_τ, for $|X| = 5$.

Theorem 3.14: Let $|X| = n(n \geq 4)$ and G_τ be a discrete topological graph. Then, G_τ has bi-dominating set and $\gamma_{bi}(G_\tau) = \sum_{i=1}^{n-1} \binom{n}{i} - 4$.

Proof: Let $D = \{u, v\}$ such that u and v are two vertices having singleton element. Then, u and v having elements. Assume that $u = \{1\}$ and $v = \{2\}$, so $V - D = \{u_1, u_2, u_3, \ldots, u_{n-1}\}$ and $\gamma_{bi}(G_\tau) = \sum_{i=1}^{n-1} \binom{n}{i} - 2(n-1)$. Let S_i be a set of all vertices have i elements such that $|S_i| = \binom{n}{i}$ for all i. Let $u \in D$, where u be a vertex have i elements, and $u \nsubseteq v$ and $v \nsubseteq u$ for all $v \in V - D$. Thus, there are only two vertices in $V - D$ are not subset of u and u is not subset of them. $\square$

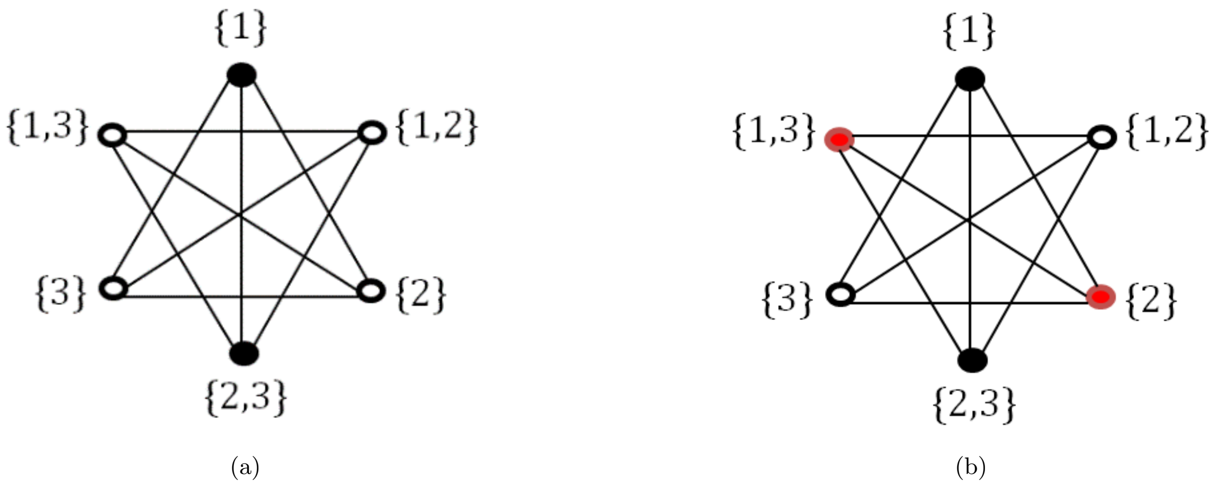


Figure 6: D and D^{-1} of bi-domination for $\overline{C_6}$. (a) $D = \{\{1\},\{2\}\}$ is the bi-dominating set. (b) $D^{-1} = \{\{1,3\},\{2\}\}$ is the inverse bi-dominating set.

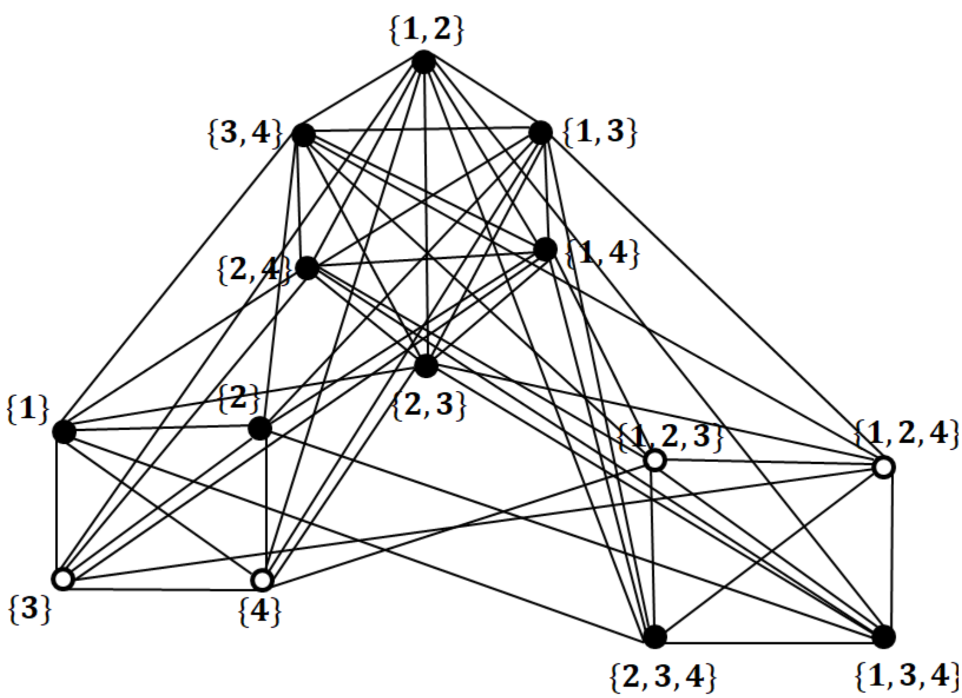


Figure 7: A minimum bi-dominating set in G_τ, where $|X| = 4$. (Note: Black nodes form D, white nodes form $V - D$.)

Because there exist one or more elements in u and this elements are not exist in only two vertices of $V - D$. Then, u is adjacent with only two vertices from $V - D$. To be D is bi-dominating set must be for all $u \in D$. Where u dominates only two vertices in $V - D$. To prove D is a minimum bi-dominating set, let D_1 is a minimum bi-dominating set where $|D_1| < |D|$. So, there is at least one vertex in D_1 dominates three or more vertices in $V - D_1$, and this contrary with definition of bi-domination. From prove above we get as D has all vertices of G_τ unless four vertices of $V-D$. Since the order of G_τ is $\sum_{i=1}^{n-1}\binom{n}{i}$ by Corollary 2.6. Then, $\gamma_{bi}(G_\tau)=\sum_{i=1}^{n-1}\binom{n}{i}-4$. As an example, see Figure 7.

Corollary 3.15: Let G_τ be a discrete topological graph defined on a set X, and $|X|=n(n\geq 4)$, then G_τ has bi-domination where $\gamma_{bi}(G_\tau)=2^n-6$.

Proof: From proof of Theorem 3.14, since D has all vertices of G_τ unless four vertices of $V-D$, and the order of G_τ which is 2^n-2, by Proposition 2.5. Thus, $\gamma_{bi}(G_\tau)=2^n-6$. □

Proposition 3.16: Let $|X|=n(n\geq 4)$ and G_τ be a discrete topological graph. Then, G_τ has no inverse bi-dominating set.

Proof: Since the order of G_τ is 2^n-2 by Proposition 2.5 and $\gamma_{bi}(G_\tau)=2^n-6>\frac{2^n-2}{2}$, then by Observation 3.3. The graph has no inverse bi-domination. □

4. Conclusions

Three types of domination parameters with their inverse are applied on the topological graph. Several properties of the topological graph are introduced.

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[1] *College of Education for Women, Shatrah University, Thi-Qar, 64001 Iraq.*
E-mail address: mmhd@shu.edu.iq

and

[2] *Department of Mathematics, College of Education for Pure Sciences, University of Thi-Qar, Thi-Qar, Iraq.*
E-mail address: Haifaa_muhsan@utq.edu.iq, lkalzaki@utq.edu.iq, zainab78_neem.math@utq.edu.iq

and

[3] *Department of Computer Science and Engineering, National Institute of Technology, Tiruchirappalli, Tamilnadu, India.*
E-mail address: pavan@nitt.edu