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A Comparative Analysis of Reliability and Availability of a Degraded Three-Unit Redundant System Under Various Distribution

Diksha Mangla, Shilpa Rani *, Shakuntla Singla†, Savita Garg

ABSTRACT: For current high reliability systems, particularly those with redundancy and repair, predicting system performance over time is of prime importance. The classical reliability modeling by means of the exponential distribution often does not reflect realistic failure and up-repair behavior. A comparative study of three statistical distributions such as Exponential, Weibull and Gamma to estimate the system reliability and availability under different transition states has been discussed for a series type three-units system, which operates based on degraded unit concept and the presence of a stand-unit. The degradation of a unit takes place in two modes, firstly degraded to 70-75% and then to 50%. A detailed state-transition model is developed, incorporating full working, degraded, and failed states, with transitions governed by failure and repair rates. Each distribution is applied to model time-dependent behaviours of system components, including transitions from full functionality to standby, degradation, and complete failure. The exponential distribution, mathematically convenient as it is, requires constant failure rate and it often produces unreasonably low reliability predictions. Weibull distribution includes shape and scale as parameters in order to include the aspect of aging and repair and predict a more gradual and realistic degradation curve. The accumulated failure times are modelled by a Gamma distribution and the framework is extended to systems with multiple series failure processes. Overall, this study demonstrates that while exponential models are computationally efficient, they are inadequate for capturing complex system dynamics. The Weibull distribution offers a balance between realism and tractability, making it the preferred approach for reliability engineering in systems with repair, redundancy, and aging components. The insights gained from this comparison can guide the selection of appropriate modelling techniques for accurate performance assessment of safety-critical systems.

Keywords: Reliability, availability, exponential distribution, Gamma and Weibull distribution, degraded units and preventive maintenance.

Contents

1 Introduction **2**

2 Model Description and Notation **3**

3 Mathematic Modelling **4**

4 Availability and Reliability Analysis with Various Distribution **5**
4.1 Availability Analysis . . . 5
4.1.1 Exponential distribution . . . 5
4.1.2 Weibull distribution: . . . 6
4.1.3 Gamma distribution . . . 7
4.2 Reliability Analysis . . . 8
4.2.1 Exponential distribution: . . . 8
4.2.2 Weibull distribution: . . . 9
4.2.3 Gamma distribution . . . 9

5 Comparative Analysis **10**

6 Result Discussion **12**

7 Conclusion **12**

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1. Introduction

In reliability engineering, evaluating the long-term performance of complex systems is essential for operational safety, cost-effectiveness, and customer satisfaction. Systems such as communication networks, medical equipment, industrial automation, and defense platforms are expected to function reliably over extended periods, often under varying conditions. These systems typically include redundancy (standby components) and repair mechanisms to mitigate the effects of failures and maintain service continuity. To accurately model the behaviour of such systems, it is important to understand how individual components fail and recover over time. This modelling is typically done using probabilistic distributions that represent time-to-failure and time-to-repair characteristics. Among the most commonly used are the Exponential, Weibull, and Gamma distributions. The Exponential distribution is widely known for its simplicity and mathematical convenience. It assumes a constant failure rate, which makes it analytically tractable but often unrealistic especially in systems, where, failure likelihood changes with age or usage. The Weibull distribution, on the other hand, introduces shape and scale parameters to model increasing or decreasing failure rates, offering a more flexible and accurate representation of aging or wear-out processes. Lastly, the Gamma distribution is effective in modelling accumulated failure time from multiple independent events, capturing the impact of compounding risks across a system.Pinho et al. [1] has study introduces the Gamma-Exponentiated Weibull distribution, encompassing several well-known lifetime models. Key properties such as moments, entropies, and order statistics are derived. The model's effectiveness is demonstrated through maximum likelihood estimation and application to real failure time data. Cordeiro et al. [2] study introduces the exponential–Weibull distribution, a flexible three-parameter lifetime model encompassing several known distributions. It explores key mathematical properties, including moments, mean deviations, and generating functions. Parameter estimation via maximum likelihood and real data application demonstrate the model's effectiveness and adaptability. Das. [3] study presents a CMS design approach that selects processing routes to maximize system reliability while minimizing cost, enhancing overall system performance. It accounts for various machine failure characteristics (increasing, decreasing, or constant rates) and includes rerouting to handle breakdowns. A practical example demonstrates the model's effectiveness. Jain et al. [4] study models a fault-tolerant system with active and standby components using a Markovian machine repair approach under imperfect recovery and server interruptions. Performance indices are derived via Laplace transforms and matrix methods, with cost optimization performed using direct search and PSO. Results highlight the model's effectiveness in achieving optimal reliability and cost-efficiency. Singla et al. [5] has study Recent technological advancements have made it difficult to maintain complex industrial systems efficiently. Reliability, availability, and maintainability (RAM) are key indicators used to assess system performance under defined conditions. Singla et al. [6] focused overall four metaheuristic algorithms—GSA, SCA, GWO, and RGA—for parameter estimation in Software Reliability Growth Models (SRGMs). RGA and GWO demonstrated superior accuracy and better R^2 values compared to others. The results confirm their effectiveness in handling complex software reliability optimization tasks. Singla et al. [7] This study evaluates five algorithm based on metaherustic approach and identifies the Coati Optimization Algorithm (COA) as the most effective for reliability and cost optimization. COA outperforms MFO, WOA, DA, and GOA by offering faster, more efficient solutions for complex distributed system architectures. The findings highlight COA's potential for enhancing reliability while minimizing costs. Singla et al. [8] has study analyzes a two-unit system's performance using reliability metrics such as working itime before a failure and total working performance of system under varying failure and repair rates. By applying Particle Swarm Optimization and Markov-based modeling, it optimizes profit and system efficiency through preventive maintenance. The findings highlight that well-maintained systems significantly enhance reliability and reduce operational costs. Gupta et al. [9] has studied a exponential distributed function worked with three parameter, a special case of the exponentiated Weibull model, capable of modeling both increasing and decreasing hazard rates. The authors demonstrate its superior fit over gamma and Weibull models using a real dataset. Mohammed et al. [10] study proposes a model worked withree components under combination of three distribution i.e. Exponential, Gamma, and Weibull to analyze heterogeneous survival data. Using the EM algorithm, the model parameters were estimated and compared with individual and mixture models based on LL and AIC criteria. Results show the proposed model offers superior fit and flexibility, making it a better choice for modeling heterogeneous survival data. Ranjan et al. [11] study Bayesian competing risk model based on the minimum of gamma (aging) and exponential (accidental) failure times, using weak priors and MCMC methods, including the Gibbs sampler. Model performance is evaluated through simulated data, showing the model's ability to capture both failure types effectively. Comparative analysis with Weibull-exponential models confirms the proposed model's suitability and robustness. Raqab et al. [12] study explores Bayesian estimation and prediction for the generalized exponential (GE) distribution, suitable for modeling skewed nonnegative data. Using informative priors, importance sampling, and MCMC methods (Gibbs and Metropolis), the authors estimate parameters and reliability functions. The approach is validated with two real data sets, demonstrating the model's practical applicability. An ABC algorithm has been applied to optimize the maximum efficiency of computer devices by Anu et al. [13].

This paper constructs a detailed multi-state system model, accounting for full, degraded, and failed states, each governed by different failure and repair transitions. By applying each of the three distributions to this system, we evaluate and compare their effectiveness in predicting availability and reliability over time. Through analytical expressions and numerical simulations, we show how the choice of distribution significantly impacts performance metrics. The goal is to guide system designers in selecting the most appropriate model for accurate and realistic system behaviour forecasting. The complete paper is divided into 7 sections. The 1st and 2nd sections contain the introduction with some relevant review and model description. The third section includes the mathematical formulation of the presented model. The availability and reliability measures correspond to different distribution is obtained in section fourth. The comparison is made in section fifth. The section six and seven includes the result discussion and conclusion about the study.

2. Model Description and Notation

The studied model worked with three components, in which a stand by is used with 1st component i.e. A and the third component undergo two types of degradation which result in the reducing efficiency of performance. Component B worked with a perfect repair policy without any degradation. The preventive maintenance concept is implemented in two degraded states: D_1 (first degraded state) and D_2 (second degraded state).

Following are the states of the system:

S_0 = ABD, S_1=(A)BD, S_2=ABD_1,

S_3 = ABD_2, S_4= (A)BD_1, S_5=(A)BD_2,

S_6=(A)bD, S_7=AbD,

S_8=(A)bD_1, S_9=AbD_1, S_{10}=AbD_2,

S_{11}=(A)bD_2, S_{12}=(A)Bd, S_{13}=ABd

Notation:

A/a →Full working unit / Malfunctioned unit.

(A) →Stand by unit.

α → Malfunction time from S_0 to S_1 and A to (A).

α_2 → Malfunction time from B to b.

α_3 / α_4 → Malfunction time from D to D_1, and D_1 to D_2 respectively.

α_5 → Malfunction time from D_2 to d.

β → Repair rate from S_1 to S_0 and (A) to A.

β_2 → Repair rate from b to B.

β_3 / β_4 / β_5→ Repair rate from D_1 to D, D_2 to D_1, and d to D respectively.

A(t) → Availability of the system.

R(t) → Reliability of the system.

The state transition diagram based on the description is presented in figure 1.

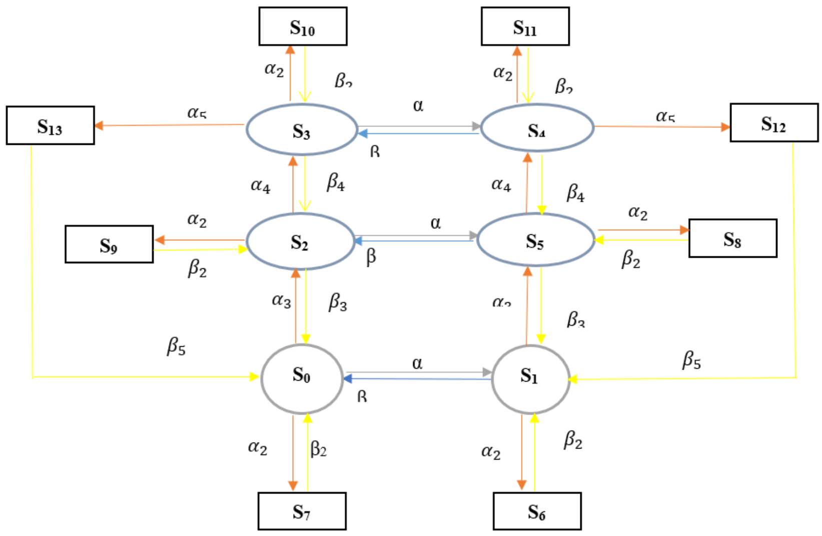

Figure 1: State transition Diagram for three-unit redundant system

3. Mathematic Modelling

The Markov birth process concept is taken into consideration when designing the mathematical representation of the arrangement that is being presented. The mnemonic rule is used to create the first-order Chapman-Kolmogorov differential equations that correspond to the transition diagram and yield the reliability parameters. The probability that the system will be presented in state S_i at time $t \geq 0$ is defined as $P_i(t)$ and with a beginning condition.

$$P_i = \begin{cases} 1, & \text{if } i = 0 \\ 0, & \text{if } i \neq 0 \end{cases}$$

$$\frac{dP_i(t)}{dt} = \sum_{j \neq i} P_j(t) \cdot q_{j \to i} - P_i(t) \cdot \sum_{j \neq i} q_{i \to j} \tag{3.1}$$

Where

$$\frac{dP_i(t)}{dt} = \text{Change in the probability of being in state } S_i \text{ over time.}$$

Where:

$$P_i(t) = \text{Probability that the system is in state } S_i \text{ at time } t.$$

$$q_{i \to j} = \text{Transition rate from state } i \text{ to } j.$$

$$\frac{dP_0(t)}{dt} = \beta P_1(t) + \beta_3 P_2(t) + \beta_5 P_{13}(t) - P_0(t)(\alpha + \alpha_2 + \alpha_3) \tag{3.2}$$

$$\frac{dP_1(t)}{dt} = \alpha P_0(t) + \beta_3 P_5(t) + \beta_2 P_6(t) + \beta_5 P_{12}(t) - P_1(t)(\beta + \alpha_2 + \alpha_3) \tag{3.3}$$

$$\frac{dP_2(t)}{dt} = \alpha_3 P_0(t) + \beta P_5(t) + \beta_2 P_9(t) + \beta_4 P_3(t) - P_2(t)(\beta_3 + \alpha + \alpha_2 + \alpha_4) \tag{3.4}$$

$$\frac{dP_3(t)}{dt} = \alpha_4 P_2(t) + \beta P_4(t) + \beta_2 P_{10}(t) - P_3(t)(\beta_4 + \alpha + \alpha_2 + \alpha_5) \tag{3.5}$$

$$\frac{dP_4(t)}{dt} = \beta_2 P_{11}(t) + \alpha P_3(t) + \alpha_4 P_5(t) - P_4(t)(\beta_4 + \beta + \alpha_2 + \alpha_5) \tag{3.6}$$

$$\frac{dP_5(t)}{dt} = \alpha P_2(t) + \alpha_3 P_1(t) + \beta_4 P_4(t) + \beta_2 P_8(t) - P_5(t)(\beta + \beta_4 + \alpha_2 + \beta_3) \tag{3.7}$$

$$\frac{dP_6(t)}{dt} = \alpha_2 P_1(t) - \beta_2 P_6(t) \tag{3.8}$$

$$\frac{dP_7(t)}{dt} = \alpha_2 P_0(t) - \beta_2 P_7(t) \tag{3.9}$$

$$\frac{dP_8(t)}{dt} = \alpha_2 P_5(t) - \beta_2 P_8(t) \tag{3.10}$$

$$\frac{dP_9(t)}{dt} = \alpha_2 P_2(t) - \beta_2 P_9(t) \tag{3.11}$$

$$\frac{dP_{10}(t)}{dt} = \alpha_2 P_3(t) - \beta_2 P_{10}(t) \tag{3.12}$$

$$\frac{dP_{11}(t)}{dt} = \alpha_2 P_4(t) - \beta_2 P_{11}(t) \tag{3.13}$$

$$\frac{dP_{12}(t)}{dt} = \alpha_5 P_4(t) - \beta_5 P_{12}(t) \tag{3.14}$$

$$\frac{dP_{13}(t)}{dt} = \alpha_5 P_3(t) - \beta_5 P_{13}(t) \tag{3.15}$$

4. Availability and Reliability Analysis with Various Distribution

4.1. Availability Analysis

4.1.1. Exponential distribution. Availability is the probability that the system is fully working or degraded

$$A(t) = \sum_{i \in \{0,1,2,3,4,5\}} P_i(t) = P_0(t) + P_1(t) + P_2(t) + P_3(t) + P_4(t) + P_5(t) \tag{4.1}$$

With the normalized condition

$$\sum_{i=0}^{13} P_i(t) = 1$$

setting the derivative of equations (3.2) to (3.15) equal to 0, and calculating the equations to find the values of $P_i(t)$ for $i = 0$ to 13.

After solving we get long term availability:

$$A(\infty) = \frac{\beta\alpha(\beta_3 + \beta_4)(\alpha_4 + \beta_2) + \alpha_3\beta_3(\alpha + \beta_2)(\alpha_4 + \beta_4)}{D} \tag{4.2}$$

$$D = (\alpha + \alpha_2 + \alpha_3)(\beta + \alpha_2 + \alpha_3)(\alpha + \alpha_2 + \alpha_4 + \beta_3)(\alpha + \alpha_2 + \alpha_5 + \beta_4) \tag{4.3}$$

The availability using the exponential distribution for the values $\alpha_2 = 0.1$, $\beta_2 = 0.1$, $\alpha_3 = 0.2$, $\beta_3 = 0.2$, $\alpha_4 = 0.3$, $\beta_4 = 0.3$, $\alpha_5 = 0.4$, $\beta_5 = 0.4$ is given in Table 1 by varying the parameters α and β.

The availability is affected by various failure and repair rates, represented by Table 1, and its variation is shown in Figure 2. The figure shows that increasing failure rates cause a decrease in availability, while increasing repair rates lead to an increase in availability.

Table 1: The availability analysis vs. failure and repair rates.

α	$\beta = 0.55$	$\beta = 0.65$	$\beta = 0.75$	$\beta = 0.85$	$\beta = 0.95$
0.55	0.067845	0.069477	0.070799	0.071891	0.072809
0.65	0.061151	0.062661	0.063884	0.064894	0.065742
0.75	0.055099	0.056485	0.057608	0.058535	0.059314
0.85	0.049729	0.050998	0.052026	0.052875	0.053588
0.95	0.045003	0.046164	0.047105	0.047881	0.048534

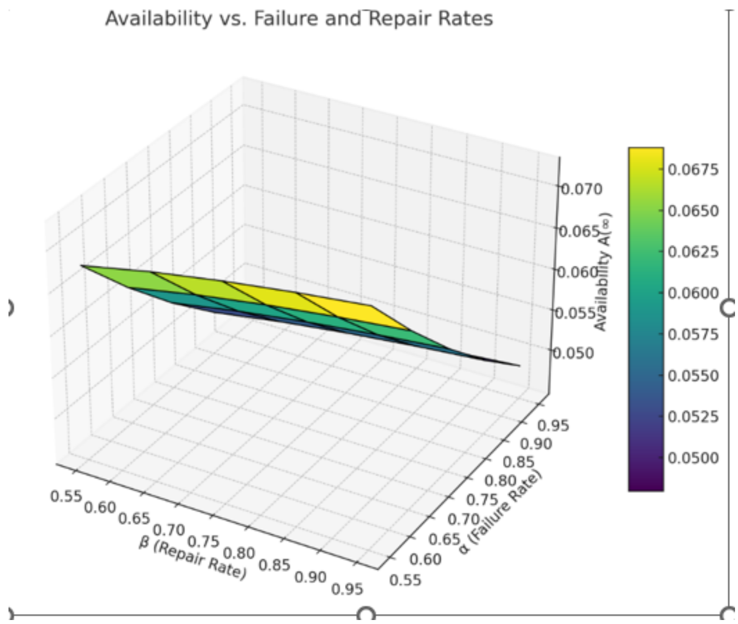

Figure 2: The availability vs. failure (α) and repair rates (β)

4.1.2. Weibull distribution: Let all failure or repair rates are of type Weibull hazard function:

$$h(t) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} \tag{4.4}$$

Where $h(t)$ denotes the failure rate, degraded rate, or repair rate.

where k = shape parameter and λ = scale parameter. The values for the shape parameter k and the scale parameter λ are selected based on how each part behaves in real life.

$\alpha : A \to (A)$ — This failure becomes more likely over time, so $k = 1.5$.
$\alpha_2 : B \to b$ — B fails slowly over time, so a moderate $k = 1.3$ is used.
$\alpha_3 : D \to D1$ — D degrades earlier, so $k = 1.2$ with shorter life.
$\alpha_4 : D1 \to D2$ — As degradation increases, failure speeds up ($k = 1.5$).
$\alpha_5 : D2 \to d$ — Final failure occurs faster ($k = 1.4$).
$\beta : (A) \to A$ — A is quickly repaired, so $k = 2.0$ and a small $\lambda = 50$.
$\beta_2 : b \to B$ — Repair is slightly slower, $k = 1.8, \lambda = 60$.
$\beta_3 : D1 \to D$ — Fast recovery, $k = 2.0, \lambda = 40$.
$\beta_4 : D2 \to D1$ — Very quick fix, so $k = 2.2, \lambda = 30$.
$\beta_5 : d \to D$ — Recovering from total failure is a bit slower, so $k = 1.9, \lambda = 35$.

These values are chosen to match how each part might realistically fail or be fixed based.

Availability is affected by the passage of time. The variation in availability with respect to time, in the range 0 to 1000 hours, is presented in Table 2.

The curve shown in Figure 3 indicates that the system starts with full availability (1.0). It dips slightly

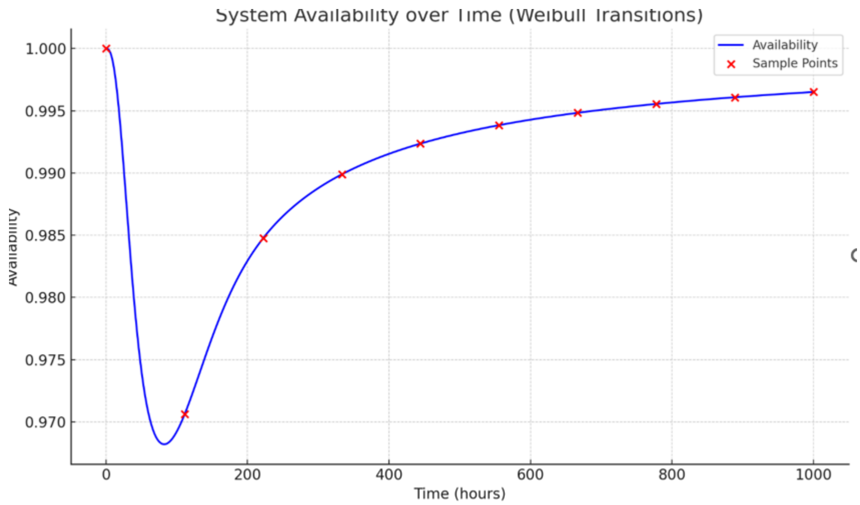

Figure 3: Availability vs. time (hours) under Weibull distribution.

due to early failures, then rebounds and stabilizes as a result of repairs modeled by Weibull-distributed rates. Eventually, An availability settles around 0.9965, demonstrating high long-term system availability.

Table 2: The availability vs. time.

Time (hours)	Availability
0	1.0000
111.11	0.9706
222.22	0.9848
333.33	0.9899
444.44	0.9924
555.56	0.9938
666.67	0.9948
777.78	0.9955
888.89	0.9961
1000	0.9965

4.1.3. Gamma distribution. Time-Dependent Availability Equation

In the Gamma distribution model, availability at time t accounts for all failure and repair transitions in the system. Let:

- $F = \{\alpha, \alpha_2, \alpha_3, \alpha_4, \alpha_5\} \rightarrow$ failure transitions
- $R = \{\beta, \beta_2, \beta_3, \beta_4, \beta_5\} \rightarrow$ repair transitions
- The generalized availability expression is:

$$A(t) = \frac{\text{Operational Time}}{\text{Total Time}} \tag{4.5}$$

$F_{\alpha_i}(t) = \text{Gamma CDF}(t; k_i, \lambda_i)$: cumulative distribution function for failure α_i

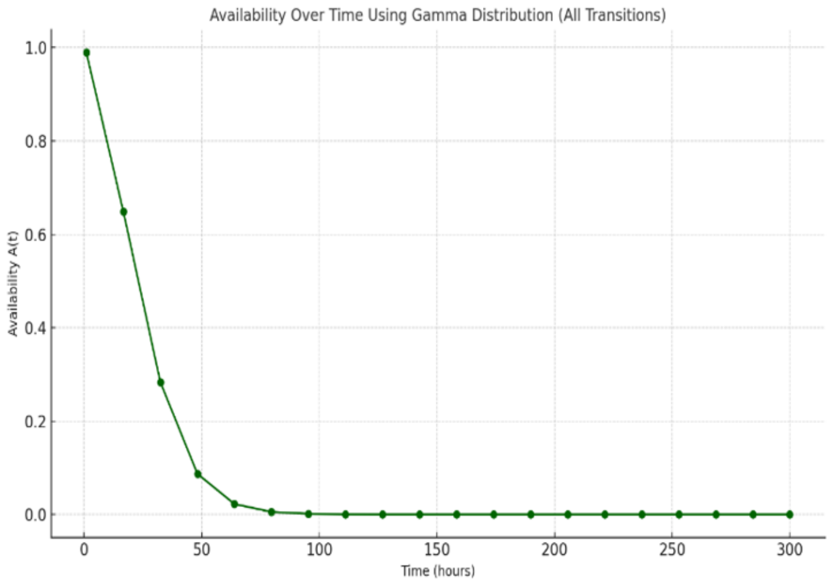

Figure 4: The variation in availability w.r.t time.

$R_{\beta_j}(t) = 1 - F_{\beta_j}(t)$: survival function for repair β_j

$$A(t) = \frac{\prod_{\beta_j \in R} \left(1 - F_{\beta_j}(t)\right)}{\prod_{\beta_j \in R} \left(1 - F_{\beta_j}(t)\right) + \sum_{\alpha_i \in F} F_{\alpha_i}(t)} \tag{4.6}$$

Table 3: The time vs. availability $A(t)$.

Time (hours)	Availability $A(t)$
1.00	0.989995
16.74	0.648706
32.47	0.283180
48.21	0.086840
63.95	0.022175
79.68	0.005213
95.42	0.001169
111.16	0.000253
126.89	0.000053
142.63	0.000011
158.37+	$\approx$ 0.000000

The variation in availability with the time period is given by Table 3 and depicted graphically by Figure 4

4.2. Reliability Analysis

4.2.1. Exponential distribution: If the system failure rate is constant and denoted as λ_{sys}, the reliability is:

$$R(t) = e^{-\lambda_{\text{sys}} t} \tag{4.7}$$

Both $S_0 = ABD$ and $S_1 = (A)BD$ are fully working states (i.e., standby A provides uninterrupted operation). Therefore, the system's reliability must include both states.

Let the total failure rate from both S_0 and S_1 to degraded or failed states be:

$$\lambda_{\text{sys}} = \alpha_2 + \alpha_3 \tag{4.8}$$

Then, reliability is:

$$R(t) = e^{-(\alpha_2+\alpha_3)t} \tag{4.9}$$

with α_2=0.1, α_3=0.2: $R(t) = e^{-0.3t}$

4.2.2. Weibull distribution: Let the system survive as long as it remains in S_0 or S_1. The cumulative Weibull-based reliability is:

$$R(t) = e^{-\left(\left(\frac{t}{\lambda_2}\right)^{k_2}+\left(\frac{t}{\lambda_3}\right)^{k_3}\right)} \tag{4.10}$$

Using α_2: $k_2 = 1.3,\ \lambda_2 = 120$

$$\alpha_3: \quad k_3 = 1.2,\ \lambda_3 = 80 \tag{4.11}$$

$$R(t) = e^{-\left(\left(\frac{t}{120}\right)^{1.3}+\left(\frac{t}{80}\right)^{1.2}\right)} \tag{4.12}$$

Table 4: Reliability of the system w.r.t. time under Exponential and Weibull distribution

Time (hours)	**Exponential** $R(t)$	**Weibull** $R(t)$
0.00	1.000000	1.000000
33.11	0.000049	0.585996
66.22	0.000000	0.284010
99.33	0.000000	0.125094
132.44	0.000000	0.051408
166.56	0.000000	0.019405
199.67	0.000000	0.007189
232.78	0.000000	0.002557
265.89	0.000000	0.000877
300.00	0.000000	0.000281

The reliability of the system decreases with time, which is presented in Figure 5 and Table 4. Weibull Curve shows how reliability gradually decreases as time increased and it reflects a realistic aging process. At the beginning ($t = 0$), the system is fully reliable. Over time, the probability of staying in working states S_0 or S_1 reduced.The rate of decline accelerates due to the increasing failure rate in Weibull distribution.

The exponential curve drops sharply in the early hours and flattens near zero. It assumes a constant failure rate at all times, regardless of age or usage. Because of that, reliability becomes very low quickly, even when that may not be realistic. Comparison in both. The Weibull model better reflects real system behavior (especially with standby like in S_1), as it captures progressive degradation. The Exponential model is simpler but tends to underestimate reliability for systems with redundancy or repair strategy.

4.2.3. Gamma distribution. The system is composed of multiple failure transitions: F = α, α_2, α_3, α_4, α_5

Assuming these are independent failure processes, the overall system reliability is the product of the survival functions of each failure transition:

$$R(t) = \prod_{i\in F} [1 - F_i(t)] = \prod_{i\in F} \left[\frac{\Gamma(k_i,\, t/\lambda_i)}{\Gamma(k_i)}\right] \tag{4.13}$$

$$F_i(t) = \frac{1}{\Gamma(k_i)} \int_0^{t/\lambda_i} x^{\,i} e^{-x}\, dx \tag{4.14}$$

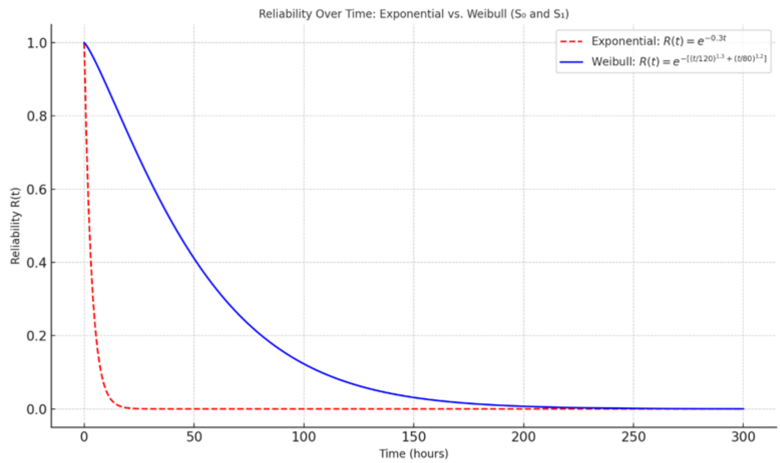

Figure 5: The reliability of the system with time under exponential and Weibull distribution.

This gives the complete mathematical expression:

$$R(t) = \prod_{i=1}^{5} \frac{\Gamma(k_i,\, t/\lambda_i)}{\Gamma(k_i)} \tag{4.15}$$

k_i and λ_i are the shape and scale parameters for each α_i transition.
$\Gamma(k_i,\, \mathrm{t}/\lambda_i)$: upper incomplete gamma function
The product runs over all failure transitions in the system.

$$R(t) = \frac{\Gamma\left(1.5,\, \frac{t}{100}\right)}{\Gamma(1.5)} \cdot \frac{\Gamma\left(1.3,\, \frac{t}{120}\right)}{\Gamma(1.3)} \cdot \frac{\Gamma\left(1.2,\, \frac{t}{80}\right)}{\Gamma(1.2)} \cdot \frac{\Gamma\left(1.5,\, \frac{t}{90}\right)}{\Gamma(1.5)} \cdot \frac{\Gamma\left(1.4,\, \frac{t}{70}\right)}{\Gamma(1.4)} \tag{4.16}$$

The reliability decreases with passage of time under Gamma distribution as shown in figure 6.

5. Comparative Analysis

Each of these distributions have its own nature to perform its function. An exponential distribution is mathematically simple, assumes a constant failure rate, and usually Each of these distributions have its own nature and capabilities. The Exponential Distribution is mathematically simple, assumes a constant failure rate, and usually gives conservative estimates. The Weibull distibution allows for shape and scale parameters and hence, capable of representing systems with aging elements or changing failure rate. Contrarily, the Gamma distribution is quite fit to representing systems experiencing more than one and sequential or cumulative failure processes, enjoying the power of a wider mathematics structure.

This analysis emphasizes the useful consequences of distribution selection on system availability and reliability by applying each of these distributions to the same system model, which is defined by full, degraded, and failed states. By using this comparative method, we hope to determine which modeling approach best captures the dynamics of real-world systems and facilitates the best possible decision-making in applications that prioritize reliability. Table 5 describes how the three distributions compare in terms of the availability and dependability analysis of the model that is being provided.

The comparison based on the study that Weibull provides the most realistic availability and reliability modelling for system, especially due to standby units and progressive failures whereas Exponential is overly pessimistic and useful only for rough estimates. Gamma offers mathematical generality, but can underestimate availability and overestimate failure without careful calibration. The variation of reliability and availability with respect to time regarding to distribution exponential, Weibull and gamma is presented in Figure 7 to understand the comparison.

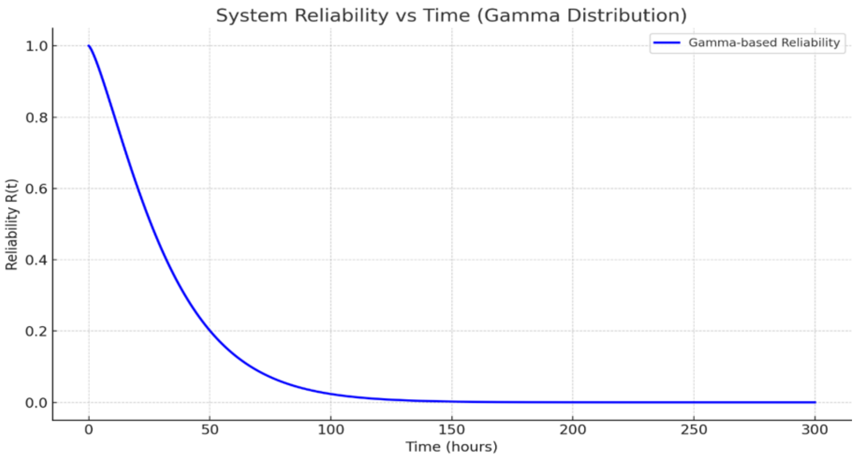

Figure 6: The Reliability vs. time under gamma distribution

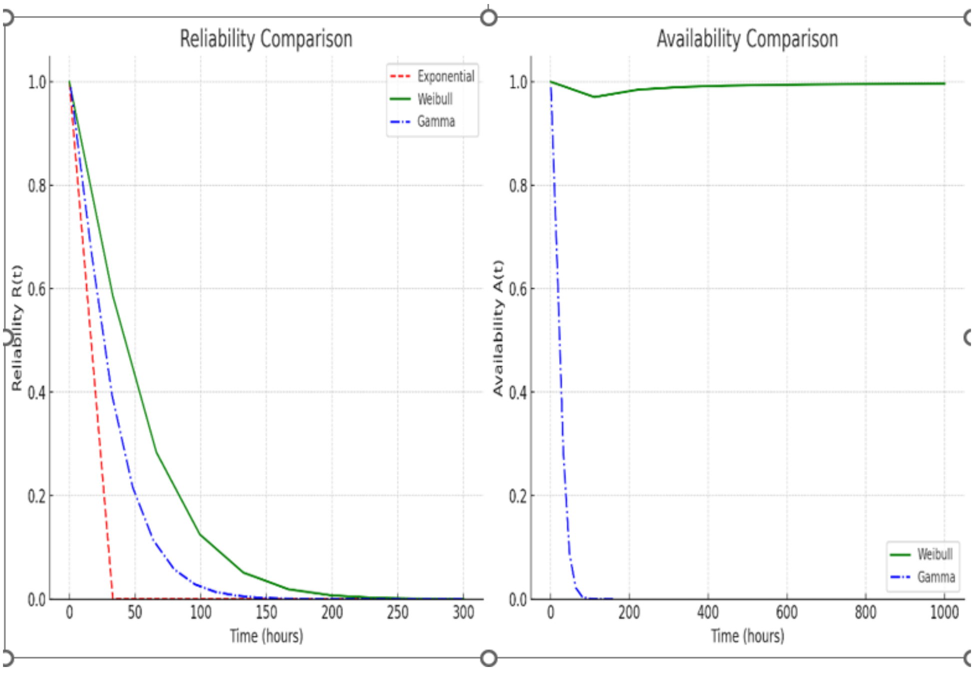

Figure 7: The variation of reliability and availability with respect to time regarding to distribution

Table 5: Comparison analysis of distributions corresponding to various rates, availability and reliability

Metric	Exponential	Weibull	Gamma
Failure Rate	Constant	Variable (time-based)	Variable (shape-based)
Initial Availability	High	High	High
Long-Term Availability	Low (0.01–0.05)	High (0.9965)	Near Zero (0)
Reliability Decay	Sharp & Fast	Gradual & Realistic	Moderate Decay
Best Use Case	Basic systems, short-term	Real-world aging systems	Systems with broad failure behaviours
Complexity	Low	Moderate	High

6. Result Discussion

The results obtained from modeling the availability and reliability of the degraded three-unit redundant system using Exponential, Weibull, and Gamma distributions reveal significant differences in the system performance predictions. Each distribution reflect the different failure and repair behaviour, and their impact on long-term availability and reliability is analyzed through numerical simulations and graphical representations. As observed in the availability plots, the Exponential distribution shows a rapid decline in availability due to its assumption of constant failure rates, making it suitable only for short-term or highly simplified systems. In contrast, the Weibull distribution demonstrates a more gradual and realistic availability curve, capturing the effects of aging and repair more accurately. It regulates high availability even over extended time periods, consistent with systems having effective standby units and repair mechanisms. Meanwhile, the Gamma distribution portrays moderate performance; although it reflects cumulative failure behaviour well, it tends to underestimate long-term availability, especially in highly reliable systems. The reliability results further confirm these trends. The Weibull model shows a smoother and more realistic decay, indicating its strength in modeling systems with progressive degradation. Whereas, An Exponential model predicts a very steep drop in reliability, while the Gamma model lies between the two, capturing sequential failure but less effectively representing recovery mechanisms. These outcomes highlight that, while the Exponential model offers analytical simplicity, it lacks accuracy in complex systems. The Weibull distribution proves to be the most realistic for systems with redundancy and repair, and the Gamma distribution, though mathematically general, requires careful parameter tuning for precise modelling.

7. Conclusion

The analysis focused on a complex three-unit redundant system incorporating standby support, repair mechanisms, and multiple degradation levels. The system's behaviour was governed by various failure and repair transitions, each influenced by time-dependent probabilistic parameters. These parameters were modeled using three different statistical distributions—Exponential, Weibull, and Gamma—to study their effect on system availability and reliability. The results show that the Weibull-based model achieves the highest long-term availability (0.9965) and provides a smooth, realistic reliability decay curve. In contrast, the exponential model predicts rapid system failure, while the Gamma model falls between the two, though it underestimates availability in long-term analysis. Graphical comparisons reinforce that Weibull modelling is more suitable for systems with standby units and progressive degradation. In conclusion, for systems involving standby components, repair mechanisms, and degradation stages, the Weibull distribution provides the best balance between realism and computational efficiency. It is recommended as the preferred choice for reliability engineers aiming to design and evaluate safety-critical systems with greater accuracy and confidence.

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