



Optimization of Reliability Indices of Repairable Systems Using RPGT and Nature-Inspired Algorithms

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ABSTRACT: Reliability of manufacturing system is a crucial determinant of their efficiency, safety and long-term availability. This study develops a comprehensive reliability model of a repairable system using the Regenerative Point Graphical Technique (RPGT). The system’s stochastic behavior is represented through state transition diagrams, from which governing equations are derived for evaluating reliability indices such as Mean Time to System Failure (MTSF), Steady-State Availability (A_0), and the Expected Number of Inspections (V_0). These measures are critical for assessing system performance under varying failure and repair rates. Further, To obtain optimal parameter values and improve system efficiency, three population-based metaheuristic techniques—Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Cuckoo Search Algorithm (CSA)—are applied. The optimization results are compared to highlight the relative strengths of each method. The findings indicate that CSA achieves better availability outcomes, while GA and PSO provide competitive results for MTSF and inspection measures. The proposed methodology demonstrates the potential of integrating RPGT-based reliability modeling with nature-inspired optimization algorithms to achieve improved performance evaluation and decision-making in repairable systems. This framework can be effectively extended to complex industrial and engineering applications where reliability and maintainability are critical.

Keywords: RPGT, reliability modeling, repairable systems, MTSF, availability, expected inspections, GA, PSO, CSA.

Contents

1 Introduction	1
2 Symbols and Nomenclature	3
3 Mathematical Formulation	3
3.1 Mean time to system failure (MTSF)	5
3.2 Availability of System	5
3.3 Expected number of inspections by the repair man	5
4 Optimization of Various Reliability Metrics Using Metaheuristic Algorithm	5
4.1 Procedure of CSA	6
4.2 Procedure of GA	6
4.3 Procedure of PSO	6
5 Result Analysis	6
6 Conclusion	9

1. Introduction

In modern industrial and engineering environments, repairable systems are widely used to ensure continuous operation despite frequent failures and uncertainties in repair. The performance of such systems depends on their ability to recover from failures quickly while maintaining high availability and efficiency. To analyze these behaviors, mathematical reliability models are developed that can evaluate key performance indices such as Mean Time to System Failure (MTSF), Steady-State Availability (A_0), and the Expected Number of Inspections (V_0). In this study, a repairable system model consisting of

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multiple subsystems is considered, where each unit is subject to random failures and repair activities. The system configuration is represented through a Regenerative Point Graphical Technique (RPGT), which uses state transition diagrams to capture possible system states (working, degraded, or failed) and the transitions between them based on failure rates (φ_i) and repair rates (λ_i). This modeling framework allows the derivation of governing equations for reliability indices, providing a comprehensive picture of the system's stochastic behavior. To further enhance the performance of the proposed model, nature-inspired optimization algorithms such as the Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and the Cuckoo Search Algorithm (CSA) are applied. These algorithms optimize the parameters of failure and repair processes to achieve the best trade-off between reliability and maintainability. A comparative analysis highlights the effectiveness of each algorithm in maximizing system availability, extending MTSF, and minimizing inspection requirements. The integration of RPGT-based modeling with optimization not only improves the accuracy of reliability evaluation but also demonstrates its potential for application in real-world industrial systems where downtime, maintenance costs, and safety are critical concerns. Singla et.al [1] examines a system composed of four units, where optimal performance is achieved when all units function, reduced capacity occurs with three operational units, and system failure happens if two or more units fail. Each unit has distinct failure and repair costs, and a single server is available continuously to restore failed units. Singla et.al [2] has study the reliability and profitability of a system with two active units and one backup unit. An external system manages maintenance and repairs, using a single server that can also fail. The supporting system oversees maintenance and repairs to keep operations continuous. The study models various reliability metrics to assess how failure and repair costs affect overall performance. Singla et.al [3] examines the reliability of a three-unit system with parallel subcomponents, where failure and repair rates differ based on operational states. The system's performance is analyzed using Regenerative Point Graphical Technique (RPGT) and optimized through deep learning methods like Adam and SGD. Fuzzy logic classifies system states, with graphical comparisons highlighting the impact of failure and repair rates. Singla et.al [4] study evaluates the structural interplay between reliability and warranty optimization in automated industries, where extended plant capacity enhances productivity yet demands substantial investment. It examines maintenance methodologies, fault analysis, and decision-making strategies for ensuring long-term system efficiency. Advanced modeling techniques like fault tree analysis and petri-nets assess system dependability across various industries. Singla et.al [5] has examines the reliability of a three-unit system and how preventive maintenance affects performance over time. Using the Markov process and Laplace transformation, key system parameters like failure time and profit are analyzed. Sensitivity analysis explores the impact of deterioration and repair rates on overall system efficiency. Singla et.al [6] has examines the behavior of a repairable 2-out-of-4 system, a configuration commonly used in engineering, by integrating evolutionary algorithms to optimize parameters such as redundancy allocation and maintenance scheduling. The proposed methodology provides insights for system designers and engineers to enhance reliability and availability. Singla et.al [7] has study the reliability of a poly-tube manufacturing system using a Markov-based model and particle swarm optimization (PSO). Failure and repair rates influence the RAM index, identifying critical components affecting system performance. Ordinary differential equations are used to analyze various operational states. To improve the working efficiency of the computer devices, an Artificial bee colony algorithm has been used to enhanced the work, studied by Thind et al. [8]. Singla et.al [9] has study the application of the Teaching-Learning-Based Optimization (TLBO) algorithm, a nature-inspired technique, to both constrained and unconstrained optimization problems. By addressing two distinct optimization scenarios, the research highlights TLBO's adaptability and effectiveness in solving diverse optimization challenges. John.et.al [10] has discussed Hardware and software reliability significantly impact device performance, with failures affecting each other. This study analyzes a multi-hardware–software system with different failure interactions, using exponential distributions and differential equations to derive key reliability metrics. Software fault prediction (SFP) is essential for maintaining quality in complex software systems and reducing maintenance costs. Z Dang [11] has study explores nature-inspired metaheuristic algorithms like particle swarm, genetic, and ant colony optimization for improving SFP. It compares approaches, highlights challenges, and suggests future research directions. Load balancing in cloud computing is a complex NP-hard problem affecting service quality and performance. J Zhou [12] study compares metaheuristic algorithms based on factors like makespan, response time, and resource utilization. Results

show that particle swarm optimization performs best in improving key performance metrics. HA Khorshidi [13] study optimizes reliability and cost in multi-state weighted k-out-of-n systems using a dynamic model. It compares genetic and imperialist competitive algorithms, showing GA achieves better solutions while ICA is faster. Parameter analysis of ICA is also conducted. Sharma. p [14] has study presents the Modified Grey Wolf Optimization (MGWO) algorithm for early detection of Parkinson's disease. Using feature selection with classifiers like random forest and decision tree, MGWO achieves 94.83 percent accuracy. Results show it outperforms the Optimized Cuttlefish Algorithm in accuracy and feature selection efficiency. SA Gorji[15] study analyzes the green hydrogen supply chain, covering production, storage, transportation, and consumption. It highlights challenges and explores metaheuristic optimization for improving efficiency and sustainability. Multi-objective optimization approaches are examined to enhance safety, capacity, and performance. The study focused on the optimization of reliability indices after applying the process of RPGT with optimization which is not focused by most of study. Unlike traditional reliability evaluation approaches that depend solely on analytical derivations or single optimization methods, this research introduces a comparative hybrid methodology by combining RPGT with three different metaheuristic optimizers. Each algorithm—PSO for fast convergence, GA for global search capability, and CSA for balanced exploration—brings unique strengths, enabling a more comprehensive reliability evaluation. This comparative optimization framework enhances accuracy, reduces computational complexity, and provides better adaptability to real-world engineering systems, making it distinct from earlier works that rely on deterministic or single-technique methods. This study sectioned in six parts. The introductory part and literature review has been covered in section 1. The section 2 described the Symbols and notations. The mathematical formulation related to transition diagram is described in section 3. The optimization using algorithms are covered in section 4. The result and conclusion are included in Section 5 and 6 respectively.

2. Symbols and Nomenclature

The transition diagram shows all the possible states of the system working with three units in series configuration and how it moves between them. Each state represents whether the subsystems are fully working or failed. The arrows in the diagram indicate the chances of moving from one state to another, depending on the failure rates (ϕ_i) and repair rates (λ_i).

This diagram is very useful because it provides the base structure for calculating probabilities, sojourn times, and reliability measures of the repairable system.

The following represents the set of symbols and notations used in the transition diagram

Circle Represents the system operating in a fully functional state.

Rectangle Shows the system has entered a failure state

A,B,C Shows the subsystems in complete operational condition

a,b,c Shows the subsystems in breakdown condition

$P_0(t)$ The chance of the system functioning at complete capacity at time t

$P_1(t)$ The probability that the system remains under cold standby mode

$(P_2(t), P_3(t), P_4(t))$ The measure of the system's presence in a failure state

(ϕ_i) , for $i=1-3$ Average rates of failure for elements A, B, C, and D

(λ_i) , for $i=1-3$. The mean repair rate values for elements A, B, C, and D, respectively.

d/dt Indicates the rate of change concerning time (t)

Av. Steady-state measure of the system's availability.

3. Mathematical Formulation

The mathematical formulation of the system is developed to describe how it behaves over time under random failures and repairs. Since the system can shift between working, degraded, and failed conditions, it is important to represent these changes using a clear mathematical framework. The Regenerative Point Graphical Technique (RPGT) is used here because it helps convert the system's practical behavior into equations that can measure key reliability parameters such as availability, failure time, and inspections.

The various reliability indices based on RPGT technique using the table 1 and 2 are given below.

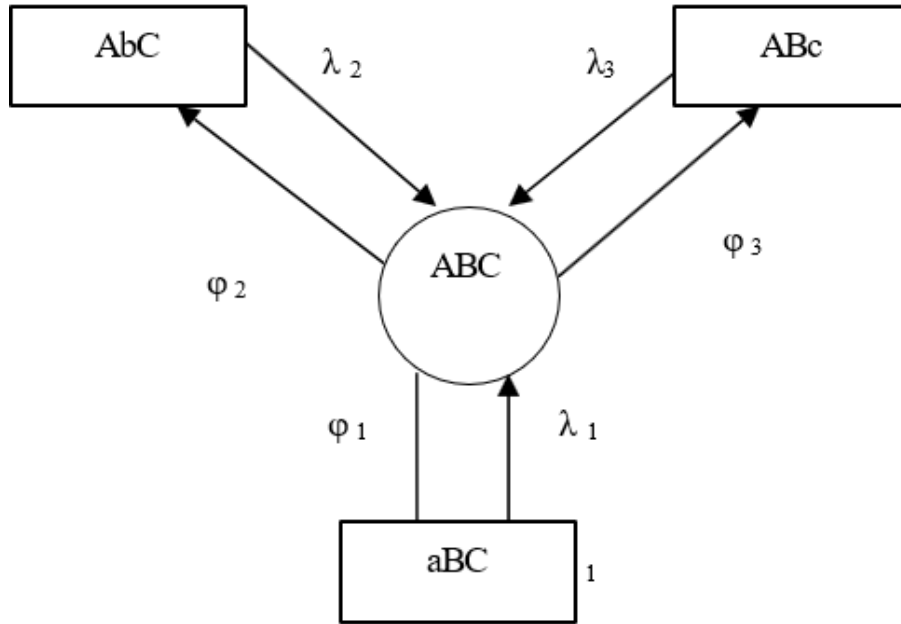


Figure 1: Transition diagram of three units

Table 1: Table 1: The various probabilities from state i to j .

From $i \rightarrow j$	Symbol	Probability $p_{i,j}$
$0 \rightarrow 1$	$p_{0,1}$	$\frac{\phi_1}{\phi_1 + \phi_2}$
$0 \rightarrow 2$	$p_{0,2}$	$\frac{\phi_2}{\phi_1 + \phi_2}$
$1 \rightarrow 0$	$p_{1,0}$	$\frac{\lambda_1}{\lambda_1 + \phi_2}$
$1 \rightarrow 4$	$p_{1,4}$	$\frac{\phi_2}{\lambda_1 + \phi_2}$
$2 \rightarrow 0$	$p_{2,0}$	$\frac{\lambda_2}{\lambda_2 + \phi_1 + \phi_3}$
$2 \rightarrow 3$	$p_{2,3}$	$\frac{\lambda_2}{\lambda_2 + \phi_1 + \phi_3}$
$2 \rightarrow 4$	$p_{2,4}$	$\frac{\phi_3}{\lambda_2 + \phi_1 + \phi_3}$
$3 \rightarrow 1$	$p_{3,1}$	$\frac{\lambda_2 + \lambda_1 + \phi_3}{\lambda_2}$
$3 \rightarrow 2$	$p_{3,2}$	$\frac{\lambda_2 + \lambda_1 + \phi_3}{\lambda_2}$
$3 \rightarrow 5$	$p_{3,5}$	$\frac{\phi_3}{\lambda_2 + \lambda_1 + \phi_3}$

Table 2: Table 2: Mean Sojourn Time

State i	Mean Sojourn Time
0	$\frac{1}{\phi_1 + \phi_2}$
1	$\frac{1}{\lambda_1 + \phi_2}$
2	$\frac{1}{\lambda_2 + \phi_1 + \phi_3}$
3	$\frac{1}{\lambda_2 + \lambda_1 + \phi_3}$

3.1. Mean time to system failure (MTSF)

$$T_0 = \frac{\sum_{i, sr} \frac{\Pr(\xi = i) \mu_i}{\prod_{m_1 \neq \xi} (1 - \bar{V}_{m_1 m_1})}}{1 - \sum_{sr} \frac{\Pr(\xi = \xi)}{\prod_{m_2 \neq \xi} (1 - \bar{V}_{m_2 m_2})}}.$$

$$T_0 = \frac{r_0 + p_{0,1}r_1 + p_{2,0}r_2 + p_{0,2}p_{2,3}r_3 + p_{0,2}p_{2,3}p_{3,1}r_1 - p_{2,3}p_{3,2}r_0 p_{0,1}p_{2,3}p_{3,2}r_1}{1 - p_{0,1}p_{1,0} - p_{0,2}p_{2,0} - p_{2,3}p_{3,2} + p_{0,2}p_{1,0}p_{2,3}p_{3,1} + p_{0,1}p_{1,0}p_{2,3}p_{3,2}}.$$

3.2. Availability of System

$$A_0 = \frac{\sum_{j, sr} \frac{\Pr(\xi = j) f_j \mu_j}{\prod_{m_1 \neq \xi} (1 - \bar{V}_{m_1 m_1})}}{\sum_{i, sr} \frac{\Pr(\xi = i) \mu_i^1}{\prod_{m_2 \neq \xi} (1 - \bar{V}_{m_2 m_2})}}.$$

$$N = (1 - p_{2,3}p_{3,2})r_0 + (p_{1,0} - p_{1,0}p_{2,3}p_{3,2})r_1 + (p_{2,0} + p_{1,0}p_{2,3}p_{3,1})r_2 + (p_{1,0}p_{3,1} + p_{2,0}p_{2,3})r_3.$$

$$D = N + \left[(p_{1,0} - p_{0,1}p_{2,3}p_{3,2})p_{1,4} + (p_{2,0} + p_{1,0}p_{2,3}p_{3,1})p_{2,4} \right] r_4 + (p_{1,0}p_{3,1} + p_{2,0}p_{3,2})p_{3,5}r_5.$$

3.3. Expected number of inspections by the repair man

$$V_0 = \frac{\sum_{j, sr} \frac{\Pr(\xi_j^{(sr \rightarrow)})}{\prod_{k_1 \neq \xi} (1 - \bar{V}_{k_1 k_1})}}{\sum_{i, sr} \frac{\Pr(\xi_i^{(sr \rightarrow)}) \mu_i^1}{\prod_{k_2 \neq \xi} (1 - \bar{V}_{k_2 k_2})}}$$

$$V_0 = \frac{((p_{1,0} - p_{0,1}p_{2,3}p_{3,2})p_{1,4} + (p_{2,0} + p_{1,0}p_{2,3}p_{3,1})p_{2,4} + (p_{1,0}p_{3,1} + p_{2,0}p_{3,2})p_{3,5})}{(1 - p_{0,1}p_{1,0} - p_{0,2}p_{2,0} - p_{2,3}p_{3,2} - p_{0,2}p_{1,0}p_{2,3}p_{3,1} + p_{0,1}p_{1,0}p_{2,3}p_{3,2})}$$

4. Optimization of Various Reliability Metrics Using Metaheuristic Algorithm

In this research, CSA, PSO and GA is used to optimize the various reliability parameters to have an optimum value for smooth and efficient running of the system

4.1. Procedure of CSA

In 2009, Xin-She Yang and Suash Deb introduced the Cuckoo Search (CS) Algorithm, a nature-inspired optimization method. The concept is modeled on the breeding strategy of certain cuckoo species that lay their eggs in the nests of other birds. The algorithm begins by creating a set of randomly generated solutions, referred to as nests, where each nest corresponds to a possible solution to the optimization problem. New solutions are then produced using a random search mechanism called Lévy flights, which enhances the exploration of the search space. Each newly generated solution is compared with an existing one, and if it proves to be better, it replaces the older solution. To maintain diversity, a fraction of the poorest solutions (nests) are abandoned and substituted with fresh random solutions. Throughout the process, the best solution identified so far is always retained. These steps are repeated iteratively until the stopping condition is satisfied, leading to the identification of the optimal solution.

4.2. Procedure of GA

Genetic Algorithm (GA) is an optimization method inspired by the process of natural evolution. It is designed to solve both constrained and unconstrained problems efficiently. GA begins with an initial set of possible solutions, known as a population. Through multiple cycles—called generations—it improves this population by applying genetic operations such as selection, crossover, and mutation. These operations mimic natural processes like survival of the fittest, reproduction, and genetic diversity. Over time, these steps help identify the best or most suitable solution.

4.3. Procedure of PSO

Particle Swarm Optimization (PSO) is a well-known meta-heuristic algorithm that draws inspiration from the collective behavior of swarms, such as flocks of birds or schools of fish. It is a stochastic, population-based optimization technique where individuals (particles) adjust their movement based on their own experience and the knowledge gained from others in the group, ultimately converging toward the best solution. The concept can be illustrated through a flock of birds searching for food. None of the birds know the exact location of the food, but each bird is aware of its progress during every attempt. As they explore different directions, they adapt according to the PSO strategy—following the bird that is closest to the food source. In this analogy, each particle (or bird) starts from a randomly chosen position in the search space and updates its movement using both its own best historical position and the best position found by its neighbors, thereby moving toward the global best solution. When this optimization approach is applied to reliability parameters, the results are obtained in both tabular and graphical form, as presented in the following sections.

Table 3: Optimize value of MTSF under various rates

Algorithm	ϕ_1	ϕ_2	ϕ_3	λ_1	λ_2	MTSF T_0
GA	1.085702	1.295894	3.668902	0.215784	0.768119	0.999847
PSO	3.992328	1.385218	1.426364	0.808066	1.000000	1.000000
CSA	2.902595	2.509360	0.500000	0.488480	0.833506	0.998641

Table 4: Optimize value of availability under various rates

Algorithm	ϕ_1	ϕ_2	ϕ_3	λ_1	λ_2	A_0
GA	1.085702	1.295894	3.668902	0.215784	0.768119	0.694956
PSO	3.992328	1.385218	1.426364	0.808066	1.000000	0.578828
CSA	2.902595	2.509360	0.500000	0.488480	0.833506	0.732596

5. Result Analysis

The optimization of reliability indices was performed using three metaheuristic algorithms—Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Cuckoo Search Algorithm (CSA)—on the

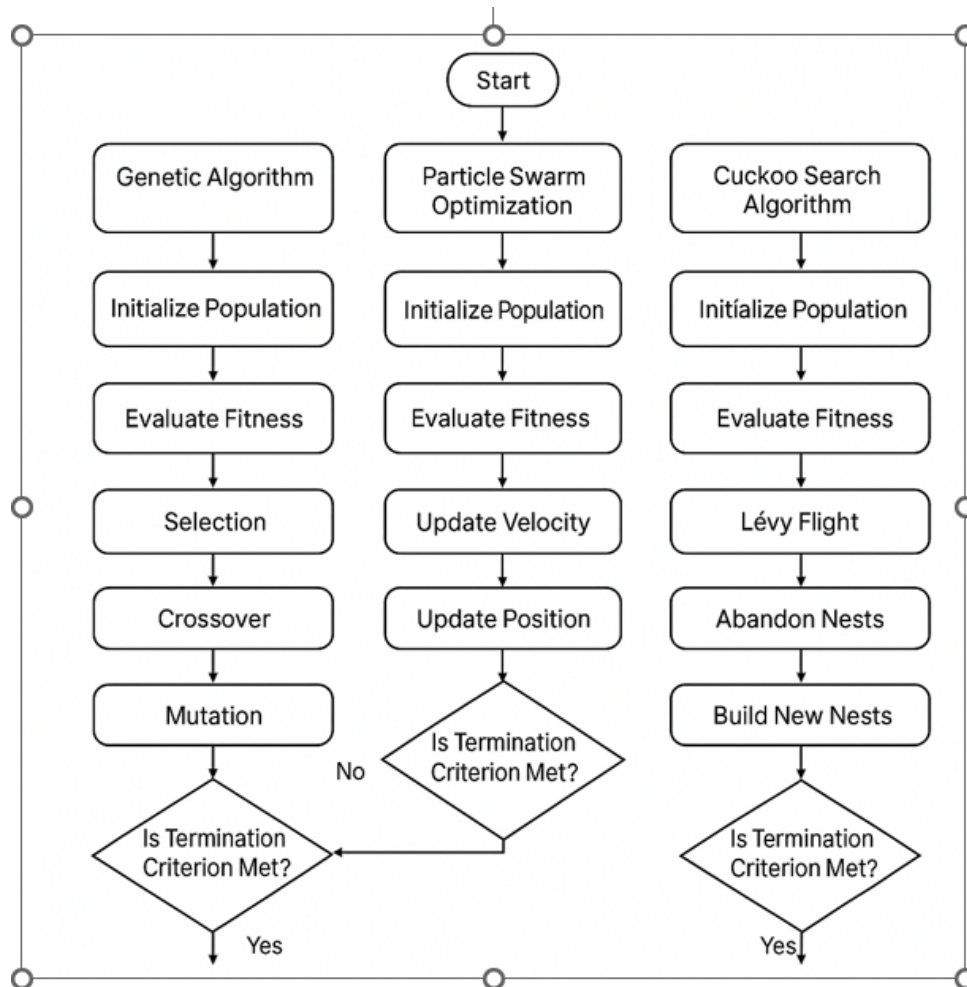


Figure 2: The flow chart for all three algorithms

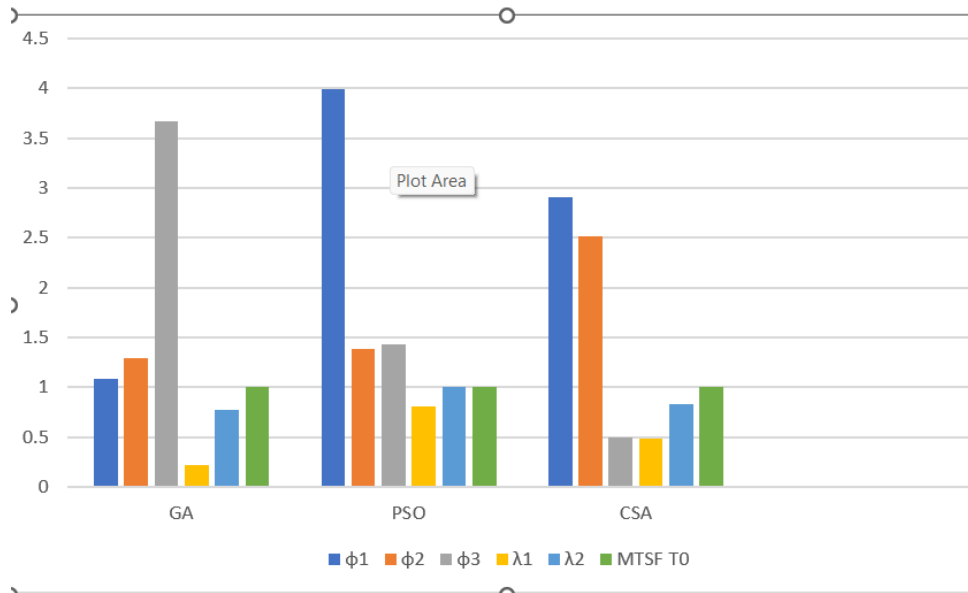


Figure 3: The Optimize MTSF w.r.t to various rates

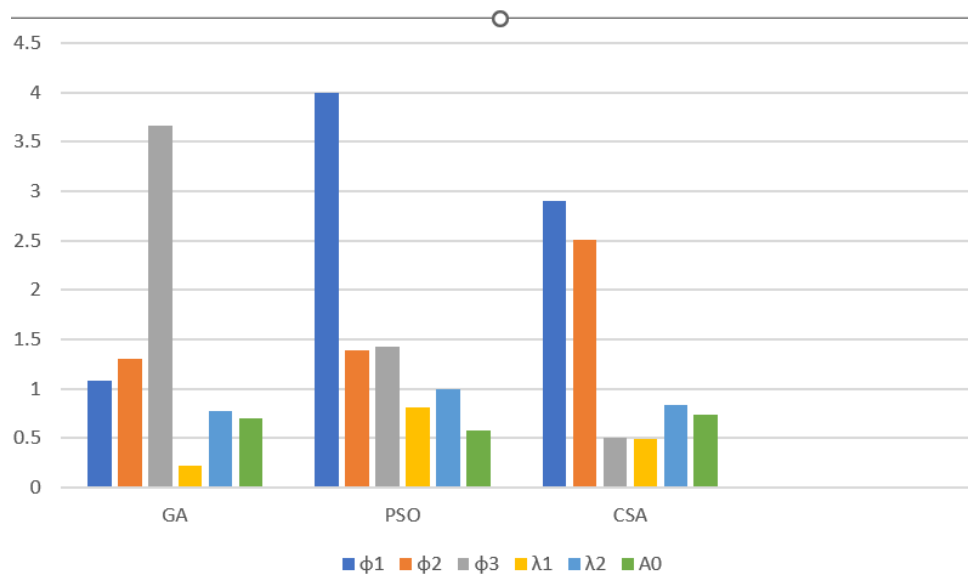


Figure 4: The Optimize availability w.r.t to various rates

Table 5: Optimize value of Expected number of inspection by repairman under various rates

Algorithm	ϕ_1	ϕ_2	ϕ_3	λ_1	λ_2	V_0
GA	1.085702	1.295894	3.668902	0.215784	0.768119	0.283418
PSO	3.992328	1.385218	1.426364	0.808066	1.000000	0.566083
CSA	2.902595	2.509360	0.500000	0.488480	0.833506	0.278590

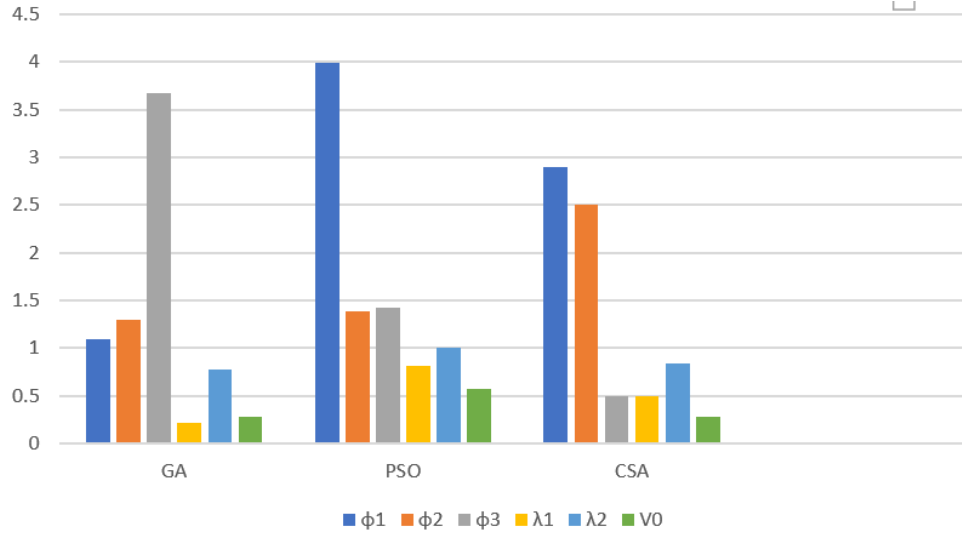


Figure 5: The Optimize expected number of inspections by repairman w.r.t to various rates

repairable system modelled through RPGT. The comparative findings are:

About Mean Time to System Failure (MTSF):

o PSO achieved the best performance (MTSF- 1.000000), showing its strong ability to maximize system lifetime.

o GA also provided very close results (MTSF- 0.999847).

o CSA yielded slightly lower MTSF (- 0.998641), but still competitive

About Steady-State Availability :

o CSA achieved the highest availability (-0.732596).

o GA performed moderately (- 0.694956).

o PSO resulted in the lowest availability (0.578828).

About Expected Number of Inspections :

o CSA gave the best result (≈ 0.278590), showing fewer inspections required.

o GA was slightly higher (≈ 0.283418).

o PSO required significantly more inspections (≈ 0.566083).

6. Conclusion

The analysis shows that no single algorithm dominates across all indices.

- If the goal is longer system lifetime, PSO is most effective.
- For continuous availability and maintenance efficiency, CSA is the preferred choice.
- GA remains a reliable all-rounder with stable optimization but slightly lower outcomes.

This comparative insight highlights the strength of integrating RPGT with metaheuristic optimization to enhance decision-making for repairable systems. The study demonstrates that combining RPGT-based reliability modelling with nature-inspired optimization algorithms provides an effective framework for analysing and improving the performance of repairable systems. By applying GA, PSO, and CSA, the research shows that each algorithm contributes uniquely—some are better at extending system lifetime,

others at improving availability, or reducing maintenance demands. Overall, the analysis indicates that there is no one-size-fits-all algorithm, and the most suitable method depends on the operational priorities of the system. (e.g., higher availability, longer life, or reduced inspections). This validates the importance of hybrid and comparative approaches in reliability engineering, offering flexibility for real-world industrial applications where efficiency, safety, and maintainability must be balanced

References

1. S. Singla and S. Rani, "Performance Optimization of 3: 4: Good System," in Proceedings of the 2023 Second International Conference on Informatics (ICI), 2023, 1-4. IEEE.
2. S. Singla. and S. Rani, "Profit analysis of linear consecutive 2:3:G system in the presence of supportive system and repairable system," in Proceedings of the 2023 Second International Conference on Informatics (ICI), 2023, 274-280.
3. S. Singla, S. Rani, D. Mangla, and U. M. Modibbo, "Behavior Analysis Presented System With Failure And Maintenance Rate With Using Deep Learning Algorithms," Reliability: Theory Applications, vol. 19, no. 3(79), pp. 476-485, 2024.
4. S. Singla and S. Rani, "An Analysis Of Reliability In Manufacturing Industries," Reliability: Theory Applications, vol. 19, no. 4(80), pp. 552-559, 2024.
5. S. Singla, D. Mangla, S. Rani, and U. M. Modibbo, "Impact Of Preventive Maintenance And Failure Rate On A Complexly Configured System: A Sensitive Analysis," Reliability: Theory Applications, vol. 19, no. 4(80), pp. 774-791, 2024
6. S. Singla, S. Rani, and D. Mangla, "Behavior Analysis of a Repairable 2-Out-of-4 System Using Evolutionary Algorithm," Journal of Mechanics of Continua and Mathematical Sciences, vol. 19, no. 7, pp. 126-137, July 2024.
7. S. Singla, D. Mangla, U. M. Modibbo, and A. K. Lal, "Critical Analysis of Failure and Repair Rates of Poly-Tube Manufacturing Plant Using PSO," Reliability: Theory Applications, vol. 19, no. 2(78), pp. 351-364, 2024.
8. Thind A., Gagandeep, Singh H., Kaushal P., Kumari N., Singh H., Lamba S., and Sharma A., Enhancement of virtual machine migration using artificial bee colony algorithm, 11th international conference on Reliability, Infocom Technologies and optimization (Trends and future directions), IEEE, pp. 1-6,2024.
9. R. Khurana, S. Singla, V. Garg, D. Mangla, S. Kumar, M. Pathak, and P. Joshi, "Application of Laplacian Teaching Learning Algorithm for Constrained and Unconstrained Optimization Problems," in Proceedings of the 2023 Second IEEE International Conference on Measurement, Instrumentation, 2023, 1-5.
10. John, Y. M., Sanusi, A., Yusuf, I., and Modibbo, U., M. (2023). Reliability Analysis of Multi-Hardware-Software System with Failure Interaction. Journal of Computational and Cognitive Engineering, 2(1), 38-46.
11. Dang, Z., Wang, H. (2024). Leveraging meta-heuristic algorithms for effective software fault prediction: a comprehensive study. Journal of Engineering and Applied Science, 1- 34.
12. Zhou, J., Lilhore, U. K., Poongodi, M., Hai, T., Simaiya, S., Jawawi, D, N. A., Alsekait, D., Ahuja, S., Biamba, C and Hamdi, M. (2023). Comparative analysis of meta-heuristic load balancing algorithms for efficient load balancing in cloud computing. Journal of Cloud Computing, 1-13.
13. Khorshidi, H. A., and Nikfalazar, S. (2015). Comparing two meta-heuristic approaches for solving complex system reliability optimization. Applied and Computational Mathematics, 4(2-1), 1-6.
14. Sharma, P., Sundaram, S., Sharma, M., Sharma, A., and Gupta, D. (2019). Diagnosis of Parkinson's disease using modified grey wolf optimization. Cognitive Systems Research, 100-115.
15. Gorji, S. A. (2023). Challenges and opportunities in green hydrogen supply chain through meta-heuristic optimization. Journal of Computational Design and Engineering, 1143- 1157.

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