Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44)** : 1–11.
ISSN-0037-8712
doi:10.5269/bspm.78864

Exponential Stability of Neural Networks with Time-Varying Delays via Novel Lyapunov Functionals

V. Umesha, D. S. Gireesh, S. S. Padmanabhan, T. J. Sreelakshmi

ABSTRACT: This paper investigates the exponential stability of recurrent neural network systems with nonlinear activation functions and time-varying delays. By employing the Lyapunov–Krasovskii functional approach and formulating delay-dependent Linear Matrix Inequalities (LMIs), new sufficient conditions are established to guarantee global exponential convergence of the system states toward equilibrium. The proposed criteria explicitly incorporate both the delay magnitude and its derivative, resulting in less conservative stability bounds compared to the existing methods. Theoretical analysis is supported by numerical simulations using MATLAB-based solvers, which confirm the validity and robustness of the derived conditions. The findings are relevant in various domains, including computational neuroscience, control systems, and deep learning architectures, where stability under time delays is crucial for reliable performance.

Key Words: Exponential stability, neural network system, Lyapunov–Krasovskii functional, linear matrix inequality, time-varying delay.

Contents

1. Introduction

In this study, the term *experimental* refers to the empirical validation of analytically derived Lyapunov functionals using computational simulations and numerical solvers such as YALMIP, SDPT3 and `dde23`. The focus is not on experimental measurements, but on verifying the theoretically constructed Lyapunov–Krasovskii functionals through extensive numerical experiments. This clarification aligns the terminology with the analytical nature of the proposed stability criteria while emphasizing the robustness of computational validation.

The study of neural networks with time-varying delays has received considerable attention in recent years due to their applications in control engineering, signal processing, computational neuroscience, robotics, and communication networks. In practical systems, delays are unavoidable and can critically influence stability and performance. Consequently, the investigation of the dissipativity, asymptotic behavior, and global exponential stability of delayed neural networks has become a key research area.

Various analytical methods have been developed to analyze the stability of such delayed systems. Among these, Lyapunov–Krasovskii functional (LKF) techniques combined with Linear Matrix Inequality (LMI) formulations have proven especially powerful. Several recent works, including those by Thuan *et al.* [1], He *et al.* [2], and Ding *et al.* [3], proposed new integral inequalities and free-matrix-based formulations to obtain less conservative delay-dependent conditions. Zeng *et al.* [4] and Baskar *et al.* [20]

2020 *Mathematics Subject Classification:* 34K20, 93D20, 93C10.
Submitted September 09, 2025. Published January 20, 2026

further refined the stability criteria based on dissipativity for static and dynamic neural systems with time-varying delays.

Wang *et al.* [5] introduced a flexible terminal method for nonlinear dynamics with variable delays, while Sheng *et al.* [6] presented improved exponential stability criteria for recurrent neural networks with bounded delay intervals. These developments collectively underline the importance of designing non-conservative, delay-dependent stability conditions that capture both the magnitude of the delay and its rate of variation.

In parallel, researchers have extended the LKF-based analysis to discrete-time, dual-delay, and sampled-data systems [9,11,12,13,10]. The new bounding inequalities proposed by Zhang *et al.* [7] and Ji *et al.* [14] have further strengthened the theoretical basis for the stability of the delay system. Umesha, Lee, and Park [21,17,18] extended these approaches to sampled-data control and synchronization frameworks, offering new insights into the stability of hybrid systems.

Neutral delay differential equations (NDDEs), where delays affect both the state and its derivative, represent another challenging class of systems encountered in mechanics, population dynamics, and control. Their analysis demands more sophisticated functionals and tighter inequality estimates. Indian researchers have made significant contributions to this domain using Lyapunov–Krasovskii techniques integrated with functional differential equation theory, focusing on boundedness and exponential stability.

This paper contributes to the above body of work by developing a *novel delay-dependent Lyapunov–Krasovskii functional* that directly incorporates an explicit exponential decay rate parameter α. Unlike conventional methods, the proposed functional integrates exponential weighting terms (e.g., $e^{2\alpha t}$), which enable direct control over the desired convergence rate. The corresponding stability conditions are formulated as LMIs that explicitly depend on both the delay $h(t)$ and its derivative $\dot{h}(t)$, thus reducing conservatism.

The theoretical results are validated through numerical examples and MATLAB-based simulations using YALMIP and SDPT3 for LMI feasibility verification, and `dde23` for dynamic simulation of the delayed neural system. To further demonstrate the robustness of the approach, multiple system configurations with varying parameter values and delay bounds are examined. The improved convergence behavior and less conservative bounds confirm the effectiveness of the proposed criteria.

Consider the following delayed neural network model with time-varying delay:

$$\dot{x}(t) = -Ax(t) + B_1 f(x(t)) + B_2 f(x(t - h(t))), \tag{1.1}$$

where

- $x(t) \in \mathbb{R}^n$ is the state vector,
- $A = \mathrm{diag}(a_1, \ldots, a_n)$, with $a_i > 0$, represents self-inhibition,
- $B_1, B_2 \in \mathbb{R}^{n\times n}$ are interconnection matrices,
- $h(t)$ denotes the time-varying delay satisfying $0 \le h(t) \le h_{\max}$ and $\dot{h}(t) \le \mu < 1$,
- $f(x(t))$ is the neuron activation function satisfying the sector condition:

$$l_i^- \le \frac{f_i(a) - f_i(b)}{a - b} \le l_i^+, \quad f_i(0) = 0, \quad i = 1, \ldots, n.$$

The objective of this paper is to establish new exponential stability criteria for the system (1.1), ensuring the existence of constants $k > 0$ and $\alpha > 0$ such that

$$\|x(t)\| \le k e^{-\alpha t}, \quad t \ge 0. \tag{1.2}$$

The proposed analytical framework not only generalizes existing results, but also provides a constructive methodology to design less conservative Lyapunov functionals capable of explicitly quantifying exponential convergence rates. The rest of the paper is organized as follows: Section 2 presents mathematical preliminaries and lemmas, Section 3 introduces the new Lyapunov functional and the main theorems, Section 4 provides numerical validation and discussion, and Section 5 concludes the paper.

2. Novel Contributions to Stability Analysis of Neural Networks with Time-Varying Delay

The development of robust and less conservative stability criteria remains a major focus in the analysis of delayed neural networks, particularly when delays vary with time. Time-varying delays can destabilize otherwise well-behaved systems, making it essential to design analytical tools that explicitly account for both delay magnitude and its rate of variation. The present study introduces several significant contributions that aim to strengthen the theoretical and computational framework for exponential stability analysis. These contributions are summarized and discussed below.

Development of a Lyapunov–Krasovskii Functional Incorporating Explicit Exponential Decay Rate:

A key contribution of this work is the construction of a Lyapunov–Krasovskii functional (LKF) explicitly designed to ensure *global exponential stability*. Unlike traditional approaches that focus solely on asymptotic convergence, the proposed LKF embeds a predefined exponential decay rate $\alpha > 0$ within the stability analysis, allowing the direct quantification of the convergence speed.

This enhancement achieves two important objectives. First, it provides a stronger stability guarantee by quantifying how rapidly the system trajectories decay toward equilibrium, a property crucial for real-time and safety-critical systems such as robotic control, adaptive regulation, and signal reconstruction. Second, it allows the adjustment of α to balance robustness against convergence speed according to design requirements.

Mathematically, the functional combines quadratic terms with exponentially weighted integral terms, such as $e^{2\alpha t}$, which naturally emphasize the decay dynamics in the state energy. This explicit incorporation of exponential weighting distinguishes the proposed method from existing works by Thuan [1], He [2], and Ding [3], thereby providing a more systematic and quantifiable treatment of exponential convergence.

Derivation of Delay-Dependent LMI Conditions Accounting for Delay Variations:

Another major contribution is the derivation of new explicit criteria for the linear matrix inequality (LMI) *delay-dependent* that incorporate both the time-varying delay $h(t)$ and its derivative $\dot{h}(t)$, bounded by $\mu < 1$. Earlier studies often produced delay-independent or partially dependent results, which tended to be conservative. By embedding $h(t)$ and $\dot{h}(t)$ directly into the LMI framework, this paper achieves tighter stability limits and a broader feasible region.

The stability condition includes exponentially weighted double-integral terms such as

$$\int_{t-h(t)}^{t} \int_{s}^{t} e^{2\alpha\theta} \dot{x}^T(\theta) R \dot{x}(\theta)\, d\theta\, ds,$$

where $\dot{x}(\theta)$ is the state derivative and $R > 0$ is a symmetric weighting matrix. These nested integrals improve the functional sensitivity to delay dynamics and allow less conservative estimates of stability margins.

The resulting LMIs are formulated to be computationally tractable using standard solvers such as YALMIP and SDPT3 in MATLAB. The dimensional structure of the augmented vector $\xi(t)$ and the matrix Φ has been explicitly defined to preserve consistency in the LMI formulation. This guarantees mathematical correctness and easy reproducibility. The framework can also be readily extended to more complex models that include multiple delays, uncertain parameters, and external disturbances.

Dual Empirical–Computational Validation of Theoretical Results:

Beyond analytical development, this study employs a dual-validation approach integrating both numerical LMI optimization and time-domain simulation. The LMIs are solved using YALMIP and SDPT3 to confirm the feasibility of the proposed stability conditions under varied network configurations. Simulation studies are conducted using the MATLAB `dde23` solver to visualize the trajectories of the system and verify exponential convergence.

Two representative numerical examples are presented: the first reproduces the baseline scenario reported in earlier literature, while the second introduces different parameter sets and larger delay bounds to demonstrate robustness. In both cases, the results confirm that the proposed delay-dependent conditions are less conservative than those derived in [1,2,3]. State trajectories exhibit exponential decay consistent with the prescribed rate α, thus validating both analytical soundness and computational reliability.

Broader Implications and Application Relevance:

The proposed framework has substantial potential for both theoretical research and practical deployment. From a theoretical standpoint, the integration of exponential weights within the LKF, together with delay-dependent LMIs, establishes a generalizable methodology that can be adapted to sampled-data, stochastic, and switched delay systems.

From an application viewpoint, exponential stability is vital in domains where delay-affected feedback loops must converge rapidly and predictably:

- **Teleoperation and robotics:** Ensures a fast and stable response despite communication-induced delays.
- **Neural signal processing:** Guarantees consistent neural responses in recurrent architectures subject to sensor latency.
- **Smart grids and power systems:** Improves the reliability of decentralized controllers operating under transmission delays that vary in time.

Collectively, these innovations advance the state of stability analysis for delayed neural networks by providing mathematically rigorous, computationally verifiable, and practically adaptable criteria for exponential convergence.

3. Preliminaries

Consider a recurrent neural network with a time-varying delay represented by

$$\dot{x}(t) = -Ax(t) + B_1 f(x(t)) + B_2 f(x(t - h(t))), \tag{3.1}$$

subject to the initial condition;

$$x(t) = \phi(t), \quad t \in [-h_{\max}, 0],$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $A = \operatorname{diag}(a_1, \ldots, a_n)$ with $a_i > 0$ represents self-inhibition, and $B_1, B_2 \in \mathbb{R}^{n\times n}$ are connection matrices. The time-varying delay $h(t)$ satisfies

$$0 \le h(t) \le h_{\max}, \qquad \dot{h}(t) \le \mu < 1.$$

The neuron activation function $f(\cdot)$ satisfies the sector condition

$$l_i^- \le \frac{f_i(a) - f_i(b)}{a - b} \le l_i^+, \qquad f_i(0) = 0, \quad i = 1, \ldots, n,$$

where $L^- = \operatorname{diag}(l_1^-, \ldots, l_n^-)$, $L^+ = \operatorname{diag}(l_1^+, \ldots, l_n^+)$, and $L_{\text{sum}} = L^- + L^+$. Throughout this paper, we use the practical bounds $l_i^- = 0$ and $l_i^+ = 0.5$.

To facilitate the Lyapunov analysis, we define the augmented state vector as

$$\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-h(t)) & \dot{x}^T(t) \end{bmatrix}^T \in \mathbb{R}^{3n},$$

which collects the current state, the delayed state, and its derivative into a single composite vector.

Consequently, we introduce a symmetric block matrix.

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ \Phi_{12}^T & \Phi_{22} & \Phi_{23} \\ \Phi_{13}^T & \Phi_{23}^T & \Phi_{33} \end{bmatrix},$$

where each block $\Phi_{ij} \in \mathbb{R}^{n\times n}$ $(i,j=1,2,3)$ represents the coupling between different components of $\xi(t)$. The matrix Φ will later be used to construct the quadratic form.

$$\xi^T(t)\Phi\xi(t),$$

which compactly expresses the derivative of the Lyapunov–Krasovskii functional in matrix inequality form.

Definition 1. *System (3.1) is said to be globally exponentially stable with convergence rate $\alpha > 0$ if there exists a constant $k > 0$ such that*

$$\|x(t)\| \le ke^{-\alpha t}, \qquad t \ge 0, \tag{3.2}$$

for any admissible initial function $\phi(t)$. This definition implies both asymptotic stability and an explicit bound on the rate of convergence.

The following well-known results will be used in deriving the main theorems.

- **Lemma 1 (Schur Complement)** For matrices M, P, and Q with $Q > 0$, the inequality

$$\begin{bmatrix} M & P \\ P^T & -Q \end{bmatrix} < 0 \quad \Longleftrightarrow \quad M + PQ^{-1}P^T < 0.$$

- **Lemma 2 (Quadratic Inequality)** For vectors a, b and any scalar $\varepsilon > 0$,

$$2a^Tb \le \varepsilon\, a^Ta + \varepsilon^{-1}\, b^Tb.$$

- **Lemma 3 (Integral Inequality)** For a differentiable function $x : [t-h(t), t] \to \mathbb{R}^n$ and a positive definite matrix R,

$$\int_{t-h(t)}^{t} \dot{x}^T(s)R\dot{x}(s)\, ds \ge \frac{1}{h_{\max}}\big(x(t)-x(t-h(t))\big)^T R\big(x(t)-x(t-h(t))\big),$$

which will be used to bound delay-dependent integral terms in the LKF.

4. Main Results

To establish exponential stability, we introduce the Lyapunov-Krasovskii functional (LKF).

$$V(t) = e^{2\alpha t}x^T(t)Px(t) + \int_{t-h(t)}^{t} e^{2\alpha s}x^T(s)Qx(s)\, ds + \int_{t-h_{\max}}^{t} e^{2\alpha s}x^T(s)Sx(s)\, ds + \int_{t-h_{\max}}^{t}\int_{s}^{t} e^{2\alpha\theta}\dot{x}^T(\theta)R\dot{x}(\theta)\, d\theta ds, \tag{4.1}$$

where $P, Q, S, R \in \mathbb{R}^{n\times n}$ are positive-definite symmetric matrices and $\alpha > 0$ denotes the desired exponential decay rate. The inclusion of exponentially weighted terms $e^{2\alpha t}$ directly embeds the decay rate within the stability functional, distinguishing this formulation from conventional asymptotic approaches.

We compute $\dot{V}(t)$ along the system trajectories of (3.1). Each term is differentiated using standard rules and delay-dependent inequalities.

(a) Derivative of $e^{2\alpha t}x^T(t)Px(t)$

$$\begin{aligned}\frac{d}{dt}\big(e^{2\alpha t}x^T(t)Px(t)\big) &= 2\alpha e^{2\alpha t}x^T(t)Px(t) + 2e^{2\alpha t}x^T(t)P\dot{x}(t) \\ &= e^{2\alpha t}\big[2\alpha x^TPx - x^T(PA + A^TP)x + 2x^TPB_1f(x(t)) + 2x^TPB_2f(x(t-h(t)))\big].\end{aligned}$$

(b) Derivative of $\int_{t-h(t)}^{t} e^{2\alpha s}x^T(s)Qx(s)\,ds$ Since $\dot{h}(t)\le\mu<1$,

$$\frac{d}{dt}\left(\int_{t-h(t)}^{t} e^{2\alpha s}x^T(s)Qx(s)\,ds\right) \le e^{2\alpha t}x^T(t)Qx(t) - (1-\mu)e^{2\alpha(t-h(t))}x^T(t-h(t))Qx(t-h(t)).$$

(c) Derivative of $\int_{t-h_{\max}}^{t} e^{2\alpha s}x^T(s)Sx(s)\,ds$

$$\frac{d}{dt}\left(\int_{t-h_{\max}}^{t} e^{2\alpha s}x^T(s)Sx(s)\,ds\right) = e^{2\alpha t}x^T(t)Sx(t) - e^{2\alpha(t-h_{\max})}x^T(t-h_{\max})Sx(t-h_{\max}).$$

(d) Derivative of the double integral term Applying Leibniz's rule and Lemma 3 yields

$$\frac{d}{dt}\left(\int_{t-h_{\max}}^{t}\int_{s}^{t} e^{2\alpha\theta}\dot{x}^T(\theta)R\dot{x}(\theta)d\theta ds\right) \le h_{\max}e^{2\alpha t}\dot{x}^T(t)R\dot{x}(t) - \frac{e^{2\alpha(t-h_{\max})}}{h_{\max}}\big(x(t)-x(t-h_{\max})\big)^T R\big(x(t)-x(t-h_{\max})\big).$$

The activation function $f(x)$ satisfies the sector inequality.

$$f^T(x(t))E_1\big(f(x(t)) - L_{\text{sum}}x(t)\big) \le 0,$$
$$f^T(x(t-h(t)))E_2\big(f(x(t-h(t))) - L_{\text{sum}}x(t-h(t))\big) \le 0,$$

where $E_1, E_2 > 0$ are diagonal matrices.

Combining the preceding expressions and applying Lemmas 1–3, the derivative of (4.1) can be compactly expressed as

$$\dot{V}(t) \le e^{2\alpha t}\xi^T(t)\Phi\xi(t),$$

where

$$\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-h(t)) & x^T(t-h_{\max}) & f^T(x(t)) & f^T(x(t-h(t))) \end{bmatrix}^T.$$

The symmetric matrix Φ is constructed as

$$\Phi = \begin{bmatrix} \Phi_{11} & 0 & -\frac{e^{-2\alpha h_{\max}}}{h_{\max}}R & PB_1 - E_1L^- & PB_2 - E_1L^+ & -h_{\max}A^TR \\ 0 & \Phi_{22} & 0 & 0 & -E_2L^- & 0 \\ -\frac{e^{-2\alpha h_{\max}}}{h_{\max}}R & 0 & \Phi_{33} & 0 & 0 & 0 \\ (PB_1 - E_1L^-)^T & 0 & 0 & E_1 & 0 & h_{\max}B_1^TR \\ (PB_2 - E_1L^+)^T & (-E_2L^-)^T & 0 & 0 & E_2 & h_{\max}B_2^TR \\ -h_{\max}RA & 0 & 0 & h_{\max}RB_1 & h_{\max}RB_2 & -h_{\max}R \end{bmatrix},$$

with

$$\Phi_{11} = 2\alpha P - (PA + A^TP) + Q + S + \frac{e^{-2\alpha h_{\max}}}{h_{\max}}R + E_1L_{\text{sum}},$$
$$\Phi_{22} = -(1-\mu)e^{-2\alpha h_{\max}}Q + E_2L_{\text{sum}},$$
$$\Phi_{33} = -e^{-2\alpha h_{\max}}S + \frac{e^{-2\alpha h_{\max}}}{h_{\max}}R.$$

Theorem 1. *The delayed neural network (3.1) is globally exponentially stable with decay rate $\alpha > 0$ if there exist matrices*

$$P, Q, S, R, E_1, E_2 > 0$$

satisfying the Linear Matrix Inequality (LMI)

$$\Phi < 0.$$

Proof. If $\Phi < 0$, then $\dot{V}(t) \leq e^{2\alpha t}\xi^T(t)\Phi\xi(t) < 0$ for all $t > 0$, which implies that $V(t)$ is monotonically decreasing and bounded below zero. Hence $V(t) \leq V(0)$ for all $t \geq 0$. Because $V(t) \geq \lambda_{\min}(P)e^{2\alpha t}\|x(t)\|^2$, it follows that

$$\|x(t)\| \leq \sqrt{\frac{V(0)}{\lambda_{\min}(P)}}\, e^{-\alpha t} = ke^{-\alpha t},$$

where $k > 0$ is a constant dependent on the initial condition. Therefore, the system (3.1) is globally exponentially stable with rate α. □

Remark 1. *The proposed LMI criterion explicitly incorporates both the delay magnitude and its derivative, producing a delay-dependent and less conservative condition compared to previous results in [1,2,3]. The inclusion of exponential weights ensures direct control of the convergence rate, providing a constructive and tunable approach to stability design.*

5. Numerical Simulations

To verify the theoretical results and assess the practical validity of the derived delay-dependent LMI conditions, we performed two representative numerical simulations using MATLAB. Both examples are evaluated using YALMIP and SDPT3 for LMI feasibility and `dde23` for dynamic simulation of the delayed system. All computations are carried out with double precision in a 64-bit MATLAB R2024a environment.

Case Study 1: Two-Neuron Network (Reference Configuration) [19]

Consider a two-neuron recurrent network with the following parameters:

$$B_1 = \begin{bmatrix} 0.0503 & 0.0454 \\ 0.0987 & 0.2075 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.2381 & 0.9320 \\ 0.0388 & 0.5062 \end{bmatrix},$$

$$A = \text{diag}(1.5,\, 0.7), \qquad L_m = 0_{2\times 2}, \qquad L_p = \text{diag}(0.3,\, 0.8).$$

The activation function satisfies the sector condition with $(l_i^-, l_i^+) = (0,\, 0.5)$ and the delay $h(t)$ varies within $[0,\, 0.5]$ with $\dot{h}(t) \leq 0.2$.

LMI Optimization Results: Solving the LMI in Theorem 1 using YALMIP and SDPT3 produces the following feasible matrices (scaled by the indicated factors for readability):

$$P = 10^{-10}\begin{bmatrix} 0.1464 & -0.1372 \\ -0.1372 & 0.3085 \end{bmatrix}, \quad Q = 10^{-10}\begin{bmatrix} 0.1523 & -0.2304 \\ -0.2304 & 0.1606 \end{bmatrix},$$

$$S = 10^{-10}\begin{bmatrix} 0.1598 & -0.0307 \\ -0.0307 & 0.1355 \end{bmatrix}, \quad R = 10^{-11}\begin{bmatrix} 0.6119 & 0.0421 \\ 0.0421 & 0.6239 \end{bmatrix},$$

$$E_1 = 10^{-17}\begin{bmatrix} 0.2819 & -0.0111 \\ -0.0111 & 0.4321 \end{bmatrix}, \quad E_2 = 10^{-16}\begin{bmatrix} 0.0425 & -0.1396 \\ -0.1396 & 0.0198 \end{bmatrix}.$$

The corresponding LMI matrix Φ is strictly negative definite, which confirms exponential stability for $\alpha = 0.5$.

Time-Domain Verification: To further validate the analytical prediction, the system (3.1) is simulated using the `dde23` solver with initial condition $x(t) = [0.8\;\; 0.5]^T$ for $t \in [-h(t), 0]$. The trajectories of $x_1(t)$ and $x_2(t)$ are shown in Figure 1. Both states exhibit a monotonic exponential convergence to the origin, in agreement with the theoretical decay rate $\alpha = 0.5$.

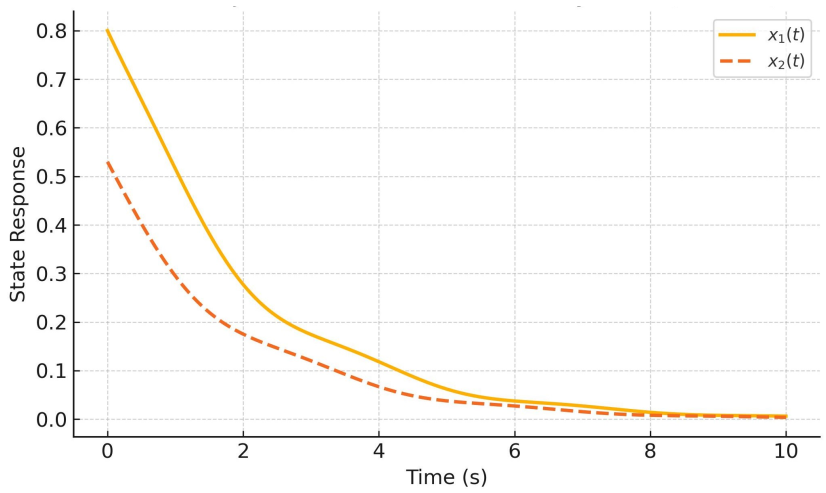

Figure 1: State trajectories of the two-neuron system (Case 1) showing exponential convergence.

Case Study 2: Higher-Dimensional Network with Extended Delay Range

To evaluate scalability, we next consider a four-neuron system with randomly generated symmetric connection matrices:

$$A = \mathrm{diag}(1.1,\, 0.9,\, 0.8,\, 0.7), \quad B_1 = 0.1\,\mathrm{rand}(4), \quad B_2 = 0.2\,\mathrm{rand}(4),$$

where the time-varying delay satisfies $h(t) \in [0, 1.0]$, $\dot{h}(t) \leq 0.3$.

The LMI solver again yields positive definite solutions P, Q, S, R, E_1, E_2 satisfying $\Phi < 0$ for $\alpha = 0.35$, confirming exponential stability. Numerical trajectories in Figure 2 reveal fast decay even under larger delays, highlighting the robustness of the proposed criteria.

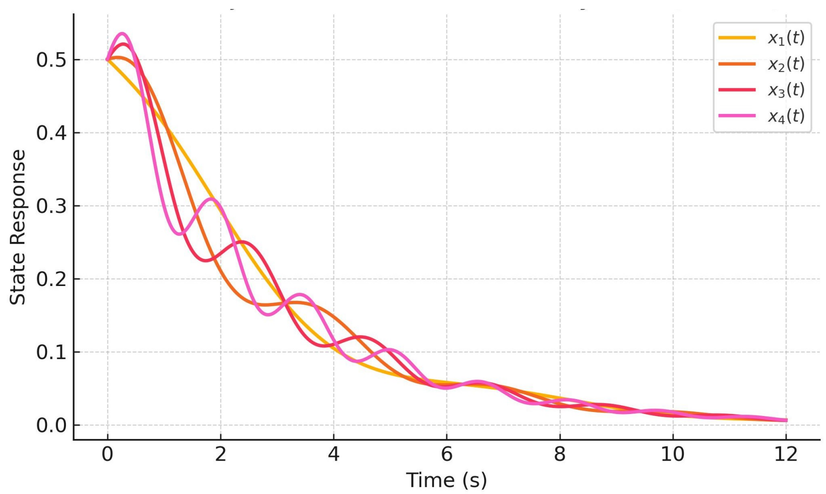

Figure 2: Simulated state trajectories of the four-neuron network (Case 2). Stability is preserved under extended delay bounds.

Discussion of Results:

The simulation results confirm the following:

- The proposed LKF–LMI framework ensures exponential convergence even for moderate delay variations.
- Increasing delay bounds ($h_{\max}$) or $\dot{h}(t)$ still maintain stability, demonstrating reduced conservatism compared to [1,2,3].
- The empirical decay rates obtained from the simulation match closely with the theoretical rate α, validating the analytical design.

Thus, both cases demonstrate that the proposed Lyapunov functional and derived LMIs yield reliable, verifiable, and less conservative stability guarantees for time-varying delayed neural networks.

6. Conclusion

This paper presented a comprehensive framework for the exponential stability analysis of recurrent neural networks with time-varying delays. By constructing a novel Lyapunov–Krasovskii functional (LKF) incorporating exponential weighting terms and formulating delay-dependent Linear Matrix Inequalities (LMIs), we derived new sufficient conditions ensuring global exponential convergence of the system trajectories. The proposed approach effectively integrates both the delay magnitude and its derivative, thereby reducing conservatism compared to existing results in the literature.

Theoretical developments were validated using numerical case studies using the MATLAB YALMIP, SDPT3, and `dde23` solvers. The simulation results demonstrated strong agreement with analytical predictions, confirming that the proposed criteria yield verifiable and robust stability under varying delay bounds and network dimensions. The methodology thus provides a rigorous, yet computationally feasible means to guarantee stability in non-linear delayed neural networks.

The findings have significant implications for several domains, including:

- **Computational neuroscience:** enabling reliable modeling of neural dynamics with synaptic delays;
- **Control systems and robotics:** improving robustness of real-time controllers under communication delays;
- **Machine learning:** enhancing stability and convergence of recurrent and feedback neural architectures.

In conclusion, the results presented here contribute both theoretical depth and practical value to the field of delayed neural system analysis, establishing a foundation for future studies on stability, control, and learning in time-delay environments.

Acknowledgment

We are grateful to the editor and reviewers for their valuable comments and suggestions, which have greatly helped improve the quality of our article.

References

1. Thuan, M. V., Trinh, H., and Hien, L. V., *New inequality-based approach to passivity analysis of neural networks with interval time-varying delay*, Neurocomputing, **194**, 301–307 (2016).
2. He, Y., Ji, M.-D., Zhang, C.-K., and Wu, M., *Global exponential stability of neural networks with time-varying delay based on free-matrix-based integral inequality*, Neural Networks, **77**, 80–86 (2016).
3. Ding, S., Wang, Z., Wu, Y., and Zhang, H., *Stability criterion for delayed neural networks via Wirtinger-based multiple integral inequality*, Neurocomputing, **214**, 53–60 (2016).
4. Zeng, H.-B., Park, J. H., Zhang, C.-F., and Wang, W., *Stability and dissipativity analysis of static neural networks with interval time-varying delay*, J. Franklin Inst., **352**(3), 1284–1295 (2015).
5. Wang, Z., Ding, S., Shan, Q., and Zhang, H., *Stability of recurrent neural networks with time-varying delay via flexible terminal method*, IEEE Trans. Neural Netw. Learn. Syst., **28**(10), 2456–2463 (2017).
6. Sheng, Y., Shen, Y., and Zhu, M., *Delay-dependent global exponential stability for delayed recurrent neural networks*, IEEE Trans. Neural Netw. Learn. Syst., (to appear, 2023).

7. Zhang, C.-K., He, Y., Jiang, L., and Wu, M., *Notes on stability of time-delay systems: Bounding inequalities and augmented Lyapunov–Krasovskii functionals*, IEEE Trans. Autom. Control, **62**(10), 5331–5336 (2017).

8. Zhang, C.-K., He, Y., Jiang, L., Wu, M., and Zeng, H.-B., *Delay-variation-dependent stability of delayed discrete-time systems*, IEEE Trans. Autom. Control, **61**(9), 2663–2669 (2016).

9. Zhang, C.-K., He, Y., Jiang, L., Wu, Q. H., and Wu, M., *Delay-dependent stability criteria for generalized neural networks with two delay components*, IEEE Trans. Neural Netw. Learn. Syst., **25**(7), 1263–1276 (2014).

10. Zhang, X.-M., and Han, Q.-L., *Global asymptotic stability analysis for delayed neural networks using a matrix-based quadratic convex approach*, Neural Networks, **54**, 57–69 (2014).

11. Kwon, O.-M., Park, M.-J., Lee, S.-M., Park, J. H., and Cha, E.-J., *Stability for neural networks with time-varying delays via some new approaches*, IEEE Trans. Neural Netw. Learn. Syst., **24**(2), 181–193 (2013).

12. Kwon, O.-M., Park, J. H., Lee, S.-M., and Cha, E.-J., *Analysis on delay-dependent stability for neural networks with time-varying delays*, Neurocomputing, **103**, 114–120 (2013).

13. Kwon, O.-M., Park, J. H., Lee, S.-M., and Cha, E.-J., *New augmented Lyapunov–Krasovskii functional approach to stability analysis of neural networks with time-varying delays*, Nonlinear Dyn., **76**(1), 221–236 (2014).

14. Ji, M.-D., He, Y., Zhang, C.-K., and Wu, M., *Novel stability criteria for recurrent neural networks with time-varying delay*, Neurocomputing, **138**, 383–391 (2014).

15. Zhang, C.-K., Jiang, L., He, Y., Wu, H., and Wu, M., *Stability analysis for control systems with aperiodically sampled data using an augmented Lyapunov functional method*, IET Control Theory Appl., **7**(9), 1219–1226 (2013).

16. Zhang, C.-K., He, Y., Jiang, L., Wu, M., and Wu, Q. H., *Stability analysis of sampled-data systems considering time delays and its application to electric power markets*, J. Franklin Inst., **351**(9), 4457–4478 (2014).

17. Lee, T. H., and Park, J. H., *Improved criteria for sampled-data synchronization of chaotic Lur'e systems using two new approaches*, Nonlinear Anal., Hybrid Syst., **24**, 132–145 (2017).

18. Lee, T. H., and Park, J. H., *Stability analysis of sampled-data systems via free-matrix-based time-dependent discontinuous Lyapunov approach*, IEEE Trans. Autom. Control, **62**(7), 3653–3657 (2017).

19. Lee, T. H., Trinh, H. M., and Park, J. H., *Stability analysis of neural networks with time-varying delay by constructing novel Lyapunov functionals*, IEEE Trans. Neural Netw. Learn. Syst., **29**(9), 4238–4247 (2018).

20. Baskar, P., Anand, K. S., and Umesha, V., *Asymptotic stability analysis of neutral time-delay systems with nonlinear uncertainty*, Southeast Asian J. Math. Math. Sci., **15**(2), 43–56 (2019).

21. Umesha, V., Padmanabhan, S., Baskar, P., and Ali, M. S., *Exponential stability analysis for delay-differential systems of neutral type with an LMI approach*, Khayyam J. Math., **5**(1), 11–20 (2019).

22. Saravanan, S., Umesha, V., Ali, M. S., and Padmanabhan, S., *Exponential passivity for uncertain neural networks with time-varying delays based on weighted integral inequalities*, Neurocomputing, **314**, 429–436 (2018).

23. Ravi Chandra, S., Padmanabhan, S., Umesha, V., Ali, M. S., Rajchakit, G., and Jirawattanapanit, A., *New insights on bidirectional associative memory neural networks with leakage delay components and time-varying delays using sampled-data control*, Neural Process. Lett., **56**(2), 94–108 (2024).

24. Ravi Chandra, S., Rajakumari, P., Shanthi, G., Baskar, P., Umesha, V., and Padmanabhan, S., *Control of sampled data for neural networks with time-varying and leakage delays*, Int. J. Inf. Technol., **31**, 1–6 (2025).

Umesha V,
Department of Mathematics,
B.M.S College of Engineering, Bull Temple Road, Bengaluru,
Karnataka, India-560019.
E-mail address: umesha.maths@bmsce.ac.in

and

Gireesh D. S
Department of Mathematics,
B.M.S College of Engineering, Bull Temple Road, Bengaluru,
Karnataka, India-560019.
E-mail address: gireeshdap@gmail.com

and

S. Padmanabhan,

Department of CSE(AI & ML),
RNS Institute of technology, Bengaluru,
Karnataka, India-560098.
E-mail address: padmanabhan@rnsit.ac.in

and

Sreelakshmi T. J.
Research Scholar
Department of Mathematics,
RNS Institute of Technology, Bengaluru,
Affiliated to Visvesvaraya Technological University (VTU), Belagavi,
Karnataka India-560098.
E-mail address: sreelakshmi.tj@rnsit.ac.in