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On Neutrosophic Ideals of B-Algebras

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ABSTRACT: This paper presents a comprehensive study of neutrosophic concepts in B-algebras, focusing on several types of neutrosophic ideals including neutrosophic ideals, neutrosophic near ideals, neutrosophic Ns-ideals, neutrosophic power ideals, and neutrosophic near power ideals. The paper establishes and proves multiple results, such as: every neutrosophic B-algebra is both a neutrosophic near ideal and a neutrosophic Ns-ideal; every neutrosophic ideal and every neutrosophic near ideal of a B-algebra is a neutrosophic N_s-ideal; every neutrosophic regular set in a B-algebra is a neutrosophic Ns-ideal; every neutrosophic B-algebra is also a neutrosophic power ideal; every neutrosophic ideal, near ideal, and regular set in a B-algebra is a neutrosophic power ideal; every neutrosophic power ideal is a neutrosophic Ns-power ideal; and similarly, every neutrosophic B-algebra, near ideal, and regular set is a neutrosophic N_s-power ideal. In addition to these findings, the paper explores the structural relationships and properties among these different types of neutrosophic ideals, providing a deeper understanding of their significance within the framework of B-algebras. These results contribute to the theoretical development of neutrosophic algebraic structures and may serve as a foundation for future applications in logic, information systems, and decision-making models involving indeterminacy.

Keywords: Neutrosophic B-algebra, Neutrosophic regular set, Neutrosophic ideal, Neutrosophic N_s-ideal, Neutrosophic power ideal.

Contents

1. Introduction

In 2002, J. Neggers and H.S. Kim [1] introduced the concept of B-algebras, which are structurally related to algebraic systems such as BCH/BCI/ BCK-algebras. They also demonstrated a strong connection between B-algebras and group theory. Earlier, in 1965, L.A. Zadeh [2] introduced the theory of fuzzy sets, which became a powerful tool in the generalization of algebraic structures. In the same year B-algebras were introduced, Jun et al. [3] applied the theory of fuzzy sets to B-algebras, laying the groundwork for further studies on fuzzy extensions. Subsequently, Ahn and Bang [4] presented findings on fuzzy subalgebras within B-algebras. In 2011, T. Senavati, M. Bhowmik, and M. Pal [5] extended this work by applying fuzzy set theory to the study of ideals and closed ideals in B-algebras. In 2021, the notions of Soft Quantum B-Algebras and Fuzzy Soft Quantum B-Algebras were introduced by Xiongtheyi, Sultan, and Ahmed [6], marking a novel direction in the study of quantum fuzzy systems. In 2022, R. Rasuli [7] proposed the concepts of T-fuzzy B-subalgebras and normal T-fuzzy B-subalgebras, further enriching the structure theory of fuzzy B-algebras. That same year, P. Muralikrishna, R. Vinod Kumar, and G. Palani [8] introduced the idea of cubic fuzzy $\beta-$ideals in $\beta-$algebras. Also in 2022, Ma. Rizal V. Dicen, Katrina E., and Belleza-Fuentes [9] studied the Russification process of dual B-algebras. In 2023, Dian Kartika Amandani, Noor Hidayat, and Abdul Rouf [10] explored the structure of m-polar fuzzy B-ideals within B-algebras. A broader generalization of fuzzy sets was proposed by Atanassov in 1986 in

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the form of intuitionistic fuzzy sets. Building on this, Royyan A., Noor H., and Vira H. Krisnawati [11,12] investigated intuitionistic fuzzy ideals in B-algebras. Further applications of fuzzy and intuitionistic fuzzy frameworks to various algebraic structures have been reported in recent years [13,14]. In addition, neutrosophic logic, introduced by Smarandache [15], provided new perspectives on uncertainty modeling in algebraic contexts. There are many applications in pure mathematics using some types of non-classical sets such as soft sets [16,17], fuzzy sets [18,19], neutrosophic sets [20,21] and others [22,23,24] have been studied and addressed, also, including the introduction of new neutrosophic ideals such as the neutrosophic Ns-ideal and the neutrosophic power ideal. These structures exhibit properties similar to classical ideals in B-algebras, but within the neutrosophic framework. However, a major challenge in previous studies is the lack of a unified or comparative framework that integrates these different types of fuzzy and neutrosophic ideals within B-algebras. For example, while T-fuzzy B-subalgebras [7], cubic fuzzy $\beta-$ideals [8], and m-polar fuzzy B-ideals[10] have been introduced, they were often developed in isolation, without sufficient comparison or classification under a common algebraic lens. Similarly, intuitionistic fuzzy and neutrosophic ideals were explored independently, and there has been limited effort to analyze their interrelations or their impact on structural properties of B-algebras such as closure, or identity behavior. This fragmented development makes it difficult to identify general patterns, common properties, or potential generalizations. This work includes several novel algebraic ideas as well as some intriguing findings regarding their use in the non-classical field of neutrosophic sets theory. Furthermore, some of these neutrosophic ideals' details and relationships are shown. This work includes several novel algebraic ideas as well as some intriguing findings regarding their use in the non-classical field of neutrosophic sets theory. Furthermore, some of these neutrosophic ideals' details and relationships are shown. The objective of this paper is to further advance the theoretical development of B-algebras by integrating fuzzy set theory, intuitionistic fuzzy sets, and neutrosophic logic. The research specifically focuses on introducing and examining new types of fuzzy and neutrosophic ideals in B-algebras, generalizing existing concepts and establishing new structural properties. This contributes to the expansion of modern algebraic theory and provides a foundational basis for future applications in mathematical and computational fields.

2. Preliminaries

Here in this part, the general notions of B-algebra are recalled P.

Definition 2.1 *[1] A B-algebra is a non-empty set P with a constant 0 and a binary operation* satisfying the following axioms:*
(1) $b_1 * b_1 = 0$
(2) $b_1 * 0 = b_1$
(3) $(b_1 * b_2) * b_3 = b_1 * (b_3 * (0 * b_2))$, $\forall b_1, b_2, b_3 \in P$.

Proposition 2.1 *[1] If $(P, *, 0)$ is a B-algebra, then*
(1) $(b_1 * b_2) * (0 * b_2) = b_1$,
(2) $b_1 * (b_2 * b_3) = (b_1 * (0 * b_3)) * b_2$,
(3) $b_1 * b_2 = 0$ *implies* $b_1 = b_2$,
(4) $0 * (0 * b_1) = b_1$,
(5) $(b_1 * b_3) * (b_2 * b_3) = b_1 * b_2$,
(6) $0 * (b_1 * b_2) = b_2 * b_1, \forall b_1, b_2, b_3 \in P$

Definition 2.2 *[1] A B-algebra $(P, *, 0)$ is said to be commutative if $b_1 * (0 * b_2) = b_2 * (0 * b_1)$ $\forall b_1, b_2, b_3 \in P$.*

Proposition 2.2 *[1] If $(P, *, 0)$ is a commutative B-algebra, then*
(1) $(0 * b_1) * (0 * b_2) = b_2 * b_1$.
(2) $(b_3 * b_2) * (b_3 * b_1) = b_1 * b_2$.
(3) $(b_1 * b_2) * b_3 = (b_1 * b_3) * b_2$.
(4) $[b_1 * (b_1 * b_2)] * b_2 = 0$.
(5) $(b_1 * b_3) * (b_2 * b_4) = (b_4 * b_3) * (b_2 * b_1)$, $\forall b_1, b_2, b_3, b_4 \in P$.

Definition 2.3 *[15] The definition of a neutrosophic set N on the universal P is $N = \{(x, T_N(x), I_N(x), F_N(x))|x \in P\}$, where $T_N(x), I_N(x), F_N(x) : P \longrightarrow [0,1]$ are maps, with $T_N(x), I_N(x)$ and $F_N(x)$they are actual numbers whose values indicate the degree of membership, non-membership, and indeterminacy of x to N, respectively.*

3. Results and Discussion

Some classes of neutrosophic B-algebras, neutrosophic regular sets, neutrosophic ideals are shown and discussed. Moreover, B-algebras and their ideals are studied in pure mathematics, but in this work their extensions using neutrosophic are studied in computational mathematics. Also, the neutrosophic sets is one of the most important sets, since it can handle more uncertainty than fuzzy sets and intuitionistic fuzzy sets, so has a wider range of applications.

Definition 3.1 *If a neutrosophic set N in P meets the following inequality and it is referred to neutrosophic B-algebra as $(NB - A)$:*
(1) $T_N(b_1 * b_2) \geq min\{T_N(b_1), T_N(b_2)\}, \forall b_1, b_2 \in P$,
(2) $I_N(b_1 * b_2) \leq max\{I_N(b_1), I_N(b_2)\}, \forall b_1, b_2 \in P$,
(3) $F_N(b_1 * b_2) \geq min\{F_N(b_1), F_N(b_2)\}, \forall b_1, b_2 \in P$.

Definition 3.2 *If a neutrosophic set N in P meets the inequality, it is considered a neutrosophic regular set (NRS).*
(1) $T_N((b_1 * b_3) * (b_2 * b_4)) \geq min\{T_N(b_1 * b_2), T_N(b_3 * b_4)\}, \forall b_1, b_2, b_3, b_4 \in P$,
(2) $I_N((b_1 * b_3) * (b_2 * b_4)) \leq max\{I_N(b_1 * b_2), I_N(b_3 * b_4)\}, \forall b_1, b_2, b_3, b_4 \in P$,
(3)$F_N((b_1 * b_3) * (b_2 * b_4)) \geq min\{F_N(b_1 * b_2), F_N(b_3 * b_4)\}, \forall b_1, b_2, b_3, b_4 \in P$.

Proposition 3.1 *If N A neutrosophic regular set then N is a $(NB - A)$.*

Proof: For any $b_1, b_2 \in P$ $T_N(b_1 * b_2) = T_N((b_1 * b_2) * (0 * 0)) \geq min\{T_N(b_1 * 0), T_N(b_2 * 0)\} = min T_N(b_1), T_N(b_2)$, $I_N(b_1 * b_2) = I_N((b_1 * b_2) * (0 * 0)) \leq max\{I_N(b_1 * 0), I_N(b_2 * 0)\} = max I_N(b_1), I_N(b_2)$, $F_N(b_1 * b_2) = F_N((b_1 * b_2) * (0 * 0)) \geq min\{F_N(b_1 * 0), F_N(b_2 * 0)\} = min F_N(b_1), F_N(b_2)$. Hence N is a $(NB - A)$.

□

Proposition 3.2 *If N is a (NB-A) in commutative B-algebra, then N is (NRS).*

Proof: Let $b_1, b_2, b_3, b_4 \in P$. $T_N((b_1 * b_3) * (b_2 * b_4)) = T_N((b_4 * b_3) * (b_2 * b_1)) = T_N(b_4 * [(b_2 * b_1) * (0 * b_3)]) = T_N(b_4 * [(b_3 * b_1) * (0 * b_2)]) = T_N(b_4 * [b_3 * ((0 * b_2) * (0 * b_1))]) = T_N(b_4 * [b_3 * (b_1 * b_2)]) = T_N([b_4 * (0 * (b_1 * b_2))] * b_3) = T_N([b_4 * (b_2 * b_1)] * b_3) = T_N([(b_4 * (0 * b_1)) * b_2] * b_3) = T_N((b_4 * (0 * b_1)) * (b_3 * (0 * b_2))) = T_N([(0 * b_2) * (0 * b_1)] * (b_3 * b_4)) = T_N((b_1 * b_2) * (b_3 * b_4)) \geq min\{T_N(b_1 * b_2), T_N(b_3 * b_4)\}$ In similarity; $I_N((b_1 * b_3) * (b_2 * b_4)) \leq max\{I_N(b_1 * b_2), I_N(b_3 * b_4)\}$, $F_N((b_1 * b_3) * (b_2 * b_4)) \geq min\{F_N(b_1 * b_2), F_N(b_3 * b_4)\}$. Hence N is a (NRS).

□

Definition 3.3 *A neutrosophic set N of B–algebra P is said to be a neutrosophic ideal (NI) if:*
(1) $T_N(0) \geq T_N(b), \forall b \in P$,
(2) $T_N(b_1) \geq min\{T_N(b_1 * b_2), T_N(b_2)\}, \forall b_1, b_2 \in P$,
(3) $I_N(0) \leq I_N(b), \forall b \in P$,
(4) $I_N(b_1) \leq max\{I_N(b_1 * b_2), I_N(b_2)\}, \forall b_1, b_2 \in P$,
(5) $F_N(0) \geq F_N(b), \forall b \in P$,
(6) $F_N(b_1) \geq min\{F_N(b_1 * b_2), F_N(b_2)\}, \forall b_1, b_2 \in P$.

Proposition 3.3 *Every $(NB - A)$ is a (NI).*

Proof: Let N be $(NB-A)$, then $T_N(0)=T_N(b_1*b_1)\geq min\{T_N(b_1),T_N(b_1)\}=T_N(b_1)$, thus $T_N(0)\geq T_N(b_1)$. Now, $T_N(b_1)=T_N((b_1*b_2)*(0*b_2))\geq min\{T_N(b_1*b_2),T_N(0*b_2)\}\geq min\{T_N(b_1*b_2),min\{T_N(0),T_N(b_2)\}\}=min\{T_N(b_1*b_2),T_N(b_2)\}$. Thus, $T_N(b_1)\geq min\{T_N(b_1*b_2),T_N(b_2)\},I_N(0)=I_N(b_1*b_1)\leq max\{I_N(b_1),I_N(b_1)\}=I_N(b_1)$, then, $I_N(0)\leq I_N(b_1)$. Now, $I_N(b_1)=I_N((b_1*b_2)*(0*b_2))\leq max\{I_N(b_1*b_2),I_N(0*b_2)\}\leq max\{I_N(b_1*b_2),max\{I_N(0),I_N(b_2)\}\}=max\{I_N(b_1*b_2),I_N(b_2)\}$. Thus, $I_N(b_1)\leq max\{I_N(b_1*b_2),I_N(b_2)\}$, $F_N(0)=F_N(b_1*b_1)\geq min\{F_N(b_1),F_N(b_1)\}=F_N(b_1)$. Thus, $F_N(0)\geq F_N(b_1)$. Now, $F_N(b_1)=F_N((b_1*b_2)*(0*b_2))\geq min\{F_N(b_1*b_2),F_N(0*b_2)\}\geq min\{F_N(b_1*b_2),min\{F_N(0),F_N(b_2)\}\}=min\{F_N(b_1*b_2),F_N(b_2)\}$. Thus, $F_N(b_1)\geq min\{F_N(b_1*b_2),F_N(b_2)\}$, hence, N is a neutrosophic ideal. □

Corollary 3.1 *Every (NRS) in B-algebra is a (NI).*

Proof: By Proposition (3.1) and Proposition (3.3). □

Remark 3.1 *As demonstrated by the following example, the converse of Corollary (3.0A) is generally untrue.*

Example 3.1 *Let $P=\{e,a,b,c,d,h\}$ be a set with the following table 1:*

Table 1: $(B,*,0)$ a B-algebra

*	**0**	**1**	**2**	**3**	**4**	**5**
0	0	2	1	3	4	5
1	1	0	2	4	5	3
2	2	1	0	5	3	4
3	3	4	5	0	2	1
4	4	5	3	1	0	2
5	5	3	4	2	1	0

They define a neutrosophic set $N:P\longrightarrow[0,1]$ by

$$T_N(x)=\begin{cases}0.4\ , & if \quad x=e,c.\\ 0.2\ , & if \quad x=a,b,d,h.\end{cases}$$

$$I_N(x)=\begin{cases}0.2\ , & if \quad x=e,c.\\ 0.5\ , & if \quad x=a,b,d,h.\end{cases}$$

$$F_N(x)=\begin{cases}0.3\ , & if \quad x=e,c.\\ 0.1\ , & if \quad x=a,b,d,h.\end{cases}$$

Then N is a neutrosophic ideal, but N it is not necessary a (NRS), Since, $T_N((b*h)*(d*a))=T_N(b)=0.2<0.4=T_N(c)=min\{T_N(b*d),T_N(h*a)\}$.

Definition 3.4 *A neutrosophic set N of B –algebra P is called a neutrosophic near ideal (NNI) if:*
(1) $T_N(0)\geq T_N(b),\forall b\in P$,
(2) $T_N(b_1)\geq min\{T_N(b_1*b_2),T_N(b_2)\},\forall b_1,b_2\in P$,
(3) $T_N(0*b)\geq T_N(b),\forall b\in P$,
(4) $I_N(0)\leq I_N(b),\forall b\in P$,
(5) $I_N(b_1)\leq max\{I_N(b_1*b_2),I_N(b_2)\},\forall b_1,b_2\in P$,
(6) $I_N(0*b)\leq I_N(b),\forall b\in P$,

(7) $F_N(0) \geq F_N((b), \forall b \in P,$
(8) $F_N(b_1) \geq min\{F_N(b_1 * b_2), F_N(b_2)\}, \forall b_1, b_2 \in P,$
(9) $F_N(0 * b) \geq F_N((b), \forall b \in P.$

Remark 3.2 *Let P be (NNI) of a B-algebra, then we have P is a (NI).*

Proposition 3.4 *Every a $(NB - A)$ of B-algebra P is a (NNI).*

Proof:

Let N be $(NB - A)$ then, $T_N(0) = T_N(b_1 * b_1) \geq min\{T_N(b_1), T_N(b_1)\} = T_N(b_1)$.Thus, $T_N(0) \geq T_N(b_1)$. Now, $T_N(b_1) = T_N((b_1 * b_2) * (0 * b_2)) \geq min\{T_N(b_1 * b_2), T_N(0 * b_2)\} \geq min\{T_N(b_1 * b_2), min\{T_N(0), T_N(b_2)\} = min\{T_N(b_1 * b_2), T_N(b_2)\}$. Thus, $T_N(b_1) \geq min\{T_N(b_1 * b_2), T_N(b_2)\}, T_N(0 * b_1) \geq min\{T_N(0), T_N(b_1)\} = T_N(b_1)$. Now, $I_N(0) = I_N(b_1 * b_1) \leq max\{I_N(b_1), I_N(b_1)\} = I_N(b_1)$. Thus, $I_N(0) \leq I_N(b_1)$. Now, $I_N(b_1) = I_N((b_1 * b_2) * (0 * b_2)) \leq max\{I_N(b_1 * b_2), I_N(0 * b_2)\} \leq max\{I_N(b_1 * b_2), max\{I_N(0), I_N(b_2)\}\} = max\{I_N(b_1 * b_2), I_N(b_2)\}$. Thus, $I_N(b_1) \leq max\{I_N(b_1 * b_2), I_N(b_2)\}, I_N(0 * b_1) \leq max\{I_N(0), I_N(b_1)\} = I_N(b_1), F_N(0) = F_N(b_1 * b_1) \geq min\{F_N(b_1), F_N(b_1)\} = F_N(b_1)$,Thus, $F_N(0) \geq F_N(b_1)$.Now, $F_N(b_1) = F_N((b_1 * b_2) * (0 * b_2)) \geq min\{F_N(b_1 * b_2), F_N(0 * b_2)\} \geq min\{F_N(b_1 * b_2), min\{F_N(0), F_N(b_2)\}\} = min\{F_N(b_1 * b_2), F_N(b_2)\}$. Thus, $F_N(b_1) \geq min\{F_N(b_1 * b_2), F_N(b_2)\}$, $F_N(0 * b_1) \geq min\{F_N(0), F_N(b_1)\} = F_N(b_1)$. Hence N is (NNI).

□

4. A Neutrosophic N_S-Ideal

This section defies classes of neutrosophic N_S-ideals and examines how they relate to other classes of (NI).

Definition 4.1 *Let S be subset of B-algebra P. A neutrosophic set N is said to be a neutrosophic N_s-ideal $(NN_S - I)$ if :*
(1) $T_N(0) \geq T_N(b), \forall b \in P,$
(2) $T_N(b_1) \geq min\{T_N(b_1 * b_2), T_N(b_2)\}, \forall b_1, b_2 \in P,$
(3) $I_N(0) \leq I_N(b), \forall b \in P,$
(4) $I_N(b_1) \leq max\{I_N(b_1 * b_2), I_N(b_2)\}, \forall b_1, b_2 \in P,$
(5) $F_N(0) \geq F_N(b), \forall b \in P,$
(6) $F_N(b_1) \geq min\{F_N(b_1 * b_2), F_N(b_2)\}, \forall b_1, b_2 \in P.$

Example 4.1 *Let $P = \{0, 1, 2, 3\}$ be a set with the Table 2:*

Table 2: $(B, *, 0)$ is a B-algebra

*	**0**	**1**	**2**	**3**
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

$$T_N(x) = \begin{cases} 0.6, & if \quad x = 0. \\ 0.2, & if \quad x = 1, 3. \\ 0.1, & if \quad x = 2. \end{cases}$$

$$I_N(x) = \begin{cases} 0.2\ , & if \quad x = 0. \\ 0.4\ , & if \quad x = 1, 3. \\ 0.3\ , & if \quad x = 2. \end{cases}$$

$$F_N(x) = \begin{cases} 0.6\ , & if \quad x = 0. \\ 0.2\ , & if \quad x = 1, 3. \\ 0.1\ , & if \quad x = 2. \end{cases}$$

Then N is a neutrosophicN_S-ideal in P. Since
$T_N(0) = 0.6 = min\{T_N(0*0), T_N(0)\} = 0.6,$
$T_N(0) = 0.6 > min\{T_N(0*2), T_N(2)\} = 0.1,$
$T_N(1) = 0.2 = min\{T_N(1*0), T_N(0)\} = 0.2,$
$T_N(1) = 0.2 = min\{T_N(1*2), T_N(2)\} = 0.2,$
$T_N(2) = 0.1 = min\{T_N(2*0), T_N(0)\} = 0.1,$
$T_N(2) = 0.1 = min\{T_N(2*2), T_N(2)\} = 0.1,$
$T_N(3) = 0.2 = min\{T_N(3*0), T_N(0)\} = 0.2,$
$T_N(3) = 0.2 > min\{T_N(3*2), T_N(2)\} = 0.1.$

Also
$I_N(0) = 0.2 = maxI_N(0*0), I_N(0) = 0.2,$
$(0) = 0.2 < max\{I_N(0*2), I_N(2)\} = 0.3,$
$I_N(1) = 0.4 = max\{I_N(1*0), I_N(0)\} = 0.4,$
$I_N(1) = 0.4 = max\{I_N(1*2), I_N(2)\} = 0.4,$
$I_N(2) = 0.3 = max\{I_N(2*0), I_N(0)\} = 0.3,$
$I_N(2) = 0.3 = max\{I_N(2*2), I_N(2)\} = 0.3,$
$I_N(3) = 0.4 = max\{I_N(3*0), I_N(0)\} = 0.4,$
$I_N(3) = 0.4 = max\{I_N(3*2), I_N(2)\} = 0.4.$

And
$F_N(0) = 0.6 = min\{F_N(0*0), F_N(0)\} = 0.6,$
$F_N(0) = 0.6 > min\{F_N(0*2), F_N(2)\} = 0.1,$
$F_N(1) = 0.2 = min\{F_N(1*0), F_N(0)\} = 0.2,$
$F_N(1) = 0.2 = min\{F_N(1*2), F_N(2)\} = 0.2,$
$F_N(2) = 0.1 = min\{F_N(2*0), F_N(0)\} = 0.1,$
$F_N(2) = 0.1 = min\{F_(N_S)(2*2), F_N(2)\} = 0.1,$
$F_N(3) = 0.2 = min\{F_N(3*0), F_N(0)\} = 0.2,$
$F_N(3) = 0.2 > min\{F_N(3*2), F_N(2)\} = 0.1.$

Proposition 4.1 *Every a (NI) of B-algebra P is a neutrosophic N_S-ideal.*

Proof: Let S be subset of P and let N be a (NI), then by Definition (4.1) we have $T_N(0) \geq T_N(b_1), \forall b_1 \in P$. $T_N(b_1) \geq min\{T_N(b_1 * b_2), T_N(b_2)\}, \forall b_1, b_2 \in P$, since $S \subseteq P$, then $T_N(0) \geq T_N(b_1), T_N(b_1) \geq min\{T_N(b_1*b_2), T_N(b_2)\}, \forall b_2 \in S, insimilarity I_N(0) \leq I_N(b_1), \forall b_1 \in P, I_N(b_1) \leq max\{I_N(b_1*b_2), I_N(b_2)\}$, $\forall b_2 \in S$. $F_N(0) \geq F_N(b_1), \forall b_1 \in P, F_N(b_1) \geq \min F_N(b_1 * b_2), F_N(b_2), \forall b_2 \in S$. thus N is a $(NN_S - I)$.

Note that: In general, the converse of Proposition (4.1) is not true, from Example (4.1), N is a $(NN_S - I)$ but N is not a neutrosophic ideal, since $T_N(2) = 0.1 \ngeq min\{T_N(2*1), T_N(1)\} = 0.3$. □

Corollary 4.1 *Every a $(NB - A)$ is neutrosophic N_S-ideal.*

Proof: It comes straight from Propositions (3.3) and (4.1). □

Take note that, in general terms, the opposite of Corollary (4.0A) is not true, as demonstrated by Example (3.1), $S = \{0, 1, 3\} \subseteq P$ they define:

$$T_N(x) = \begin{cases} 0.9\ , if \quad x = 0. \\ 0.2\ \ , if \quad x = 1, 2, 4. \\ 0.1\ \ , if \quad x = 3, 5. \end{cases}$$

$$I_N(x) = \begin{cases} 0.2\ , if \quad x = 0. \\ 0.4\ \ , if \quad x = 1, 2, 4. \\ 0.3\ \ , if \quad x = 3, 5. \end{cases}$$

$$F_N(x) = \begin{cases} 0.6\ , if \quad x = 0. \\ 0.3\ \ , if \quad x = 1, 2, 4. \\ 0.2\ \ , if \quad x = 3, 5. \end{cases}$$

N is a neutrosophic N_S-ideal, but N is not a neutrosophic B-algebra, since, $T_N(1 * 4) = T_N(5) = 0.1 \ngeq min\{T_N(1), T_N(4)\} = 0.2$

Corollary 4.2 *Every a (NNI)of a B-algebra P is* $(NN_S - I)$.

Proof: They obtain from Remark (3.2) & Proposition (4.1). □

Corollary 4.3 *Every a (NRS) of B-algebra is* $(NN_S - I)$.

Corollary 4.4 *They obtain from Corollary (3.0A) & Proposition (4.1).*

Definition 4.2 *A* $(NN_S - I)$ *of B-algebra P is called a neutrosophic near* N_S*-ideal* $(NNN_S - I)$ *if it is a* $(NB - A)$.

Example 4.2 *In Example (4.1),* $S = \{0, 1, 2, 3\}$ *subset of* P*, and they define by;*

$$T_N(x) = \begin{cases} 0.8\ , \quad if \quad x = 0. \\ 0.3\ \ , \quad if \quad x = 1, 2, 3, 4, 5. \end{cases}$$

$$I_N(x) = \begin{cases} 0.3\ , if \quad x = 0. \\ 0.8\ \ , if \quad x = 1, 2, 3, 4, 5. \end{cases}$$

$$F_N(x) = \begin{cases} 0.8\ , \quad if \quad x = 0. \\ 0.3\ \ , \quad if \quad x = 1, 2, 3, 4, 5. \end{cases}$$

Then N neutrosophic near N_S*-ideal, Note that in general it is not necessary every neutrosophic* N_S*-ideal is* $(NB - A)$*, from Example (3,8) ,* $S = \{0, 1, 3\}$ *is subset of* P*, let N be a neutrosophic set defined as the following*

$$T_N(x) = \begin{cases} 0.9\ , if \quad x = 0. \\ 0.2\ \ , if \quad x = 1, 2, 4. \\ 0.1\ \ , if \quad x = 3, 5. \end{cases}$$

$$I_N(x) = \begin{cases} 0.1\ , if \quad x = 0. \\ 0.3\ \ , if \quad x = 1, 2, 4. \\ 0.2\ \ , if \quad x = 3, 5. \end{cases}$$

$$F_N(x) = \begin{cases} 0.9\ , if \quad x = 0. \\ 0.2\ \ , if \quad x = 1, 2, 4. \\ 0.1\ \ , if \quad x = 3, 5. \end{cases}$$

then N is $(NN_S - I)$*, but N is not* $(NB - A)$ *since,* $T_N(1 * 4) = T_N(5) = 0.1 \ngeq min\{T_N(1), T_N(4)\} = 0.2$.

5. Neutrosophic Power Ideals

This section explains their link to a neutrosophic ideal and defines a neutrosophic power ideal.

Definition 5.1 *A neutrosophic set N of a B-algebra B is said to be a neutrosophic power ideal (NPI), if:*
(1) $T_N(0) \geq T_N(b)$,$\forall b \in B$.
(2) $\forall b_1, b_2 \in B$, $\exists n \in Z^+, b_1^n \neq 0$, such that $T_N({b_1}^n) \geq min\{T_N(b_1 * b_2), T_N(b_2)\}$.
(3) $I_N(b) \leq I_N(0)$,$\forall b \in B$.
(4)$\forall b_1, b_2 \in B$, $\exists n \in Z^+, b_1^n \neq 0$, such that $I_N({b_1}^n) \leq max\{I_N(b_1 * b_2), I_N(b_2)\}$.
(5)$F_N(0) \geq F_N(b)$,$\forall b \in B$.
(6) $\forall b_1, b_2 \in B$, $\exists n \in Z^+, {b_1}^n \neq 0$, such that $F_N({b_1}^n) \geq min\{F(b_1 * b_2), F_N(b_2)\}$.
Where $b_1^n = ((b_1 * b_1) * b_1) * b_1 * \ldots * b_1$ (product b_1 n-times using $*$)

Example 5.1 *If B a set such that $B = \{0, 1, 2\}$ with Table 3:*

Table 3: $(B; *, 0)$ is a B-algebra.

$*$	**0**	**1**	**2**
0	0	2	1
1	1	0	2
2	2	1	0

We define a neutrosophic set $N : B \longrightarrow [0, 1]$ by

$$T_N(x) = \begin{cases} 0.9 \ , if \quad x = 0, 2. \\ 0.4 \ \ , if \quad x = 1. \end{cases}$$

$$I_N(x) = \begin{cases} 0.4 \ , if \quad x = 0, 1. \\ 0.9 \ \ , if \quad x = 2. \end{cases}$$

$$F_N(x) = \begin{cases} 0.9 \ , if \quad x = 0, 2. \\ 0.4 \ \ , if \quad x = 1. \end{cases}$$

Then $T_N(0) = 0.9 > T_N(1), T_N(2) = 0.4$,
$T_N(0) = 0.9 > min\{T_N(0 * 1), T_N(1)\} = 0.4$,
$T_N(0) = 0.9 > min\{T_N(0 * 2), T_N(2)\} = 0.4$,
$T_N(1) = 0.4 = min\{T_N(1 * 1), T_N(1)\} = 0.4$,
$T_N(1) = 0.4 = min\{T_N(1 * 0), T_N(0)\} = 0.4$,
$T_N(1) = 0.4, min\{T_N(1 * 2), T_N(2)\} = 0.9$,
$\exists n = 3 \in Z^+, 1^3 \neq 0, 1^3 = 2$ *such that,*
$T_N(1^3) = T_N(2) = 0.9 = min\{T_N(1 * 2), T_N(2)$
$\} = min\{T_N(2), T_N(2)\} = 0.9$,
$T_N(2) = 0.9 = min\{T_N(2 * 0), T_N(0)\} = 0.9$,
$T_N(2) = 0.9 = min\{T_N(2 * 2), T_N(2)\} = 0.9$,
$T_N(2) = 0.9 > min\{T_N(2 * 1), T_N(1)\} = 0.4$.
Also,
$I_N(0) = 0.4 < I_N(1), I_N(2) = 0.9$,
$I_N(0) = 0.4 < max\{I_N(0 * 1), I_N(1)\} = 0.9$,
$I_N(0) = 0.4 < max\{I_N(0 * 2), I_N(2)\} = 0.9$,
$I_N(1) = 0.4 = max\{I_N(1 * 1), I_N(1)\} = 0.4$,
$I_N(1) = 0.4 = max\{I_N(1 * 0), I_N(0)\} = 0.4$,
$I_N(1) = 0.4, max\{I_N(1 * 2), I_N(2)\} = 0.9$,
$\exists n = 3 \in Z^+, 1^3 \neq 0, 1^3 = 2$ *such that,*
$I_N(13) = I_N(2) = 0.9 = max\{I_N(1 * 2), I_N(2)\} = max\{I_N(2), I_N(2)\} = 0.9$,
$I_N(2) = 0.9 = max\{I_N(2 * 0), I_N(0)\} = 0.9$,

$I_N(2) = 0.9 = max\{I_N(2 * 2), I_N(2)\} = 0.9,$
$I_N(2) = 0.9, max\{I_N(2 * 1), I_N(1)\} = 0.4,$
$\exists n = 3 \in Z^+, 2^3 \neq 0, 2^3 = 1$ *such that,*
$I_N(2^3) = I_N(1) = 0.4 = max\{I_N(2 * 1), I_N(1)\} = max\{I_N(1), I_N(1)\} = 0.4,$
And
$F_N(0) = 0.9 > F_N(1), F_N(2) = 0.4,$
$F_N(0) = 0.9 > min\{F_N(0 * 1), F_N(1)\} = 0.4,$
$F_N(0) = 0.9 > min\{F_N(0 * 2), F_N(2)\} = 0.4,$
$F_N(1) = 0.4 = min\{F_N(1 * 1), F_N(1)\} = 0.4,$
$F_N(1) = 0.4 = min\{F_N(1 * 0), F_N(0)\} = 0.4,$
$F_N(1) = 0.4, min\{F_N(1 * 2), F_N(2)\} = 0.9,$
$\exists n = 3 \in Z^+, 1^3 \neq 0, 1^3 = 2$ *such that,*
$F_N(1^3) = F_N(2) = 0.9 = min\{F_N(1 * 2), F_N(2)\} = min\{F_N(2), F_N(2)\} = 0.9,$
$F_N(2) = 0.9 = min\{F_N(2 * 0), F_N(0)\} = 0.9,$
$F_N(2) = 0.9 = min\{F_N(2 * 2), F_N(2)\} = 0.9,$
$F_N(2) = 0.9 = min\{F_N(2 * 1), F_N(1)\} = 0.4,$
Hence N is a neutrosophic power ideal.

Proposition 5.1 *In B-algebra B every a $(NI)$is a neutrosophic power ideal.*

Proof: Obviously, the proof is held. □

Remark 5.1 *As demonstrated by Example (5.1), the opposite of Proposition (5.1) is generally not true.* $T_N(1) = 0.4 \ngeq min\{T_N(1 * 2), T_N(2)\} = min\{T_N(2), T_N(2)\} = 0.9.$

Corollary 5.1 *Every $(NB - A)$ is a neutrosophic power ideal.*

Proof: By Proposition (3.3) and Proposition (5.1). □

Corollary 5.2 *Every (NNI) of a B-algebra B is neutrosophic power ideal.*

Proof: By Remark (3.3) and Proposition (5.1). □

Corollary 5.3 *Every (NRS)in B-algebra is a neutrosophic power ideal.*

Proof: By Proposition (3.1) and Corollary (5.0A). □

Definition 5.2 *A (NPI) N in B-algebra B is said to be a neutrosophic near power ideal $(NNPI)$ if it also is $(NB - A)$.*

Proposition 5.2 *Let N be a (NPI) of B-algebra. If $b_1, b_2 \in B$, $b_1 \neq 0$ then $T_N({b_1}^n) \geq T_N(b_2)$, $I_N({b_1}^n) \leq I_N(b_2)$ and $F_N({b_1}^n) \geq F_N(b_2)$ for some $n \in Z^+$ s.t. ${b_1}^n \neq 0$, where $b_1 * b_2 = 0$.*

Proof:

Let $b_1, b_2 \in B$. Since, N a (NPI), then, $\exists n \in Z^+$, such that ${b_1}^n \neq 0$ and $T_N({b_1}^n) \geq min\{T_N(b_1 * b_2), T_N(b_2)\} = min\{T_N(0), T_N(b_2)\} = T_N(b_2)$, Thus $T_N({b_1}^n) \geq T_N(b_2)$, $I_N({b_1}^n) \leq max\{I_N(b_1 * b_2), I_N(b_2)\} = max\{I_N(0), I_N(b_2)\} = I_N(b_2)$, Thus $I_N({b_1}^n) \leq I_N(b_2)$, and $F_N({b_1}^n) \geq min\{F_N(b_1 * b_2), F_N(b_2)\} = min\{F_N(0), F_N(b_2)\} = F_N(b_2)$, Thus $F_N({b_1}^n) \geq F_N(b_2)$
□

Corollary 5.4 *For all $b_1 \in B$ there exists $n \in Z^+$, such that $T_N({b_1}^n) \geq T_N(b_1)$, $I_N({b_1}^n) \leq I_N(b_1)$, and $F_N({b_1}^n) \geq F_N(b_1)$.*

Definition 5.3 *Let S be subset of B-algebra B. A neutrosophic N said to be a neutrosophic N_S- power ideal ($NN_S - PI$) if:*
(1) $T_N(0) \geq T_N(b_1)$,$\forall b_1 \in B$.
(2) $\forall b_1 \in B$, $\forall b_2 \in S, \exists n \in Z^+, {b_1}^n \in 0$, such that $T_N({b_1}^n) \geq min\{T_N(b_1 * b_2), T_N(b_2)\}$.
(3)$I_N(b_1) \leq I_N(0)$,$\forall b_1 \in B$.
(4) $\forall b_1 \in B$, $\forall b_2 \in S, \exists n \in Z^+, {b_1}^n \neq 0$, such that $I_N({b_1}^n) \leq max\{I_N(b_1 * b_2), I_N(b_2)\}$
(5) $F_N(0) \geq F_N(b_1)$, $\forall b_1 \in B$.
(6) $\forall b_1 \in B$, $\forall b_2 \in S, \exists n \in Z^+, {b_1}^n \neq 0$, such that $F_N({b_1}^n) \geq min\{F_N(b_1 * b_2), F_N(b_2)\}$.

Example 5.2 *Let $B = \{0, 1, 2, 3, 4, 5\}$ be a set with the Table 4:*

Table 4: (B, $*$,0) a B-algebra

$*$	**0**	**1**	**2**	**3**	**4**	**5**
0	0	4	5	3	1	2
1	1	0	2	5	4	3
2	2	1	0	4	3	5
3	3	2	1	0	5	4
4	4	5	3	2	0	1
5	5	3	4	1	2	0

Take $S = \{0, 3\}$ and they define a neutrosophic set $N : B \longrightarrow [0, 1]$

$$T_N(x) = \begin{cases} 0.7 \ , if \quad x = 0. \\ 0.4 \ \ , if \quad x = 1, 2, 3. \\ 0.2 \ \ , if \quad x = 4, 5. \end{cases}$$

$$I_N(x) = \begin{cases} 0.2 \ , if \quad x = 0. \\ 0.7 \ \ , if \quad x = 1, 2, 3. \\ 0.4 \ \ , if \quad x = 4, 5. \end{cases}$$

$$F_N(x) = \begin{cases} 0.7 \ , if \quad x = 0. \\ 0.4 \ \ , if \quad x = 1, 2, 3. \\ 0.2 \ \ , if \quad x = 4, 5. \end{cases}$$

Then N is a ($NN_S - PI$) in B. Since $T_N(0) \geq T_N(x) \forall x \in B$ $T_N(1) = 0.4 = min\{T_N(1*0), T_N(0)\} = min\{T_N(1), T_N(0)\} = 0.4$,
$T_N(1) = 0.4 > min\{T_N(1 * 3), T_N(3)\} = min\{T_N(5), T_N(3)\} = 0.2$,
$T_N(2) = 0.4 = min\{T_N(2 * 0), T_N(0)\} = min\{T_N(2), T_N(0)\} = 0.4$,
$T_N(2) = 0.4 > min\{T_N(2 * 3), T_N(3)\} = min\{T_N(4), T_N(3)\} = 0.2$,
$T_N(3) = 0.4 = min\{T_N(3 * 0), T_N(0)\} = min\{T_N(3), T_N(0)\} = 0.4$,
$T_N(3) = 0.4 = min\{T_N(3 * 3), T_N(3)\} = min\{T_N(0), T_N(3)\} = 0.4$,
$T_N(4) = 0.2 = min\{T_N(4 * 0), T_N(0)\} = min\{T_N(4), T_N(0)\} = 0.2$,
$T_N(4) = 0.2, min\{T_N(4 * 3), T_N(3)\} = min\{T_N(2), T_N(3)\} = 0.4$,
$\exists n = 3 \in Z^+, 4^3 \neq 0, 4^3 = 1$ *such that*

$$T_N(4^3) = T_N(1) = 0.4 = min\{T_N(4 * 3), T_N(3)\} = min\{T_N(2), T_N(3)\} = 0.4.$$

$$T_N(5) = 0.2 = min\{T_N(5 * 0), T_N(0)\} = min\{T_N(5), T_N(0)\} = 0.2$$

$T_N(5) = 0.2, min\{T_N(5 * 3), T_N(3)\} = min\{T_N(1), T_N(3)\} = 0.4,$
$\exists n = 3 \in Z^+, 5^3 \neq 0, 5^3 = 2$ *such that*

$$T_N(5^3) = T_N(2) = 0.4 = min\{T_N(5 * 3), T_N(3)\} = min\{T_N(1), T_N(3)\} = 0.4.$$

Also,

$I_N(0) < I_N(x), \forall x \in B$ $I_N(1) = 0.7 = max\{I_N(1 * 0), I_N(0)\} = max\{I_N(1), I_N(0)\} = 0.7,$
$I_N 1 = 0.7 = max\{I_N(1 * 3), I_N(3)\} = max\{I_N(5), I_N(3)\} = 0.7,$
$I_N(2) = 0.7 = max\{I_N(2 * 0), I_N(0)\} = max\{I_N(2), I_N(0)\} = 0.7,$
$I_N 2 = 0.7 = max\{I_N(2 * 3), I_N(3)\} = max\{I_N(4), I_N(3)\} = 0.7,$
$I_N(3) = 0.7 = max\{I_N(3 * 0), I_N(0)\} = max\{I_N(3), I_N(0)\} = 0.,$
$I_N(3) = 0.7 = max\{I_N(3 * 3), I_N(3)\} = max\{I_N(0), I_N(3)\} = 0.7,$
$I_N(4) = 0.4 = max\{I_N(4 * 0), I_N(0)\} = max\{I_N(4), I_N(0)\} = 0.4,$
$I_N(4) = 0.4, max\{I_N(4 * 3), I_N(3)\} = max\{I_N(2), I_N(3)\} = 0.7,$
$\exists n = 3 \in Z^+$, $4^3 \neq 0, 4^3 = 1$ *such that* $I_N(4) = I_N(1) = 0.7 = max\{I_N(4 * 3), I_N(3)\} = max\{I_N(2), I_N(3)\} = 0.7$, $I_N(5) = 0.4 = max\{I_N(5 * 0), I_N(0)\} = max\{I_N(5), I_N(0)\} = 0.4,$
$I_N(5) = 0.4, max\{I_N(5 * 3), I_N(3)\} = max\{I_N(1), I_N(3)\} = 0.7$, $\exists n = 3 \in Z^+, 5 \neq 0, 5 = 2$ *such that* $I_N(5) = I_N(2) = 0.7 = max\{I_N(5 * 3), I_N(3)\} = max\{I_N(1), I_N(3)\} = 0.7.$
And, $F_N(0) \geq F_N(x) \forall x \in B$ $F_N(1) = 0.4 = min\{F_N(1 * 0), F_N(0)\} = min\{F_N(1), F_N(0)\} = 0.4,$
$F_N(1) = 0.4 > min\{F_N(1 * 3), F_N(3)\} = min\{F_N(5), F_N(3)\} = 0.2,$
$F_N(2) = 0.4 = min\{F_N(2 * 0), F_N(0)\} = min\{F_N(2), F_N(0)\} = 0.4,$
$F_N(2) = 0.4 > min\{F_N(2 * 3), F_N 3\} = min\{F_N 4, F_N 3\} = 0.2,$
$F_N(3) = 0.4 = min\{F_N(3 * 0), F_N(0)\} = min\{F_N(3), F_N(0)\} = 0.4,$
$F_N(3) = 0.4 = min\{F_N(3 * 3), F_N(3)\} = min\{F_N(0), F_N(3)\} = 0.4,$
$F_N(4) = 0.2 = min\{F_N(4 * 0), F_N(0)\} = min\{F_N(4), F_N(0)\} = 0.2,$
$F_N(4) = 0.2, min\{F_N(4 * 3), F_N(3)\} = min\{F_N(2), F_N(3)\} = 0.4,$
$\exists n = 3 \in Z^+, 4 \neq 0, 4 = 1$ *such that* $F_N(4) = F_N(1) = 0.4 = min\{F_N(4 * 3), F_N(3)\} = min\{F_N(2), F_N(3)\} = 0.4,$
$F_N(5) = 0.2 = min\{F_N(5 * 0), F_N(0)\} = min\{F_N(5), F_N(0)\} = 0.2,$
$F_N(5) = 0.2, min\{F_N(5 * 3), F_N(3)\} = min\{F_N(1), F_N(3)\} = 0.4,$
$\exists n = 3 \in Z^+, 5 \neq 0, 5 = 2$ *such that* $F_N(5) = F_N(2) = 0.4 = min\{F_N(5*3), F_N(3)\} = min\{F_N(1), F_N(3)\} = 0.4,$

Proposition 5.3 *In B-algebra B, every a (NPI) is a $(NN_S - PI)$.*

Proof: Let N be a (NPI) & S subset of B . From Definition (5.1) we have:
(1) $T_N(0) \geq T_N(b_1), \forall b_1 \in B.$
(2) Let $b_1 \in B$.Then $\forall b_2 \in S, \exists n_{b_2} \in Z^+$ s.t ${b_1}^n \neq 0 \& T_N({b_1}^{n_{b_2}}) \geq min\{T_N(b_1 * b_2), T_N(b_2)\}$. Thus $max\{T_N({b_1}^{n_{b_2}}) : b_2 \in S\} \geq min\{T_N(b_1 * b_2), T_N(b_2)\}, \forall b_2 \in S.$
(3) $I_N(b_1) \leq I_N(0), \forall b_1 \in B.$
(4) Let $b_1 \in B$.Then $\forall b_2 \in S, \exists n_{b_2} \in Z^+ s.t {b_1}^n \neq 0 \& I_N({b_1}^{n_{b_2}}) \leq max\{I_N(b_1 * b_2), I_N(b_2)\}$. Thus $min\{I_N({b_1}^{n_{b_2}}) : b_2 \in S\} \leq max\{I_N(b_1 * b_2), T_N(b_2)\}, \forall b_2 \in S.$
(5) $F_N(0) \geq F_N(b_1), \forall b_1 \in B.$
(6) Let $b_1 \in B$ .Then $\forall b_2 \in S$, $\exists n_{b_2} \in Z^+$ s.t ${b_1}^n \neq 0 \& F_N({b_1}^{n_{b_2}}) \geq min\{F_N(b_1 * b_2), F_N(b_2)\}$.
Thus $max\{F_N({b_1}^{n_{b_2}}) : b_2 \in S\} \geq min\{F_N(b_1 * b_2), F_N(b_2)\}, \forall b_2 \in S$. Hence N is a $(NN_S - PI)$. □

Corollary 5.5 *In B-algebra, every a (NI) is a $(NN_S - PI)$.*

Proof: It satisfies from Proposition (5.1) & Proposition (5.3). □

Corollary 5.6 *In B-algebra B, every a $(NB - A)$ is a $(NN_S - PI)$.*

Proof: From Corollary (5.0A) and Proposition (5.3). □

Corollary 5.7 *In B-algebra B, every (NNI) is a* $(NN_S - PI)$*.*

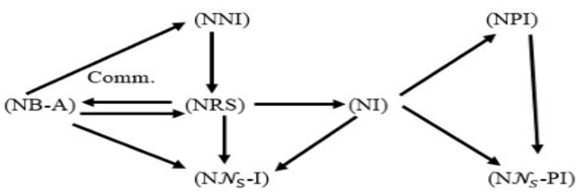
Proof: From Corollary(5.0B) and Proposition (5.3). □

Corollary 5.8 *In B-algebra, every a (NRS) is a* $(NN_S - PI)$*.*

Proof: They obtain from Corollary (5.0C) and Proposition (5.3). □

Remark 5.2 *From the following diagram can be observe the relationship between these types of neutrosophic ideals.*

Remark 5.3 *From the following diagram can be observe the relationship between these types of neutrosophic ideals.*



6. Conclusion

In this paper, several types of neutrosophic ideals in B-algebras were introduced and studied, namely: neutrosophic ideal, neutrosophic near ideal, neutrosophic Ns-ideal, neutrosophic power ideal, and neutrosophic near power ideal. A number of propositions were proposed to clarify the relationships among these types, representing a novel theoretical contribution to the field. These findings highlight the potential of neutrosophic logic in analyzing algebraic structures involving indeterminacy and open the door to broader applications in areas that require flexible models for uncertainty. However, this study faces some limitations, most notably its theoretical nature without providing concrete applications or numerical examples, and its restriction to B-algebras without examining generalizations to other algebraic structures. This work plans to employ soft set theory to further explore these types of ideals from a more adaptable perspective, integrating them into the framework of neutrosophic soft sets. Expanding the study to include various algebraic structures and developing practical applications and algorithms based on these concepts in fields such as artificial intelligence and data analysis are also recommended. Thus, this research serves as a foundational step toward establishing a comprehensive theoretical framework for neutrosophic ideals with potential for wide mathematical and practical relevance.

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