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The Construction of 3-TEPC Labeling for Stellation of Octagonal Grid Graph

M. Basher

ABSTRACT: Consider G is a finite and simple graph with vertex set $V(G)$ and edge set $E(G)$. An edge labeling $\rho : E(G) \to \{0, 1, ..., r\}$, where r is an integer, $2 \leq r \leq |E(G)|$, induces a vertex labeling $\rho^* : V(G) \to \{0, 1, ..., r-1\}$, defined in such a way that $\rho^*(v) = \prod_{i=1}^{n} \rho(e_i)$ (mod r), where e_i are the edges incident to the vertex v. The mapping ρ is called an r-total edge product cordial labeling, r-TEPC labeling, of G if $| e_\rho(i) + v_{\rho^*}(i) - e_\rho(j) + v_{\rho^*}(j) | \leq 1$ for every $0 \leq i, j \leq r-1$, where the numbers of edges and vertices labeled with integer i are denoted by $e_\rho(i)$ and $v_{\rho^*}(i)$ respectively. In this paper we have shown that the stellation of an octagonal grid graph admits a 3-TEPC labeling.

Keywords: Cordial labeling, 3-TEPC labeling, octagonal grid graph.

Contents

1. Introduction

Suppose that G is a basic, connected finite graph with vertex set $V(G)$ and an edge set $E(G)$. A graph labeling is a mapping of integers to vertices (or edges) or both subject to specific conditions. Consider the domain of the function is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling). If the domain is $V(G) \cup E(G)$ thus the labeling is known as a total labeling. Consider $\rho : V(G) \to \{0, 1\}$ is a vertex labeling that induces edge labeling $\rho^* : E(G) \to \{0, 1\}$ defined by $\rho^*(uv) = |\rho(u) - \rho(v)|$. The labeling ρ is said to be cordial if $|V_\rho(0) - V_\rho(1)| \leq 1$ and $|e_\rho(0) - e_\rho(1)| \leq 1$. Cahit was the first to propose the concept of cordial labeling [1]. A significant amount of work has gone into cordial labeling. For most recent results, see [2,3,4,5,6,7,8]. A vertex labeling $\rho : V(G) \to \{0, 1\}$ induces an edge labeling $\rho^* : E(G) \to \{0, 1\}$ defined by $\rho^*(xy) = \rho(x)\rho(y)$ which is known as cordial product labeling if $|V_\rho(0) - V_\rho(1)| \leq 1$ and $|e_\rho(0) - e_\rho(1)| \leq 1$ where $V_\rho(0)$ and $V_\rho(1)$ indicate the number of vertices that are marked by 0 and 1 respectively. Whenever $e_\rho(0)$ and $e_\rho(1)$ indicate the number of edges that are marked by 0 and 1 respectively. The idea of product cordial labeling introduced by Sundaram et al. [9]. In [10] Barasara and Vaidya proposed a variation on the cordial theme known as edge product cordial labeling and the TEPC labeling. Suppose $2 \leq r \leq | E(G) |$ is an integer. An edge labeling $\rho : \{0, 1, ..., r-1\}$ induces a vertex labeling $\rho^* : V(G) \to \{0, 1, ..., r-1\}$ defined by $\rho^*(v) = \prod_{e_\ell} \rho(e_\ell)$ (mod r), where $e_\ell, 1 \leq \ell \leq m$ are edges incident to v. The labeling ρ is considered to be r-TEPC labeling of G if $|e_\rho(i) + v_{\rho^*}(i) - (e_\rho(j) + v_{\rho^*}(j))| \leq 1$ for each i, j, $0 \leq i, j \leq r-1$, where $e_\rho(i)$ and $v_\rho(i)$ are the numbers of edges and vertices that are marked with integer i under the labeling ρ, $1 \leq i \leq r-1$, A zaizah et al. proposed the notion of r-TEPC labeling [11]. A graph G which admits r-total edge product cordial labeling is called a r-TEPC graph. Some results on the r-TEPC labeling can be found in [12,13,14,15,16,17].

The octagonal grid graph is denoted by O_ℓ^k where k and ℓ refers to the numbers of octagons in a row and column respectively [18]. This construction contains $4k\ell + 2(k+\ell)$ vertices and $6k\ell + \ell + k$ edges. The process of stellation for O_ℓ^k involves the addition of a vertex to each face of O_ℓ^k, followed by connecting this new vertex to every vertex of the corresponding face. The stellation of O_ℓ^k is represented as $St(O_\ell^k)$, as illustrated in Figure 1. Graph labeling is a critical region of graph theory that is applicable in a variety of disciplines, including computer science, coding theory, medicine, communication networking,

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and chemistry, in addition to mathematics. Vertex labeling is employed in the chemistry field to construct valence isomers, while transition labeling is employed to investigate chemical reaction networks. One of the most important chemical structures derived from an octagonal grid O_ℓ^k called nanosheets $C_4C_8(R)$. They are engaged in nanotechnology, nanomedicine, and gene transfer. Nanosheets are distinct from their bulk counterparts due to their exceptionally thin structures. They are optimally designed for the delivery of diverse drugs, including therapeutic DNA and RNAs, owing to their elevated surface-to-volume ratio [19]. In this paper, we prove that the stellation of octagonal graph $St(O_\ell^k)$ admits 3-TEPC labeling.

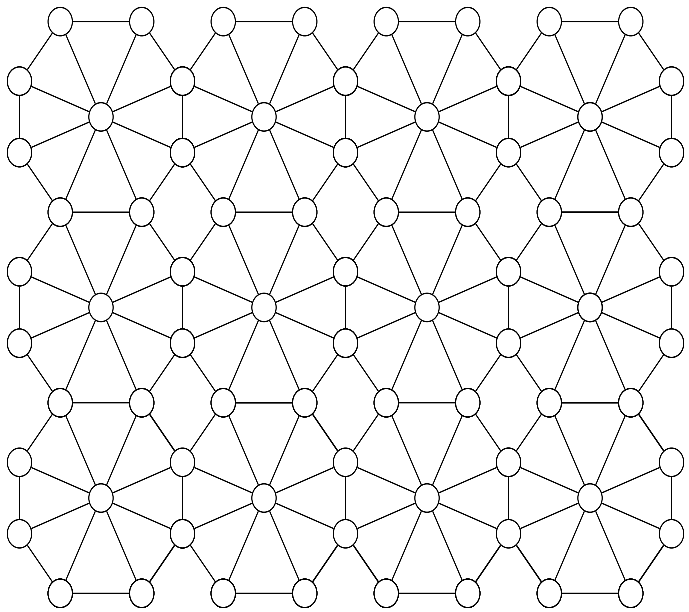

Figure 1: Stellation of octagonal grid graph $St(O_4^3)$.

2. Main Results

The stellation of an octagonal grid graph is represented by $St(O_\ell^k)$, in which k represents the number of octagons in a row and ℓ represents the number of octagons in a column. The sum of vertices and edges in the stellation of an octagonal grid graph is $19k\ell + 3\ell + 3k$.
In this article, the terms " free edge" and "separated edge" will be used to denote to the edges with just one end vertex and no other end vertices respectively. The segments will be affixed using two different processes, the symbol $\oplus^{\uparrow}$ stand for a process that affixes two segments uprightly, and the symbol $\underset{\rightarrow}{\oplus}$ stand for a process that affixes two segments horizontally.

Theorem 2.1 *The graph $St(O_1^k)$ is 3-total edge product cordial for $k \geq 1$.*

Proof: The 3-TEPC labeling of $St(O_1^1)$, $St(O_1^2)$ and $St(O_1^3)$ is illustrated in Figure 2. Table 1 shows the list of the all numbers having the labels 0,1 and 2 under the given labeling. By the symbol A_1^3, we refer

Table 1: Number of 0s, 1s and 2s utilized in $St(O_1^k)$ for $k = 1, 2, 3$

$St(O_1^k)$	$e_\rho(0) + v_{\rho^*}(0)$	$e_\rho(1) + v_{\rho^*}(1)$	$e_\rho(2) + v_{\rho^*}(2)$
$k = 1$	9	8	8
$k = 2$	18	19	19
$k = 3$	23	23	23

the graph created from $St(O_1^3)$ by removing the two vertices u_1^4, v_1^4, then the two uprightly free edges are

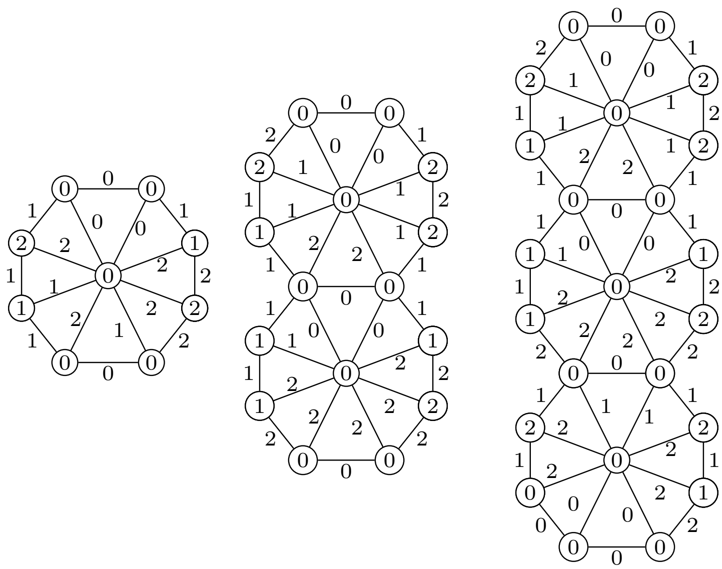

Figure 2: The 3-TEPC labeling of $St(O_1^k), k = 1, 2, 3$.

formed. The graph A_1^3 is called segment. Figure 3 shows the A_1^3 segment and its labeling. Observe that the free edges are labeled with 1. In this labeling we note that the numbers of 0,1 and 2 are used only 22 times.

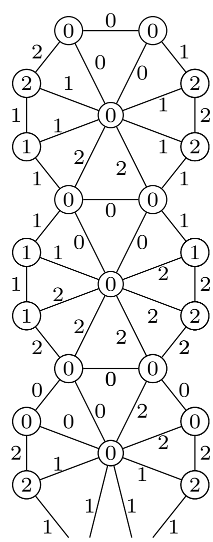

Figure 3: The 3-TEPC labeling of segment A_1^3.

Case 1. If $k = 3s$, $s \geq 1$, then to get the graph $St(O_1^k)$, we affix $(s-1)$ segments A_1^3 together in the upright direction, since the free edges in the segment A_1^3 are labeled with number 1 it implies that by affixing these segments, we maintain the vertex labels in the segment $A_1^3 \oplus^{\uparrow} A_1^3 \oplus^{\uparrow} ... \oplus^{\uparrow} A_1^3 = (s-1)A_1^3$.

We affix $(s-1)A_1^3$ uprightly to $St(O_1^3)$ get $St(O_1^k)$, i.e.,

$$St(O_1^k) = \begin{bmatrix} (s-1)A_1^3 \\ \oplus^{\uparrow} \\ St(O_1^3) \end{bmatrix}$$

It is obvious that $St(O_1^k)$ contains number of $0, 1$ and 2 precisely $22s+1$.

Case 2. If $k = 3s+1$, $s \geq 1$, then to get the graph $St(O_1^k)$, we affix s segments A_1^3 jointly and the segment $St(O_1^1)$ in the upright direction. Hence we get.

$$St(O_1^k) = \begin{bmatrix} sA_1^3 \\ \oplus^{\uparrow} \\ St(O_1^1) \end{bmatrix}$$

It is obvious that $St(O_1^k)$ contains numbers 1 and 2 precisely $22s+8$ while 0 is used precisely $22s+9$.

Case 3. If $k = 3s+2$, $s \geq 1$, then to get the graph $St(O_1^k)$, we affix s segments A_1^3 jointly and the segment $St(O_1^2)$ in the upright direction. Hence we get:

$$St(O_1^k) = \begin{bmatrix} sA_1^3 \\ \oplus_{\uparrow} \\ St(O_1^2) \end{bmatrix}$$

It is obvious that $St(O_1^k)$ contains numbers 1 and 2 precisely $22s+19$ while 0 is used precisely $22s+18$. □

Theorem 2.2 *The graph $St(O_2^k)$ is 3-total edge product cordial for $k \geq 1$.*

Proof: The graph $St(O_1^2)$ is isomorphic to $St(O_2^1)$, and Figure 2 depicts the 3-TEPC labeling of $St(O_2^1)$. For $k = 2, 3$ the Figure 4 depicts the appropriate 3-TEPC labeling of $St(O_2^k)$, where $k = 2, 3$. Table 2 displays the number of 0, 1 and 2 utilized in $St(O_2^k)$ for $k = 2, 3$.

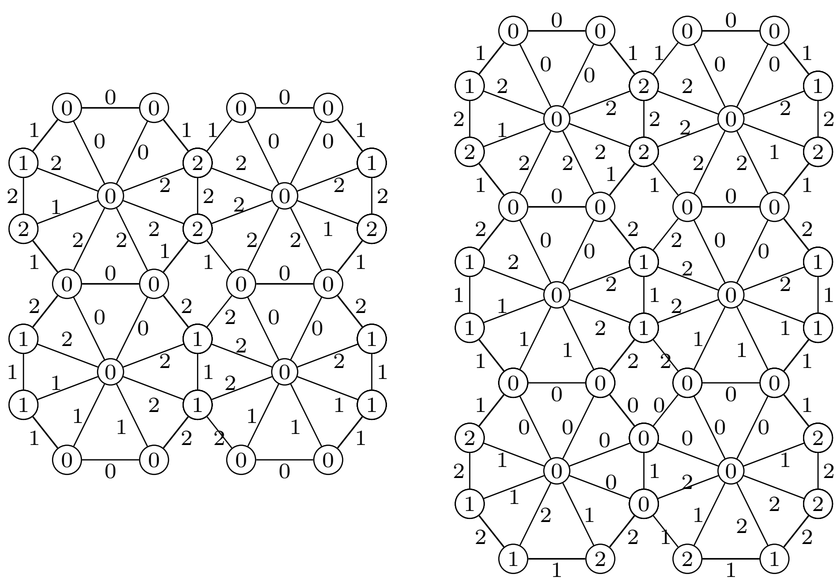

Figure 4: The 3-TEPC labeling of $St(O_2^k), k = 2, 3$.

Figure 5 shows the segment A_2^3 and its labeling. Observe that the free edges are labeled with 1 and each

Table 2: Number of 0s, 1s and 2s utilized in $St(O_2^k)$ for $k = 2, 3$

$St(O_2^k)$	$e_\rho(0) + v_{\rho^*}(0)$	$e_\rho(1) + v_{\rho^*}(1)$	$e_\rho(2) + v_{\rho^*}(2)$
$k = 2$	30	29	29
$k = 3$	43	43	43

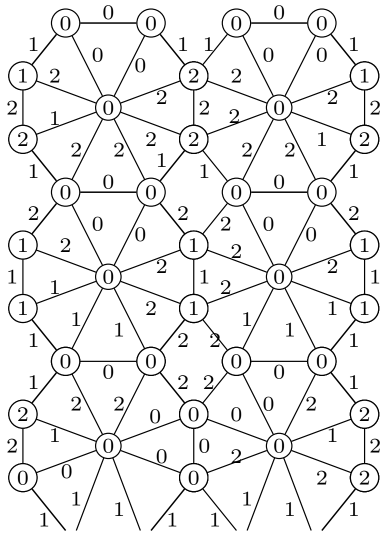

Figure 5: The 3-TEPC labeling of segment A_2^3.

number of 0, 1 and 2 are utilized 41 times exactly.

Case 1. If $k = 3s$, $s \geq 1$, then to get the $St(O_2^k)$, we affix $(s-1)$ segments A_2^3 jointly and the segment $St(O_2^3)$ in the upright direction. Thus we get:

$$St(O_2^k) = \begin{bmatrix} (s-1)A_2^3 \\ \oplus^{\uparrow} \\ St(O_2^3) \end{bmatrix}$$

Obviously, $St(O_2^k)$ contains number of 0,1 and 2 precisely $41s + 2$.

Case 2. If $k = 3s + 1$, $s \geq 1$, then to get $St(O_2^k)$, we affix s segments A_2^3 jointly in the upright direction. Since the free edges in A_2^k are labeled with number 1, then the vertex labels in the segment $A_2^k = A_2^3 \oplus^{\uparrow} A_2^3 \oplus^{\uparrow} ... \oplus^{\uparrow} A_2^3 = sA_2^3$ remain fixed. Hence we affix sA_2^3 to $St(O_2^1)$ in the upright direction to get $sA_2^k \oplus^{\uparrow} St(O_2^1) = St(O_2^k)$, i.e.,

$$St(O_2^k) = \begin{bmatrix} sA_2^3 \\ \oplus^{\uparrow} \\ St(O_2^1) \end{bmatrix}$$

It is obvious that $St(O_2^k)$ contains number of 1 and 2 precisely $41s + 19$ times and 0 precisely $41s + 18$ times.

Case 3. If $k = 3s + 2$, $s \geq 1$, then we get the $St(O_2^k)$ by affix s segments A_2^k jointly and the segment $St(O_2^2)$ in the upright direction. Hence we get:

$$St(O_2^k) = \begin{bmatrix} sA_2^3 \\ \oplus^{\uparrow} \\ St(O_2^2) \end{bmatrix}$$

It is obvious that $St(O_2^k)$ contains number of 1 and 2 precisely $41s + 29$ times and 0 precisely $41s + 30$ times. □

Theorem 2.3 *The graph $St(O_3^k)$ is 3-total edge product cordial for $k \geq 1$.*

Proof: The graphs $St(O_3^1)$ and $St(O_s^2)$ are 3-TEPC graphs because graph $St(O_3^1)$ is isomorphic to $St(O_1^3)$ and graph $St(O_s^2)$ is isomorphic to $St(O_2^3)$. The 3-TEPC labeling of these graphs are depicted in Figure 2 and Figure 4 respectively. The 3-TEPC labeling of $St(O_3^3)$ is depicted in Figure 6. Table 3. illustrated

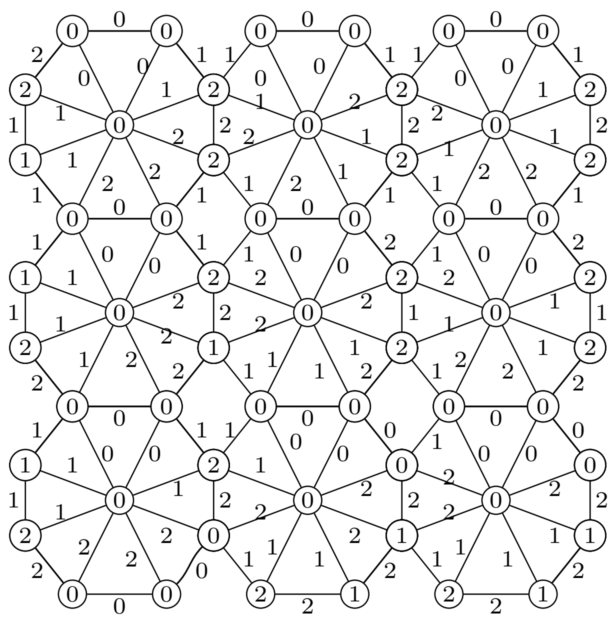

Figure 6: The 3-TEPC labeling of $St(O_3^3)$

the number of 0, 1 and 2 utilized in $St(O_3^k)$ for $k = 3$.

Table 3: Number of 0s, 1s and 2s utilized in $St(O_3^3)$

$St(O_3^k)$	$e_\rho(0) + v_{\rho^*}(0)$	$e_\rho(1) + v_{\rho^*}(1)$	$e_\rho(2) + v_{\rho^*}(2)$
$k = 3$	63	63	63

The segment A_3^3 and its labeling are shown in Figure 7. Observe that this labeling has the characteristics that the free edges are labeled with 1 and each number of 0, 1 and 2 is utilized precisely 60 times.

Case 1. If $k = 3s$, $s \geq 1$, then to get the graph $St(O_3^k)$, we affix $(s-1)$ segments A_3^3 with each other and the segment $St(O_3^3)$ in the upright direction. The vertex labels in the segment $A_3^3 \oplus^{\uparrow} A_3^3 \oplus^{\uparrow} ... \oplus^{\uparrow} A_3^3 = (s-1)A_3^3$ remain fixed because the free edges are labeled with number 1. Then we affix $(s-1)A_3^3$ uprightly to $St(O_3^3)$ to get $(s-1)A_3^3 \oplus^{\uparrow} St(O_3^3) = St(O_3^k)$ i.e.,

$$St(O_3^k) = \begin{bmatrix} (s-1)A_3^3 \\ \oplus^{\uparrow} \\ St(O_3^3) \end{bmatrix}$$

It is obvious that O_3^k contains number of 0, 1 and 2 precisely $60s + 3$ times.

Case 2. If $k = 3s + 1$, $s \geq 1$, then to get the graph $St(O_3^k)$, we affix s segments A_3^3 jointly and the segment $St(O_3^1)$ in the upright direction. Thus we get:

$$St(O_3^k) = \begin{bmatrix} sA_3^3 \\ \oplus^{\uparrow} \\ St(O_3^1) \end{bmatrix}$$

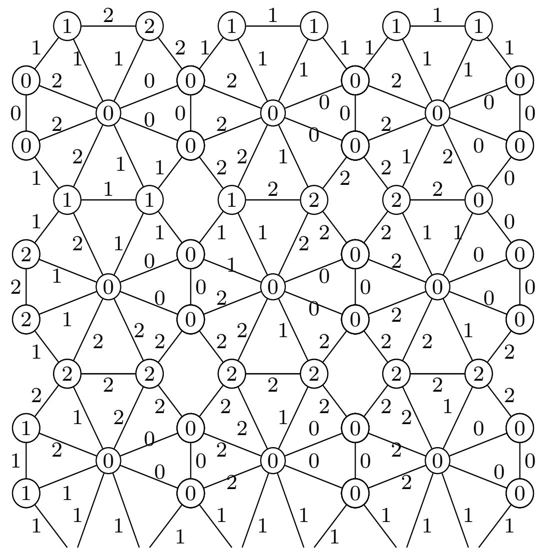

Figure 7: The 3-TEPC labeling of A_3^3

Clearly, we can notice that $St(O_3^k)$ contains number of 0, 1 and 2 precisely $60s + 23$ times.
Case 3. If $k = 3s + 2$, $s \geq 1$, so to construct the graph $St(O_3^k)$, we affix s segments A_3^3 jointly and the segment $St(O_3^2)$ in the upright direction. The results:

$$St(O_3^k) = \begin{bmatrix} sA_3^3 \\ \oplus^{\uparrow} \\ St(O_3^2) \end{bmatrix}$$

Obviously, the graph $St(O_3^k)$ contains number of 0, 1 and 2 precisely $60s + 43$ times.

□

Theorem 2.4 *The graph $St(O_\ell^k)$ is 3-total edge product cordial for $k, \ell \geq 1$.*

Proof: To generate a 3-total edge product cordial labeling of $St(O_\ell^k)$,we require a new labeled segment D_3^3. The segment D_3^3 has ten free edges and one separated edge. Figure 8 depicts the segment D_3^3 and its labeling. Observe that, in this segment the free edges and separated edge are labeled with 1 and each number of 0, 1 and 2 is used precisely 57 times.
Firstly, to get the 3-TEPC labeling of $St(O_\ell^k)$ for $k = 1, 2, 3$ and for all ℓ, we utilized the theorems from 1 to 3. Furthermore, we will utilize the segments $\overrightarrow{St(O_3^3)}$,$\overrightarrow{St(O_2^3)}$, $\overrightarrow{St(O_1^3)}$ which produced by rotating the segments $St(O_3^3)$, $St(O_2^3)$, $St(O_1^3)$ clockwise through the angle 90° about its center. According to the values of k and ℓ there are several cases.
Case 1. If $k = 3s$, $s \geq 1$.
Here, we affix the segment D_3^3 uprightly $s - 1$ times. Since the free edges in the segment are labeled by number 1, then by joining these segments together does not modifying the vertex labels of the segment $D_3^3 \oplus^{\uparrow} D_3^3 \oplus^{\uparrow} ... \oplus^{\uparrow} D_3^3 = (s - 1)D_3^3$. After that, we affix $(s - 1)D_3^3$ and $\overrightarrow{St(O_3^3)}$ uprightly to get

$$A = \begin{bmatrix} (s-1)D_3^3 \\ \oplus^{\uparrow} \\ \overrightarrow{St(O_3^3)} \end{bmatrix}$$

Note that, free edges of the segment A have label 1 and each number 0,1 and 2 is used precisely $57s + 6$.
Subcase i. When $\ell = 3t$, $t \geq 1$. We affix the segment A horizontally $t - 1$ times and furthermore, we

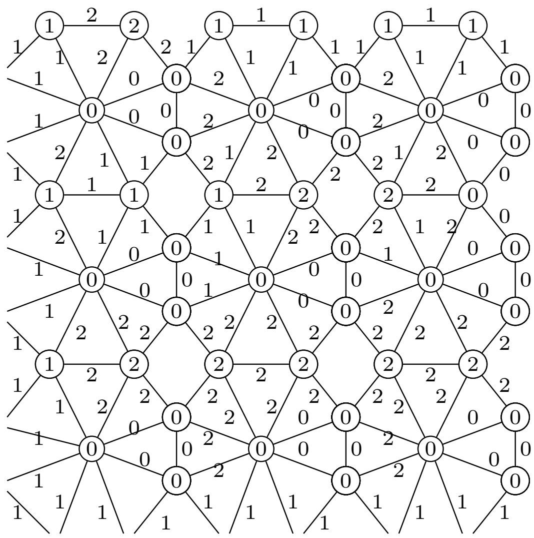

Figure 8: The 3-TEPC labeling of segment D_3^3

affix horizontally the segment $St(O_3^k)$ and we get:

$$St(O_\ell^k) = [St(O_3^k) \underset{\rightarrow}{\oplus} (t-1)A]$$

Subcase ii. When $\ell = 3t+1$, $t \geq 1$.
The 3-TEPC labeling of the stellation of octagonal grid $St(O_\ell^k)$ is obtained as:

$$St(O_\ell^k) = [St(O_1^k) \underset{\rightarrow}{\oplus} tA]$$

Subcase iii. When $\ell = 3t+2$, $t \geq 1$.
We affix the segment A horizontally t times and furthermore, we affix horizontally the segment $St(O_2^k)$, hence we get:

$$St(O_\ell^k) = [St(O_2^k) \underset{\rightarrow}{\oplus} tA]$$

Case 2. If $k = 3s+1$, $s \geq 1$.
The segment B is formed by affixing uprightly the segment $\overrightarrow{St(O_1^3)}$ with uprightly s times of the segment D_3^3, i.e.,

$$B = \begin{bmatrix} \overrightarrow{St(O_1^3)} \\ \oplus^{\uparrow} \\ sD_3^3 \end{bmatrix}$$

Clearly, the free edges of the segment B are labeled by label 1 and each of the numbers 0 through 2 has a multiplicity of $57s+23$.
Subcase i. When $\ell = 3t$, $t \geq 1$.
We affix the segment B horizontally $t-1$ times and furthermore, we affix horizontally the segment $St(O_3^k)$, then we get:

$$St(O_\ell^k) = [St(O_3^k) \underset{\rightarrow}{\oplus} (t-1)B]$$

Subcase ii. When $\ell = 3t+1$, $t \geq 1$.
The 3-TEPC labeling of the stellation of octagonal grid $St(O_\ell^k)$ is obtained as:

$$St(O_\ell^k) = [St(O_1^k) \underset{\rightarrow}{\oplus} tB]$$

Subcase iii. When $\ell = 3t + 2$, $t \geq 1$.
Clearly, the segment B is joined horizontally t times, and the segment $St(O_2^k)$ is also joined horizontally, resulting in:

$$St(O_\ell^k) = [St(O_2^k) \underset{\rightarrow}{\oplus} tB]$$

Case 3. If $k = 3s + 2$, $s \geq 1$.
The segment C is constructed by affixing uprightly the segment $\overrightarrow{St(O_2^3)}$ with uprightly s times of the segment D_3^3, i.e.,

$$C = \begin{bmatrix} \overrightarrow{St(O_2^3)} \\ \oplus\uparrow \\ sD_3^3 \end{bmatrix}$$

Observe that the free edges of the segment are labeled by label 1 and each the numbers 0 through 2 has multiplicity $57s + 43$.
Subcase i. When $\ell = 3t$, $t \geq 1$.
To get the stellation of octagonal grid, we affix the segment C horizontally $t - 1$ times and then affix the resultant segment horizontally to $St(O_3^k)$, thus

$$St(O_\ell^k) = [St(O_3^k) \underset{\rightarrow}{\oplus} (t-1)C]$$

Subcase ii. When $\ell = 3t + 1$, $t \geq 1$.
The 3-TEPC labeling of the stellation of octagonal grid $St(O_\ell^k)$ is derived as follows:

$$St(O_\ell^k) = [St(O_1^k) \underset{\rightarrow}{\oplus} tC]$$

Subcase iii. When $\ell = 3t + 2$, $t \geq 1$.
Now, the segment C is affixed horizontally t times, and the segment $St(O_2^k)$ is likewise attached horizontally. The result is as follows:

$$St(O_\ell^k) = [St(O_2^k) \underset{\rightarrow}{\oplus} tC]$$

Table 4. describes all possibilities for obtaining the grid $St(O_\ell^k)$ for $k, \ell \geq 1$ and displays how often the

Table 4: Number of 0s, 1s and 2s utilized in $St(O_\ell^k)$

Case	Grid $St(O_\ell^k)$	$e_\rho(0) + v_{\rho^*}(0)$	$e_\rho(1) + v_{\rho^*}(1)$	$e_\rho(2) + v_{\rho^*}(2)$
1.	$k = 3s, s \geq 1$	$m = 57st + 3s + 6t - 3$		
i.	$\ell = 3t, t \geq 1$	m	m	m
ii.	$\ell = 3t+1, t \geq 1$	$m + 19s + 4$	$m + 19s + 4$	$m + 19s + 4$
iii.	$\ell = 3t+2, t \geq 1$	$m + 38s + 5$	$m + 38s + 5$	$m + 38s + 5$
2.	$k = 3s+1, s \geq 1$	$m = 57st + 3s + 23t - 20$		
i.	$\ell = 3t, t \geq 1$	m	m	m
ii.	$\ell = 3t+1, t \geq 1$	$m + 19s + 29$	$m + 19s + 28$	$m + 19s + 28$
iii.	$\ell = 3t+2, t \geq 1$	$m + 38s + 38$	$m + 38s + 39$	$m + 38s + 39$
3.	$k = 3s+2, s \geq 1$	$m = 57st + 3s + 43t$		
i.	$\ell = 3t, t \geq 1$	m	m	m
ii.	$\ell = 3t+1, t \geq 1$	$m + 19s + 18$	$m + 19s + 19$	$m + 19s + 19$
iii.	$\ell = 3t+2, t \geq 1$	$m + 38s + 30$	$m + 38s + 29$	$m + 38s + 29$

numbers 0,1, and 2 are used as edge and vertex labels. As can be seen, the resultant grid $St(O_\ell^k)$ in each of the preceding situations achieves the property of possessing a 3-TEPC labeling. □

3. Conclusions

This study presents the construction of 3-TEPC labeling for the stellation of octagonal grid graph $St(O_\ell^k)$. For $k, \ell \geq 1$, we demonstrated that $St(O_\ell^k)$ is 3-TEPC. Future work may focus on extending the labeling to other stellated grid families like hexagonal grid and rhombus grid.

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M. Basher,
[1]*Department of Mathematics, College of Science,*
Qassim University, P. O. Box 6644, Buraydah,51452, Saudi Arabia.
[2]*Department of Mathematics and Computer Science,*
Faculty of Science, Suez University, P.O.Box:43221, Suez, Egypt.

E-mail address: m.basher@qu.edu.sa; mohamed.basher@sci.suezuni.edu.eg