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Antisymmetric Gravitational Lorentz Force Tensor Predicts Gravitational Force Theory

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ABSTRACT: Three-dimensional gravitational Lorentz force $m[\mathbf{g}+(\mathbf{g}t\times\boldsymbol{\omega})]$ predicts the antisymmetric gravitational Lorentz force (GLF) tensor $G^{\mu\nu}$, whose components are Newton's gravitational force $m\mathbf{g}$ and the gravitational Coriolis force $m(\mathbf{g}t\times\boldsymbol{\omega})$ in the language of standard electrodynamics. It obeys the principle of relativity, the conservation law, and symmetry. The model consists of gravitational force equations, gravitational force Maxwell's equations, the conservation law, the wave equation of Newtonian gravitational force, and the wave equation of gravitational Coriolis force. The framework of the dual of the GLF tensor, $*G^{\mu\nu}$, possesses the same results, and both the matrix method and Einstein's summation convention method (ESCM) are valid. The transformation of GLF laws in a non-inertial coordinate metric, based on the single transformation law (STL) for four-vectors and tensors, reveals the existence of zero-point gravitational energy, gravitational torque, and zero-point gravitational Maxwell's equations as gravitational power and force. The conservation law obtained by the matrix method gives zero, but by ESCM it predicts a seven-dimensional wave of gravitational energy accompanied by the classical conservation law. This framework reduces to its classical limit when extra terms are subjected to zero, and it provides a simple representation of gravitational physics for ready applications not only in daily life but also in the study of astrophysics and cosmology.

Key Words:: Antisymmetric gravitational Lorentz force tensor, gravitational Maxwell's equations, gravitational conservation law, waves of gravitational forces, 7D wave of gravitational energy.

Contents

1. Introduction

Einstein's theory of gravitation is based on a symmetric tensor having 10 independent components, whereas our model is based on an antisymmetric tensor having 6 independent components, analogous to standard electrodynamics. Since electrodynamics is an empirically established theory, we expect our model of gravitation will give consistent results. Our previous model on the antisymmetric electromagnetic force tensor [1] did predict the existence of an antisymmetric gravitational Lorentz force tensor. It has rightly been said that a consistent theory of gravitation will emerge as an analogy of electrodynamics [2]. In the contemporary literature, the search for such a model is ongoing [3, 4] via the concept of the gravitational Lorentz force. Fedosin's model [5] contains the framework of gravitation theory as an analogy of Maxwellian electrodynamics, which is half force theory and half field theory, whereas our model is based completely on forces. Melvin Schwartz [6] points out the importance of the antisymmetric tensor as the natural choice of Nature as a medium of expression, which supports our approach.

The antisymmetric gravitational Lorentz force (GLF) tensor behaves as a guidance force field, and its nature is attractive. In the contemporary world, physicists are looking for a Maxwell-type theory of gravitation via the well-known framework of gravitoelectromagnetism [7-12]. There has been no attempt

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to develop a Maxwell-type theory for inertia. We have developed a theory of inertia and gravitation as an analogy of electrodynamics [13], but again these are partially force and partially field theories. Their transformations in a non-inertial metric in terms of velocity predicted Lorentz force for inertia and gravitation.

The antisymmetric GLF tensor $G^{\mu\nu}$ consists of Newton's second law of motion for gravitational force mg and the gravitational Coriolis force $m(\mathbf{g}t \times \boldsymbol{\omega})$. It constitutes a complete framework of gravitation theory expressed in the language of electrodynamics via gravitational force field equations, gravitational Maxwell's equations, conservation laws, and wave equations of gravitational forces. The dual of the GLF tensor has the same structure of equations. Looking at the electrodynamic paradigm of gravitation theory, the temporal component of the GLF tensor corresponds to Newton's law of gravitation, while the spatial component represents the gravitational Coriolis force minus the gravitational Lorentz force. The antisymmetry of the GLF tensor reflects the difference of GLF, which represents action and reaction, or the balance of GLF. Gravitational Maxwell's equations are straightforward to interpret: the Gaussian force law is the divergence of Newton's force of gravitation, and the Amperian force law is the curl of the gravitational Coriolis force together with the time-varying Newtonian force of gravitation. The conservation law of the gravitational tensor holds as usual.

The transformation of antisymmetric GLF electrodynamics, based on the STL for four-vectors and tensors in a non-inertial coordinate metric, predicts new symmetry of GLF laws in tensor notation represented in matrix form along the diagonals of force field equations, gravitational Maxwell's equations, and the conservation law. The temporal component of the GLF tensor is transformed into Newtonian gravitational force, zero-point gravitational energy, and gravitational Coriolis torque, while the spatial component of the GLF tensor transforms into minus gravitational force. The antisymmetric GLF tensor is transformed into gravitational Coriolis torque, and the dual of the GLF tensor is transformed into gravitational torque with a minus sign. From this dual structure, we obtain the dual of gravitational Gauss's law and Faraday's law in terms of gravitational forces. These new symmetry terms represent the zero-point structure of gravitational electrodynamics, including zero-point gravitational energy balance and zero-point gravitational Maxwell's equations expressed as gravitational power plus gravitational force. The conservation law obtained by the matrix method is equal to zero, but by the usual method it gives a seven-dimensional wave of gravitational energy along with the classical conservation law.

Notations in this model are adopted according to the modern approach of relativity. Greek alphabets $\mu, \nu, \alpha, \beta, \dots$ run from 0 to 3, and Latin letters $i, j, k, \dots$ run from 1 to 3. A comma (,) denotes partial differentiation. For example, $F_{,0} = \frac{\partial F}{\partial t}$ represents the partial derivative of the force field with respect to time, $F_{,1} = \frac{\partial F}{\partial x}$ represents the partial derivative with respect to the x-axis, $F_{,2} = \frac{\partial F}{\partial y}$ represents the partial derivative with respect to the y-axis, and $F_{,3} = \frac{\partial F}{\partial z}$ represents the partial derivative with respect to the z-axis. Moreover, the notation $f_{\mu\nu,\nu}$ denotes the four-dimensional (spacetime) partial derivative of the Lorentz force tensor.

1.1. Formulation of Antisymmetric Gravitational Lorentz Force Tensor Dynamics

Structure of Gravitational Lorentz Force Tensor

$$G^{\mu\nu} = m \begin{bmatrix} 0 & g^1 & g^2 & g^3 \\ -g^1 & 0 & (gt\times\omega)^3 & -(gt\times\omega)^2 \\ -g^2 & -(gt\times\omega)^3 & 0 & (gt\times\omega)^1 \\ -g^3 & (gt\times\omega)^2 & -(gt\times\omega)^1 & 0 \end{bmatrix} \tag{1.1}$$

The tensor components can also be expressed as

$$G^{0i} = mg^i, \qquad G^{ij} = m\,\epsilon^{ijk}(gt\times\omega)_k. \tag{1.2}$$

Newtonian Gravitational Force Field

$$G^{0i} = mg^i. \tag{1.3}$$

Spatial Component of Gravitational Lorentz Force

$$G^{i\nu} = m(gt\times\omega) - m\big[\mathbf{g} + (gt\times\omega)\big]. \tag{1.4}$$

Gravitational Lorentz Force

$$G^{\mu\nu} = m\big[\mathbf{g} + (gt \times \omega)\big] - m\big[\mathbf{g} + (gt \times \omega)\big] = 0. \tag{1.5}$$

The difference of gravitational Lorentz force implies the antisymmetry of the gravitational Lorentz force tensor.

Lorentz Force Field Maxwell's Equations

Gaussian Force Law for Gravitation.

$$G^{0i}{}_{,i} = \nabla \cdot (m\mathbf{g}). \tag{1.6}$$

Amperian Force Law for Gravitation.

$$G^{i\nu}{}_{,\nu} = m\left[\nabla \times (gt \times \omega) - \mathbf{g}_{,0}\right]. \tag{1.7}$$

The curl of the gravitational Coriolis force is related to the time-varying gravitational force as

$$\nabla \times m(gt \times \omega) = G^{i\nu}{}_{,\nu} + \frac{\partial (m\mathbf{g})}{\partial t}. \tag{1.8}$$

This shows that the curl of the gravitational Coriolis force is equal to the time-varying gravitational force.

Gravitational Maxwell's Equations in Tensor Form The generalized form of the gravitational Maxwell's equations is given by

$$G^{\mu\nu}{}_{,\nu} = \nabla \cdot (m\mathbf{g}) + m\left[\nabla \times (gt \times \omega) - \mathbf{g}_{,0}\right]. \tag{1.9}$$

Conservation Law of Gravitational Lorentz Force Tensor The conservation law of the antisymmetric gravitational Lorentz force tensor is expressed as

$$G^{\mu\nu}{}_{,\nu\mu} = \big(\nabla \cdot m\mathbf{g}\big)_{,0} + m\left[\nabla \cdot \nabla \times (gt \times \omega) - \mathbf{g}_{,0}\right] = 0. \tag{1.10}$$

This relation demonstrates the internal consistency of the framework, since the divergence of a curl vanishes identically, and the conservation law holds.

1.2. Dual of Antisymmetric Gravitational Lorentz Force Tensor Dynamics

Einstein's original theory of gravitation does not contain a dual structure of gravitation. Later, physicists introduced the Hodge dual of the curvature tensor. Our formulation of the dual of the gravitational tensor is analogous to the standard dual formulation in electrodynamics.

Structure of Dual of Antisymmetric Gravitational Lorentz Force Tensor

$$*G^{\mu\nu} = m \begin{bmatrix} 0 & (gt \times \omega)^1 & (gt \times \omega)^2 & (gt \times \omega)^3 \\ -(gt \times \omega)^1 & 0 & -g^3 & g^2 \\ -(gt \times \omega)^2 & g^3 & 0 & -g^1 \\ -(gt \times \omega)^3 & -g^2 & g^1 & 0 \end{bmatrix} \tag{1.11}$$

$$mg^i \;\longrightarrow\; m(gt \times \omega), \qquad m(gt \times \omega) \;\longrightarrow\; -mg^i. \tag{1.12}$$

The gravitational Coriolis force is obtained as

$$*G^{0i} = m(gt \times \omega). \tag{1.13}$$

The dual of Newtonian gravitational force is gravitational Coriolis force. Spatial Component of Dual Gravitational Force

$$*G^{i\nu} = m(gt \times \omega) - m\big[\mathbf{g} + (gt \times \omega)\big]. \tag{1.14}$$

Gravitational Lorentz force field

$$*G^{\mu\nu} = m\big[\mathbf{g} + (gt \times \omega)\big] - m\big[\mathbf{g} + (gt \times \omega)\big] = 0, \tag{1.15}$$

The difference of gravitational Lorentz force implies the antisymmetry of gravitational Lorentz force tensor.

Dual of Gaussian Force of Gravitation:

$$ {}^*G^{0i}{}_{,i} = \nabla \cdot \big(m(\mathbf{g}t \times \boldsymbol{\omega})\big) \tag{1.16}$$

Divergence of gravitational Coriolis force is nonzero.

Faraday's Law for Gravitation:

$$ {}^*G^{i\nu}{}_{,\nu} = -(\nabla \times m\mathbf{g}) - \frac{\partial}{\partial t} m\big[(\mathbf{g}t \times \boldsymbol{\omega})\big] \tag{1.17}$$

$$ (\nabla \times m\mathbf{g}) = -{}^*G^{i\nu}{}_{,\nu} - \frac{\partial}{\partial t} m\big[(\mathbf{g}t \times \boldsymbol{\omega})\big] \tag{1.18}$$

Curl of gravitational force is equal to negative of time varying gravitational Coriolis force Gravitational Force Maxwell's Equations in Tensor Form

$$ {}^*G^{\mu\nu}{}_{,\nu} = \nabla \cdot (m\mathbf{g}) + m\big[\nabla \times (\mathbf{g}t \times \boldsymbol{\omega}) - g_{,0}\big] \tag{1.19}$$

Conservation Law for Dual of Gravitational Force Field:

$$ {}^*G^{\mu\nu}{}_{,\nu\mu} = (\nabla \cdot m\mathbf{g})_{,0} + m\big[\nabla \cdot \nabla \times (\mathbf{g}t \times \boldsymbol{\omega}) - g_{,0}\big] = 0 \tag{1.20}$$

Gravitational Force Wave Equation:

$$ \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) m\mathbf{g} = 0 \tag{1.21}$$

The gravitational Coriolis force wave equation is

$$ \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) \big[m(g_t \times \omega)\big] = 0. \tag{1.22}$$

Adding Eqs.(1.11) and (1.12), we obtain the gravitational Lorentz force wave equation:

$$ \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) m\big[\mathbf{g} + (g_t \times \omega)\big] = 0. \tag{1.23}$$

We have summarized all the above results in Table 1 to facilitate the readers.

Table 1: Summary of Results of Antisymmetric Gravitational Lorentz Force Dynamics and its Dual Dynamics

Antisymmetric Gravitational Lorentz Force Tensor	**Dual Antisymmetric Gravitational Lorentz Force Tensor**
Gravitational Lorentz Force Tensor $G^{\mu\nu}$ $= m \begin{bmatrix} 0 & g^1 & g^2 & g^3 \\ -g^1 & 0 & (gt\times\omega)^3 & -(gt\times\omega)^2 \\ -g^2 & -(gt\times\omega)^3 & 0 & (gt\times\omega)^1 \\ -g^3 & (gt\times\omega)^2 & -(gt\times\omega)^1 & 0 \end{bmatrix}$ $G^{0i} = mg^i \quad G^{ij} = m\,\epsilon^{ijk}(g_t\times\omega)_k$	Dual of Gravitational Lorentz Force Tensor $*G^{\mu\nu}$ $= m \begin{bmatrix} 0 & (gt\times\omega)^1 & (gt\times\omega)^2 & (gt\times\omega)^3 \\ -(gt\times\omega)^1 & 0 & -g^3 & g^2 \\ -(gt\times\omega)^2 & g^3 & 0 & -g^1 \\ -(gt\times\omega)^3 & -g^2 & g^1 & 0 \end{bmatrix}$ $mg^i \to m(gt\times\omega), \quad m(gt\times\omega) \to -mg^i$
Newtonian Gravitational Force: $G^{0i} = mg^i$	Gravitational Coriolis Force: $*G^{0i} = m(g_t\times\omega)$
Spatial Component of Gravitational Lorentz Force : $G^{i\nu} = m(gt\times\omega) - m[\mathbf{g} + (gt\times\omega)]$	Spatial Component of Dual of Gravitational Force : $*G^{i\nu} = m(gt\times\omega) - m[\mathbf{g} + (gt\times\omega)]$
Antisymmetry of Gravitational Lorentz Force : $G^{\mu\nu} = m[\mathbf{g} + (gt\times\omega)] - m[\mathbf{g} + (gt\times\omega)] = 0$	Antisymmetry of Dual of Gravitational Lorentz Force : $*G^{\mu\nu} = m[\mathbf{g} + (g_t\times\omega)] - m[\mathbf{g} + (gt\times\omega)] = 0$
Gravitational Force Maxwell's Equations	Dual of Gravitational Force Maxwell's Equations
Gaussian Force Law for Gravitation: $G^{0i}{}_{,i} = \nabla\cdot(m\mathbf{g})$	Dual of Gaussian Force of gravitation: $*G^{0i}{}_{,i} = \nabla\cdot m(gt\times\omega)$
Amperian Force Law for Gravitation: $G^{i\nu}{}_{,\nu} = m[(\nabla\times(gt\times\omega)) - \mathbf{g}_{,0}]$	Faraday's Law for Gravitation: $*G^{i\nu}{}_{,\nu} = -(\nabla\times m\mathbf{g}) - \frac{\partial}{\partial t}m(g_t\times\omega)$
Gravitational Maxwell's Equations in Tensor Form: $G^{\mu\nu}{}_{,\nu} = \nabla\cdot(m\mathbf{g}) + m[(\nabla\times(gt\times\omega)) - \mathbf{g}_{,0}]$	Dual Grav. Force Maxwell's Equations in Tensor : $*G^{\mu\nu}{}_{,\nu} = \nabla\cdot(m\mathbf{g}) + m[\nabla\times(gt\times\omega) - \mathbf{g}_{,0}]$
Gravitational Force Wave Equation: $\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) m\mathbf{g} = 0$	Gravitational Coriolis Force Wave Equation: $\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) m(gt\times\omega) = 0$
Gravitational Coriolis Force Wave Equation: $\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) m(gt\times\omega) = 0$	Gravitational Force Wave Equation: $\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) m\mathbf{g} = 0$
Gravitational Lorentz Force Wave Equation: $\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) m[\mathbf{g} + (gt\times\omega)] = 0$	Gravitational Lorentz Force Wave Equation: $\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) m[\mathbf{g} + (gt\times\omega)] = 0$
Obeys principle of relativity	Obeys principle of relativity

2. Antisymmetric Gravitational Electrodynamics in Noninertial Coordinate Metric

In our earlier work, we derived the full transformation of antisymmetric mechanical electrodynamics in noninertial coordinate metrics. To avoid repetition, we directly present the summarized results here in Table 2. For detailed step-by-step calculations, readers are referred to our previous paper.
In the table, we have employed a simple color scheme: indicates the classical contributions from the inertial framework, while highlights the modifications due to noninertial effects.

Table 2: Gravitational Lorentz Force Dynamics and its Dual Dynamics in Noninertial Coordinate Metric

Noninertial Coordinate Metric Transformation Matrix
$[g^{\mu}_{\alpha}] = \begin{bmatrix} 1 & -x_1 & -x_2 & -x_3 \\ -x_1 & 1 & 0 & 0 \\ -x_2 & 0 & 1 & 0 \\ -x_3 & 0 & 0 & 1 \end{bmatrix}$

Table 3: Gravitational Lorentz Force Dynamics and its Dual Dynamics in Noninertial Coordinate Metric

Gravitational Lorentz Force Dynamics	**Dual Gravitational Lorentz Force Dynamics**
Transformation of GLF Tensor: $f^{\mu'\nu'} = g^{\mu'}_{\alpha} f^{\alpha\nu'}$	Transformation of Dual GLF Tensor: ${}^*f^{\mu'\nu'} = g^{\mu'}_{\alpha}{}^* f^{\alpha\nu'}$
Zero-point gravitational energy: $G^{0'0'} = x \cdot mg$	Zero-point gravitational Coriolis energy: ${}^*G^{0'0'} = x \cdot m(gt \times \omega)$
Zero-point negative gravitational energy: $G^{i'i'} = -x \cdot mg$	Zero-point negative gravitational Coriolis energy: ${}^*G^{i'i'} = x \cdot m(gt \times \omega) - x \cdot m(gt \times \omega)$
Zero-point action and reaction of gravitational energy: $G^{\mu'\mu'} = x \cdot mg - x \cdot mg = 0$	Zero-point action and reaction of Coriolis energy: ${}^*G^{\mu'\mu'} = x \cdot m(gt \times \omega) - x \cdot m(gt \times \omega)$
Temporal component as 4D Newtonian force): $G^{0'\nu'} = mg + m\big[x \cdot g + x \times (gt \times \omega)\big]$	Temporal component of 4D GL Coriolis force: ${}^*G^{0'\nu'} = m(gt \times \omega) - m\big[(x \times g) - mx \cdot (gt \times \omega)\big]$
Spatial component of GLF tensor: $G^{i'\nu'} = m\big[(gt \times \omega) - g\big] - m\big[x \cdot g + (gt \times \omega)\big]$	Spatial component of dual GLF tensor: ${}^*G^{i'\nu'} = m\big[g + (gt \times \omega)\big] - m\big[x \cdot (gt \times \omega) + g\big]$
Gravitational Lorentz Force Tensor as Coriolis Torque: $G^{\mu'\nu'} = x \times m(gt \times \omega)$	Dual of Gravitational Lorentz Force Tensor as torque: ${}^*G^{\mu'\nu'} = -x \times mg$
Transformation law for GLF Maxwell's equations: $f^{\mu'\nu'}{}_{,\nu'} = g^{\mu'}_{\alpha} f^{\alpha\nu'}{}_{,\nu'}$	Transformation law for dual GLF Maxwell's equations: ${}^*f^{\mu'\nu'}{}_{,\nu'} = g^{\mu'}_{\alpha}{}^* f^{\alpha\nu'}{}_{,\nu'}$
Zero-Point Gravitational Maxwell's Equations : $G^{\mu'\mu'}{}_{,\mu'} = x \cdot (mg)_{,0} - \nabla[x \cdot mg]$	Zero-Point Max. Eqs.as Coriolis power-Coriolis force: ${}^*G^{\mu'\mu'}{}_{,\mu'} = x \cdot (mgt \times \omega)_{,0} - \nabla[x \cdot (mgt \times \omega)]$

Gravitational Force Gauss's law: $G^{0'\nu'}{}_{,\nu'} = \nabla \cdot (mg) - x \cdot m\big[\nabla \times (gt \times \omega) - (mg)_{,0}\big]$	Dual of Gravitational Gaussian Force law: ${}^{*}G^{0'\nu'}{}_{,\nu'} = \nabla \cdot m(gt \times \omega) + x \cdot m\big[(\nabla \times g) + (gt \times \omega)_{,0}\big]$
Gravitational Ampere's Force law: $G^{i'\nu'}{}_{,\nu'} = m\big[(\nabla \times gt \times \omega) - (mg)_{,0}\big] - \nabla[x \cdot mg]$	Faraday's Law of Forces: ${}^{*}G^{i'\nu'}{}_{,\nu'} = -m\big[(\nabla \times g) + (gt \times \omega)_{,0}\big] - \nabla[x \cdot (mg_t \times \omega)]$
Transformation law for conservation law: $f^{\mu'\nu'}{}_{,\nu'\mu'} = g^{\mu'}_{\alpha} f^{\alpha\nu'}{}_{,\nu'\mu'}$	Transformation law for dual conservation law: ${}^{*}f^{\mu'\nu'}{}_{,\nu'\mu'} = g^{\mu'}_{\alpha}{}^{*}f^{\alpha\nu'}{}_{,\nu'\mu'}$
GLF Conservation Law by Matrix Method: $G^{\mu'\nu'}{}_{,\nu'\mu'} = \nabla[x \cdot (mg)_{,0}] - \nabla[x \cdot (mg)_{,0}] = 0$	Dual of GLF Conservation Law by Matrix method: ${}^{*}G^{\mu'\nu'}{}_{,\nu'\mu'} = \nabla[x \cdot (mgt \times \omega)_{,0}] - \nabla[x \cdot (mgt \times \omega)_{,0}] = 0$
Conservation Law by Usual Method 7D wave of Gravitational Energy in Red Color : $G^{\mu'\nu'}{}_{,\nu'\mu'} = G^{\mu\nu}{}_{,\nu\mu} + [\Box - \nabla^2][x \cdot mg]$	Dual Conservation Law by Usual Method 7D wave of Grav. Coriolis Energy in Red Color : ${}^{*}G^{\mu'\nu'}{}_{,\nu'\mu'} = {}^{*}G^{\mu\nu}{}_{,\nu\mu} + [\Box - \nabla^2][x \cdot (mgt \times \omega)]$

3. Discussion and Comparison

A consistent model of gravitation theory based on the antisymmetric gravitational Lorentz force tensor is developed in the language of electrodynamics. All the laws are gravitational force laws. We have successfully constructed three models of physical theories, namely:

1. Antisymmetric electromagnetic Lorentz force tensor formulation of electrodynamics,
2. Antisymmetric mechanical Lorentz force tensor formulation of mechanical electrodynamics, and
3. Antisymmetric gravitational Lorentz force tensor formulation of gravitation theory.

All of these possess the same mathematical structure but are physically different. The superposition of three antisymmetric tensorial forces governs our physical universe from micro to cosmic scales. The nature of electrodynamics is attractive and repulsive depending on the nature of charges. The nature of the mechanical or inertial force tensor is repulsive, while that of the gravitational tensorial force is attractive. The temporal part of force tensors governs linear or tangential motion, while the spatial part governs the geodesic motion. The spiral motion of galaxies is governed by Coriolis force. All these tensorial forces govern our universe as a single unit. The form invariance of these models can be verified by employing the framework of [14, 15], where it is shown that time and space component laws are relative, but spacetime laws of physics as a whole remain the same for all observers in their original form after transformation.

If we compare our gravitational force theory with that of Einstein's theory of gravitation, then it is obvious that our model is consistent with standard electrodynamics. Besides this, physicists are looking for a gravitation theory analogous to electrodynamics via linearization of Einstein's field equations in terms of gravitoelectromagnetism (GEM). Still, they are missing the antisymmetric gravitational Lorentz force tensor. Anyhow, we appreciate the efforts of Einstein and the contemporary physicists working in the field of gravitation physics.

We present the results of our model consisting of Gravitational Maxwell's equations and wave equations. Gravitational Maxwell's equations in noninertial coordinate metrics are obtained along with an outstanding conservation law containing the 7D wave of gravitational energy.

3.1. Gravitational Maxwell's Equations

Gaussian Force Law for Gravitation:

$$G^{0i}{}_{,i} = \nabla \cdot \mathbf{mg} \tag{3.1}$$

Amperian Force Law for Gravitation:

$$\nabla \times (\mathbf{mg}t \times \boldsymbol{\omega}) = G^{i\nu}{}_{,\nu} + \frac{\partial \mathbf{mg}}{\partial t} \tag{3.2}$$

Dual of Gaussian Force of Gravitation:

$$^{*}G^{0i}{}_{,i} = \nabla \cdot (\mathbf{mgt} \times \boldsymbol{\omega}) \tag{3.3}$$

Faraday's Law for Gravitation:

$$\nabla \times \mathbf{m}g = -^{*}G^{i\nu}{}_{,\nu} - \frac{\partial}{\partial t}(\mathbf{mgt} \times \boldsymbol{\omega}) \tag{3.4}$$

Gravitational Force Wave Equation:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{mg} = 0 \tag{3.5}$$

Coriolis Force Wave Equation:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)(\mathbf{mgt} \times \boldsymbol{\omega}) = 0 \tag{3.6}$$

Gravitational Lorentz Force Wave Equation:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\left[\mathbf{mg} + (\mathbf{mgt} \times \boldsymbol{\omega})\right] = 0 \tag{3.7}$$

3.2. Gravitational Maxwell's Equations in Noninertial Coordinate Metric

Gravitational Force Gauss's Law:

$$\nabla \cdot \mathbf{mg} = G^{0'\nu'}{}_{,\nu'} + \mathbf{x} \cdot [\nabla \times (\mathbf{mgt} \times \boldsymbol{\omega}) - \mathbf{mg}, \mathbf{0}] \tag{3.8}$$

Gravitational Force Ampere's Law:

$$\nabla \times (\mathbf{mgt} \times \boldsymbol{\omega}) = G^{i'\nu'}{}_{,\nu'} + \frac{\partial}{\partial t}\mathbf{mg} + \nabla(\mathbf{x} \cdot \mathbf{mg} \tag{3.9}$$

Dual of Gravitational Gaussian Force Law:

$$\nabla \cdot (\mathbf{mgt} \times \boldsymbol{\omega}) = {}^{*}G^{0'\nu'}{}_{,\nu'} - \mathbf{x} \cdot [(\nabla \times \mathbf{mg} + (\mathbf{mgt} \times \boldsymbol{\omega})_{,0}] \tag{3.10}$$

Faraday's Law of Gravitational Forces:

$$\nabla \times \mathbf{mg} = -^{*}G^{i'\nu'}{}_{,\nu'} - \frac{\partial}{\partial t}(\mathbf{mgt} \times \boldsymbol{\omega}) + \nabla(\mathbf{x} \cdot (\mathbf{mgt} \times \boldsymbol{\omega})) \tag{3.11}$$

Conservation Law by Usual Method:

$$G^{\mu'\nu'}{}_{,\mu'} = G^{\mu\nu}{}_{,\mu} + \left[\Box^2 - \nabla^2\right](\mathbf{x} \cdot \mathbf{mg} \tag{3.12}$$

The conservation law obtained by the usual technique gives the classical conservation law plus the 7D wave of gravitational energy. The 7D wave operator $\left(\Box^2 - \nabla^2\right)$ is an invariant quantity.

4. Conclusion

A very simple and outstanding model of gravitation theory in terms of Newtonian gravitational force and gravitational Coriolis force as components of the antisymmetric gravitational Lorentz force tensor $G^{\mu\nu}$ is developed, which has no counterexample in the contemporary world. It obeys the principle of relativity, conservation law, and symmetry principle. In other words, it is a consistent framework of general relativity in the language of standard electrodynamics.

This model, along with the antisymmetric mechanical Lorentz force tensor and the antisymmetric Lorentz force electrodynamic tensor, provides us with a mathematical unification of three theories. Our next project is on the formulation of the antisymmetric Lorentz position tensor, which itself possesses electrodynamic structure, whose time derivative gives the antisymmetric Lorentz momentum tensor. Furthermore, the above-mentioned models can be derived from it in a simple way. These models can be applied by physicists and engineers to explore astrophysics, cosmology, magnetohydrodynamics, and daily life physical experiments.

Dedicated to: H. A. Lorentz, Isaac Newton, and Albert Einstein.

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