



Individual Component Failures Role in an Imperfect One-out-of-Three Cold and Warm Standby System

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ABSTRACT: This study investigates the reliability and failure behavior of a non-repairable one-out-of-three standby system incorporating both cold and warm redundancy under imperfect switching conditions. Motivated by real-world challenges in hospital emergency power systems, the model includes three critical components: the primary power grid, a cold standby generator, and a warm standby UPS, along with a fallible switching mechanism. A continuous-time Markov chain (CTMC) framework is constructed to evaluate system dynamics, where each component is subject to exponentially distributed failures. System performance is quantified via key reliability metrics including reliability function $R(t)$, the failure probability $F(t)$, hazard rate $h(t)$, mean time to failure (MTTF), and transients probabilities. Numerical solutions are derived through matrix exponential computations and linear system solvers in MATLAB (R2023b). Results indicate that reliability degrades exponentially with time, the hazard rate stabilizes around a constant value, and the system has an expected operational lifespan of approximately 43.19 hours. The steady-state analysis of the transient probabilities shows that operational states 6, 9, 10, 13, and 14 are visited most frequently before the system is absorbed into failure, pinpointing which components deserve priority in redundancy planning

Key Words: Standby system, Markov chain, warm redundancy, imperfect switching.

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1. Introduction

Reliability is a fundamental feature of engineering systems, which is defined as a system or component's capacity to perform its intended task under the conditions specified for a nominated period [1,2]. The reliability analysis attempts to determine the possibility of system failure in a given operational context and explains the uncertainties in system modeling and environmental variables. This discipline is crucial in many engineering applications, critical infrastructure, and high-dependability environments.

Modern engineering systems, such as power generation, production, and industrial automation, require high dependence and access to maintain operational continuity. Assessment of reliability and availability is important to ensure that these systems can work without obstacles and can be effectively cured when errors occur. Unplanned system errors, especially during operation, may have sufficient shutdowns,

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resulting in productivity losses, financial errors, and recognized damage to organizations. In extreme cases, frequent system errors can destroy consumer confidence, triggering a waterfall of economic and operating results.

Reliability with electrical systems is important in a very dependent environment in hospitals and clinics. Power outages in such settings can directly compromise patient safety, highlighting the need for strong failure analysis and flexible system design. In this study, we examine the error occurrence to the electrical power supply system in settings for the emergency chamber settings in a hospital, mainly focusing on the dynamics of failure frequency and switching to the mechanism between many energy sources. This analysis is particularly relevant in Iraq, where the national grid is often exposed to frequent, long-term interruptions. As a result, many functions depend on combining primary grid compounds, generators, and transformers to ensure uninterrupted service [12].

The complexity of modern systems, the characteristics of many components, different error modes, and mutually dependent operations present credibility and important challenges in modeling. Extensive models must capture the dependence on such systems that integrate [3] component levels, system level structure, and operating processes. The interaction between operational mobility and failure behavior requires an approach that combines the system's reliability theory with stochastic modeling techniques.

Redundancy is a widely used strategy to improve system reliability. Among redundancy schemes, standby redundancy is categorized into cold and warm types. In cold standby systems, backup units remain inactive until needed and are presumed to be in the standby condition and not to fail. In contrast, warm standby units are partially active and thus susceptible to failure even while idle. Transitioning from a failed primary unit to a standby unit requires a switching mechanism, which may function perfectly or imperfectly [4,5,8,12,13,14,15].

This study focuses on a one-out-of-three standby system comprising one primary unit, one cold standby unit, one warm standby unit, and an imperfect switch as an application for the electrical network in the operating room of a hospital in Iraq. Table 1 highlights the notations used in this paper. Figure 1 illustrates the system configuration. The primary unit is operated initially, while the cold and warm standby units remain in the reserve. The switching mechanism transfers functionality in a backup device backing unit in the primary unit. In particular, the switch and warm standby unit can fail before activation, and the reliability can show further complexity to the model.

The remainder of the paper is organized as follows: Section 2 reviews the literature and explicitly

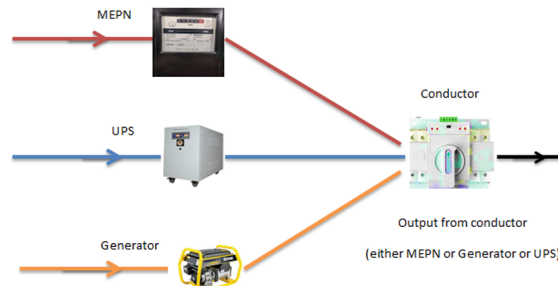


Figure 1: One-out-of-three standby system

identifies the modeling gap that motivates this work. Section 3 outlines the modeling assumptions and methodology based on Laplace transforms and Markov processes. Section 4 derives theoretical as the expression for the mean time to failure and the system's transients probabilities vector. Section 5 introduces a real application of the system to actual power outages in Iraq, while Section 6 investigates and analyzes the reliability matrices of the applied system. Finally, the paper ends with a brief conclusion.

Table 1: Notations

Symbol	Description
a	Primary unit
b	Cold standby unit
ω	Warm standby unit
s	Switching unit (changeover)
O	Operative unit
F	Failed unit
so	The switch is operating
sf	The switch is failed
λ	Constant failure rate of primary unit
β	Constant failure rate of cold standby unit
ε	Constant failure rate of warm standby unit
p	Constant failure rate of switching unit
$P_j(t)$	The probability of state j at time t
$P_0, \dots, P_{16}$	The switch is operating

2. Literature Review and Gap Identification

Standby redundancy literature can be grouped into three streams:(i) perfect-switch cold standby [9,11,12,13,15],(ii) warm standby with repairable units [5,6,15], and(iii) multi-state or Petri-net models that capture network effects [1,3].

Gap statement:To the best of our knowledge, *no closed-form reliability metrics exist for a non-repairable one-out-of-three system that simultaneously contains one primary, one cold and one warm unit together with an imperfect switch.* The present paper closes this gap by deriving exact MTTF and transient probabilities via an absorbing CTMC and validates the model against one-year outage data from an Iraqi hospital.

Asker [15] develops Markov-based reliability models for maintained/non-maintained systems with series, parallel, and r-out-of-n redundancy. He generalizes availability functions for maintained systems and derives MTTF for non-maintained cases applied to n=3 systems. Eight standby system models are analyzed, considering perfect/imperfect switching and cold/partially-loaded standby units, computing steady-state availability and MTTF via state-space methods.

Barger [1] explored modern control systems composed of multiple smart devices' components with computational and communication capabilities typically organized around a communication network. Such systems' dependability and safety assessment pose significant challenges due to their dynamic behavior. To address this, Barger focused on evaluating key dependability parameters using a pilot system modeled as a Colored Petri Net, incorporating dynamic aspects into the analysis.

Kolowrocki [3] extended the traditional two-state approach to multi-state systems for reliability evaluation, particularly for large and complex systems. By breaking the system components over time without repair, the Kholaroki demonstrated the capacity for more accurate reliability, safety, and operational efficiency analysis. This multi-state structure provides a rich representation of system behavior compared to the binary (two-state) model, enabling better insight into large systems' performance.

Sadeghi et al. [5] investigated the reliability of a warm standby repairable system under two scenarios involving imperfect switching mechanisms. Their study considered a repairable system with one active unit, one warm standby unit, and a single repairman, where failure and repair times were modeled using exponential distributions. In the first scenario, the active unit is replaced by the standby unit with a certain coverage probability. In the second scenario, the switching mechanism is repairable, with its failure and repair times also following exponential distributions. Using Markov processes and Laplace transforms, Sadeghi et al. derived analytical expressions for MTTF and steady-state availability, providing valuable insights into system design and optimization.

Batra et al. [6] examined the reliability and optimal configuration of the system with separate number of standby devices operated with a single operational device. When he used Semi-Markov processes

and a regeneration point approach, he evaluated the viability of using zero, one or two standby units. Their findings provide practical guidance for designing systems with increased reliability and operational efficiency.

Adfarati et al. [7] made a comprehensive analysis of a microgrid system operated by renewable energy resources, focusing on reliability, economic and environmental aspects. Renewable energy sources have received prominence due to their many benefits, including state incentives and public support. Adfarati et al. With a view to reducing energy costs, life cycle, annual electricity loss and lowering of greenhouse gas emissions, the dependency, cost -effectiveness and ecological benefits of integrating renewable energy into microgrids were assessed. Their work emphasizes the importance of permanent technologies to achieve financial and environmental goals.

Krishnan [9] analyzed the reliability of r -out-of- n systems, which are widely used in engineering applications requiring redundancy. Similarly, Jia et al. [10] examined the optimisation of activation sequences and reliability evaluation of non-repairable multi-state generating systems, emphasizing the role of warm standby configurations. Their study highlights the significance of optimizing system configurations to enhance reliability and operational efficiency.

Patawa et al. [11] studied a cold standby repairable system consisting of two distinct units with a waiting time feature. The failure times of the units followed exponential distributions with different rates, while their repair times adhered to a one-parameter Lindley distribution. The authors evaluated system performance metrics such as mean time to system failure (MTSF) and steady-state availability, demonstrating the impact of waiting times on system reliability.

Hindi and Asker [12,13] developed an availability estimation model for a repairable one-out-of-three cold standby system, applying it to an industrial case involving electrical power networks. They analyzed various reliability optimization problems using Markov models, including redundancy allocation, reliability allocation, and reliability-redundancy allocation. Their work provides a comprehensive overview of methodologies for solving these problems, offering valuable insights for practitioners and researchers.

3. Model Assumptions

The analysis of this redundant system consists of three units: a primary unit (a), a cold standby unit (b), and a warm standby unit (ω). All units operate with an imperfect switching mechanism. The analysis is based on the following assumptions:

- Unit a , representing the primary electrical power grid, has operational priority. Initially, unit a is active, while units b and ω remain in standby mode, and the switching mechanism is functioning properly.
- The operational unit (a), cold standby unit (b), warm standby unit (ω), and the switching device all have exponential time-to-failure distributions characterized by parameters λ , β , ε , and p , respectively.
- The switching mechanism is imperfect. When the primary unit fails, the warm standby unit is available to take over, provided the switching device is functioning. After switching to the warm standby unit, the device will activate the cold standby unit.
- Only one failure event can occur during a given time frame.
- A failure of the switching device does not necessarily result in system failure.
- The failure of the warm standby unit does not automatically cause system failure.
- System failure occurs if the primary operational unit fails while the switching device is inoperative.
- System failure also occurs when all units have failed, regardless of the status of the switching device.

The transition matrix, Q , best captures the change of a system from one state to another. The system states shown in Figure 2 and transition probability matrix $P = I + Q$, I is an identity matrix with the same size of the matrix Q , can be examined using the time derivative of state probabilities and a Markov transition diagram to get the reliability matrices [12,13,15]

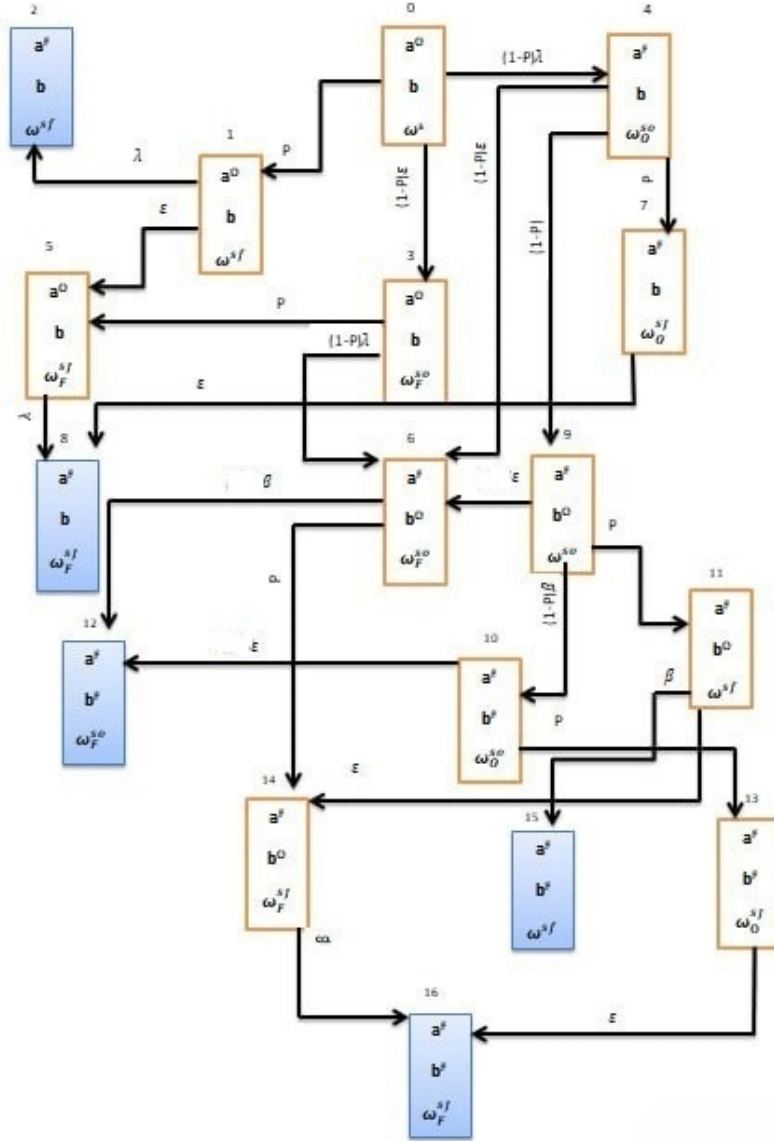


Figure 2: Markov transition diagram for one-out-of-three warm-cold standby system.

$$Q = \begin{pmatrix} -((1-p)(\varepsilon+\lambda)+p) & P & 0 & (1-P)\varepsilon & (1-p)\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -(\lambda+\varepsilon) & \lambda & 0 & 0 & \varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(p+(1-p)\lambda) & 0 & p & (1-p)\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -((1-p)\varepsilon+(1-p)+p) & 0 & (1-p)\varepsilon & p & 0 & (1-p) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -((1-p)\beta+p) & 0 & 0 & 0 & 0 & 0 & (1-p)\beta & 0 & p & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon & \varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon & 0 & 0 & -(\varepsilon+(1-p)\beta+p) & (1-p)\beta & p & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\varepsilon+p) & 0 & \varepsilon & p & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\beta+\varepsilon) & 0 & 0 & \varepsilon & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon & 0 & 0 & \varepsilon \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta & 0 & \beta \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

4. The Theoretical Aspects of the Main Results

By using Q (transition rate matrix of the system), the state's equations $\bar{P}_j(t)$ (which are set of linear first order equations), when initial state i is known, for the supposed system according to Equations (2.10) in [15] are:

$$[P_0(t), \dots, P_{16}(t)]Q = [\bar{P}_0(t), \dots, \bar{P}_{16}(t)]. \quad (4.1)$$

The transients probabilities, $P_0, \dots, P_{16}$, are determined according to Matrix Equation (2.20) in [15] by:

$$[P_0, \dots, P_{16}]Q = [0, \dots, 0]. \quad (4.2)$$

The transients vector $[P_0, \dots, P_{16}]$ will compute by solving the linear system (4.2) with the constraint $\sum_i P_i = 1$, using a numerical solver in MATLAB (R2023b) in next application section.

Also, according to Section (2.7) in [15], the MTTF of the system can be determined by considering the absorbing states of the system (states: 2, 8, 12, 15, and 16) as failed states. Now suppose that the initial state at $t = 0$ is state 0. By deleting the row and the column of the transition rate matrix corresponding to the absorbing states and taking Laplace transforms (with $s = 0$) of Equations (4.1), we obtain:

$$[P_0^*(0), P_1^*(0), P_3^*(0), \dots, P_{14}^*(0),]Q = [-1, 0, 0, \dots, 0]. \quad (4.3)$$

Thus, according to equation (2.24) in [15]: the MTTF_S is:

$$\text{MTTF}_S = R^*(0) = \sum_{\forall i} P_i^*(0), i \text{ is not absorbing state}, \quad (4.4)$$

where, $P_i^*(s)$ is the Laplace transform of the state probability $P_i(t)$.

5. Application of the System

In this paper, the value of the constant failure of the primary unit (the Iraqi electrical grid) and the constant failure of the cold standby unit (the locally generated electrical power) are taken from **actual power outages in Iraq** (see Table 2). On the other hand, the constant failure of the UPS and the switching unit is estimated because it depends on numerous variables, including batteries, the manufacturing process, product quality control, and others.

The system parameters are defined based on empirical data and engineering estimates. The **constant failure rate** for the *primary unit* is calculated as:

$$\lambda = \frac{53}{12 \times 24} \approx 0.1840 \simeq 0.2 \text{ failures/hour},$$

where the numerator represents the total number of failures observed over a 12-month period, assuming continuous operation.

Table 2: The approach failure rate parameters

months	main power outage (in hours)	locally power log-in in hours	locally power log-off
january	2	1.7	0.3
febreuary	2	1.8	0.2
march	2	1.8	0.2
april	3	2.7	0.3
may	3	2.8	0.2
june	10	7.5	2.5
july	10	7.5	2.5
august	10	7.5	2.5
septemper	4	2.7	1.3
october	3	2.8	0.2
november	2	1.8	0.2
december	2	1.8	0.2
	$\sum = 53$		$\sum = 10.6$

The *failure rate of the cold standby unit* is similarly computed:

$$\beta = \frac{10.6}{12 \times 24} \approx 0.0368 \simeq 0.04 \text{ failures/hour.}$$

The *failure rate for the warm standby unit* is estimated based on typical industry benchmarks and engineering judgment as:

$$\varepsilon = 0.03 \text{ failures/hour.}$$

Finally, the *switching failure rate*, which accounts for the probability of unsuccessful transitions between units, is assumed to be:

$$p = 0.01.$$

These values serve as the baseline parameters for modeling the system using a continuous-time Markov chain.

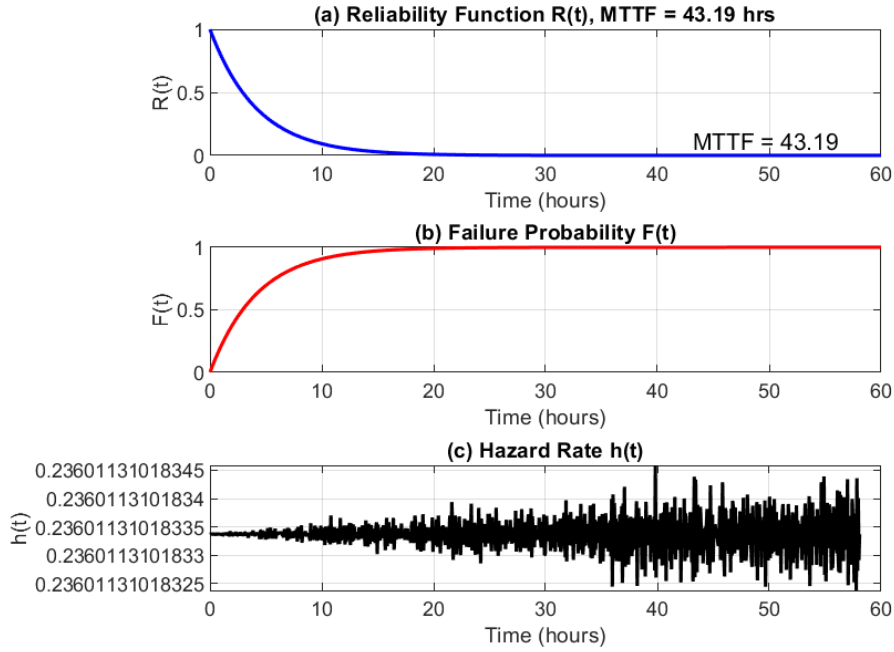
Remark 5.1 In Table 2 We take into account the generator's operational delay.

6. System Reliability, Failure, and Hazard Analysis

This section presents a comprehensive analysis of the system's time-dependent reliability characteristics, including the reliability function $R(t)$, failure probability function $F(t)$, and hazard rate $h(t)$, , and hazard rate $h(t)$ -together with an appraisal of the most frequently visited operational states via the transient probabilities of the underlying Markov model (see Table ?? and Figure 3).

Table 3: Transients Probabilities

State i	Probability P_i
0	0.015795
1	0.002576
2	0.000000
3	0.014390
4	0.006599
5	0.003279
6	0.182798
7	0.098778
8	0.000000
9	0.127701
10	0.198858
11	0.049286
12	0.000000
13	0.162864
14	0.136989
15	0.000000
16	0.000000
Sum	0.999913

Figure 3: Reliability Function $R(t)$, Failure Probability $F(t)$, and Hazard Rate $h(t)$

6.1. Analysis of Reliability Function $R(t)$

The reliability function $R(t)$, as seen in the Figure 3-a, quantifies the probability that the system remains operational at a given time t . This function is observed to decrease monotonically, which is characteristic of degrading systems. Specifically:

- At $t = 0$, the system is fully reliable, i.e., $R(0) = 1$.
- As time progresses, $R(t)$ declines exponentially toward zero, signifying an increasing likelihood of system failure.
- The Mean Time to Failure (MTTF) is calculated as

$$\text{MTTF} = \int_0^{\infty} R(t) dt, \quad (6.1)$$

and is approximately **43.19 hours**, representing the system's expected operational lifetime before failure; however, unlike a single-component exponential system, $R(\text{MTTF})/e^{-1}$, and the asymptotically constant hazard rate suggests only a quasi-memoryless behavior in the long run, not a strictly exponential failure distribution. corresponding to the point at which $R(t) \approx e^{-1} \approx 0.37$.

6.2. Analysis of Failure Probability Function $F(t)$

The cumulative failure probability function $F(t) = 1 - R(t)$ represents the cumulative probability that the system has failed by time t . The key characteristics of $F(t)$, from the Figure 3-b, are:

- Initially, $F(0) = 0$, reflecting that no failure has occurred at the outset.
- Over time, $F(t)$ increases and asymptotically approaches 1, indicating that the system is almost certain to fail in the long run.
- The rapid growth of $F(t)$ in early time periods highlights a high initial failure risk.
- The complementary relationship between $R(t)$ and $F(t)$ confirms the consistency of the reliability model.

6.3. Analysis of Hazard Rate Function $h(t)$

The hazard rate $h(t)$ describes the instant failure frequency of time t , conditional on the surviving system to that point. Along with the the Figure 3-c, The behavior of $h(t)$ provides the following insights:

- The hazard rate stabilizes to a relatively constant value as time increases, indicating the system follows a memoryless (exponential) failure distribution.
- The estimated value of the hazard rate is $h(t) \approx 0.02349$ failures/hour, which aligns with the MTTF via the relation $\text{MTTF} = 1/h(t) \approx 43.19$ hours.

This constancy supports modeling the failure process using an exponential distribution, suitable for systems with no aging effects or time-dependent degradation beyond the initial phase.

6.4. Transient Probabilities

To understand the system's long-term behavior, transient probabilities were computed for each of the 17 discrete states in the underlying Markov model, are listed in Table 3.

- The transient probabilities P_i were calculated for all states $i = 0, 1, \dots, 16$.
- States such as **6, 9, 10, 13 and 14** exhibit comparatively higher transient probabilities, suggesting they are visited more frequently before the system reaches an absorbing state.
- Absorbing states, including **states 2, 8, 12, 15, and 16**, have zero steady-state probabilities. This indicates that once the system transitions into these states, it does not leave, consistent with the definition of absorbing states.
- The near-unity sum of all transient probabilities ($\sum P_i \approx 0.999913$) confirming proper normalization and validating the integrity of the Markov model used in the analysis.

7. Conclusion

This paper derived exact CTMC-based reliability metrics for a non-repairable 1-out-of-3 standby system that combines one cold and one warm spare with an imperfect switch. Calibrated on one-year outage data from an Iraqi hospital, the model yields an MTTF of 43.19 h and an essentially constant hazard rate, confirming quasi-memoryless behaviour. Transient probabilities identify states 6, 9, 10, 13, and 14 as the most visited before absorption, pinpointing the generator and switch as the critical components to upgrade. The closed-form expressions can be embedded in redundancy-allocation or cost-optimisation routines; extensions to repairable components or time-varying rates are straightforward.

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