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On Rational Contractions of Geraghty-Fisher Type in Controlled Fuzzy Metric Spaces with Application

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ABSTRACT: This paper presents a novel rational contraction condition of the Geraghty-Fisher type within the framework of controlled fuzzy metric spaces, thereby extending and strengthening existing fixed point results. The proposed contraction condition provides a more general and effective approach for establishing fixed points in fuzzy environments. To validate the theoretical findings, an illustrative example is included along with a graphical representation demonstrating convergence. Moreover, the applicability of the theorem is illustrated by proving the existence and uniqueness of solutions for a second-order differential equation associated with a two-point boundary value problem in electric circuit theory. The results obtained not only generalize several known fixed point theorems but also offer new mathematical tools applicable to real-world problems in engineering and applied sciences. This work contributes to the advancement of fixed point theory and opens avenues for further research in fuzzy metric spaces and their applications.

Key Words: Controlled fuzzy metric space, rational contraction, Geraghty-Fisher type, fixed point theorem, Boundary value problem.

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1. Introduction

In 1922, S. Banach [6] provided an important result in fixed point theory. Since then, this topic has been extensively studied, developed, and generalized by many researchers in various spaces. The works of Bakhtin [5], Bourbaki [7], and Czerwik [8] were among the first to expand the theory of fixed points for b-metric spaces. Furthermore, several authors proved significant fixed point theorems in b-metric spaces ([1], [2], [3]). Controlled metric spaces were introduced by Nabil Mlaiki et al. [20], who also proved some fixed point results in this context. Fisher-type fixed point results in controlled metric spaces were defined by Durdana L. [9]. Jamshaid Ahmad et al. [14] introduced Reich-type contractions and (α, F)-contractions in the class of controlled metric spaces and established new related fixed point theorems.

On the other hand, significant theoretical developments in fuzzy set theory were initiated by Zadeh [27]. Fuzzy set theory led to the concept of fuzzy metric spaces, defined by Kramosil and Michalek [16], which can be regarded as a generalization of statistical metric spaces. Subsequently, M. Grabiec [11] defined G-complete fuzzy metric spaces, extending the notion of complete fuzzy metric spaces. Following Grabiec's work, George and Veeramani [10] modified the concept of M-complete fuzzy metric spaces using continuous t-norms.

Nadaban [22] introduced the concept of fuzzy b-metric spaces, and Kim et al. [15] established fixed point results in this setting. More recently, Mehmood et al. [17] defined extended fuzzy b-metric spaces, which generalize fuzzy b-metric spaces. M"uzeyyen Sangurlu Sezen [23] introduced controlled fuzzy metric spaces, which further generalize extended fuzzy b-metric spaces.

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2. Preliminaries

Now, we begin with some basic concepts, notations and definitions. Let $\mathbb{R}$ represent the set of real numbers, $\mathbb{R}_+$ represent the set of all non-negative real numbers and $\mathbb{N}$ represent the set of natural numbers.

We start with the following definitions of a fuzzy metric space.

Definition 2.1 [10]. An ordered triple $(X, M, *)$ is called fuzzy metric space such that X is a nonempty set, $*$ defined a continuous t-norm and M is a fuzzy set on $X \times X \times (0, \infty)$, satisfying the following conditions, for all $x, y, z \in X, s, t > 0$.
(FM1) $M(x, y, t) > 0$.
(FM2) $M(x, y, t) = 1$ iff $x = y$.
(FM3) $M(x, y, t) = M(y, x, t)$.
(FM4) $(M(x, y, t) * M(y, z, s)) \leq M(x, z, t + s)$.
(FM5) $M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is left continuous.

In 2017, Nădăban [22] introduced the idea of a fuzzy b-metric space to generalize the notion of a fuzzy metric spaces introduced by Kramosil and Michalek [16].

Definition 2.2 [22]. Let X is a non-empty set and $b \geq 1$ be a given real number and $*$ be a continuous t-norm. A fuzzy set M in $X^2 \times (0, \infty)$ is called fuzzy b-metric on X if for all $x, y, z \in X$ and $s, t > 0$ the following conditions hold.
(FbM1) $M(x, y, t) = 0$.
(FbM2) $M(x, y, t) = 1$ iff $x = y$.
(FbM3) $M(x, y, t) = M(y, x, t)$.
(FbM4) $M(x, z, b(t + s)) \geq M(x, y, t) * M(y, z, s)$.
(FbM5) $M(x, y, \cdot) : (0, \infty) \to [0, 1]$ is left continuous and $lim_{t\to\infty} M(x, y, t) = 1$.

Mehmood et al. [17] introduced the notion of an extended fuzzy b-metric space following the approach of Grabiec [11].

Definition 2.3 [17]. Let X be a non-empty set, $\alpha : X \times X \to [1, \infty)$, $*$ is a continuous t-norm and M_α is a fuzzy set on $X^2 \times (0, \infty)$, is called extended fuzzy b-metric on X if for all $x, y, z \in X$ and $s, t > 0$, satisfying the following conditions.
$(FbM_\alpha 1)$ $M_\alpha(x, y, 0) = 0$.
$(FbM_\alpha 2)$ $M_\alpha(x, y, t) = 1$ iff $x = y$.
$(FbM_\alpha 3)$ $M_\alpha(x, y, t) = M_\alpha(y, x, t)$.
$(FbM_\alpha 4)$ $M_\alpha(x, z, \alpha(x, z)(t + s)) \geq M_\alpha(x, y, t) * M_\alpha(y, z, s)$.
$(FbM_\alpha 5)$ $M_\alpha(x, y, \cdot) : (0, \infty) \to [0, 1]$ is continuous.
Then $(X, M_\alpha, *, \alpha(x, y))$ is an extended fuzzy b-metric space.

In [23], Sezen introduced the controlled fuzzy metric spaces, which is a generalization of extended fuzzy b-metric spaces.

Definition 2.4 [23]. Let X be a non-empty set, $\lambda : X \times X \to [1, \infty)$, $*$ is a continuous t-norm and M_λ is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following conditions, for all $a, c, d \in X, s, t > 0$:
$(FM_\lambda 1)$ $M_\lambda(a, c, 0) = 0$.
$(FM_\lambda 2)$ $M_\lambda(a, c, t) = 1$ iff $a = c$.
$(FM_\lambda 3)$ $M_\lambda(a, c, t) = M_\lambda(c, a, t)$.
$(FM_\lambda 4)$ $M_\lambda(a, d, t + s) \geq M_\lambda(a, c, \frac{t}{\lambda(a,c)}) * M_\lambda(c, d, \frac{s}{\lambda(c,d)})$.
$(FM_\lambda 5)$ $M_\lambda(a, c, \cdot) : [0, \infty) \to [0, 1]$ is continuous.
Then, the triple $(X, M_\lambda, *)$ is called a controlled fuzzy metric space on X.

Definition 2.5 [23]. Let $(X, M_\lambda, *)$ be a controlled fuzzy metric spaces. Then

1. A sequence $\{x_n\}$ in X is said to be G-convergent to x in X, if and only if $lim_{n\to\infty} M_\lambda(x_n, x, t) = 1$ for any $n > 0$ and for all $t > 0$.
2. A sequence $\{x_n\}$ in X is said to be G-Cauchy sequence if and only if $lim_{n\to\infty} M_\lambda(x_n, x_{n+m}, t) = 1$ for any $m > 0$ and for all $t > 0$.

3. The controlled fuzzy metric space is called G-complete if every G- Cauchy sequence is convergent.

The objective of this work is to prove a fixed point theorem using new Fisher type rational contractive condition in controlled fuzzy metric spaces, which is an extension of [10], [17], [22], [21], [23]. Our result generalizes many recent fixed point theorems in the literature ([15], [17], [22], [21]). We furnish an example to validate our result. Application is also provided to show the utility of our result.

3. Main Result

In this section, a new rational contraction condition of the Geraghty–Fisher type is introduced in the setting of controlled fuzzy metric spaces, and a corresponding fixed point theorem is established.

Definition 3.1 Following [13], for a real number $b > 1$, let F_b denotes the class of all functions $\xi : [0, \infty) \to [0, \frac{1}{b})$ satisfying the following condition.

$$\xi(t_n) \to \frac{1}{b} \ \ as \ \ n \to \infty \ \ implies \ \ t_n \to 0 \ \ as \ \ n \to \infty.$$

$$F_b = \{\xi : [0, \infty) \to [0, \frac{1}{b}) \ \ such\ that \ \ \lim_{n\to\infty} \xi(t_n) \to \frac{1}{b} \ \ implies \ \ \lim_{n\to\infty} t_n \to 0\}.$$

Definition 3.2 Let $(Y, M_\lambda, *)$ is a complete controlled fuzzy metric space on Y. Let $\lambda : X \times X \to [1, \infty)$ and a mapping $S : Y \to Y$ is called a new Geraghty-Fisher type rational contraction condition, such that

$$M_\lambda(Sa, Sc, \chi(M_\lambda(a, c, t))t) \geq N_\lambda(a, c, t), \tag{3.1}$$

where

$$N_\lambda(a, c, t) = min\{(\alpha M_\lambda(a, c, t) + (1 - \alpha)M_\lambda(a, Sa, t)), (\beta M_\lambda(c, Sc, t) + (1 - \beta)\frac{M_\lambda(a, Sa, t), M_\lambda(c, Sc, t)}{M_\lambda(a, c, t)})\}$$

for all $\alpha, \beta \in [0, 1], \quad a, c \in Y, \quad \chi \in F_b$.

Theorem 3.1 *Let $(Y, M_\lambda, *)$ is a complete controlled fuzzy metric space on Y. A mapping $S : Y \to Y$ satisfies new Geraghty-Fisher type rational contraction condition (3.1), for all $\alpha, \beta \in [0, 1], \quad \chi \in F_b$, then S admits a unique fixed point.*

Proof: Let a_0 is an arbitrary point in Y, We define a sequence $\{a_n\}$ in Y by

$$a_{n+1} = Sa_n \ \ for\ all\ n \in \mathbb{N}.$$

so $Sa_{n_0} = a_{n_0}$ and the proof is finished. Suppose that $a_n \neq a_{n+1}$ for all $n \in \mathbb{N}$,

$$M_\lambda(Sa_{n-1}, Sa_n, t) < 1 \ \ for\ all\ n \in \mathbb{N} \ \ and \ \ s > 0.$$

Using Theorem 3.1, so for all $n \in \mathbb{N}$, we can write

$$M_\lambda(a_n, a_{n+1}, t) \geq M_\lambda(a_{n-1}, a_n, t),$$

for all $n \in \mathbb{N}$. Thus (3.1), we get

$$\begin{aligned} M_\lambda(a_n, a_{n+1}, t) &\geq M_\lambda(a_{n-1}, a_n, \frac{t}{\chi(M_\lambda(a_{n-1}, a_n, t))})) \\ &\geq M_\lambda(a_{n-2}, a_{n-1}, \frac{t}{\chi(M_\lambda(a_{n-1}, a_n, t)).\chi(M_\lambda(a_{n-2}, a_{n-1}, t))})) \\ &\ \ \vdots \\ &\geq M_\lambda(a_0, a_1, \frac{t}{\chi(M_\lambda(a_{n-1}, a_n, t)).\chi(M_\lambda(a_{n-2}, a_{n-1}, t)) \cdots \chi(M_\lambda(a_0, a_1, t))}). \end{aligned} \tag{3.2}$$

Thus by (3.2), we have

$$M_\lambda(a_n, a_{n+1}, t) > M_\lambda(a_0, a_1, \chi(M_\lambda(a_0, a_1, t))). \tag{3.3}$$

Consider the triangle inequality, using the condition $(FM_\lambda 4)$ of definition 2.4, we have

$$\begin{aligned} M_\lambda(a_n, a_{n+m}, t) &\geq (M_\lambda(a_n, a_{n+1}, \frac{t}{2\lambda(\chi(M_\lambda(a_n, a_{n+1}, t)})\\ &* M_\lambda(a_{n+1}, a_{n+m}, \frac{t}{2\lambda(\chi(M_\lambda(a_{n+1}, a_{n+m}, t)}))\\ &\geq (M_\lambda(a_n, a_{n+1}, \frac{t}{2\lambda(\chi(M_\lambda(a_n, a_{n+1}, t)})\\ &* M_\lambda(a_{n+1}, a_{n+2}, \frac{t}{(2)^2\lambda(\chi(M_\lambda(a_{n+1}, a_{n+m}, t)\lambda(\chi(M_\lambda(a_{n+1}, a_{n+2}, t)})\\ &* M_\lambda(a_{n+2}, a_{n+m}, \frac{t}{(2)^2\lambda(\chi(M_\lambda(a_{n+1}, a_{n+m}, t)\lambda(\chi(M_\lambda(a_{n+2}, a_{n+m}, t)}))\\ &\vdots\\ &\geq [M_\lambda(a_0, a_1, \frac{t}{2^{n-1}\lambda(\chi(M_\lambda(a_n, a_{n+1}, t)})\\ &* (*_{i=n+1}^{n+m-2}(M_\lambda(a_0, a_1, \frac{t}{(2)^{m-1}(\prod_{j=n+1}^{i}\lambda(\chi(M_\lambda(a_j, a_{n+m}, t))\lambda(\chi(M_\lambda(a_i, a_{i+1}, t)}))\\ &* (M_\lambda(a_0, a_1, \frac{t}{(2)^{m-1}(\prod_{i=n+1}^{n+m-1}\lambda(\chi(M_\lambda(a_i, a_{n+m}, t))}))]. \end{aligned} \tag{3.4}$$

Therefore, by taking limit as $n \to \infty$ in (3.4), from (3.3) together with (3.1) we have

$$\lim_{n\to\infty} M_\lambda(a_n, a_{n+m}, kt) \geq (1 * 1 * \cdots * 1) = 1,$$

for all $t > 0$ and $n, m \in \mathbb{N}$. Thus, $\{a_n\}$ is a G-Cauchy sequence in X. From the completeness of $(X, M_\lambda, *)$, there exists $u \in X$ such that

$$\lim_{n\to\infty} M_\lambda(a_n, u, kt) = 1, \tag{3.5}$$

for all $t > 0$. Now we show that u is a fixed point of h. For any $t > 0$ and from the condition $(FM_\lambda 4)$ of definition 2.4, we have

$$\begin{aligned} M_\lambda(u, Su, kt) &\geq min\{(\alpha M_\lambda(a, b, t) + (1-\alpha)M_\lambda(a, Sa, t)), (\beta M_\lambda(b, Sb, t)\\ &+ (1-\beta)\frac{M_\lambda(a, Sa, t), M_\lambda(b, Sb, t)}{M_\lambda(a, b, t)})\}\\ &\geq M_\lambda(u, a_{n+1}, \frac{t}{2\lambda(\chi(M_\lambda(u, a_{n+1}, t))}\\ &* M_\lambda(Sa_n, Su, \frac{t}{2\lambda(\chi(M_\lambda(a_{n+1}, Su, t))}\\ &\geq M_\lambda(u, a_{n+1}, \frac{t}{2\lambda(\chi(M_\lambda(u, a_{n+1}, t))}\\ &* M_\lambda(a_n, u, \frac{t}{2\lambda(\chi(M_\lambda(a_{n+1}, Su, t))}. \end{aligned} \tag{3.6}$$

Letting $n \to \infty$ in (3.6) and using (3.5), we get

$$\lim_{n\to\infty}(M_\lambda(u, Su, t)) = 1 \Leftrightarrow \lim_{n\to\infty} M_\lambda(u, Su, t) = 1,$$

for all $t > 0$, that is, $u = Su$.
Let $w \in X$ is an another fixed point of S and there exists $t > 0$ such that $u \neq w$, then it follows from (3.1) that

$$\begin{aligned} M_\lambda(u, w, t) &= M_\lambda(Su, Sw, t) \\ &\geq M_\lambda(u, w, \frac{t}{\chi(M_\lambda(u, w, t)}) \\ &\geq M_\lambda(u, w, \frac{t}{\chi(M_\lambda(u, w, t))^2}) \\ &\vdots \\ &> M_\lambda(u, w, \frac{t}{\chi(M_\lambda(u, w, t))^n}), \end{aligned} \tag{3.7}$$

for all $n \in \mathbb{N}$. By taking limit as $n \to \infty$ in (3.7), $M_\lambda(u, w, t) = 1$ for all $t > 0$, that is, $u = w$. This completes the proof. □

Remark 3.1 *If*

$$\begin{aligned} N_\lambda(a, c, t) = min\{&(\alpha M_\lambda(a, c, t) + (1 - \alpha) M_\lambda(a, Sa, t)), (\beta M_\lambda(c, Sc, t) \\ &+ (1 - \beta)\frac{M_\lambda(a, Sa, t), M_\lambda(c, Sc, t)}{M_\lambda(a, c, t)})\} \\ = M_\lambda(a, c, t) \end{aligned}$$

in Theorem 3.1, we obtain the following result.

Corollary 3.1 *Let $(Y, M_\lambda, *)$ is a new rational contraction of Geraghty-Fisher type in controlled fuzzy metric space on Y. If $S : Y \to Y$ is a mapping such that for all $a, c \in Y$, and $\lambda : Y \times Y \to [1, \infty)$, $t > 0$ and $\chi \in F_b$.*

$$M_\lambda(Sa, Sc, \chi(M_\lambda(a, c, t))t) \geq M_\lambda(a, c, t),$$

then S admits a unique fixed point.

The following example illustrates Theorem 3.1.

4. Example

Example 4.1 *Let $Y = [0, 1]$. Define the function $M_\lambda : Y \times Y \times [0, \infty) \to [0, 1]$ by*

$$M_\lambda(a, c, t) = \frac{t}{t + |a - c|}$$

with the continuous t-norm $*$ defined as $t_1 * t_2 = t_1 \cdot t_2$. Define the function $\lambda : Y \times Y \to [1, \infty)$ by

$$\lambda(a, c) = \begin{cases} 1 & \text{if } a, c \in Y, \\ \max\{a, c\} & \text{otherwise.} \end{cases}$$

Clearly, $(Y, M_\lambda, *)$ is a Geraghty-Fisher type controlled fuzzy metric space. Now, consider the mapping $S : Y \to Y$ defined by

$$S(a) = \frac{a}{3}.$$

Let F_b denote the class of functions $\chi : [0, \infty) \to [0, \frac{1}{b})$, where $\chi \in F_b$ is given by $\chi(t) = \frac{1}{3}$.

Now, we show that $(Y, M_\lambda, *)$ is a controlled fuzzy metric space. It is easy to verify conditions $(FM_\lambda 1)$, $(FM_\lambda 2)$, and $(FM_\lambda 3)$ from Definition 2.4. We only need to check condition $(FM_\lambda 4)$, for which we consider the following cases:

Case I. *If $a = c$, then $Sa = Sc$. We have*

$$\begin{aligned} M_\lambda(Sa, Sc, \chi(M_\lambda(a,c,t))t) &= \frac{\chi(M_\lambda(a,a,t))t}{\chi(M_\lambda(a,a,t))t + |S(a) - S(a)|} \\ &= 1 \geq N_\lambda(a,c,t). \end{aligned}$$

Case II. *If $a \neq c$, then $Sa \neq Sc$. In this case,*

$$\begin{aligned} M_\lambda(Sa, Sc, \chi(M_\lambda(a,c,t))t) &= \frac{\chi(M_\lambda(a,c,t))t}{\chi(M_\lambda(a,c,t))t + |S(a) - S(c)|} \\ &= \frac{\frac{1}{3}t}{\frac{1}{3}t + \frac{|a-c|}{3}} \\ &= \frac{t}{t + |a-c|} \\ &= M_\lambda(a,c,t) \geq N_\lambda(a,c,t), \end{aligned}$$

therefore, all the conditions of Theorem 3.1 are satisfied, and the mapping S has a unique fixed point at $y = 0$.

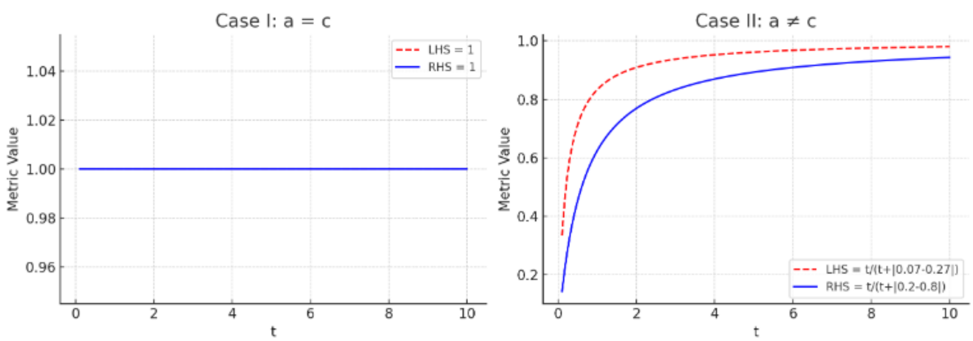

Figure 1: 2D comparison of LHS and RHS under Case I ($a = c$) and Case II ($a \neq c$). In Case I, both LHS and RHS coincide at 1, while in Case II the LHS dominates the RHS.

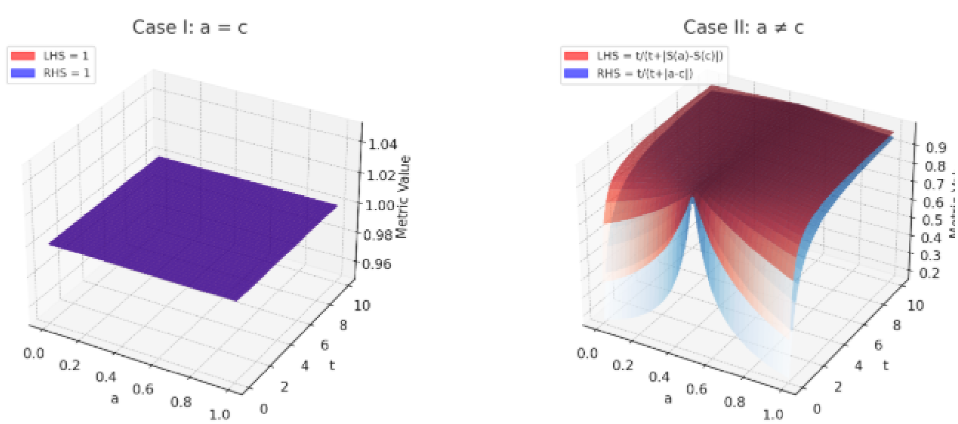

Figure 2: 3D comparison of LHS and RHS under Case I ($a = c$) and Case II ($a \neq c$). The surface plot shows how the LHS dominates RHS except when $a = c$, where they coincide at 1.

5. Application

In this section, the applicability of the developed fixed point results in fuzzy controlled metric spaces is demonstrated by considering a boundary value problem. Specifically, we establish the existence and uniqueness of solutions for a second-order differential equation arising in electric circuit theory. By constructing an appropriate operator associated with the given problem and verifying that it satisfies the proposed Geraghty–Fisher type rational contraction condition, the desired solution is obtained. This application highlights the effectiveness of the theoretical results presented in the preceding sections.

Application to Electric Circuit Differential Equation As an application of our main result, now solve an electric circuit, which is in the form of a second-order differential equation. In Tomar et al. [25] used that electric circuit containing an electromotive force E_1 supplied by a battery, a resistor R_1, an inductor L_1 and a capacitor C_1 in series. If the current I_1 is the rate of change of charge q with respect to time t, $I_1 = \frac{dq}{dt}$. let us recall the following usually formulas

1. $V = I_1 R_1$
2. $V = \frac{q}{C}$
3. $V = L_1 \frac{dI_1}{dt}$

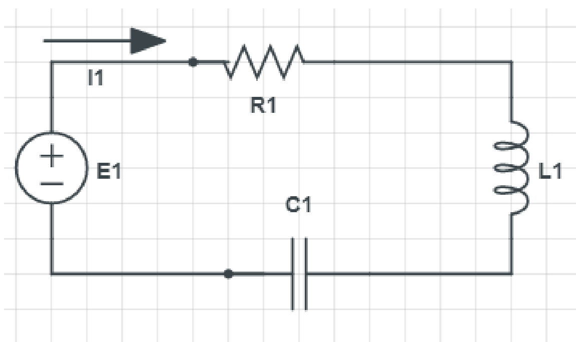

Figure 3: RLC Electric Circuit

where $V =$ voltage, now by the voltage law, the sum of these voltage drops is equal to the supplied voltage, i,e.,

$$R_1 i + \frac{q}{c} + L_1 \frac{dI}{dt} = V(t),$$

or

$$\begin{cases} L_1 \frac{d^2 q}{dt^2} + R_1 \frac{dq}{dt} + \frac{q}{c} = V(t), \\ q(0) = 0 \\ q'(0) = 0. \end{cases} \tag{5.1}$$

Green's function $\zeta(t, r)$ associated to (5.1) is given by

$$\zeta(t, r) = \begin{cases} -re^{\mu(t-r)} & \text{if } 0 \leq r \leq t \leq 1 \\ -te^{\mu(r-t)} & \text{if } 0 \leq t \leq r \leq 1. \end{cases} \tag{5.2}$$

where $\mu > 0$ is a constant, calculated in terms of R_1 and L_1. Let $Y = C([0, 1], \mathbb{R}^+)$ be the set of all non negative real valued functions defined on $[0, T]$. For $t > 0$, we define

$$M_\lambda(x, y, t) = sup_{t \in [0,1]} e^{-\frac{|x-y|}{t}} \tag{5.3}$$

for all $\alpha, \beta \in [0, I], \quad x, y \in Y, \quad \chi \in F_b$.with the continuous t-norm $*$ such that $t_1 * t_2 = min\{t_1, t_2\}$ and define $\lambda : Y \times Y \to [1, \infty)$, as

$$\lambda(x, y) = \begin{cases} 1 & \text{if } x, y \in Y \\ max\{x, y\} & \text{otherwise.} \end{cases}$$

It is easy to prove that $(Y, M_\lambda, *)$ is a complete controlled fuzzy metric space on Y. We now prove for the existence of a solution of the LCR-circuit equation of the second order differential equation.

Theorem 5.1 *Consider the following integral equation*

$$P(t) = \int_0^I \zeta(t, r) G(t, r, P(r)) dr. \tag{5.4}$$

where

1. *There exist a continuous function $\zeta : [0, I] \times [0, I] \to \mathbb{R}^+$ such that*

$$sup_{t \in [0,I]} \int_0^I \zeta(t, r) dr \geq 1$$

2. *There exist a continuous function $G : [0, I] \times [0, I] \to \mathbb{R}^+$ such that*

$$|G(t, r, x(r)) - G(t, r, y(r))| \geq -log(N_\lambda |x(r) - y(r)|) \chi(M_\lambda(x, y, t)) t$$

   where

$$N_\lambda |x(r) - y(r)| = min\{(\alpha |x(r) - y(r)| + (1 - \alpha)|x(r) - Hx(r)|, \\ (\beta |x(r) - Hy(r)| + (1 - \beta) \frac{|x(r) - Hx(r)|, |x(r) - Hy(r)|}{|x(r) - y(r)|}\}$$

for all $\alpha, \beta \in [0, 1], \quad x, y \in Y, \quad \chi \in F_b$. Then, the integral equation (5.5) has a unique solution.

Proof: For $x, y \in Y$, by using of assumptions (1)–(2), we have

$$P(t) = \int_0^1 \zeta(t, r) G(t, r, P(r)) dr. \tag{5.5}$$

Consider the mapping $H : Y \to Y$ defined by

$$HP(t) = \int_0^1 \zeta(t, r) G(t, r, P(r)) dr. \tag{5.6}$$

for all $t, r \in [0, 1]$. Now,

$$\begin{aligned} M_\lambda(Hx, Hy, \chi(M_\lambda(x, y, t))t) &= sup_{t \in [0,I]} e^{-\frac{|Hx(t) - Hy(t)|}{\chi(M_\lambda(x,y,t))t}} \\ &\geq sup_{t \in [0,1]} e^{-\frac{|\int_0^I \zeta(t,r) G(t,r,x(r)) dr - \int_0^T \zeta(t,r) G(t,r,y(r)) dr|}{\chi(M_\lambda(x,y,t))t}} \\ &\geq sup_{t \in [0,1]} e^{-\frac{\int_0^I |\zeta(t,r)||(G(t,r,x(r)) - G(t,r,y(r)))| dr}{\chi(M_\lambda(x,y,t))t}} \\ &\geq sup_{t \in [0,1]} e^{\frac{\int_0^I |\zeta(t,r)| dr log(N_\lambda |x(r) - y(r)| \chi(M_\lambda(x,y,t))t}{\chi(M_\lambda(x,y,t))t}} \\ &\geq sup_{t \in [0,1]} e^{\frac{log(N_\lambda |x(r) - y(r)|) \chi(M_\lambda(x,y,t))t \int_0^1 \zeta(t,r) dr}{\chi(M_\lambda(x,y,t))t}} \\ &\geq sup_{t \in [0,1]} e^{\frac{log(N_\lambda |x(r) - y(r)|) \chi(M_\lambda(x,y,t))t}{\chi(M_\lambda(x,y,t))t}} \\ &\geq N_\lambda |x(r) - y(r)| \\ &= min\{(\alpha |x(r) - y(r)| + (1 - \alpha)|x(r) - Hx(r)|, \\ &\quad (\beta |x(r) - Hy(r)| + (1 - \beta) \frac{|x(r) - Hx(r)|, |x(r) - Hy(r)|}{|x(r) - y(r)|}\}) \end{aligned}$$

which yields

$$M_\lambda(Hx, Hy, \chi(M_\lambda(x,y,t))t) \geq N_\lambda(x,y,t), \tag{5.7}$$

where

$$N_\lambda(x,y,t) = min\{(\alpha M_\lambda(x,y,t) + (1-\alpha)M_\lambda(x,Sx,t)), (\beta M_\lambda(y,Hy,t) + (1-\beta)\frac{M_\lambda(x,Hx,t), M_\lambda(y,Hy,t)}{M_\lambda(x,y,t)})\}$$

for all $\alpha, \beta \in [0,1]$, $x, y \in Y$, $\chi \in F_b$. therefore, all the conditions of Theorem 3.1 are satisfied. As a result, the mapping H has a unique fixed point $x \in Y$, which is a solution of the differential equation arising in electric circuit equation. □

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