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Modeling and Analyzing a Complex Feedback Queue Network Model with Priority Having Bi-Tandem and Parallel Servers in Stochastic Environment

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ABSTRACT: The current study examines a complex queueing network model having three parallel subsystems. The first subsystem comprises of biserial service channels, while the second involves parallel service channels, which are linked in series with a third common subsystem. A distinctive feature of this model is the possibility of customer feedback, allowing customers to re-enter the system after service, based on a defined transition probability tied to their satisfaction. The system prioritizes the first and second subsystems, with customer arrivals occurring independently at the other subsystems. Both arrival and service processes are modeled using poisson distributions. The steady-state behavior is analyzed through differential difference equations utilizing generating function techniques. This paper investigate the impact of feedback-priority mechanism on the system's performance and a parametric analysis is conducted to further explore the model's applicability in real-world situations. The results, are presented both numerically and graphically, effectively validating the model's behavior under different settings.

Key Words: Feedback, priority, Bi-tandem, parallel server, stochastic environment, generating function techniques, average queue length, revisiting facility.

Contents

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1. Introduction

Queueing theory plays an important role in improving the efficiency of any service organization by optimizing service delivery through priority mechanisms and customer revisits. In literature, many researchers have developed different types of queueing models incorporating these features. Cobham [1] introduced the concept of preemptive priority queues and analyzed the results by taking different priority levels. This marked the beginning of significant research on priority queues, a key area in queueing theory. Maggu [2] further discussed the phase type service queueing problems, specifically focusing on biserial system. After that, Gupta, D. et al. [3,4] did a lot of work in the field of queueing theory. He analyzed various queue network models, particularly those that involved biserial and parallel channel linked with a common server. Later on, Sharma, S. et al. [5] extended this body of work by studying a queueing problem, which contains two parallel servers and offering their solution. Chun, Y. [6] discussed Queueing problems containing two parallel servers. Agrawal. S.K. et al. [7] took this further by developing a generalized queueing model and analyzed the behavior of the model in steady state. Gupta, R. et al. [8,9] continued this line of inquiry by focusing on the queueing model consisting of biserial and parallel severs and the customers arrival occurs in fixed- size batches. In more recent years, analysis of a feedback bi-serial queue network model within fuzzy environment was discussed by Saini, V. et al. [10]. A complex semi bi-tandem feedback queue network, whose probabilities are different in each visit was analyzed by Saini, V. et al. [11]. The priority bi-serial queue model was mathematically analyzed by Saini, A. et al. [12,13] and derived the queue performance metrics. Gbadebo et al. [14] investigated the role of priority queues in an optical job's admission control systems. Adeleke et al. [15] designed a treap -based Model with single server Queue network with Poisson arrivals, offering a novel approach to queueing analysis. Later on, Tamuli et al. [16] investigated the behavior of multi server heterogeneous system with variation in the customer behavior. In particular, the study by S., P. [17] investigated biserial servers connected with a main server using a fuzzy ordering approach, providing insights into performance evaluation under uncertainty. Similarly, Saini et al. [18] applied queueing theory to assess the performance metrics of manufacturing systems, demonstrating its effectiveness in real-world industrial applications. Later on, Dayana, D. et al. [19] explored a neutrosophic queueing model with a single server, which is of finite capacity , enhancing the range of applications for queueing theory. Preeti et al. [20,21] designed a model based on both feedback and priority mechanism and evaluated its behavior in stochastic environment. The present Paper discussed a sophisticated feedback queue network model with biserial and parallel service channels linked with a common service channel. At first subsystem, priority is applied whereas general arrival of the customers takes place at other subsystems. Our primary objective is to conduct a parametric analysis of the proposed model and assessing its performance by calculating queue performance metrics by using generating function techniques. Some Special cases are also explored at the end to validate the proposed model.

2. Practical Implications of the Model

The queueing network model has broad applicability in real world setting, such as administrative offices, banks, manufacturing, megamarts, and other similar contexts. Let's take an example of a bank where customers are served by three service channels in sequence according to priority basis, where the first and second service channels have two sections i.e. biserial and parallel service channels, and the third service channel is a common service counter. This type of model is used in queueing theory to optimize customer service, decrease wait times, and improves the overall efficiency of the system. Service channel 1 is divided into two sections naming: 1A and 1B, where section 1A perform transactions like withdrawls and deposits whereas section 1B handles loan services and financial related queries. Service channels 2 is also divided into 2 sections, which are parallel in nature naming: 2A and 2B. section 2A handles general services of bank like account opening, account transfers and inquiries whereas section 2B handles services like foreign exchange, applications for foreign exchange and high-value transactions. A customer arrives at the bank and is directed to either section 1A (deposits and withdrawals) or section 1B (loans and financial advice), depending on their need. Customers waiting in a queue within each section until it is their turn to be served. After finishing their transaction at server 1, customers proceed to Server 2. Depending on their requirement, customers are directed to section 2A (general banking services) or Section 2B (special services like foreign exchange, applications for credit card etc.). The parallel nature of sections 2A and 2B allows flexibility, as customers can switch between the two sections based on availability. Once customers have completed their tasks at servers 1 and 2, they proceed to server 3. This server serves as a common counter where customers complete their banking tasks, such as document verification, account inquiries, or general assistance. If a customers become unsuccessful in providing service then they take revisit atmost once to any of the service channels. After getting services to all service channels, the customers finally exit from the system.

3. Notations Used

Table 1: Terminology used in the model

Service channel	Z_{11}	Z_{12}	Z_{21}	Z_{22}	Z_3
Arrival rates	$\lambda_{1H}, \lambda_{1L}$	$\lambda_{2H}, \lambda_{2L}$	λ'_1	λ'_2	–
Service rates	μ_{1H}, μ_{1L}	μ_{2H}, μ_{2L}	μ'_1	μ'_2	μ_3
Formation of queues	Q_{1H}, Q_{1L}	Q_{2H}, Q_{2L}	Q_3	Q_4	Q_5
Customer moving probabilities in case of first visit	β_{12}, β_{15}	β_{21}, β_{25}	β_{35}	β_{45}	$\beta_5, \beta_{51}, \beta_{52}, \beta_{53}, \beta_{54}$
Customer moving probabilities in case of second visit	β'_{12}, β'_{15}	β'_{21}, β'_{25}	β'_{35}	β'_{45}	β'_5
Customer leaving probabilities in case of first visit	p	q	r	s	t
Customer leaving probabilities in case of second visit	p'	q'	r'	s'	t'

4. Description of Mathematical Problem

In the current mathematical model, there are three subsystems Z_1,Z_2 and Z_3 as shown in figure 1. The subsystem Z_1 consists of bi-serial service channels(i.e. a service subsystem that includes two parallel channels working together to process customers) Z_{11} and Z_{12} whereas Z_2 consists of parallel service channels Z_{21} and Z_{22}. The Parallel subsystem Z_1 and Z_2 are linked in series with a third subsystem Z_3 features a common server. Priority is applied at first subsystem Z_1 and feedback is allowed at every service channel to its previous service channel with a facility of revisit is also available for customer satisfaction with service. Intially, the customers enter subsystems Z_1 and Z_2 in ahead of service channels Z_{11}, Z_{12} and Z_{21}, Z_{22} with arrival rates λ_{iH},λ_{iL} (i=1,2) and λ'_1, λ'_2 and getting service with service rates μ_{iH}, μ_{iL}(i=1,2) and μ'_1, μ'_2 respectively. After receiving service in either subsystem Z_1 or Z_2, he/she will proceed to the subsystem Z_3 for further services. In case of unsuccessful service, the customers may revisit the service channels atmost once governed by a specified transition probability.

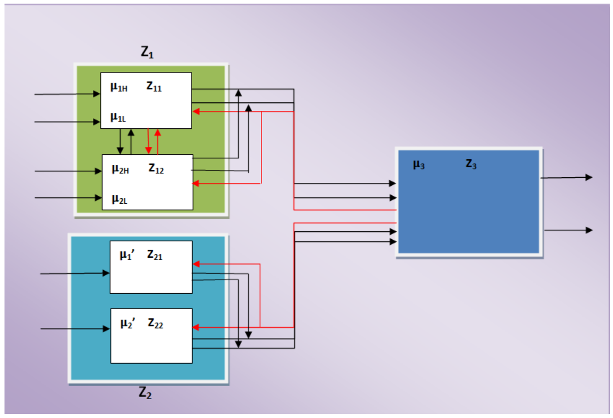

Figure 1: Description of the Proposed Mathematical Model

Justification for Parameter Selection

The arrival and service rate parameters (e.g., $\lambda_{1H}, \lambda_{1L}, \mu_{1H}, \mu_{1L}$) used in the numerical analysis were selected within the typical ranges reported in related queueing models [12,13]. These parameter choices correspond to a moderate traffic intensity ($\rho = \lambda/\mu < 1$), ensuring that the system remains stable and reaches a steady-state condition. Furthermore, a sensitivity analysis was performed over varying parameter combinations to verify the robustness and consistency of the obtained performance measures.

4.1. Possible States of leaving the service channels

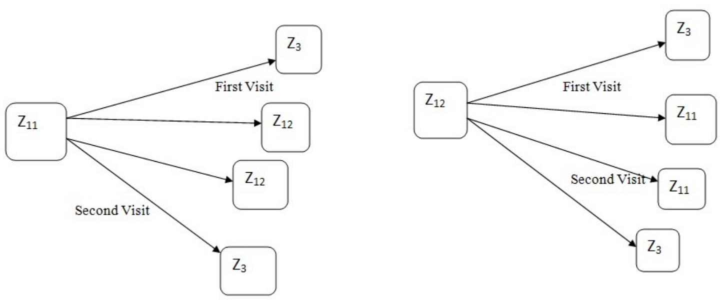

Figure 2: Possible states of leaving the service channels Z_{11} and Z_{12}

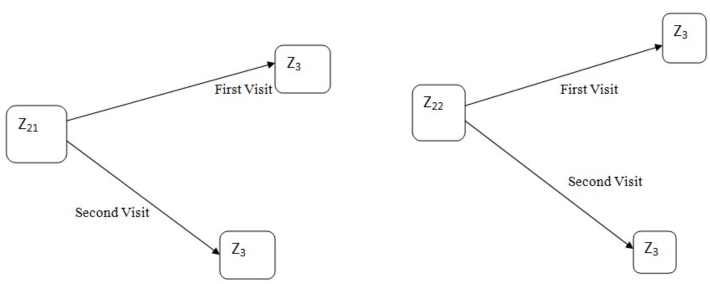

Figure 3: Possible states of leaving the service channels Z_{21} and Z_{22}

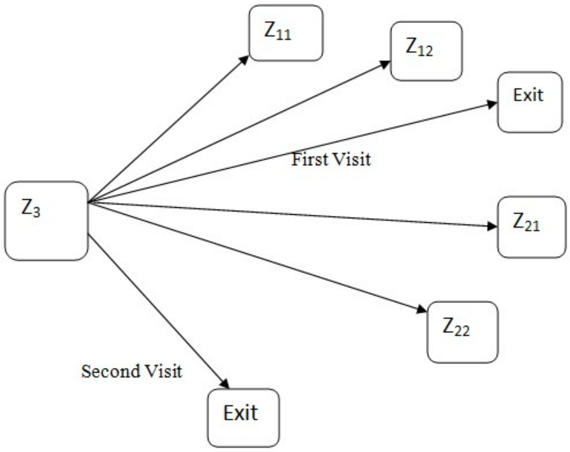

Figure 4: Possible states of leaving the service channels Z_3

After receiving service at Z_{11}, the customers may either leave the system or visit the service channels Z_{12} or Z_3 with probabilities p or β_{12} or β_{15} respectively, such that $\beta_{12} + \beta_{15} = 1$. In the event that the customers becomes unsuccessful in providing service have the option to revisit service channel Z_{11}, then after leaving the system with probability p', they may revisit either service channels Z_{12} or Z_3 with Probabilities β'_{12} or β'_{15} such that the equation $\beta'_{12} + \beta'_{15} = 1$ holds(as shown in figure 2). Moreover, the equation $p\beta_{12} + p\beta_{15} + p'\beta'_{12} + p'\beta'_{15} = 1$ must also satisfy.
At Service channel Z_{12}, the customers may either leave the system or go to service channels Z_{11} or Z_3 with probabilities q or β_{21} or β_{25} such that $\beta_{21} + \beta_{25} = 1$. If a revisit to the service channels occurs, then after leaving the system with probability q', they may revisit service channels Z_{11} or Z_3 with probability β'_{21} or β'_{25} and $\beta'_{21} + \beta'_{25} = 1$ must holds. However, the conditions $q\beta_{21} + q\beta_{25} + q'\beta'_{21} + q'\beta'_{25} = 1$ must true(as shown in figure 2).
At Service channel Z_{21} (as shown in figure 3), The customers will either exit the system or will move to the service channel Z_3 with probabilities r or β_{35}. In case of revisit probability is β'_{35} such that $r\beta_{35} + r'\beta'_{35} = 1$, Similarly, at service channel Z_{22} (as shown in figure 3), it follows the same structure with the equation $s\beta_{45} + s'\beta'_{45} = 1$ must satisfy. After getting service to Z_{22} (as shown in figure 4), the customers will proceed to service channel Z_3 . At this point, customers may either leave the system or move to one of the service channels Z_{11}, Z_{12}, Z_{21}, Z_{22} or exit from the sytem with probabilities t or β_{51}, β_{52},β_{53},β_{54} or β_5 such that $\beta_{51} + \beta_{52} + \beta_{53} + \beta_{54} + \beta_5 = 1$, and if revisit occurs, then the customers may either leave the system or exit from the system with probabilities t' or β'_5, then the equation $t\beta_{51} + t\beta_{52} + t\beta_{53} + t\beta_{54} + t\beta_5 + t'\beta'_5 = 1$ must also hold.

5. Analysis of Differential Difference Equations in Steady State

The Probability Function $P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H},Q_3,Q_4,Q_5}$ in which $Q_{1L}, Q_{1H}, Q_{2L}, Q_{2H}, Q_3, Q_4, Q_5$ are the numbers of unit waiting in lines in ahead of the service channels Z_{11},Z_{12}, Z_{21}, Z_{22}, Z_3 (as mention in figure 1) as given below:

$$
\begin{aligned}
&(\lambda_{1L} + \lambda_{1H} + \lambda_{2L} + \lambda_{2H} + \lambda'_1 + \lambda'_2 + \mu_{1H} + \mu_{2H} + \mu'_1 + \mu'_2 + \mu_3) \\
&P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H},Q_3,Q_4,Q_5} = \lambda_{1L} P_{Q_{1L}-1,Q_{1H},Q_{2L},Q_{2H},Q_3,Q_4,Q_5} \\
&+ \lambda_{1H} P_{Q_{1L},Q_{1H}-1,Q_{2L},Q_{2H},Q_3,Q_4,Q_5} + \lambda_{2L} P_{Q_{1L},Q_{1H},Q_{2L}-1,Q_{2H},Q_3,Q_4,Q_5} \\
&+ \lambda_{2H} P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H}-1,Q_3,Q_4,Q_5} + \lambda'_1 P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H},Q_3-1,Q_4,Q_5} \\
&+ \lambda'_2 P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H},Q_3,Q_4-1,Q_5} + \mu_{1H}(p\beta_{12} + p'\beta'_{12}) P_{Q_{1L},Q_{1H}+1,Q_{2L},Q_{2H}-1,Q_3,Q_4,Q_5} \\
&+ \mu_{1H}(p\beta_{15} + p'\beta'_{15}) P_{Q_{1L},Q_{1H}+1,Q_{2L},Q_{2H},Q_3,Q_4,Q_5-1} \\
&+ \mu_{2H}(q\beta_{21} + q'\beta'_{21}) P_{Q_{1L},Q_{1H}-1,Q_{2L},Q_{2H}+1,Q_3,Q_4,Q_5} + \mu_{2H}(q\beta_{25} + q'\beta'_{25}) \\
&P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H}+1,Q_3,Q_4,Q_5-1} + \mu'_1(r\beta_{35} + r'\beta'_{35}) P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H},Q_3+1,Q_4,Q_5-1} \\
&+ \mu'_2(s\beta_{45} + s'\beta'_{45}) P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H},Q_3,Q_4+1,Q_5-1} \\
&+ \mu_3(t\beta_5 + t'\beta'_5) P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H},Q_3,Q_4,Q_5+1} + \mu_3(t\beta_{51}) P_{Q_{1L},Q_{1H}-1,Q_{2L},Q_{2H},Q_3,Q_4,Q_5+1} \\
&+ \mu_3(t\beta_{52}) P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H}-1,Q_3,Q_4,Q_5+1} + \mu_3(t\beta_{53}) P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H},Q_3-1,Q_4,Q_5+1} \\
&+ \mu_3(t\beta_{54}) P_{Q_{1L},Q_{1H},Q_{2L},Q_{2H},Q_3,Q_4-1,Q_5+1}
\end{aligned}
\tag{5.1}
$$

6. Mathematical Solution of the Model

By taking all the necessary combinations of $Q_{1L}, Q_{1H}, Q_{2L}, Q_{2H}, Q_3, Q_4, Q_5$, total (5.128) equations has been obtained and the Generating function techniques is used to find the solution of these differential difference equations, then we get the steady state solution in reduced form given in equation (6.1):
Firstly, let us assume that

$$
\begin{aligned}
&p\beta_{12} + p'\beta'_{12} = A, p\beta_{15} + p'\beta'_{15} = B, q\beta_{21} + q'\beta'_{21} = C, \\
&q\beta_{25} + q'\beta'_{25} = D, r\beta_{35} + r'\beta'_{35} = G, s\beta_{45} + s'\beta'_{45} = I, \\
&t\beta_5 + t'\beta'_5 = J, t\beta_{51} = K, t\beta_{52} = L, t\beta_{53} = M, t\beta_{54} = N
\end{aligned}
$$

$$
\begin{aligned}
&H(T_1, T_2, T_3, T_4, T_5, T_6, T_7) \\
&= \frac{H_5(\mu_3(1 - \frac{J}{T_7} - \frac{KT_2}{T_7} - \frac{LT_4}{T_7} - \frac{MT_5}{T_7} - \frac{NT_6}{T_7})}{\alpha} \\
&+ \frac{H_4(\mu'_2(1 - \frac{IT_7}{T_6})) + H_3(\mu'_1(1 - \frac{GT_7}{T_5}))}{\alpha} \\
&+ \frac{H_2(\mu_{2H}(1 - \frac{CT_2}{T_4} - \frac{DT_7}{T_4}) - \mu_{2L}(1 - \frac{CT_1}{T_3} - \frac{DT_7}{T_3})))}{\alpha} \\
&+ \frac{H_7(\mu_{2L}(1 - \frac{CT_1}{T_3} - \frac{DT_7}{T_3})) + H_6(\mu_{1L}(1 - \frac{AT_3}{T_1} - \frac{BT_7}{T_1}))}{\alpha} \\
&+ \frac{H_1(\mu_{1H}(1 - \frac{AT_4}{T_2} - \frac{BT_7}{T_2}) - \mu_{1L}(1 - \frac{AT_3}{T_1} - \frac{BT_7}{T_1}))}{\alpha}
\end{aligned}
\tag{6.1}
$$

where

$$
\begin{aligned}
&\alpha = \lambda_{1L}(1 - T_1) + \lambda_{1H}(1 - T_2) + \lambda_{2L}(1 - T_3) + \lambda_{2H}(1 - T_4) \\
&+ \lambda'_1(1 - T_5)) + \lambda'_2(1 - T_6) + \mu_{1H}(1 - \frac{AT_4}{T_2} - \frac{BT_7}{T_2}) \\
&+ \mu_{2H}(1 - \frac{CT_2}{T_4} - \frac{DT_7}{T_4}) + \mu'_1(1 - \frac{GT_7}{T_5}) + \mu'_2(1 - \frac{IT_7}{T_6}) \\
&+ \mu_3(1 - \frac{J}{T_7} - \frac{KT_2}{T_7} - \frac{LT_4}{T_7} - \frac{MT_5}{T_7} - \frac{NT_6}{T_7})
\end{aligned}
$$

and

$$H_1 = H_0(T_1,T_3,T_4,T_5,T_6,T_7), H_2 = H_0(T_1,T_2,T_3,T_5,T_6,T_7),$$
$$H_3 = H_0(T_1,T_2,T_3,T_4,T_6,T_7), H_4 = H_0(T_1,T_2,T_3,T_4,T_5,T_7),$$
$$H_5 = H_0(T_1,T_2,T_3,T_4,T_5,T_6), H_6 = H_{0,0}(T_3,T_4,T_5,T_6,T_7),$$
$$H_7 = H_{0,0}(T_1,T_2,T_5,T_6,T_7)$$

For

$$|T_1|+|T_2|+|T_3|+|T_4|+|T_5|+|T_6|+|T_7| = 1, \quad H(1,1,1,1,1,1,1) = 1 \tag{6.2}$$

Using equation (6.2) in (6.1), we get the indeterminate form $\frac{0}{0}$ and L'Hospital rule for limits is applied to resolve this form. By Differentating both numerator and denominator using this rule w.r.t relevant variables, the results are obtained as outlined below:

$$\mu_{2L}C(H_2 - H_7) + \mu_{1L}(H_6 - H_1) = -\lambda_{1L} \tag{6.3}$$

$$\mu_{1H}C(H_1 - 1) + \mu_{2H}C(1 - H_2) + \mu_3 K(1 - H_5) = -\lambda_{1H} \tag{6.4}$$

$$\mu_{2L}(H_7 - H_2) + \mu_{1L}A(H_1 - H_6) = -\lambda_{2L} \tag{6.5}$$

$$\mu_{1H}A(1 - H_1) + \mu_{2H}(H_2 - 1) + \mu_3 L(1 - H_5) = -\lambda_{2H} \tag{6.6}$$

$$\mu_3 M(1 - H_5) + \mu_1'(H_3 - 1) = -\lambda_1' \tag{6.7}$$

$$\mu_3 N(1 - H_5) + \mu_2'(H_4 - 1) = -\lambda_2' \tag{6.8}$$

$$\begin{aligned}&\mu_3(H_5 - 1) + \mu_2' I(1 - H_4) + \mu_1' G(1 - H_3) + \mu_{2H}D(1 - H_2)\\&+ \mu_{2L}D(H_2 - H_7) + \mu_{1L}B(H_1 - H_6) + \mu_{1H}B(1 - H_1) = 0\end{aligned} \tag{6.9}$$

Solving above equations for the values of $H_1, H_2, H_3, H_4, H_5, H_6, H_7$ and the following results are obtained:

$$\begin{aligned}H_1 =& 1 - \frac{1}{\mu_{1H}}[\lambda_{1H}(1 - NI - MG - LD) + \lambda_{2H}(C - CNI -\\&CMG + KD) + \lambda_2' I(LC + K) + \lambda_1' G(LC + K)\\&+ \frac{(LC + K)}{(1 - AC)}(\lambda_{1L}(AD + B) + \lambda_{2L}(D + BC))]\end{aligned} \tag{6.10}$$

$$\begin{aligned}H_2 =& 1 - \frac{1}{\mu_{2H}(LC + K)}[\lambda_{1H}((L + AK)(1 - NI - MG\\&- LD) - L) + \lambda_{2H}((L + AK)(C - CNI - CMG\\&+ KD) + K) + \lambda_2' I(LC + K) + \lambda_1' G(LC + K)\\&+ \frac{(LC + K)(L + AK)}{(1 - AC)}(\lambda_{1L}(AD + B) + \lambda_{2L}(D + BC))]\end{aligned} \tag{6.11}$$

$$\begin{aligned}H_3 =& 1 - \frac{1}{\mu_1'}[\lambda_1' + \frac{M}{(LC + K)}[\lambda_{1H}((1 - AC)(1 - NI - MG\\&- LD) - 1) + \lambda_{2H}((1 - AC)(C - CNI - CMG\\&+ KD) - C) + \lambda_2' I(LC + K) + \lambda_1' G(LC + K)\\&+ (LC + K)(\lambda_{1L}(AD + B) + \lambda_{2L}(D + BC))]\end{aligned} \tag{6.12}$$

$$\begin{aligned} H_4 =& 1-\frac{1}{\mu_2'}[\lambda_2' + \frac{N}{(LC+K)}[\lambda_{1H}((1-AC)(1-NI-MG \\ & -LD)-1)+\lambda_{2H}((1-AC)(C-CNI-CMG+KD) \\ & -C)+\lambda_2' I(LC+K)+\lambda_1' G(LC+K)+(LC+K) \\ & (\lambda_{1L}(AD+B)+\lambda_{2L}(D+BC))] \end{aligned} \tag{6.13}$$

$$\begin{aligned} H_5 =& 1-\frac{1}{\mu_3(LC+K)}[\lambda_{1H}((1-AC)(1-NI-MG-LD)-1) \\ & +\lambda_{2H}((1-AC)(C-CNI-CMG+KD)-C) \\ & +\lambda_2' I(LC+K)+\lambda_1' G(LC+K) \\ & +(LC+K)(\lambda_{1L}(AD+B)+\lambda_{2L}(D+BC))] \end{aligned} \tag{6.14}$$

$$\begin{aligned} H_6 =& 1-[\frac{(\lambda_{1L}+\lambda_{2L}C)}{\mu_{1L}(1-AC)}+\frac{1}{\mu_{1H}}[\lambda_{1H}(1-NI-MG-LD) \\ & +\lambda_{2H}(C-CNI-CMG+KD) \\ & +\lambda_2' I(LC+K)+\lambda_1' G(LC+K) \\ & +\frac{(LC+K)}{(1-AC)}(\lambda_{1L}(AD+B)+\lambda_{2L}(D+BC)))] \end{aligned} \tag{6.15}$$

$$\begin{aligned} H_7 =& 1-[\frac{(\lambda_{1L}A+\lambda_{2L})}{\mu_{2L}(1-AC)}+\frac{1}{\mu_{2H}(LC+K)}[\lambda_{1H}((L+AK) \\ & (1-NI-MG-LD)-L)+\lambda_{2H}((L+AK) \\ & (C-CNI-CMG+KD)+K)+\lambda_2' I(LC+K) \\ & +\lambda_1' G(LC+K)+\frac{(LC+K)(L+AK)}{(1-AC)}(\lambda_{1L}(AD+B) \\ & +\lambda_{2L}(D+BC))])] \end{aligned} \tag{6.16}$$

The value of Utilization factors at different service channels are outlined as below:

$$\begin{aligned} \gamma_1 =& \frac{1}{\mu_{1H}}[\lambda_{1H}(1-NI-MG-LD)+\lambda_{2H}(C-CNI-CMG \\ & +KD)+\lambda_2' I(LC+K)+\lambda_1' G(LC+K) \\ & +\frac{(LC+K)}{(1-AC)}(\lambda_{1L}(AD+B)+\lambda_{2L}(D+BC))] \end{aligned} \tag{6.17}$$

$$\begin{aligned} \gamma_2 =& \frac{1}{\mu_{2H}(LC+K)}[\lambda_{1H}((L+AK)(1-NI-MG-LD)-L) \\ & +\lambda_{2H}((L+AK)(C-CNI-CMG+KD)+K) \\ & +\lambda_2' I(LC+K)+\lambda_1' G(LC+K)+\frac{(LC+K)(L+AK)}{(1-AC)} \\ & (\lambda_{1L}(AD+B)+\lambda_{2L}(D+BC))] \end{aligned} \tag{6.18}$$

$$\begin{aligned}\gamma_3 =&\frac{1}{\mu_1'}[\lambda_1' + \frac{M}{(LC+K)}[\lambda_{1H}((1-AC)(1-NI-MG-LD)-1)\\&+\lambda_{2H}((1-AC)(C-CNI-CMG+KD)-C)+\lambda_2' I(LC+K)\\&+\lambda_1' G(LC+K)+(LC+K)(\lambda_{1L}(AD+B)+\lambda_{2L}(D+BC))]\end{aligned} \tag{6.19}$$

$$\begin{aligned}\gamma_4 =&\frac{1}{\mu_2'}[\lambda_2' + \frac{N}{(LC+K)}[\lambda_{1H}((1-AC)(1-NI-MG-LD)-1)\\&+\lambda_{2H}((1-AC)(C-CNI-CMG+KD)-C)+\lambda_2' I(LC+K)\\&+\lambda_1' G(LC+K)+(LC+K)(\lambda_{1L}(AD+B)+\lambda_{2L}(D+BC))]\end{aligned} \tag{6.20}$$

$$\begin{aligned}\gamma_5 =&\frac{1}{\mu_3(LC+K)}[\lambda_{1H}((1-AC)(1-NI-MG-LD)-1)\\&+\lambda_{2H}((1-AC)(C-CNI-CMG+KD)-C)\\&+\lambda_2' I(LC+K)+\lambda_1' G(LC+K)\\&+(LC+K)(\lambda_{1L}(AD+B)+\lambda_{2L}(D+BC))]\end{aligned} \tag{6.21}$$

$$\begin{aligned}\gamma_6 =&[\frac{(\lambda_{1L}+\lambda_{2L}C)}{\mu_{1L}(1-AC)}+\frac{1}{\mu_{1H}}[\lambda_{1H}(1-NI-MG-LD)+\lambda_{2H}(C\\&-CNI-CMG+KD)\\&+\lambda_2' I(LC+K)+\lambda_1' G(LC+K)+\frac{(LC+K)}{(1-AC)}(\lambda_{1L}\\&(AD+B)+\lambda_{2L}(D+BC)))]\end{aligned} \tag{6.22}$$

$$\begin{aligned}\gamma_7 =&[\frac{(\lambda_{1L}A+\lambda_{2L})}{\mu_{2L}(1-AC)}+\frac{1}{\mu_{2H}(LC+K)}[\lambda_{1H}((L+AK)(1-NI\\&-MG-LD)-L)+\lambda_{2H}((L+AK)(C-CNI-CMG\\&+KD)+K)+\lambda_2' I(LC+K)+\lambda_1' G(LC+K)\\&+\frac{(LC+K)(L+AK)}{(1-AC)}(\lambda_{1L}(AD+B)+\lambda_{2L}(D+BC))])]\end{aligned} \tag{6.23}$$

The solution in steady state can be exit only when the conditions $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7 < 1$ must be satisfied.

7. Queue Performance Indicator

- Average Queue Length

$$L = L_{Q_{1\mathrm{H}}} + L_{Q_{1\mathrm{L}}} + L_{Q_{2\mathrm{H}}} + L_{Q_{2\mathrm{L}}} + L_{Q_3} + L_{Q_4} + L_{Q_5}$$

Where

$$\begin{aligned}&L_{Q_{1\mathrm{H}}} = \frac{\gamma_1}{(1-\gamma_1)}, L_{Q_{1\mathrm{L}}} = \frac{\gamma_6}{(1-\gamma_6)}, L_{Q_{2\mathrm{H}}} = \frac{\gamma_2}{(1-\gamma_2)},\\&L_{Q_{2\mathrm{L}}} = \frac{\gamma_7}{(1-\gamma_7)}, L_{Q_3} = \frac{\gamma_3}{(1-\gamma_3)}, L_{Q_4} = \frac{\gamma_4}{(1-\gamma_4)},\\&L_{Q_5} = \frac{\gamma_5}{(1-\gamma_5)}\end{aligned}$$

are the partial and average queue length of the system.

8. Parametric Study Through Numerical and Graphical Representation of the Model

Arrival and Service rates:

$$\lambda_{1H} = 5, \lambda_{1L} = 7, \lambda_{2H} = 6, \lambda_{2L} = 6, \lambda'_1 = 4, \lambda'_2 = 5, \mu_{1H} = 50,$$
$$\mu_{1L} = 56, \mu_{2H} = 48, \mu_{2L} = 43, \mu'_1 = 70, \mu'_2 = 75, \mu_3 = 89.$$

First Visit Probabilities:

$$\beta_{12} = 0.6, \beta_{15} = 0.4, \beta_{21} = 0.3, \beta_{25} = 0.7, \beta_{35} = 1, \beta_{45} = 1, \beta_5 = 0.2,$$
$$\beta_{51} = 0.1, \beta_{52} = 0.3, \beta_{53} = 0.1, \beta_{54} = 0.3, p = 0.6, q = 0.3, r = 0.2,$$
$$s = 0.1, t = 0.5.$$

Second Visit Probabilities:

$$\beta'_{12} = 0.4, \beta'_{15} = 0.6, \beta'_{21} = 0.2, \beta'_{25} = 0.8, \beta'_{35} = 1, \beta'_{45} = 1, p' = 0.4,$$
$$q' = 0.7, r' = 0.8, s' = 0.9, t' = 0.5.$$

Table 2: Partial and average queue lengths for various values of arrival rate λ_{1H} at Z_{11}.

λ_{1H}	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total Queue Length
1.0	0.0841	0.4248	0.8192	0.8162	0.4339	0.3290	1.2361	4.1432
1.8	0.0971	0.4747	1.1359	1.1074	0.5790	0.3486	1.3615	5.1043
2.8	0.1138	0.5423	1.7300	1.6355	0.8077	0.3740	1.5397	6.7430
3.5	0.1259	0.5934	2.3900	2.1962	1.0117	0.3923	1.6812	8.3907
4.3	0.1399	0.6561	3.6843	3.2230	1.3096	0.4139	1.8637	11.2904
5.0	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
6.0	0.1710	0.8072	23.8275	12.3073	2.3698	0.4620	2.3478	42.2925

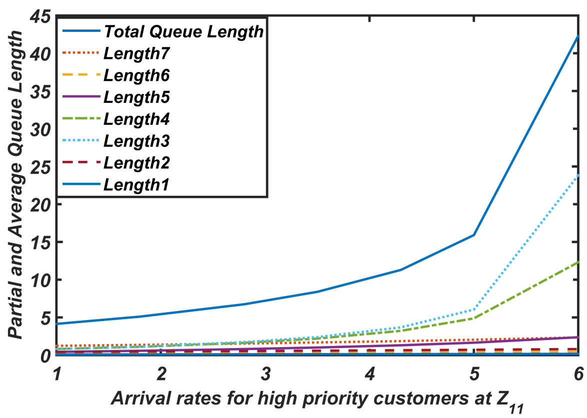

Figure 5: Variation of queue lengths with arrival rate λ_{1H}.

Table 3: Partial and average queue lengths for various values of arrival rate λ_{1L} at Z_{11}.

λ_{1L}	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total Queue Length
1.0	0.1392	0.6528	3.5951	3.1551	1.2923	0.2055	1.3104	10.3504
3.5	0.1447	0.6782	4.3711	3.7321	1.4301	0.2910	1.5686	12.2158
7.0	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
9.0	0.1570	0.7370	7.5466	5.8139	1.8003	0.5296	2.4060	18.9903
13.5	0.1673	0.7882	15.5539	9.6456	2.1991	0.8022	3.6449	32.8012
15.0	0.1708	0.8060	23.0714	12.1014	2.3585	0.9160	4.2858	43.7099
18.0	0.1777	0.8426	261.3549	23.3239	2.7303	1.1930	6.3003	295.9228

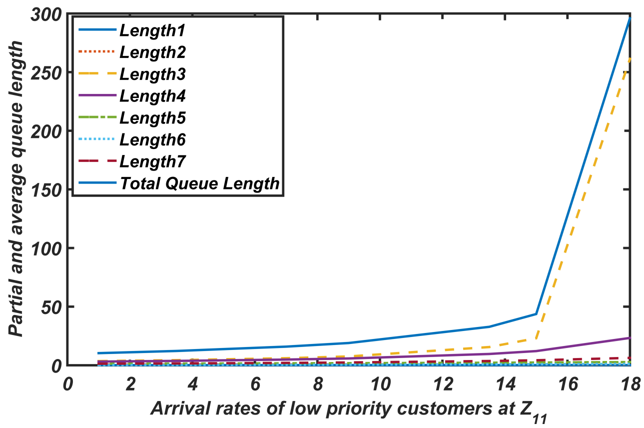

Figure 6: λ_{1L} vs Queue Lengths

Table 4: Partial and average queue lengths for various values of arrival rate λ_{2H} at Z_{12}.

λ_{2H}	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total Queue Length
1.0	0.1237	0.5840	2.2494	2.0790	0.9719	0.3890	1.6548	8.0517
4.0	0.1408	0.6601	3.7986	3.3093	1.3311	0.4152	1.8759	11.5311
6.0	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
7.5	0.1614	0.7588	9.8127	7.0728	1.9602	0.4471	2.1855	22.3985
8.3	0.1663	0.7830	14.1537	9.0862	2.1548	0.4546	2.2658	29.0645
9.0	0.1705	0.8048	22.3600	11.9018	2.3473	0.4612	2.3395	40.3852
10.0	0.1767	0.8368	102.1897	20.4597	2.6671	0.4708	2.4508	129.2515

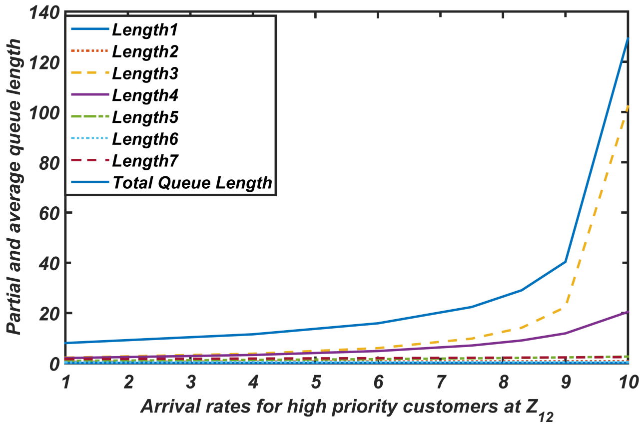

Figure 7: Variation of queue lengths with arrival rate λ_{2H}.

Table 5: Partial and average queue lengths for different values of arrival rate λ_{2L} at Z_{12}.

λ_{2L}	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total Queue Length
2.0	0.1436	0.6730	4.1956	3.6042	1.4012	0.3829	1.2285	11.6291
4.2	0.1485	0.6959	5.0680	4.2256	1.5336	0.4102	1.6140	13.6958
6.0	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
7.0	0.1548	0.7260	6.7171	5.3092	1.7248	0.4465	2.3520	17.4303
10.0	0.1616	0.7594	9.8871	7.1115	1.9646	0.4875	3.8058	24.1774
14.5	0.1719	0.8120	27.3652	13.1928	2.4153	0.5536	12.7528	57.2634
16.0	0.1754	0.8302	60.0174	17.9213	2.5976	0.5769	35.2450	117.3638

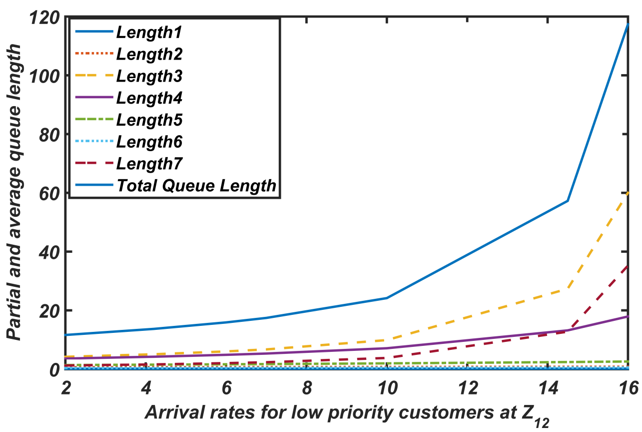

Figure 8: Variation of queue lengths with arrival rate λ_{2L}.

Table 6: Partial and average queue lengths for different values of arrival rate λ_1' at Z_{21}.

λ_1'	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total Queue Length
1.5	0.1469	0.5745	3.6819	3.9123	1.4693	0.4247	1.6282	11.8377
2.5	0.1492	0.6279	4.4049	4.2566	1.5397	0.4281	1.7804	13.1868
3.0	0.1503	0.6560	4.8571	4.4475	1.5765	0.4298	1.8634	13.9806
4.0	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
5.5	0.1559	0.8123	9.0702	5.6561	1.7775	0.4385	2.3652	20.2757
7.0	0.1593	0.9211	16.7161	6.6783	1.9139	0.4438	2.7607	29.5931
8.0	0.1616	1.0011	34.8751	7.5540	2.0126	0.4473	3.0804	49.1321

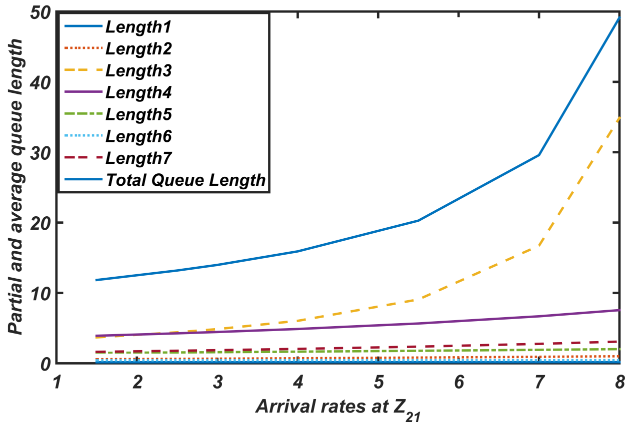

Figure 9: Variation of queue lengths with arrival rate λ_1'.

Table 7: Partial and average queue lengths for various values of arrival rate λ_2' at Z_{22}.

λ_2'	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total Queue Length
2.0	0.1458	0.5491	4.4049	2.9962	1.4355	0.4229	1.5582	11.5126
3.5	0.1492	0.6279	5.1129	3.7565	1.5397	0.4281	1.7804	13.3947
5.0	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
6.0	0.1548	0.7787	6.8202	5.9652	1.7348	0.4368	2.2513	18.1417
8.5	0.1604	0.9603	9.8507	12.0033	1.9624	0.4455	2.9140	28.2966
9.0	0.1616	1.0011	10.7623	14.7306	2.0126	0.4473	3.0804	32.1959
11.0	0.1661	1.1832	16.7161	96.6839	2.2313	0.4544	3.9162	121.3513

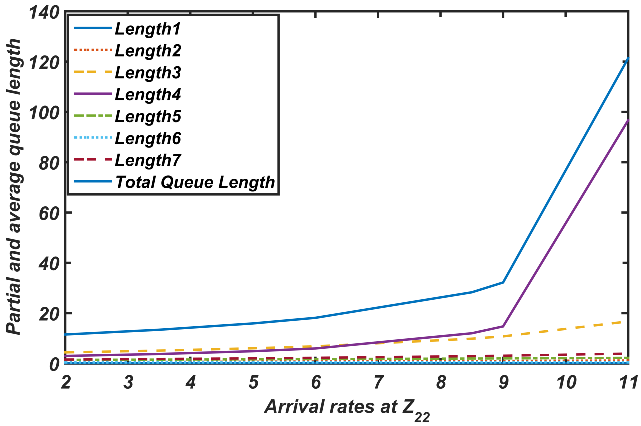

Figure 10: Variation of queue lengths with arrival rate λ_2'.

Table 8: Partial and average queue lengths for various service rates μ_{1H} of high priority customers at Z_{11}.

μ_{1H}	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total queue length
10	1.9555	0.7151	6.0343	4.8742	1.6533	4.9390	2.0450	22.2164
25	0.3599	0.7151	6.0343	4.8742	1.6533	0.7688	2.0450	16.4506
34	0.2416	0.7151	6.0343	4.8742	1.6533	0.5737	2.0450	16.1373
45	0.1724	0.7151	6.0343	4.8742	1.6533	0.4641	2.0450	15.9584
50	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
70	0.1044	0.7151	6.0343	4.8742	1.6533	0.3596	2.0450	15.7859
89	0.0803	0.7151	6.0343	4.8742	1.6533	0.3233	2.0450	15.7255

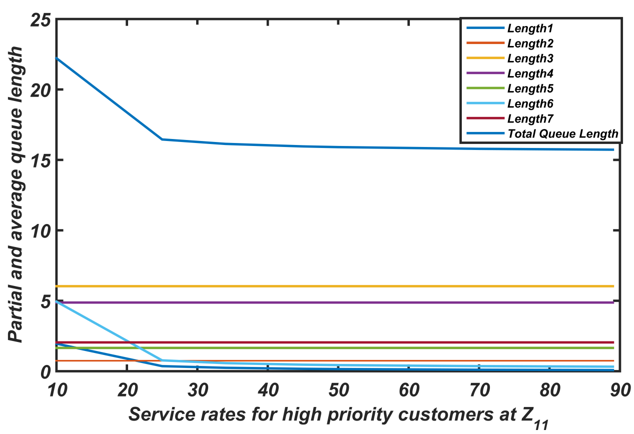

Figure 11: Service rate μ_{1H} versus queue lengths.

Table 9: Partial and average queue lengths for various service rates μ_{1L} of low priority customers at Z_{11}.

μ_{1L}	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total queue length
20	0.1525	0.7151	6.0343	4.8742	1.6533	1.5526	2.0450	17.0271
35	0.1525	0.7151	6.0343	4.8742	1.6533	0.6787	2.0450	16.1531
40	0.1525	0.7151	6.0343	4.8742	1.6533	0.5880	2.0450	16.0625
56	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
72	0.1525	0.7151	6.0343	4.8742	1.6533	0.3597	2.0450	15.8341
85	0.1525	0.7151	6.0343	4.8742	1.6533	0.3233	2.0450	15.7977
99	0.1525	0.7151	6.0343	4.8742	1.6533	0.2961	2.0450	15.7706

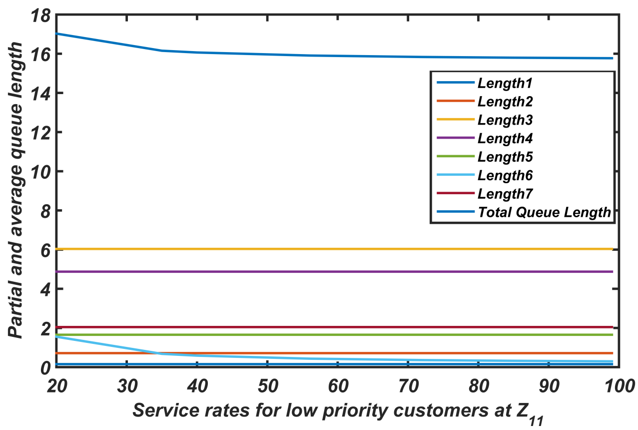

Figure 12: Service rate μ_{1L} versus queue lengths.

Table 10: Partial and average queue lengths for various service rates μ_{2H} of high priority customers at Z_{12}.

μ_{2H}	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total queue length
30	0.1525	2.0041	6.0343	4.8742	1.6533	0.4333	11.7818	26.9335
35	0.1525	1.3355	6.0343	4.8742	1.6533	0.4333	4.7624	19.2454
48	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
57	0.1525	0.5411	6.0343	4.8742	1.6533	0.4333	1.5365	15.2252
69	0.1525	0.4086	6.0343	4.8742	1.6533	0.4333	1.1963	14.7525
75	0.1525	0.3640	6.0343	4.8742	1.6533	0.4333	1.0898	14.6014
86	0.1525	0.3033	6.0343	4.8742	1.6533	0.4333	0.9507	14.4016

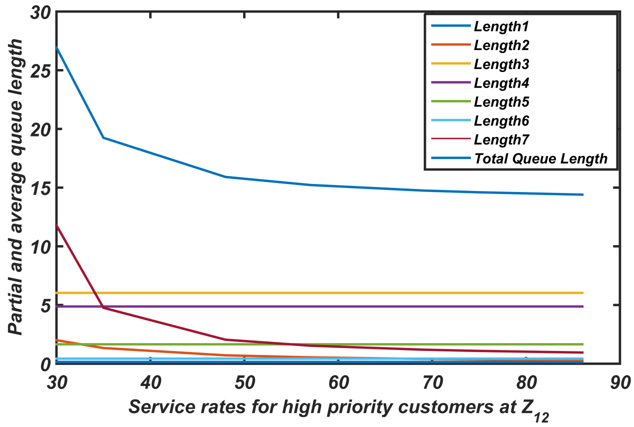

Figure 13: Relationship between service rate μ_{2H} and queue lengths.

Table 11: Partial and average queue lengths for various service rates μ_{2L} of low priority customers at Z_{12}.

μ_{2L}	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total queue length
27	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	4.6335	18.4962
36	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.5856	16.4483
43	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
58	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	1.5364	15.3991
71	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	1.3319	15.1946
81	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	1.2328	15.0955
95	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	1.1377	15.0004

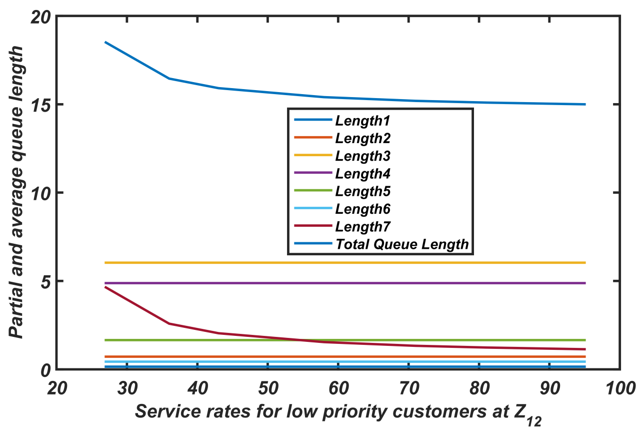

Figure 14: Effect of service rate μ_{2L} on queue lengths.

Table 12: Partial and average queue lengths for various service rates μ_1' at Z_{21}.

μ_1'	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total queue length
65	0.1525	0.7151	12.1281	4.8742	1.6533	0.4333	2.0450	22.0015
70	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
76	0.1525	0.7151	3.7645	4.8742	1.6533	0.4333	2.0450	13.6379
85	0.1525	0.7151	2.4066	4.8742	1.6533	0.4333	2.0450	12.2800
94	0.1525	0.7151	1.7687	4.8742	1.6533	0.4333	2.0450	11.6421
96	0.1525	0.7151	1.6703	4.8742	1.6533	0.4333	2.0450	11.5437
100	0.1525	0.7151	1.5031	4.8742	1.6533	0.4333	2.0450	11.3764

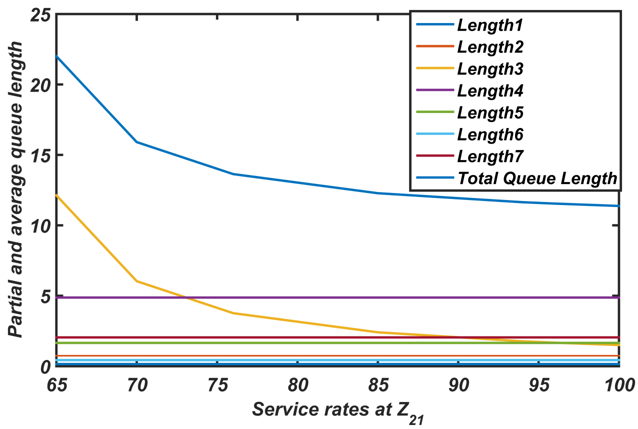

Figure 15: Effect of service rate μ_1' on queue lengths.

Table 13: Partial and average queue lengths for various service rates μ_2' at Z_{22}.

μ_2'	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total queue length
65	0.1525	0.7151	6.0343	22.4845	1.6533	0.4333	2.0450	33.5181
69	0.1525	0.7151	6.0343	9.1954	1.6533	0.4333	2.0450	20.2289
75	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
86	0.1525	0.7151	6.0343	2.6183	1.6533	0.4333	2.0450	13.6519
89	0.1525	0.7151	6.0343	2.3249	1.6533	0.4333	2.0450	13.3584
93	0.1525	0.7151	6.0343	2.0226	1.6533	0.4333	2.0450	13.0562
99	0.1525	0.7151	6.0343	1.6926	1.6533	0.4333	2.0450	12.7261

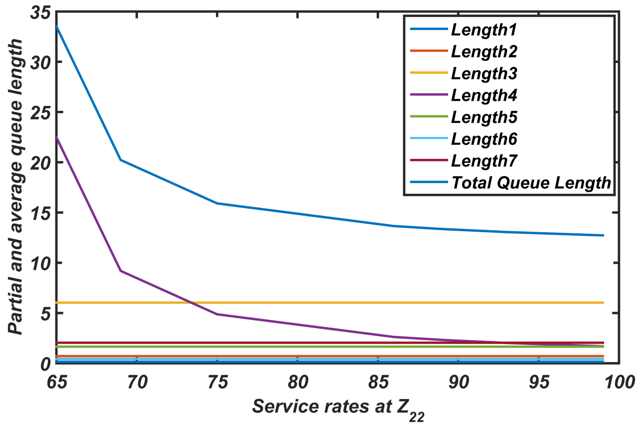

Figure 16: Effect of service rate μ_2' on queue lengths.

Table 14: Partial and average queue lengths for various service rates μ_3 at Z_3.

μ_3	Length 1	Length 2	Length 3	Length 4	Length 5	Length 6	Length 7	Total Queue Length
60	0.1525	0.7151	6.0343	4.8742	12.2073	0.4333	2.0450	26.4617
67	0.1525	0.7151	6.0343	4.8742	4.8044	0.4333	2.0450	19.0588
77	0.1525	0.7151	6.0343	4.8742	2.5743	0.4333	2.0450	16.8287
89	0.1525	0.7151	6.0343	4.8742	1.6533	0.4333	2.0450	15.9077
94	0.1525	0.7151	6.0343	4.8742	1.4388	0.4333	2.0450	15.6932
97	0.1525	0.7151	6.0343	4.8742	1.3349	0.4333	2.0450	15.5893
99	0.1525	0.7151	6.0343	4.8742	1.2736	0.4333	2.0450	15.5280

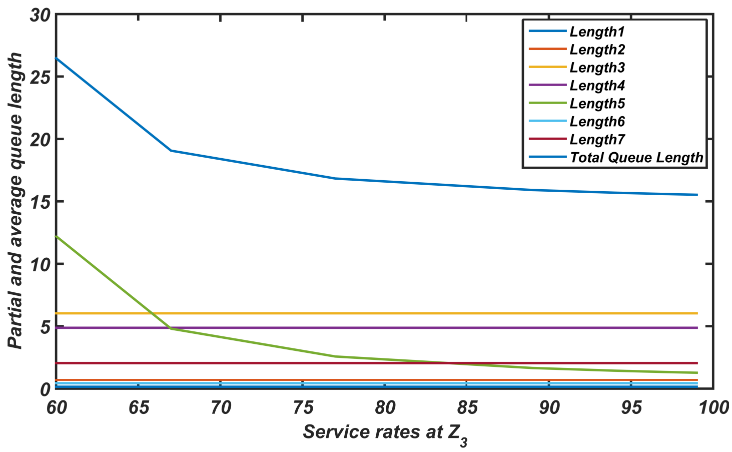

Figure 17: Effect of service rate μ_3 on queue lengths.

9. Results and Discussion

In the current queueing model, there are three subsystems. The first subsystem consists of bi-serial service channels, while the second subsystem contains parallel service channels, which are linked in series with a third subsystem features a shared service channel. In section 2, Practical application of the model has been discussed. The terminology used in this model represented by different symbols has been discussed in section 3. A thorough discussion of this model, accompanied by its graphical representation is presented in section 4. In section 5, the emergence of several mathematical steady state equations is obtained which have been used to calculate the average queues length which is the key characteristics of this model. The following interpretation has been made from above Numerical and Graphical analysis of the model:

- From Table 2 and Figure 5 provides a relationship between the mean arrival rate and both the partial and average queue length using exactly the same parameters as presented in Table 2. It is evident that as the arrival rate of high priority customers increases, so do the partial and average line length of the customers. A notable increase in the average queue length occurs when $\lambda_{1H} = 6, \lambda_{1L} = 18, \lambda_{2H} = 10, \lambda_{2L} = 16, \lambda_1' = 8, \lambda_2' = 11$. This is consistent with empirical facts, as an increase in number of individuals at a particular server leads to a larger queue length. A similar conclusion can be establish in Tables 3-7 and Figures 6-10.

- From Table 8 and Figure 11 shows the variations in the average queue length with change in the mean service rate assuming all the input parameters remains consistent with those in Table 8. The result demonstrates that with an increase in service rate , then the corresponding average and partial queues length decreases. The mean service rate is plotted against both average and partial queue length. A significant reduction can be seen in the average and partial queue length with increase in the mean service rates. A sharp decrease can be seen in average queue length when $\beta_{1H} = 25, \beta_{2H} = 35, \beta_1' = 70, \beta_2' = 69, \beta_3 = 67$ and minimal changes in Queue length occurs for low priority customers. This aligns with both theoretical and practical observations, as an increase in the service rate, the customers at various servers will be processed more quickly or fastly, thereby decreasing the queue length. This trend can be further confirmed in Tables 9-14 and Figures 12-17.

10. Particular Cases

- If we removed both priority and feedback mechanisms, then the resulting model matches with [3]

- if we removed the concept of priority, then the resulting model aligns with the one proposed by [11].

- if we removed the concept of feedback in this model, then the system aligns with the model given by [13].

11. Concluding Remarks

In this article, a complex feedback queue network model with bi-tandem and parallel service channels has been analyzed in a stochastic environment and and offers significant insights into the dynamic behavior of such systems.The model incorporates feedback and priority mechanisms, along with the facility of revisit at most once is available for the customers so that satisfaction with service increases. In comparison with systems employing only priority discipline, incorporating feedback improves the system's throughput performance but can result in increased queue lengths when the feedback intensity is high. The inclusion of priority and feedback mechanisms presents a more realistic network for addressing real world problems in the fields such as telecommunications, health care centers and service systems. From results, it is clearly indicate that as the arrival rate increases, the corresponding queue line length also increases while a higher service rates leads to a decrease in queue length. The model can be extended to incorporate multiple service channels, that are arranged in either biserial or parallel configurations, to increase applicability in real world systems.

12. Future Scope

- The model could be explained within a fuzzy environment.

- The phenomenon of balking and reneging could be considered in this model for more comprehensive analysis.

- The model can be further studied by considering three parallel server inspite of two.

13. Compliance with Ethical Standards

- There were no conflicts of interest in this Paper.
- The authors have no relevant financial or non-financial interests to disclose.

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