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Antisymmetric Mechanical Lorentz Force Tensor Constitutes Mechanical Electrodynamics

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ABSTRACT: Mechanical Lorentz force $[F + (P \times \omega)]$ predicts a complete framework of classical dynamics in terms of an antisymmetric second rank tensor $f^{\mu\nu}$ whose components are Newtonian force F and Coriolis force $(P \times \omega)$ that behaves like standard electrodynamics. We call this tensor the Lorentz force tensor (LFT). It consists of mechanical force equations (MFE), mechanical Maxwell's equations (MME), mechanical conservation law (MCL), and mechanical wave equations (MWE). Calculations are performed by Einstein's summation method and written in matrix form for simplicity. The transformation of LFT laws in a noninertial coordinate metric, based on a single transformation law (STL) for 4-vectors and tensors, predicts new symmetry of MFE, MME, and MCL along the diagonals as the zero-point structure of LFT. Zero-point terms validate the conservation law, symmetry, and principle of relativity. It reduces to its classical limit when noninertial effects are subjected to zero. The emergence of a 7D wave of mechanical energy supplemented with the classical conservation law is the most astonishing product of this model. The spectrum of application of this model is very wide and includes mechanical engineering, astrophysics, cosmology, and magnetohydrodynamics.

Key Words: Mechanical Lorentz force, antisymmetric Lorentz tensor, mechanical electrodynamic, 4D wave of Newtonian force, wave of Coriolis force, 7D wave of mechanical energy.

Contents

1. Introduction

Our previous model on antisymmetric electromagnetic Lorentz force electrodynamics [1] predicted the existence of antisymmetric mechanical Lorentz force. Since after the successful applications of Maxwellian electrodynamics in daily life, motivated the physicists to look for the mechanical analogue of Maxwellian electrodynamics [2,3,4,5,6,7] via different approaches. Our methodology is very simple and consistent with the standard Maxwellian electrodynamics. Mechanical electrodynamics consists of LFT, MFE, MME and MCL. The components of antisymmetric mechanical Lorentz force tensor $f^{\mu\nu}$ are Newton's 2nd law of motion $F = ma$ and Coriolis force $(P \times \omega)$ which are well-known concepts in classical dynamics. This observation provides us an elegant framework of classical dynamics in the language of electrodynamics. Its nature is relativistic by birth as it obeys the principle of relativity. Now, we look at its structure in the language of electrodynamics. Temporal component of LFT is Newton's 2nd law of motion. Spatial component is Coriolis force minus difference of LFT that validates antisymmetric nature of LFT. Gaussian force law, Amperian force law and conservation of Gaussian plus Amperian force law. Dual of Lorentz force electrodynamics behaves in the same way obeying conservation law. Inhomogeneous wave equation for Coriolis force is obtained by taking the curl of Ampere's law and using the time derivative of Faraday's force law. Similarly, wave equation for Newton's 2nd law is obtained by taking the curl of Faraday's force law and utilizing the time derivative of Ampere's law. We get the wave equation for Lorentz force by adding these wave equations. This model can readily be applied by mechanical engineers, astrophysicists,

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cosmologists and theoretical physicists. The form invariance of this model can be checked by using the framework of [8] where it is shown that time and space components of laws of physics are relative but spacetime laws as a whole remain the same for all observers in their original form after transformation.

In order to see the predictions of mechanical electrodynamics, we transform the laws of mechanical electrodynamics in noninertial coordinate metric based on STL for 4-vectors and tensors. The noninertial coordinate metric is playing the role of medium in which mechanical Lorentz force electrodynamics experiences noninertial effects which are nothing but constitutive relations in this medium. In this formulation, we get new symmetry of LFT, Lorentz force mechanical Maxwell's equations, conservation law. It predicts zero-point structure of mechanical electrodynamics that validates conservation law, symmetry and principle of relativity. The model reduces to its classical limit when the noninertial effects are neglected. The transformation of Lorentz force tensor in matrix form in noninertial coordinate metric experiences new symmetry. Zero-point Lorentz force tensor represents action and reaction of energy or balance of energy along the diagonal. Mechanical Lorentz force tensor is transformed into Coriolis torque. Newton's 2nd law of motion is transformed into itself and 4-dimensional torque consisting of zero-point energy plus Coriolis torque. The dual of Newton's 2nd law of motion is Coriolis force that is transformed into itself, usual torque and Coriolis energy. Gauss's law is transformed into itself and dot product of coordinate with Amperian force law. Similarly, Amperian force law is transformed into itself and mechanical force in terms of negative of gradient of energy. Zero-point Maxwell's equations represent unification of mechanical force and power. Conservation law by matrix method becomes equal to zero but by usual technique gives 7D wave of mechanical energy supplemented with classical conservation law. The 7D wave operator is an invariant quantity. This new conservation emerges as the consequence of new symmetry due to STL for 4-vectors and tensors. The concept of more general wave equation is present in the contemporary world [9] but our approach is simple and straightforward. The other model in the name of electromagnetism in noninertial coordinates [10] doesn't meet the claimed results but the efforts must be appreciated.

Notations in this model are adopted according to the modern approach of relativity. Greek alphabets $\mu, \nu, \alpha, \beta, \ldots$ run from 0 to 3 and Latin letters $i, j, k, \ldots$ from 1 to 3. A comma $(,)$ denotes partial differentiation, e.g. $F_{,0} = \partial F/\partial t$ is the partial derivative of the force field with respect to time, $F_{,1} = \partial F/\partial x$ is the partial derivative of force with respect to the x-axis, $F_{,2} = \partial F/\partial y$ is the partial derivative of force with respect to the y-axis, and $F_{,3} = \partial F/\partial z$ is the partial derivative of force with respect to the z-axis. Moreover, $f^{\mu\nu}{}_{,\nu}$ means the 4-dimensional or spacetime partial derivative of the Lorentz force tensor.

1.1: Formulation of Mechanical Lorentz Force Electrodynamics

The antisymmetric mechanical Lorentz force tensor is written in matrix form where components of the tensor clearly represent the structure of classical dynamics.

Structure of Lorentz Force Tensor

$$f^{\mu\nu} = \begin{bmatrix} 0 & F^1 & F^2 & F^3 \\ -F^1 & 0 & (P\times\omega)^3 & -(P\times\omega)^2 \\ -F^2 & -(P\times\omega)^3 & 0 & (P\times\omega)^1 \\ -F^3 & (P\times\omega)^2 & -(P\times\omega)^1 & 0 \end{bmatrix} \tag{1}$$

$$f^{0i} = F^i, \qquad f^{ij} = \varepsilon^{ijk}(P\times\omega)_k \tag{2}$$

Newtonian Force Field

$$f^{0i} = ma^i = F^i \tag{3}$$

Spatial Component of Lorentz Force Field

$$f^{i\nu} = (P\times\omega) - [F + (P\times\omega)] \tag{4}$$

Lorentz Force Field Tensor

$$f^{\mu\nu} = [F + (P\times\omega)] - [F + (P\times\omega)] = 0 \tag{5}$$

Thus, the mechanical Lorentz force tensor remains antisymmetric.

1.2: Mechanical Lorentz Force Maxwell's Equations

Gauss's Force Law

$$f^{0i}{}_{,i} = \nabla \cdot F \tag{6}$$

Ampere's Force Law

$$f^{i\nu}{}_{,\nu} = (\nabla \times (P \times \omega)) - F_{,0} \tag{7}$$

$$\nabla \times (P \times \omega) = f^{i\nu}{}_{,\nu} + \frac{\partial F}{\partial t} \tag{7a}$$

The curl of the Coriolis force is equal to the force density and the time-varying Newtonian force. Adding (6) and (7), we obtain the mechanical Maxwell's equations in tensor form:

Mechanical Maxwell's Equations in Tensor Form

$$f^{\mu\nu}{}_{,\nu} = \nabla \cdot F + \Big[\nabla \times (P \times \omega) - F_{,0}\Big] \tag{8}$$

1.3: Derivation of Conservation Law for Lorentz Force Tensor

We derive the conservation law of the Lorentz force tensor by taking the time derivative of Gauss's force law (6) and the divergence of Ampere's force law (7).

$$f^{0i}{}_{,i0} = (\nabla \cdot F)_{,0} \tag{9}$$

$$f^{i\nu}{}_{,\nu i} = \Big[\nabla \cdot \nabla \times (P \times \omega) - F_{,0}\Big] \tag{10}$$

Adding (9) and (10), we obtain the conservation law in tensor form:

$$f^{\mu\nu}{}_{,\nu\mu} = (\nabla \cdot F)_{,0} + \Big[\nabla \cdot \nabla \times (P \times \omega) - F_{,0}\Big] = 0 \tag{11}$$

Thus, conservation of the Lorentz force in tensor form holds as usual.

1.4: Dual Structure of Mechanical Electrodynamics

The dual structure of mechanical electrodynamics is needed to obtain the second set of Maxwell's equations, where we recover Faraday's law and Gauss's law for magnetism in Maxwellian electrodynamics. The dual of the Lorentz force tensor $f^{\mu\nu}$ is denoted by ${}^*f^{\mu\nu}$.

Dual of Mechanical Lorentz Force Tensor:

$${}^*f^{\mu\nu} = \begin{bmatrix} 0 & (P\times\omega)^1 & (P\times\omega)^2 & (P\times\omega)^3 \\ -(P\times\omega)^1 & 0 & -F^3 & F^2 \\ -(P\times\omega)^2 & F^3 & 0 & -F^1 \\ -(P\times\omega)^3 & -F^2 & F^1 & 0 \end{bmatrix} \tag{12}$$

$${}^*f^{0i} = (P \times \omega)^i, \qquad {}^*f^{ij} = -\varepsilon^{ijk} F_k \tag{13}$$

Coriolis Force Field:

$${}^*f^{0i} = (P \times \omega)^i \tag{14}$$

Dual of Spatial Component of Lorentz Force Field:

$${}^*f^{i\nu} = (P \times \omega) - \big[(P \times \omega) - F\big] \tag{15}$$

Dual of Lorentz Force Tensor:

$${}^*f^{\mu\nu} = \big[F + (P \times \omega)\big] - \big[F + (P \times \omega)\big] = 0 \tag{16}$$

Thus, the dual of the mechanical Lorentz force tensor remains antisymmetric.

1.5: Dual of Lorentz Force Maxwell's Equations

Dual of Gauss's Force Law (Divergence of Coriolis force):

$$^*f^{0i}{}_{,i} = \nabla \cdot (P \times \omega) \tag{17}$$

Faraday's Force Law:

$$^*f^{i\nu}{}_{,\nu} = -(\nabla \times F) - \frac{\partial}{\partial t}(P \times \omega) \tag{18}$$

$$\nabla \times F = -{}^*f^{i\nu}{}_{,\nu} - \frac{\partial}{\partial t}(P \times \omega) \tag{18a}$$

Thus, the curl of the Newtonian force is equal to the negative of the time-varying Coriolis force.
—

1.6: Derivation of Dual Conservation Law for Lorentz Force

By taking the divergence of Faraday's force law (18) and substituting the divergence of the time-varying Coriolis force from the dual of Gauss's force law (17), we obtain

$$^*f^{0i}{}_{,i0} = \nabla \cdot \frac{\partial}{\partial t}(P \times \omega) \tag{19}$$

Faraday's Force Law :

$$\nabla \cdot \left({}^*f^{i\nu}{}_{,\nu}\right) = -\nabla \cdot (\nabla \times F) - \frac{\partial}{\partial t}\nabla \cdot (P \times \omega) \tag{20}$$

Adding (19) and (20), we get:

$$^*f^{\mu\nu}{}_{,\nu\mu} = \nabla \cdot \frac{\partial}{\partial t}(P \times \omega) - \nabla \cdot (\nabla \times F) - \nabla \cdot \frac{\partial}{\partial t}(P \times \omega) = 0 \tag{21}$$

1.7: Derivation of Lorentz Force Wave Equations

By taking the negative time derivative of Ampere's force law (7) and the curl of Faraday's force law (18), we obtain the wave equation of the Newtonian force. For simplicity, we consider the source-free version:

Newtonian Force Wave Equation:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) F = -\frac{\partial}{\partial t} f^{i\nu}{}_{,\nu} \tag{22}$$

Similarly, by taking the curl of Ampere's force law (7) and the time derivative of Faraday's force law (18), we obtain the wave equation of the Coriolis force:

Coriolis Force Wave Equation:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) (P \times \omega) = 0 \tag{23}$$

Adding equations (22) and (23), we get the wave equation of the mechanical Lorentz force:

Mechanical Lorentz Force Wave Equation:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) \left[F + (P \times \omega)\right] = 0 \tag{24}$$

Table-1a

Summary of Results of Antisymmetric Mechanical Lorentz Force Tensor Dynamics

Table-1b

Summary of Results of Dual of Antisymmetric Mechanical Lorentz Force Tensor Dynamics

Antisymmetric Lorentz Force Dynamics	**Mathematical Representation**
Structure of Lorentz Force Field Tensor $f^{\mu\nu}$	$\begin{bmatrix} 0 & F^1 & F^2 & F^3 \\ -F^1 & 0 & (P\times\omega)^3 & -(P\times\omega)^2 \\ -F^2 & -(P\times\omega)^3 & 0 & (P\times\omega)^1 \\ -F^3 & (P\times\omega)^2 & -(P\times\omega)^1 & 0 \end{bmatrix}$ $f^{0i} = F^i \quad f^{ij} = \varepsilon^{ijk}(P\times\omega)_k$
Newtonian Force Field	$f^{0i} = F^i$
Spatial Inertial Force Field	$f^{i\nu} = (P\times\omega) - [F + (P\times\omega)]$
Antisymmetry of Mechanical Lorentz Force Tensor	$f^{\mu\nu} = [F + (P\times\omega)] - [F + (P\times\omega)] = 0$
Mechanical Lorentz Force Maxwell's Equations	Formulae of Lorentz Force Maxwell's Equations
Gaussian Force Law	$f^{0i}_{,i} = \nabla\cdot F$
Amperian Force Law	$f^{i\nu}_{,\nu} = [\nabla\cdot\nabla\times(P\times\omega) - F_{,0}]$
Conservation Law for Inertial Force	$f^{\mu\nu}_{,\nu\mu} = \nabla\cdot F_{,0} + [\nabla\cdot\nabla\times(P\times\omega) - F_{,0}] = 0$
Newtonian Force Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) F = 0$
Coriolis Force Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)(P\times\omega) = 0$
Wave Equation of Mechanical Lorentz Force	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[F + (P\times\omega)] = 0$

Dual of Antisymmetric Lorentz Force Dynamics	**Mathematical Representation**
Structure of Dual Inertia Field Tensor $^*f^{\mu\nu}$	$\begin{bmatrix} 0 & (P\times\omega)^1 & (P\times\omega)^2 & (P\times\omega)^3 \\ -(P\times\omega)^1 & 0 & -F^3 & F^2 \\ -(P\times\omega)^2 & F^3 & 0 & -F^1 \\ -(P\times\omega)^3 & -F^2 & F^1 & 0 \end{bmatrix}$ $(P\times\omega) \;\to\; -F^i, \qquad F^i \;\to\; (P\times\omega)$
Coriolis Force Field	$^*f^{0i} = (P\times\omega)$
Dual of Spatial Inertial Force Field	$^*f^{i\nu} = (P\times\omega) - [(P\times\omega) - F]$
Dual of Antisymmetric Lorentz Force Tensor	$^*f^{\mu\nu} = [F + (P\times\omega)] - [F + (P\times\omega)] = 0$

2. Mechanical Electrodynamics in Noninertial Coordinate Metric

2.1. Transformation of Lorentz Force Tensor in Noninertial Coordinate Metric

The noninertial coordinate metric plays the role of medium and the equations in this medium will represent the constitutive relations for the physical theory under consideration. The transformation of mechanical Lorentz tensor and other relations in noninertial coordinate metric will experience physical

Mechanical Lorentz Force Maxwell's Equations	Formulae of Dual Mechanical Maxwell's Equations
Dual of Gauss's Law	${}^*f^{0i}_{,i} = \nabla \cdot (P \times \omega)$
Faraday's Force Law	${}^*f^{i\nu}_{,\nu} = -(\nabla \times F) - \frac{\partial}{\partial t}(P \times \omega)$
Dual Conservation Law	${}^*f^{\mu\nu}_{,\nu\mu} = \nabla \cdot (P \times \omega)_{,0} - (\nabla \cdot \nabla \times F) - \nabla \cdot (P \times \omega)_{,0} = 0$
Coriolis Force Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)(P \times \omega) = 0$
Newtonian Force Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) F = 0$
Wave Equation of Mechanical Lorentz Force	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[F + (P \times \omega)] = 0$

effects of inertia.

Color Scheme: In order to distinguish the effects of inertia and classical relations, we use the color scheme for simplicity. Blue color represents classical theory and red color represents inertial effects.

The noninertial coordinate metric is given as

$$[g^{\mu}_{\alpha}] = \begin{bmatrix} 1 & -x_1 & -x_2 & -x_3 \\ -x_1 & 1 & 0 & 0 \\ -x_2 & 0 & 1 & 0 \\ -x_3 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$

$$f^{\mu'\nu'} = g^{\mu'}_{\alpha} f^{\alpha\nu'} \tag{26}$$

$$\begin{bmatrix} f^{0'\nu'} \\ f^{1'\nu'} \\ f^{2'\nu'} \\ f^{3'\nu'} \end{bmatrix} = \begin{bmatrix} 1 & -x_1 & -x_2 & -x_3 \\ -x_1 & 1 & 0 & 0 \\ -x_2 & 0 & 1 & 0 \\ -x_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & F^1 & F^2 & F^3 \\ -F^1 & 0 & (P \times \omega)^3 & -(P \times \omega)^2 \\ -F^2 & -(P \times \omega)^3 & 0 & (P \times \omega)^1 \\ -F^3 & (P \times \omega)^2 & -(P \times \omega)^1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} f^{0'\nu'} \\ f^{1'\nu'} \\ f^{2'\nu'} \\ f^{3'\nu'} \end{bmatrix} = \begin{bmatrix} x \cdot F & F^1 + x_2\,(P \times \omega)^3 - x_3\,(P \times \omega)^2 & F^2 + x_3\,(P \times \omega)^1 - x_1\,(P \times \omega)^3 & F^3 + x_1\,(P \times \omega)^2 - x_2\,(P \times \omega)^1 \\ -F^1 & -x_1 F^1 & -x_1 F^2 + (P \times \omega)^3 & -x_1 F^3 - (P \times \omega)^2 \\ -F^2 & -x_2 F^1 - (P \times \omega)^3 & -x_2 F^2 & -x_2 F^3 + (P \times \omega)^1 \\ -F^3 & -x_3 F^1 + (P \times \omega)^2 & -x_3 F^2 - (P \times \omega)^1 & -x_3 F^3 \end{bmatrix} \tag{27}$$

The terms appearing along the diagonal represent zero-point structure of theory.

Zero-point Energy

$$f^{0'0'} = x \cdot F \tag{28}$$

Zero-point Negative Energy

$$f^{i'i'} = -x \cdot F \tag{29}$$

Action and Reaction of Zero-point energy as balance

$$f^{\mu'\mu'} = x \cdot F - x \cdot F \tag{30}$$

$$\begin{bmatrix} f^{0'\nu'} \\ f^{i'\nu'} \end{bmatrix} = \begin{bmatrix} F + \left[x \cdot F + x \times (P \times \omega)\right] \\ -\left[F + (P \times \omega)\right] + (P \times \omega) - x \cdot F \end{bmatrix} \tag{31}$$

4-Dimensional form of Mechanical Lorentz tensor

$$f^{0'\nu'} = F + \big[x \cdot F + x \times (P \times \omega)\big] \tag{32}$$

The extra term $\tau^{\nu} = (\tau^0, \tau^i) = [x \cdot F + x \times (P \times \omega)]$ represent unification of mechanical work or energy and Coriolis torque. They have same dimensions but physically different as work is scalar and torque is a vector quantity. They both constitute 4-dimensional torque τ^{ν}. Their dimension is one coordinate time greater than the classical term force F due to contribution of coordinate metric. We retain their original form to justify the classical limit. Equation (32) can be written as

$$f^{0'\nu'} = F + \tau^{\nu} \tag{32a}$$

Newton's 2nd law of motion in noninetial coordinate metric is transformed in to itself and 4-dimensional torque which has 7 dimensions. Very very strange consequence.

Tensorial form of Spatial component of Mechanical Lorentz tensor

$$f^{i'\nu'} = (P \times \omega) - [F + (P \times \omega)] - x \cdot F \tag{34}$$

$$f^{i'\nu'} = f^{i\nu} - x \cdot F \tag{34a}$$

New Symmetry of Mechanical Lorentz tensor as Coriolis Torque

$$f^{\mu'\nu'} = [F + (P \times \omega)] - [F + (P \times \omega)] + [x \cdot F + x \times (P \times \omega)] - x \cdot F \tag{35}$$

Equation (35) shows that antisymmetric Lorentz force tensor is transformed in to difference or balance of Lorentz force, 4-D torque and energy. It other words, it contains action and reaction of Lorentz force, action and reaction of zero-point mechanical energy and the net effect is Coriolis torque.

$$f^{\mu'\nu'} = f^{\mu\nu} + [x \cdot F + x \times (P \times \omega)] - x \cdot F \tag{35a}$$

$$f^{\mu'\nu'} = x \times (P \times \omega) \tag{35b}$$

Note that the new symmetry of mechanical Lorentz force tensor as Coriolis torque emerges as the consequence of STL for 4-vectors and tensor.

2.2: Transformation of Mechanical Lorentz force Maxwell's Equations

$$f^{\mu'\nu'}_{,\nu'} = g^{\mu'}_{\alpha} f^{\alpha\nu'}_{,\nu'} \tag{36}$$

$$\begin{bmatrix} f^{0'\nu'}_{,\nu'} \\ f^{1'\nu'}_{,\nu'} \\ f^{2'\nu'}_{,\nu'} \\ f^{3'\nu'}_{,\nu'} \end{bmatrix} = \begin{bmatrix} 1 & -x_1 & -x_2 & -x_3 \\ -x_1 & 1 & 0 & 0 \\ -x_2 & 0 & 1 & 0 \\ -x_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & F^1_{,1} & F^2_{,2} & F^3_{,3} \\ -F^1_{,0} & 0 & (P \times \omega)^3_{,2} & -(P \times \omega)^2_{,3} \\ -F^2_{,0} & -(P \times \omega)^3_{,1} & 0 & (P \times \omega)^1_{,3} \\ -F^3_{,0} & (P \times \omega)^2_{,1} & -(P \times \omega)^1_{,2} & 0 \end{bmatrix}$$

$$\begin{bmatrix} f^{0'\nu'}_{,\nu'} \\ f^{1'\nu'}_{,\nu'} \\ f^{2'\nu'}_{,\nu'} \\ f^{3'\nu'}_{,\nu'} \end{bmatrix} = \begin{bmatrix} x \cdot F_{,0} & F^1_{,1} + x_2 (P \times \omega)^3_{,1} - x_3 (P \times \omega)^2_{,1} & F^2_{,2} + x_3 (P \times \omega)^1_{,2} - x_1 (P \times \omega)^3_{,2} & F^3_{,3} + x_1 (P \times \omega)^2_{,3} - x_2 (P \times \omega)^1_{,3} \\ -F^1_{,0} & -x_1 F^1_{,1} & -x_1 F^2_{,2} + (P \times \omega)^3_{,2} & -x_1 F^3_{,3} - (P \times \omega)^2_{,3} \\ -F^2_{,0} & -x_2 F^1_{,1} - (P \times \omega)^3_{,1} & -x_2 F^2_{,2} & -x_2 F^3_{,3} + (P \times \omega)^1_{,3} \\ -F^3_{,0} & -x_3 F^1_{,1} + (P \times \omega)^2_{,1} & -x_3 F^2_{,2} - (P \times \omega)^1_{,2} & -x_3 F^3_{,3} \end{bmatrix} \tag{37}$$

$$\begin{bmatrix} f^{0'\nu'}_{,\nu'} \\ f^{i'\nu'}_{,\nu'} \end{bmatrix} = \begin{bmatrix} \nabla \cdot F - x \cdot [(\nabla \times P \times \omega) - F_{,0}] \\ -\nabla(x \cdot F) + [(\nabla \times P \times \omega) - F_{,0}] \end{bmatrix} \tag{38}$$

Gaussian force Law

$$f^{0'\nu'}_{,\nu'} = \nabla \cdot F - x \cdot [(\nabla \times P \times \omega) - F_{,0}] \tag{39}$$

$$f^{0'\nu'}_{,\nu'} = f^{0i}_{,i} - x \cdot [(\nabla \times P \times \omega) - F_{,0}] \tag{39a}$$

Amperian Force Law

$$f^{i'\nu'}_{,\nu'} = \big[(\nabla \times P \times \omega) - F_{,0}\big] - \nabla[x \cdot F] \tag{40}$$

$$f^{i'\nu'}_{,\nu'} = f^{i\nu}_{,\nu} - \nabla[x \cdot F] \tag{40a}$$

Combining (39a) and (40a), we have Lorentz force Maxwell's equations in tensor form

$$f^{\mu'\nu'}_{,\nu'} = \nabla \cdot F + \big[(\nabla \times P \times \omega) - F_{,0}\big] - x \cdot \big[(\nabla \times P \times \omega) - F_{,0}\big] - \nabla[x \cdot F] \tag{41}$$

$$f^{\mu'\nu'}_{,\nu'} = f^{\mu\nu}_{,\nu} - x \cdot \big[(\nabla \times P \times \omega) - F_{,0}\big] - \nabla[x \cdot F] \tag{41a}$$

Mechanical Maxwell's equations in noninertial coordinate metric are transformed into the classical case plus the dot product of coordinate with Ampere's force law and mechanical force. Here force appears as the negative of gradient of energy as we know from mechanics.

2.3: Transformation of Conservation Law by Matrix Method

$$f^{\mu'\nu'}_{,\nu'\mu'} = g^{\mu'}_{\alpha} f^{\alpha\nu'}_{,\nu'\mu'} \tag{42}$$

$$\begin{bmatrix} f^{0'\nu'}_{,\nu'0'} \\ f^{1'\nu'}_{,\nu'1'} \\ f^{2'\nu'}_{,\nu'2'} \\ f^{3'\nu'}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} 1 & -x_1 & -x_2 & -x_3 \\ -x_1 & 1 & 0 & 0 \\ -x_2 & 0 & 1 & 0 \\ -x_3 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & F^1_{,10} & F^2_{,20} & F^3_{,30} \\ -F^1_{,01} & 0 & (P \times \omega)^3_{,21} & -(P \times \omega)^2_{,31} \\ -F^2_{,02} & -(P \times \omega)^3_{,12} & 0 & (P \times \omega)^1_{,32} \\ -F^3_{,03} & (P \times \omega)^2_{,13} & -(P \times \omega)^1_{,23} & 0 \end{bmatrix} \tag{43}$$

$$\begin{bmatrix} f^{0'\nu'}_{,\nu'0'} \\ f^{1'\nu'}_{,\nu'1'} \\ f^{2'\nu'}_{,\nu'2'} \\ f^{3'\nu'}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} \nabla(x \cdot F_{,0}) & F^1_{,10} + x_2(P \times \omega)^3_{,12} - x_3(P \times \omega)^2_{,13} & F^2_{,20} + x_3(P \times \omega)^1_{,23} - x_1(P \times \omega)^3_{,21} & F^3_{,30} + x_1(P \times \omega)^2_{,31} - x_2(P \times \omega)^1_{,32} \\ -F^1_{,01} & -x_1 F^1_{,10} & -x_1 F^2_{,20} + (P \times \omega)^3_{,21} & -x_1 F^3_{,30} - (P \times \omega)^2_{,31} \\ -F^2_{,02} & -x_2 F^1_{,10} - (P \times \omega)^3_{,12} & -x_2 F^2_{,20} & -x_2 F^3_{,30} + (P \times \omega)^1_{,32} \\ -F^3_{,03} & -x_3 F^1_{,10} + (P \times \omega)^2_{,13} & -x_3 F^2_{,20} - (P \times \omega)^1_{,23} & -x_3 F^3_{,30} \end{bmatrix} \tag{44}$$

$$f^{\mu'\nu'}_{,\nu'\mu'} = \nabla \cdot F_{,0} + \nabla \cdot (\nabla \times P \times \omega) - F_{,0} - \nabla \left\{x \cdot \big[(\nabla \times P \times \omega) - F_{,0}\big]\right\} - \nabla(x \cdot F_{,0}) = 0 \tag{45}$$

$$f^{\mu'\nu'}_{,\nu'\mu'} = f^{\mu\nu}_{,\nu\mu} - \nabla \left\{x \cdot \big[(\nabla \times P \times \omega) - F_{,0}\big]\right\} - \nabla(x \cdot F_{,0}) = 0 \tag{45a}$$

2.4: Derivation of Conservation Law in Noninertial Coordinate Metric by Usual Method

Taking the time derivative of Gauss's force law (39) and utilizing the curl of Faraday's law (64), we have

$$f^{0'\nu'}_{,\nu'0'} = \nabla \cdot F_{,0} + \Box^2[x \cdot F] \tag{46}$$

Now, performing the divergence of Ampere's law (40), gives

$$f^{i'\nu'}_{,\nu'i'} = \nabla \cdot \big[(\nabla \times P \times \omega) - F_{,0}\big] - \nabla^2[x \cdot F] \tag{47}$$

Adding equations (46) and (47),

$$f^{\mu'\nu'}_{,\nu'\mu'} = \nabla \cdot F_{,0} + \nabla \cdot \left[(\nabla \times P \times \omega) - F_{,0}\right] + \left[\Box^2 - \nabla^2\right][x \cdot F] \tag{48}$$

$$f^{\mu'\nu'}_{,\nu'\mu'} = f^{\mu\nu}_{,\nu\mu} + \left[\Box^2 - \nabla^2\right][x \cdot F] \tag{48a}$$

Conservation law by the usual method gives the classical conservation law plus a 7D wave of mechanical energy, where the 7D wave operator $\left[\Box^2 - \nabla^2\right]$ is an invariant quantity.

2.5: Dual Lorentz Force Mechanical Electrodynamics in Noninertial Coordinate Metric

As the process of calculations for the dual model is the same, we present the results directly in Table 2b.

Table-2a
Summary of Mechanical Lorentz Force (MLF) Dynamics in Noninertial Coordinate Metric

Mechanical Lorentz Force Dynamics	**Dual Mechanical Lorentz Force Dynamics**
Transformation of MLF Tensor	$f^{\mu'\nu'} = g^{\mu'}_{\alpha} f^{\alpha\nu'}$
Zero-point Mechanical Energy	$f^{0'0'} = x \cdot F$
Zero-point Negative Mechanical Energy	$f^{i'i'} = -x \cdot F$
Zero-point Action and Reaction of Mechanical Energy	$f^{\mu'\mu'} = x \cdot F - x \cdot F = 0$
Temporal Component as 4D Newtonian Force	$f^{0'\nu'} = F + [x \cdot F + x \times (P \times \omega)]$
Spatial Component of MLF Tensor	$f^{i'\nu'} = [(P \times \omega) - F] - [x \cdot F + (P \times \omega)]$
Mechanical Lorentz Force Tensor as Coriolis Torque	$f^{\mu'\nu'} = f^{\mu\nu} + x \times (P \times \omega)$
Transformation Law for MLF Maxwell's Equations	$f^{\mu'\nu'}_{,\nu'} = g^{\mu'}_{\alpha} f^{\alpha\nu'}_{,\nu'}$
Zero-point Mechanical Maxwell's Equations	$f^{\mu'\mu'}_{,\mu'} = x \cdot F_{,0} - \nabla[x \cdot F]$
Mechanical Force Gauss's Law	$f^{0'\nu'}_{,\nu'} = \nabla \cdot F - x \cdot [\nabla \times (P \times \omega) - F_{,0}]$
Mechanical Ampere's Force Law	$f^{i'\nu'}_{,\nu'} = [(\nabla \times P \times \omega) - F_{,0}] - \nabla[x \cdot F]$
Transformation Law for Conservation Law	$f^{\mu'\nu'}_{,\nu'\mu'} = g^{\mu'}_{\alpha} f^{\alpha\nu'}_{,\nu'\mu'}$
MLF Conservation Law by Matrix Method	$f^{\mu'\nu'}_{,\nu'\mu'} = \nabla[x \cdot F_{,0}] - \nabla[x \cdot F_{,0}] = 0$
Conservation Law by Usual Method	$f^{\mu'\nu'}_{,\nu'\mu'} = f^{\mu\nu}_{,\nu\mu} + [\Box^2 - \nabla^2][x \cdot F]$

Important Results from Table-2a

1. **Origin of Zero-Point Energy:**

$$f^{0'0'} = x \cdot F \tag{49}$$

2. **Generalized Newton's Second Law of Motion:**

$$f^{0'\nu'} = F + \big[x \cdot F + x \times (P \times \omega)\big] \quad (50)$$

3. **Origin of Coriolis Torque:**

$$f^{\mu'\nu'} = x \times (P \times \omega) \quad (51)$$

4. **Zero-point Mechanical Maxwell's Equations as Origin of Unified Power–Force:**

$$f^{\mu'\mu'}_{,\mu'} = x \cdot F_{,0} - \nabla[x \cdot F] \quad (52)$$

Table-2b
Summary of Dual of Mechanical Lorentz Force Dynamics in Noninertial Coordinate Metric

Mechanical Lorentz Force Dynamics	Dual Mechanical Lorentz Force Dynamics
Transformation of Dual of MLF Tensor	${}^*f^{\mu'\nu'} = g^{\mu'}_{\alpha}\, {}^*f^{\alpha\nu'}$
Zero-Point Mechanical Coriolis Energy	${}^*f^{0'0'} = x \cdot (P \times \omega)$
Zero-Point Negative Mechanical Coriolis Energy	${}^*f^{i'i'} = x \cdot (P \times \omega) - x \cdot (P \times \omega)$
Zero-Point Action and reaction of Coriolis Energy	${}^*f^{\mu'\mu'} = x \cdot (P \times \omega) - x \cdot (P \times \omega)$
Temporal Component of 4D ML Coriolis Force	${}^*f^{0'\nu'} = (P \times \omega) - \big[(x \times F) + x \cdot (P \times \omega)\big]$
Spatial Component of Dual MLF Tensor	${}^*f^{i'\nu'} = [F + (P \times \omega)] - [x \cdot (P \times \omega) + F]$
Dual of MLF Tensor as torque	${}^*f^{\mu'\nu'} = {}^*f^{\mu\nu} - x \times F$
Transformation Law Dual MLF Maxwell's Eqs.	${}^*f^{\mu'\nu'}_{,\nu'} = g^{\mu'}_{\alpha}\, {}^*f^{\alpha\nu'}_{,\nu'}$
Zero-Point Max. Eqs. as Coriolis power–Coriolis force	${}^*f^{\mu'\mu'}_{,\mu'} = x \cdot (P \times \omega)_{,0} - \nabla[x \cdot (P \times \omega)]$
Dual of Mechanical Gaussian Force law	${}^*f^{0'\nu'}_{,\nu'} = \nabla \cdot (P \times \omega) + x \cdot \big[(\nabla \times F) + (P \times \omega)_{,0}\big]$
Faraday's Law of Forces	${}^*f^{i'\nu'}_{,\nu'} = -\big[(\nabla \times F) + (P \times \omega)_{,0}\big] - \nabla[x \cdot (P \times \omega)]$
Transformation Law for Dual Conservation Law	${}^*f^{\mu'\nu'}_{,\nu'\mu'} = g^{\mu'}_{\alpha}\, {}^*f^{\alpha\nu'}_{,\nu'\mu'}$
Dual of MLF Conservation Law by Matrix method	${}^*f^{\mu'\nu'}_{,\nu'\mu'} = \nabla[x \cdot (P \times \omega)_{,0}] - \nabla[x \cdot (P \times \omega)_{,0}] = 0$
Dual Conservation Law by Usual Method	${}^*f^{\mu'\nu'}_{,\nu'\mu'} = {}^*f^{\mu\nu}_{,\nu\mu} + [\Box^2 - \nabla^2][x \cdot (P \times \omega)]$

Important Results from Table-2b

1. Origin of Zero-Point Coriolis Energy

$$ {}^*f^{0'0'} = x \cdot (P \times \omega) \quad (53)$$

2. Dual of Generalized Newton's 2nd Law of Motion as Coriolis Force and Dual of 4D Torque

$$ {}^*f^{0'\nu'} = (P \times \omega) - \big[(x \times F) + x \cdot (P \times \omega)\big] \quad (54)$$

3. Origin of Torque

$$^*f^{\mu'\nu'} = -x \times F \tag{55}$$

4. Dual Zero-Point Mechanical Maxwell's Equations as Origin of Unified Coriolis Power–Force

$$^*f^{\mu'\mu'}_{,\mu'} = x \cdot (P \times \omega)_{,0} - \nabla\big[x \cdot (P \times \omega)\big] \tag{56}$$

3. Discussion and Comparison

A comprehensive and consistent model of antisymmetric mechanical Lorentz force dynamics is developed in the language of electrodynamics. Newton's second law of motion and the Coriolis force are the components of the mechanical Lorentz force tensor. This tensorial force governs the motion of bodies linearly by Newton's law and circularly by the Coriolis force. Geodesics are caused by the Lorentz force.

Antisymmetric mechanical Lorentz force in a noninertial coordinate metric is transformed into the Coriolis torque equation (35), and its dual is transformed into the usual torque. The nature of noninertial forces is repulsive, while that of gravitation is attractive. We shall discuss the roles of both forces in the next paper on the antisymmetric gravitational Lorentz force tensor, which is ready for submission.

In our earlier paper [11], electrodynamics, inertia, and gravitation in noninertial metrics (except the noninertial coordinate metric) were presented in the form of Tables 1 to 5. Those models were partially force-theoretic and partially field-theoretic. This model is an entirely different discovery based solely on forces. Such a model is not available in our contemporary world of the relativistic paradigm.

Here, we present the Mechanical Maxwell's equations, wave equations, and conservation law in Lorentz Force Tensor (LFT) form and their transformation in the noninertial coordinate metric.

Mechanical Maxwell's Equations

Gaussian Force Law

$$f^{0i}_{,i} = \nabla \cdot F \tag{3.1}$$

Amperian Force Law

$$\nabla \times (P \times \omega) = f^{i\nu}_{,\nu} + \frac{\partial F}{\partial t} \tag{3.2}$$

Dual of Gaussian Force Law

$$^*f^{0i}_{,i} = \nabla \cdot (P \times \omega) \tag{3.3}$$

Faraday's Force Law

$$\nabla \times F = -^*f^{i\nu}_{,\nu} - \frac{\partial}{\partial t}(P \times \omega) \tag{3.4}$$

Note the symmetry of Mechanical Maxwell's equations as sources are nonzero.

Wave Equation of Newtonian Force

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) F = 0 \tag{3.5}$$

Wave Equation of Coriolis Force

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)(P \times \omega) = 0 \tag{3.6}$$

Adding wave equations (3.5) and (3.6), we get the wave equation of the Mechanical Lorentz force.

Mechanical Lorentz Force Wave Equation

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\big[F + (P \times \omega)\big] = 0 \tag{3.7}$$

4. Mechanical Maxwell's Equations in Noninertial Coordinate Metric

Gaussian Force Law

$$f^{0'\nu'}_{,\nu'} = \nabla \cdot F - x \cdot \big[(\nabla \times (P \times \omega)) - F_{,0}\big] \tag{3.8}$$

Amperian Force Law

$$f^{i'\nu'}_{,\nu'} = \big[(\nabla \times (P \times \omega)) - F_{,0}\big] - \nabla(x \cdot F) \tag{3.9}$$

Dual of Gaussian Force Law

$$^*f^{0'\nu'}_{,\nu'} = \nabla \cdot (P \times \omega) + x \cdot \Big[(\nabla \times F) + (P \times \omega)_{,0}\Big] \tag{3.10}$$

Faraday's Law for Lorentz Force

$$^*f^{i'\nu'}_{,\nu'} = -\Big[(\nabla \times F) + (P \times \omega)_{,0}\Big] - \nabla\big[x \cdot (P \times \omega)\big] \tag{3.11}$$

Mechanical Maxwell's equations in the noninertial coordinate metric reduce to the classical limit when noninertial terms are subjected to zero.

Conservation Law in Noninertial Coordinate Metric

$$f^{\mu'\nu'}_{,\nu'\mu'} = f^{\mu\nu}_{,\nu\mu} + \big[\Box^2 - \nabla^2\big](x \cdot F) \tag{3.12}$$

The 7D wave operator $\big[\Box^2 - \nabla^2\big]$ is an invariant operator and is the inner product of $(\Box, \nabla)$ and $(\Box, -\nabla)$, where $\Box^2$ is known as the d'Alembertian operator (4D wave operator) and ∇^2 is the well-known Laplacian operator.

Dual Conservation Law in Noninertial Coordinate Metric

$$^*f^{\mu'\nu'}_{,\nu'\mu'} = {^*f}^{\mu\nu}_{,\nu\mu} + \big[\Box^2 - \nabla^2\big]\big(x \cdot (P \times \omega)\big) \tag{3.13}$$

Results (3.12) and (3.13) are the marvellous predictions of this model.

5. Conclusion

Antisymmetric mechanical Lorentz force tensor in the language of electrodynamics is the dawn of a relativistic framework of classical dynamics in terms of forces. In other words, it is the spacetime physics of inertia that has been obscured under the umbrella of the equivalence principle. Newton's second law of motion is an integral part of the theory of inertia. The role of the Coriolis force is very important to understand geodesic motion. The antisymmetric Lorentz force tensor acts as a guidance force field.

This model shows that the spacetime laws of nature hold everywhere independent of charge and mass. The most beautiful outcome of this model in the noninertial coordinate metric is the existence of a 7D wave of mechanical energy as the effect of inertia, along with the classical conservation law, leading us to an 11-dimensional world without any *ad hoc* assumption. Our next paper is on the antisymmetric mechanical Lorentz momentum tensor that describes the theory of inertia containing Newton's laws of motion.

Dedicated to: H. A. Lorentz, J. C. Maxwell, Albert Einstein, and Isaac Newton

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